

Theory/Phenomenology of GPDs

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Outline

Introduction to Generalized Parton Distributions (GPDs) and Deeply Virtual Compton Scattering (DVCS)

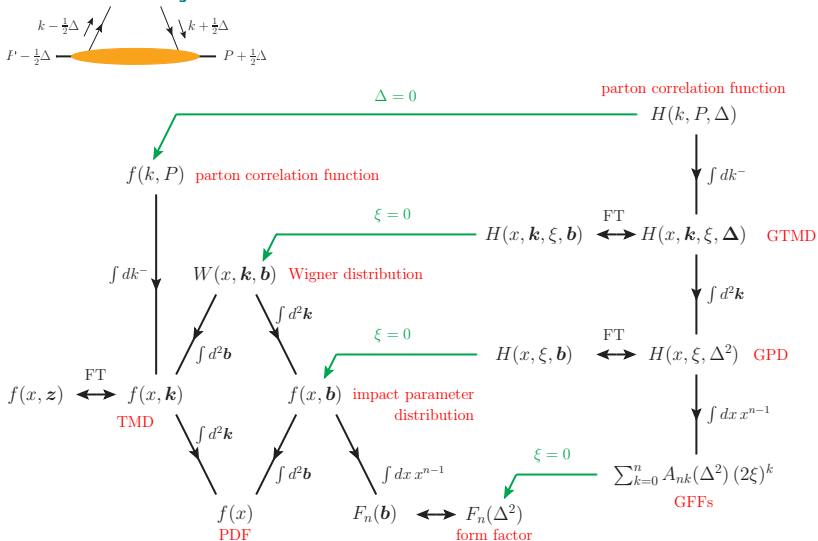
Fits to harmonics

Global fits results

Neural nets

Conclusion

Family tree of hadron structure functions



[Fig. by Markus Diehl]

($\xi \rightarrow \eta$ from now on)

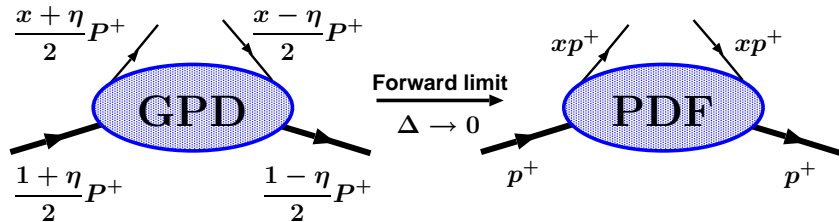
Definition of GPDs

- In QCD **GPDs** are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x, \eta, t) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) \gamma^+ q(z) | P_1 \rangle \Big|_{z^+=0, z_\perp=0}$$

$$\tilde{F}^q(x, \eta, t) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) \gamma^+ \gamma_5 q(z) | P_1 \rangle \Big|_{z^+=0, z_\perp=0}$$

(and similarly for gluons F^g and \tilde{F}^g).



$$P = P_1 + P_2; \quad t = \Delta^2 = (P_2 - P_1)^2; \quad \eta = -\frac{\Delta^+}{P^+} \text{ (skewedness)}$$

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \quad a = q, g$$

$$\tilde{F}^a = \frac{\bar{u}(P_2)\gamma^+\gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2)\gamma_5 u(P_1)\Delta^+}{2MP^+} \tilde{E}^a \quad a = q, g$$

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- “Ji’s sum rule” (related to proton spin problem)

$$J^q = \frac{1}{2} \int_{-1}^1 dx x \left[H^q(x, \eta, t) + E^q(x, \eta, t) \right]_{t \rightarrow 0} \quad [\text{Ji '96}]$$

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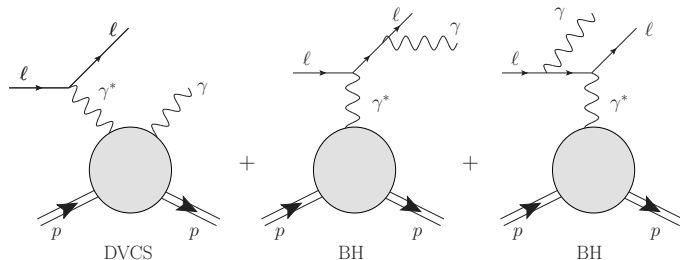
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- Distribution of partons in transversal space

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H(x, 0, t = -\vec{\Delta}_\perp^2) \quad [\text{Burkardt '00}]$$

Access to GPDs via DVCS

- **Deeply virtual Compton scattering (DVCS)** — “gold plated” process of exclusive physics
- DVCS is measured via lepton production of a photon



- **Interference** with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2 + \mathcal{I}.$$

$$\mathcal{I} \propto \frac{-e_l}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^3 [c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi)] \right\},$$

$$|\mathcal{T}_{\text{DVCS}}|^2 \propto \left\{ c_0^{\text{DVCS}} + \sum_{n=1}^2 [c_n^{\text{DVCS}} \cos(n\phi) + s_n^{\text{DVCS}} \sin(n\phi)] \right\},$$

- Choosing polarizations (and charges) we focus on particular harmonics:

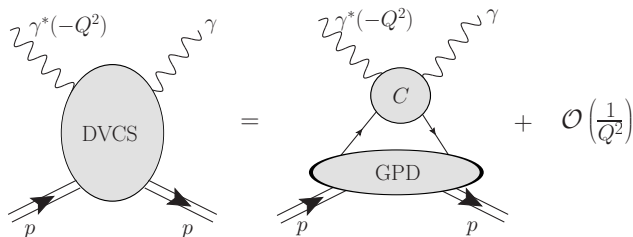
$$c_{1,\text{unpol.}}^{\mathcal{I}} \propto \left[F_1 \Re \mathcal{H} - \frac{t}{4M_p^2} F_2 \Re \mathcal{E} + \frac{x_B}{2-x_B} (F_1 + F_2) \Re \tilde{\mathcal{H}} \right]$$

[Belitsky, Müller et. al '01-'14]

- $\mathcal{H}(x_B, t, Q^2), \dots$ — four **Compton form factors** (CFFs)

Factorization of DVCS \longrightarrow GPDs

- [Collins et al. '98]



- Compton form factor is a convolution:

$${}^a\mathcal{H}(x_B, t, Q^2) = \int dx C^a\left(x, \frac{x_B}{2-x_B}, \frac{Q^2}{Q_0^2}\right) H^a\left(x, \frac{x_B}{2-x_B}, t, Q_0^2\right)$$

$a=q, G$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is “handbag” amplitude

$$\mathcal{H}(\xi, t, Q^2) \stackrel{\text{LO}}{=} \int_{-1}^1 dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, Q^2)$$

- giving access to GPD on the “cross-over” line $\eta = x$

$$\frac{1}{\pi} \Im \mathcal{H}(\xi = x, t, Q^2) \stackrel{\text{LO}}{=} H(x, x, t, Q^2) - H(-x, x, t, Q^2)$$

- while dispersion relation connects it to $\Re \mathcal{H}$

$$\Re \mathcal{H}(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im \mathcal{H}(\xi', t, Q^2) + \mathcal{C}_{\mathcal{H}}(t, Q^2)$$

Tomography? (1/3)

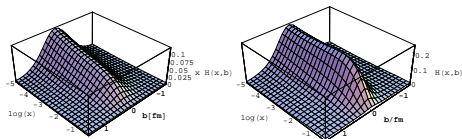
- So at LO experimentally accessible observables (CFFs, $d\sigma$) are almost* completely determined by $H(x, x, t)$
- This is **good** because phenomenology and fits are easier (one variable less)
- This is **bad** because $H(x, x, t)$ has **no** probabilistic interpretation, so results cannot be directly used for “tomography”
- To go from cross-over $\eta = x$ GPD section to $\eta = 0$ there are at least three approaches:

1. Use NLO precision and evolution to access GPDs away from $\eta = x$ line. (Need large lever arm in Q^2 ; JLab@12GeV?, EIC)

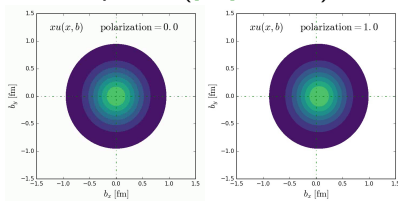
*Up to subtraction constant

Tomography? (2/3)

2. Extrapolate $H(x, x, t) \rightarrow H(x, 0, t)$ in a **model dependent** way:
- Quark and gluon sea 2D distributions $H(x, \vec{b}_\perp)$ ([KM] model)



- Sivers effect for valence quarks ([GK] model)



- See also [Dupré, Guidal, Vanderhaeghen '16]

Tomography? (3/3)

3. Don't be so focused on probabilistic interpretation.

- Like, similarly for elastic form factors . . .

$$-\frac{1}{6} \frac{dG_E(Q^2)}{dQ^2} \Big|_{\rightarrow Q^2=0} \longleftrightarrow \langle r^2 \rangle$$

. . . extrapolation to Q^2 and interpretation as nucleon 3D mean squared charge radius is also problematic [G. A. Miller '07],

- Anyway, it is form factors $F_{1,2}(Q^2)$ that are entering amplitudes of relativistic processes (like DVCS), not radii.
- So let's for a time being concentrate on measuring GPDs as best as we can.

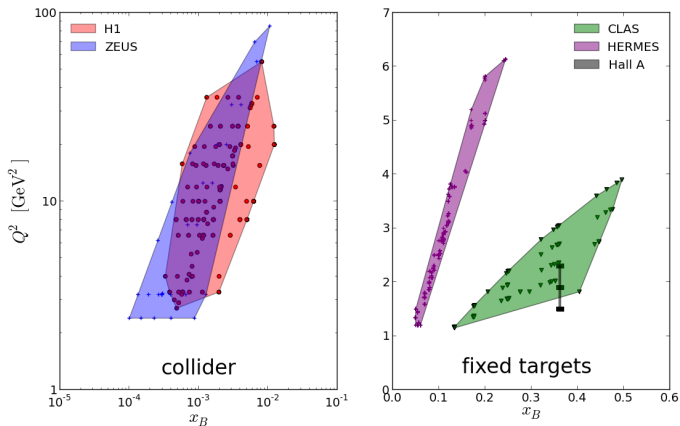
Positivity AND polynomiality

- Probabilistic interpretation is related to the positivity of state vector norm, which is reflected in GPD **positivity** constraints that are not employed enough in model building
- **Polynomiality** is easy if GPD is modeled as Radon transform of double distribution (DD)

$$H(x, \eta, t) = x \int d\beta d\alpha \delta(x - \beta - \alpha\eta) f(\beta, \alpha, t)$$

- OTOH, **positivity** can be obtained for GPDs represented as overlap of LF wave functions (but gives just DGLAP part $x > \eta$)
- So overlap of LFWFs \rightarrow DD via **inverse** Radon transform \rightarrow gives complete GPD (including ERBL part $x < \eta$). [Hwang and Müller '08, '14], [Chouika et al. '17] [Müller '17]

Experimental coverage (1/2)



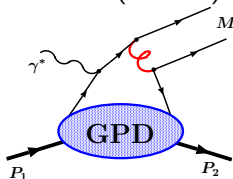
- Coming soon: COMPASS, JLab12, ... EIC
- See talks by [Van Hulse, Ferrero, Hafidi, Roche, Horn, Joo, ...]

Experimental coverage (2/2)

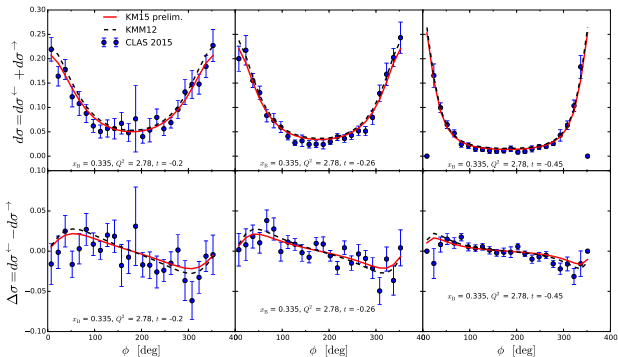
Collab.	Year	Observables	Kinematics			No. of points	
			x_B	Q^2 [GeV 2]	$ t $ [GeV 2]	total	indep.
HERMES	2001	$A_{LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_C^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi), \Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_C^{\cos(0,1)\phi}, A_{UT,DVCS}^{\sin(\phi-\phi_S)}$	0.03–0.35	1–10	<0.7	12+12+12	4+4+4
		$A_{UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$				12+12	4+4
		$A_{UT,I}^{\cos(\phi-\phi_S)\sin\phi}$				12	4
CLAS	2008	$A_{LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{LU,I}^{\sin(1,2)\phi}, A_{LU,DVCS}^{\sin\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18	6+6+6
		$A_C^{\cos(0,1,2,3)\phi}$				18+18+18+18	6+6+6+6
HERMES	2010	$A_{UL}^{\sin(1,2,3)\phi}$	0.03–0.35	1–10	<0.7	12+12+12	4+4+4
		$A_{LL}^{\cos(0,1,2)\phi}$				12+12+12	4+4+4
HERMES	2011	$A_{LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+12	4+4+4
		$A_{LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$				12+12	4+4
		$A_{LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$				12+12	4+4
		$A_{LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$				12	4
HERMES	2012	$A_{LU,I}^{\sin(1,2)\phi}, A_{LU,DVCS}^{\sin\phi}$	0.03–0.35	1–10	<0.7	18+18+18	6+6+6
		$A_C^{\cos(0,1,2,3)\phi}$				18+18+18+18	6+6+6+6
CLAS	2015	$A_{LU}(\phi), A_{UL}(\phi), A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi), \Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi), \Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

Alternative processes for GPD access

- **Deeply virtual meson production (DVMP)** $\gamma^* p \rightarrow Mp$.



- Theory more “dirty” than for DVCS (second “soft” function appears: meson distribution amplitude)
- Different mesons enable access to different flavours of GPDs [Goloskokov, Kroll]
- First global DVCS+DVMP NLO fit [Lautenschlager, Müller, Schäfer '13]
- **double DVCS** $\gamma^* p \rightarrow \gamma^* p$, **timelike DVCS**, ... see talk by [T. Horn]

Case #1: CLAS cross-section (ϕ -space)

- KM15 model (global refit including this data):
 $\chi^2/\text{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$
- looks excellent

Going from ϕ - to n -space

- It is often convenient to work with **weighted** harmonics

$$\sigma^{\sin n\phi, w} \equiv \frac{1}{\pi} \int_{-\pi}^{\pi} dw \sin(n\phi) \sigma(\phi),$$

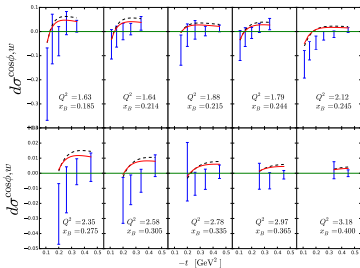
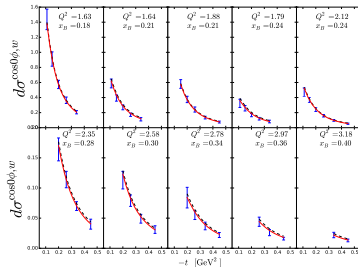
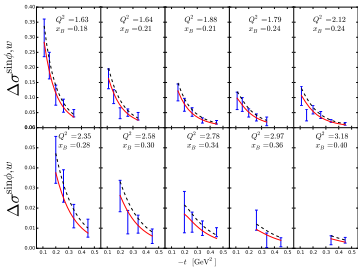
with specially weighted Fourier integral measure

$$dw \equiv \frac{2\pi \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)}{\int_{-\pi}^{\pi} d\phi \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} d\phi,$$

thus cancelling strongly oscillating factors $1/(\mathcal{P}_1(\phi)\mathcal{P}_2(\phi))$ in Bethe-Heitler and interference terms in $d\sigma$.

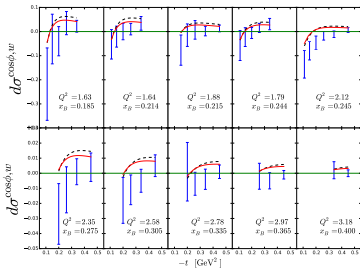
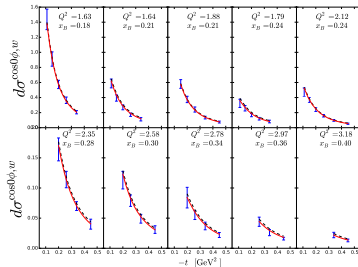
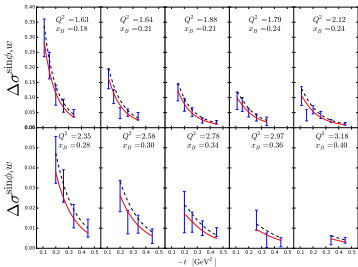
- Highest extractable harmonics:

	CLAS		Hall A	
	sine	cosine	sine	cosine
$\Delta\sigma^w$	1.6 ± 0.8	0.8 ± 1.1	1.4 ± 0.5	0.3 ± 0.8
$d\sigma^w$	0.7 ± 0.9	1.6 ± 1.2	1.2 ± 1.1	2.1 ± 0.8

Case #1: CLAS cross-section (n -space)

- $\chi^2/\text{npts} = 62.2/48$
for $d\sigma^{\cos\phi,w}$

(O.K. but not so perfect as in ϕ -space)

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- $\chi^2/\text{npts} = 62.2/48$
for $d\sigma^{\cos\phi,w}$

(O.K. but not so perfect as in ϕ -space)

In addition to $\chi^2/\text{d.o.f.}$ use:

$$\text{pull} \equiv \frac{1}{\sqrt{N}} \sum_{i=1}^N \frac{f(x_i) - y_i}{\Delta y_i}$$

Propagation of uncertainties to harmonics

- Consider three types of uncertainty:
 - uncorrelated point-to-point uncertainty (absolute size ϵ)
 - correlated normalization uncertainty (relative size ϵ)
 - correlated **modulated** (ϕ -dependent) uncertainty (e.g., relative size $\epsilon \cos(\phi)$)
- Uncorrelated uncertainty: $\Delta c_k = \sqrt{2/N} \epsilon$
- Normalization uncertainty: $\Delta c_k / c_k = \epsilon$
- Correlated modulated uncertainty: more complicated, but for hierarchical case $c_0 \gg c_1 \gg \dots$ one obtains

$$\frac{\Delta c_0}{c_0} = \frac{c_1}{2c_0} \epsilon, \quad \frac{\Delta c_1}{c_1} = \frac{c_0}{c_1} \epsilon$$

i.e. we have **enhancement of uncertainty** for subleading harmonics!

$$(c_0 + c_1 \cos \phi + \dots) \times (1 + \epsilon \cos \phi) = c_0 \left(1 + \frac{c_1}{2c_0} \epsilon \right) + c_1 \left(1 + \frac{c_0}{c_1} \epsilon \right) \cos \phi$$

Modulated correlated error in the wild

Hall A [M. Defurne et. al 2015] discussing systematic uncertainties:

tematic error from the parameter choice to be 1%.

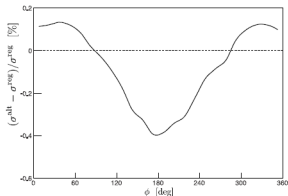
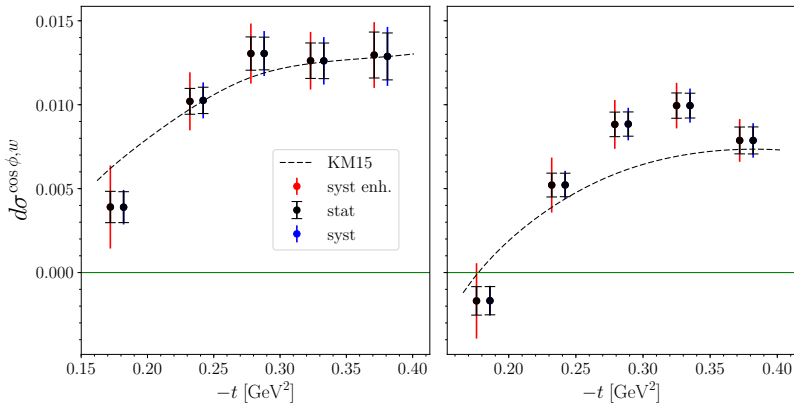


FIG. 20. Difference in % between the cross section extracted with the squared DVCS amplitude term and with the $\Re C^{Z,V}$ term for $x_B = 0.37$, $Q^2 = 2.36 \text{ GeV}^2$ and $-t = 0.33 \text{ GeV}^2$. The ϕ -profile of the difference is a consequence of the small $\cos \phi$ and $\cos 2\phi$ dependences of the $\Re C^{Z,V}$ kinematic coefficient. Both extractions give almost the same reduced $\chi^2/dof=0.94$ (nominal) and 0.93 (alternate) for the entire Kin2 setting.

Systematic uncertainty	Value	Section
HRS acceptance cut	1%	IV A
Electron ID	0.5%	IV D
HRS multitrack	0.5%	IV D
Multi-cluster	0.4%	IV D
Corrected luminosity	1%	IV D
Fit parameters	1%	VIB
Radiative corrections	2%	V
Beam polarization	2%	III A 3
<hr/>		
Total (helicity-independent)	2.8%	
Total (helicity-dependent)	3.4%	

TABLE V. Normalization systematic uncertainties in the extracted photon electroproduction cross sections. The systematic error coming from the fit parameter choice is not a normalization error per se, but we consider that 1% is an upper limit for this error on all kinematic bins. The helicity-dependent cross sections have an extra uncertainty stemming from the beam polarization measurement. The last column gives the section in which each systematic effect is discussed.

Hall A $\cos(\phi)$ harmonics

(Syst added *linearly* on top of stat.)

Hybrid GPD models for global fits

- **Sea quarks and gluons** modelled using $SO(3)$ partial wave expansion in conformal GPD moment space + Q^2 evolution.
- **Valence quarks** — model CFFs directly (ignoring Q^2 evolution):

$$\Im \mathcal{H}(\xi, t) = \pi \left[\frac{4}{9} H^{u\text{val}}(\xi, \xi, t) + \frac{1}{9} H^{d\text{val}}(\xi, \xi, t) + \frac{2}{9} H^{\text{sea}}(\xi, \xi, t) \right]$$

$$H(x, x, t) = n r 2^\alpha \left(\frac{2x}{1+x} \right)^{-\alpha(t)} \left(\frac{1-x}{1+x} \right)^b \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2} \right)^p}.$$

- $\Re \mathcal{H}$ determined by dispersion relations
- 15 free parameters in total for $H, \tilde{H}, E, \tilde{E}$.

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
$\chi^2/\text{d.o.f.}$	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
σ_{DVCS}	(45)	(45)	51	51	45		11	11
$d\sigma_{\text{DVCS}}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_C^{\cos 0\phi}$							6	6
$A_C^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w} / \sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

- [K.K., Müller, et al. '09–'15]
- Other approach: recursive fit [Goldstein, Gonzalez, Liuti '11]

KM models are available at WWW

- <http://calculon.phy.hr/gpd/> — binary code for cross sections

```
% xs.exe
```

```
xs.exe ModelID Charge Polarization Ee Ep xB Q2 t phi
```

returns cross section (in nb) for scattering of lepton of energy Ee on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of

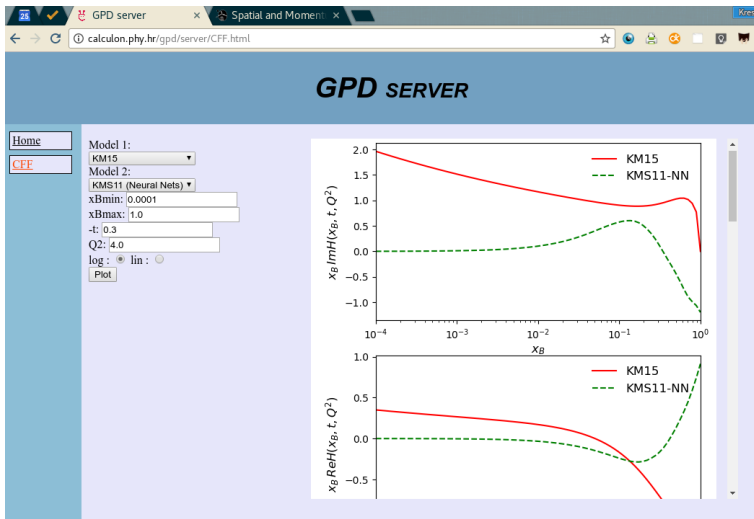
- 1 KM09a - arXiv:0904.0458 fit without Hall A data,
- 2 KM09b - arXiv:0904.0458 fit with Hall A harmonics ratio,
- 3 KM10 - arXiv:1105.0899 fit with Hall A harmonics
- 4 KM10a - arXiv:1105.0899 fit without Hall A data
- 5 KM10b - arXiv:1105.0899 fit with Hall A harmonics ratio
- 6 KMM12 - arXiv:1301.1230 fit with Hall A harmonics and polarized target
- 7 KM15 - arXiv:1512.09014 fit now includes 2015 CLAS and Hall A data

xB Q2 t phi -- usual kinematics (phi is in Trento convention)

```
% xs.exe 1 -1 1 27.6 0.938 0.111 3. -0.3 0
```

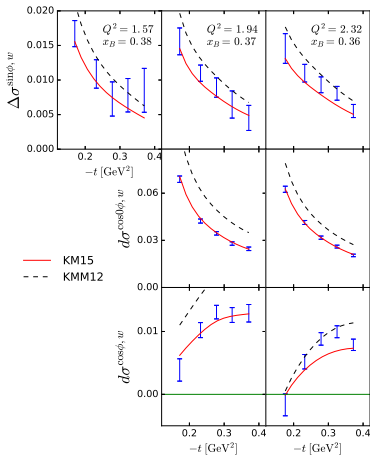
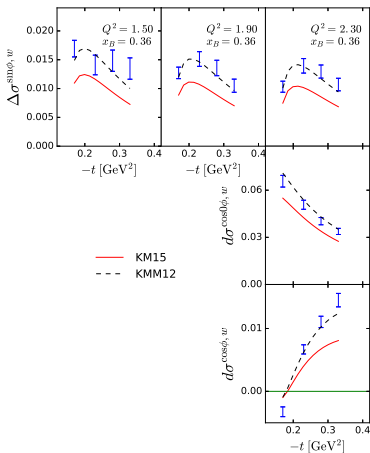
```
0.18584386497251
```

KM GPD server



- Plots of all CFFs available; numerical values soon to come ...

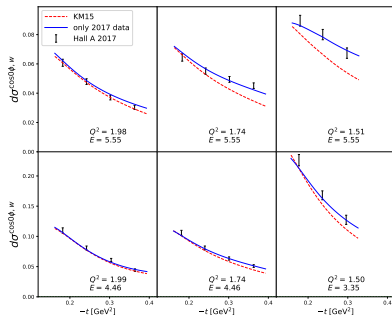
2006 vs 2015 Hall A cross-sections



- Improvement of global $\chi^2/\text{d.o.f.}$ 123.5/80 \rightarrow 240./275

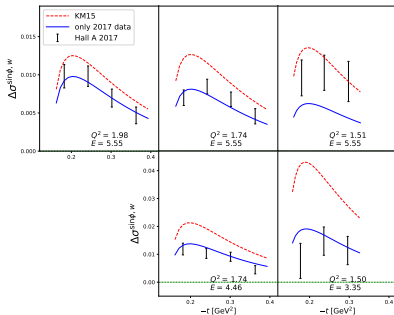
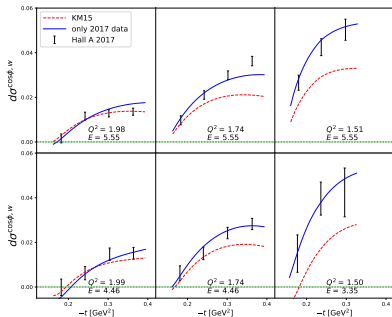
2015 vs 2017 Hall A cross-sections (1/2)

- see talk by [J. Roche]



- Predictions of KM15 model **fail** for low- Q^2 2017 Hall A measurements!

2015 vs 2017 Hall A cross-sections (2/2)



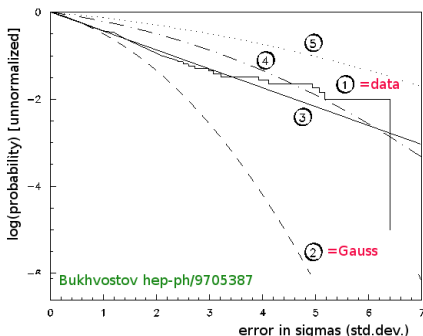
- New data are a challenge for global fit ...
- ... but have gluons really been glimpsed by Hall A?

χ^2/n_{pts} and pull values

Collaboration	Observable	n_{pts}	KMM12		KM15	
			χ^2/n_{pts}	pull	χ^2/n_{pts}	pull
ZEUS	σ_{DVCS}	11	0.49	-1.76	0.51	-1.74
ZEUS,H1	$d\sigma_{DVCS}/dt$	24	0.97	0.85	1.04	1.37
HERMES	$A_C^{\cos 0\phi}$	6	1.31	0.49	1.24	0.29
HERMES	$A_C^{\cos \phi}$	6	0.24	-0.56	0.07	-0.20
HERMES	$A_{LU,I}^{\sin \phi}$	6	2.08	-2.52	1.34	-1.28
CLAS	$A_{LU}^{\sin \phi}$	4	1.28	2.09		
CLAS	$A_{LU}^{\sin \phi}$	13			1.24	0.63
CLAS	$\Delta\sigma^{\sin \phi, w}$	48			0.41	-1.66
CLAS	$d\sigma^{\cos 0\phi, w}$	48			0.16	-0.21
CLAS	$d\sigma^{\cos \phi, w}$	48			1.16	6.36
Hall A	$\Delta\sigma^{\sin \phi, w}$	12	1.06	-2.55		
Hall A	$d\sigma^{\cos 0\phi, w}$	4	1.21	2.14		
Hall A	$d\sigma^{\cos \phi, w}$	4	3.49	-0.26		
Hall A	$\Delta\sigma^{\sin \phi, w}$	15			0.81	-2.84
Hall A	$d\sigma^{\cos 0\phi, w}$	10			0.40	0.92
Hall A	$d\sigma^{\cos \phi, w}$	10			2.52	-2.42
HERMES,CLAS	$A_{UL}^{\sin \phi}$	10	1.90	-1.89	1.10	-1.94
HERMES	$A_{LL}^{\cos 0\phi}$	4	3.44	2.17	3.19	1.99
HERMES	$A_{UT,I}^{\sin(\phi - \phi_S) \cos \phi}$	4	0.90	0.61	0.90	0.71
CLAS	$A_{UL}^{\sin \phi}$	10			0.76	0.38
CLAS	$A_{LL}^{\cos 0\phi}$	10			0.50	-0.22
CLAS	$A_{LL}^{\cos \phi}$	10			1.54	2.40

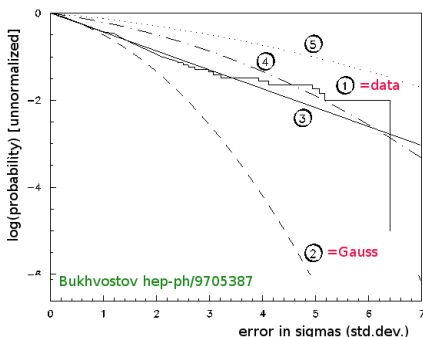
Problems with standard fitting approaches

1. Choice of fitting function introduces **theoretical bias** leading to **systematic error** which cannot be estimated (and is surely much larger for GPDs(x, η, t) than for PDFs(x)).
2. **Propagation of uncertainties** from experiment to fitted function is difficult. Errors in actual experiments are not always Gaussian.

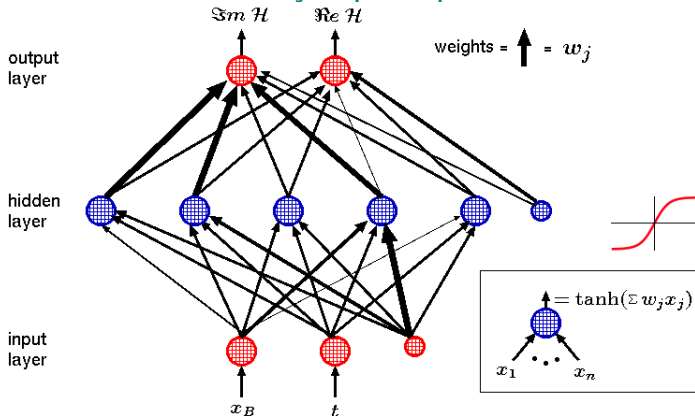


Problems with standard fitting approaches

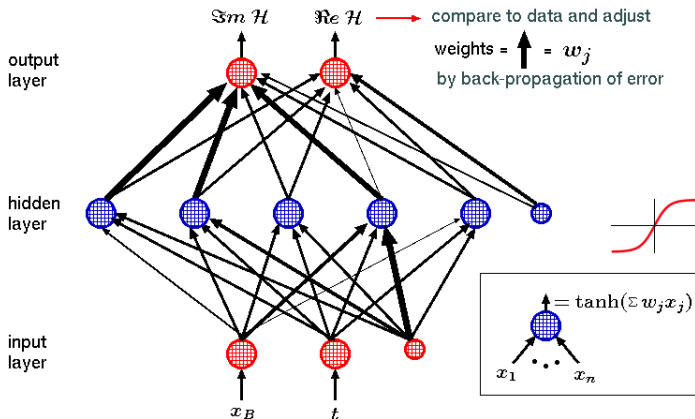
1. Choice of fitting function introduces theoretical bias leading to systematic error which cannot be estimated (and is surely much larger for GPDs(x, η, t) than for PDFs(x). → **NNets**
2. Propagation of uncertainties from experiment to fitted function is difficult. Errors in actual experiments are not always Gaussian. → **Monte Carlo error propagation**



Multilayer perceptron

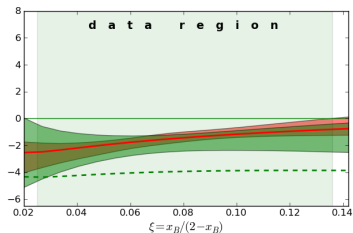
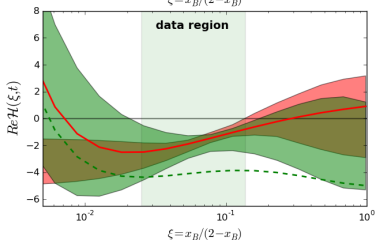
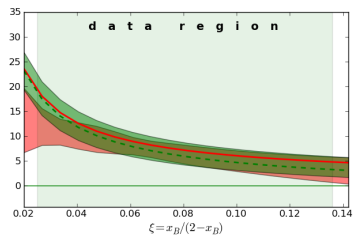
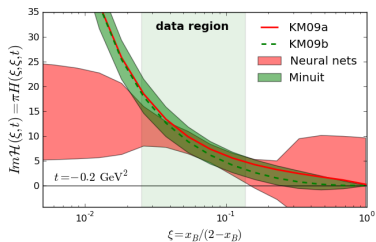


Multilayer perceptron

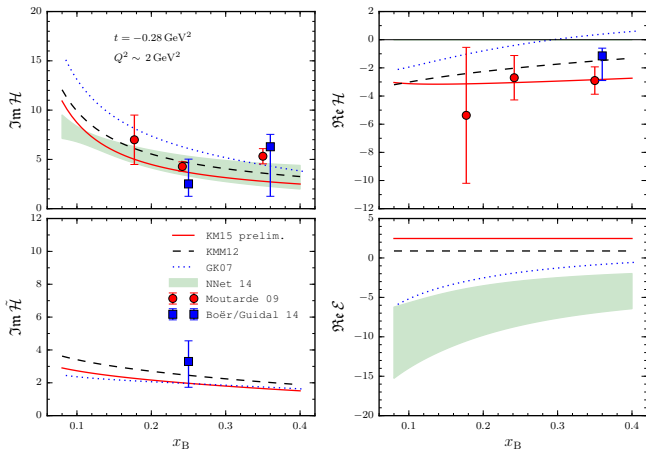


- Essentially a least-square fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \dots)) \Rightarrow$ no theory bias

Resulting neural network CFFs



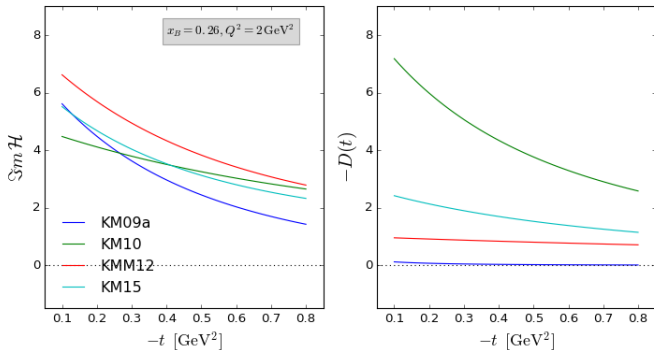
CFFs from various fits



(from [K.K., Moutarde and Liuti '16])

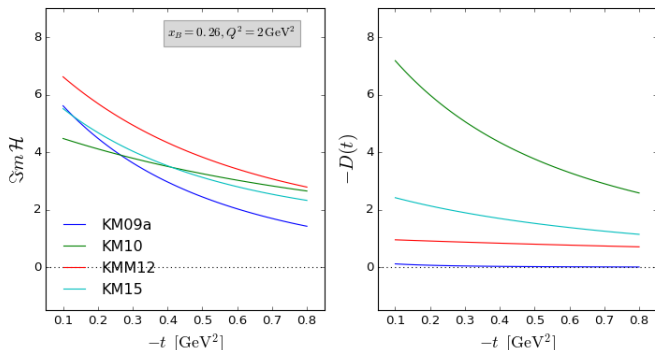
CFFs from various fits (II)

Comparing $\Im \mathcal{H}$ and D-term for a typical JLab kinematics:



CFFs from various fits (II)

Comparing $\Im \mathcal{H}$ and D-term for a typical JLab kinematics:



- We need access to ERL region. (\rightarrow double DVCS?)

Going beyond first approximations

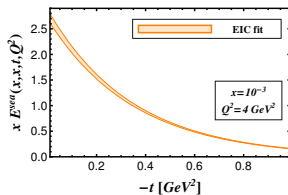
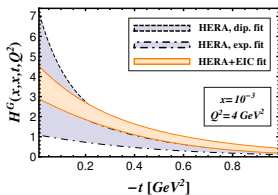
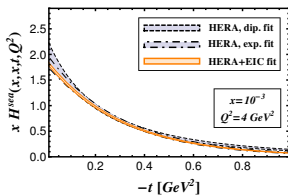
Published DVCS data (apart from new 2017 Hall A) is well described within leading order, leading twist and other approximations. Some available corrections:

- NLO evolution and NLO coefficient functions
- finite- t and target mass corrections [Braun et al. '14]
- twist-3 GPDs
- massive charm [Noritzsch '03]
- small- x resummation

can maybe be absorbed in redefinition of GPDs (think "DVCS scheme" factorization), but have to be taken into account when working with more processes (striving towards universal GPDs) and with more precise future data.

DVCS at EIC

- Future polarized electron-ion collider (EIC) will provide unique insight into sea GPDs.
- [Aschenauer, Fazio, K.K., Müller '13] fit to simulated DVCS data at $20 \text{ GeV} \times 250 \text{ GeV}$



- Improved knowledge of low- t quark and gluon GPDs H
- Improved knowledge of sea quark GPD E

Summary

- Global fits of all proton DVCS data using flexible hybrid models were in healthy shape until 2017 (some tension for first $\cos(\phi)$ harmonic of JLab cross sections).
- New Hall A 2017 data present a challenge
- Standard global model fitting and neural networks approach are complementing each other

Summary

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The End