

**Transversity 2017**

**Frascati**

**December 11 - 15, 2017**

***Phenomenological extraction of  
the Sivers distribution functions  
& Sivers sign change: an update***

***Mariaelena Boglione***



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ALMA UNIVERSITAS  
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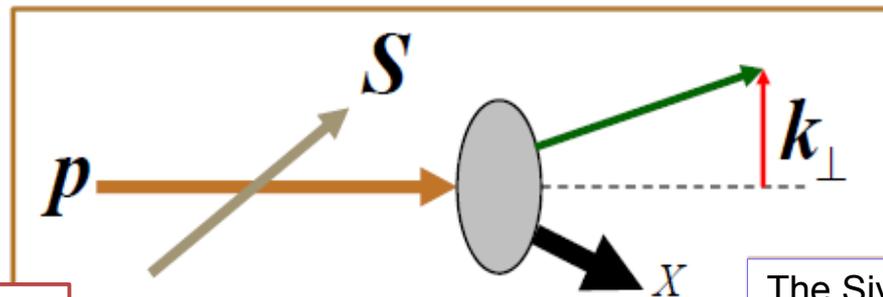
***In collaboration with:***

***M. Anselmino, U. D'Alesio, C. Flores,  
J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin***

# The *Sivers* Distribution Function

$$f_{q/p,S}(x, k_{\perp}) = f_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$
$$= f_{q/p}(x, k_{\perp}) - \frac{k_{\perp}}{M} f_{1T}^{\perp q}(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

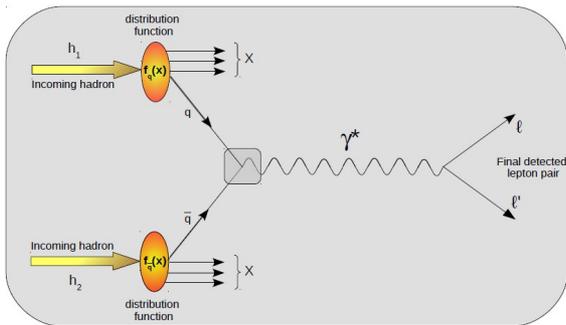


The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

The Sivers function embeds correlations between proton spin and quark transverse momentum

# Where do we learn about the **Sivers** function ?

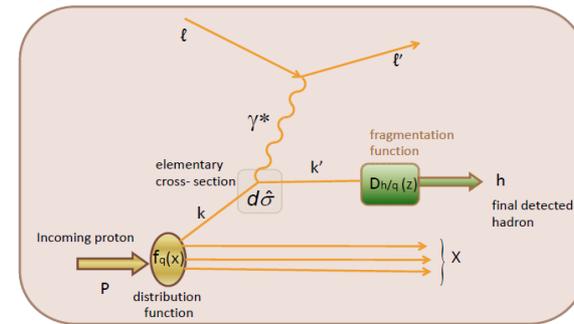
## Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x, k_{\perp}) \otimes f_{\bar{q}}(x, k_{\perp}) \otimes \hat{\sigma}^{q\bar{q} \rightarrow \ell\bar{\ell}}$$

Allows extraction of **distribution** functions

## Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

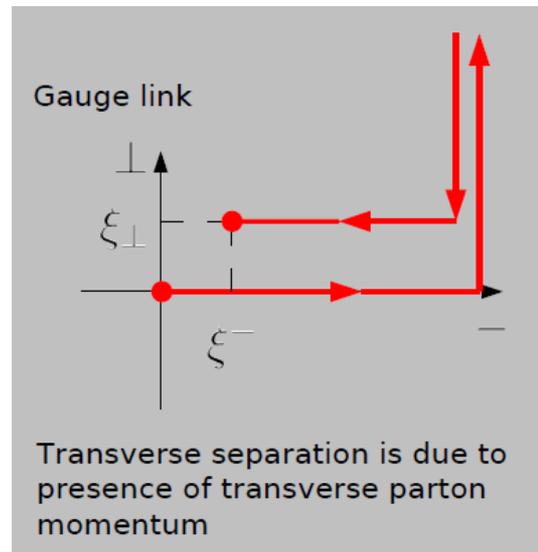
Allows extraction of **distribution** and **fragmentation** functions

# Sivers function sign change

- TMDs have to be defined in a color-gauge invariant way

$$\Phi_{ij}(x, \mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{i\mathbf{x}\mathbf{P}^+\xi^-} e^{-i\mathbf{k}_\perp\xi_\perp} \langle \mathbf{P}, \mathbf{S}_\mathbf{P} | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0}, \xi) \psi_i(\xi) | \mathbf{P}, \mathbf{S}_\mathbf{P} \rangle \Big|_{\xi^+=0}$$

- The struck quark propagates in the gauge field of the remnant and forms gauge links

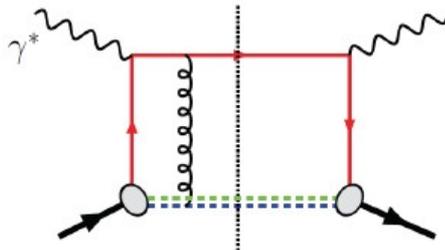


- Gauge links generate initial and final state interactions

# Sivers function sign change

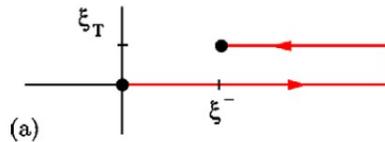
## SIDIS

- The gluon couples to the proton remnant after the quark is scattered
- Attractive final state interaction



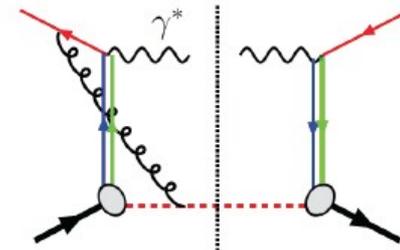
$r$    $(gb)$

Attractive



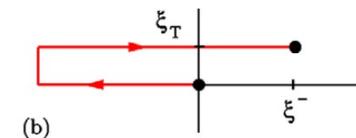
## DRELL YAN

- The gluon couples before the quark annihilates
- Repulsive initial state interaction



$r$    $r$

Repulsive



The Sivers function is process dependent: it reverses its sign when measured in SIDIS w.r.t Drell Yan processes

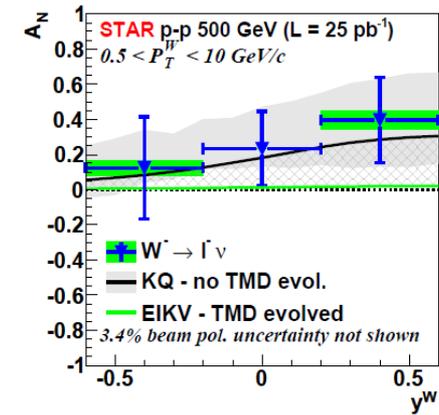
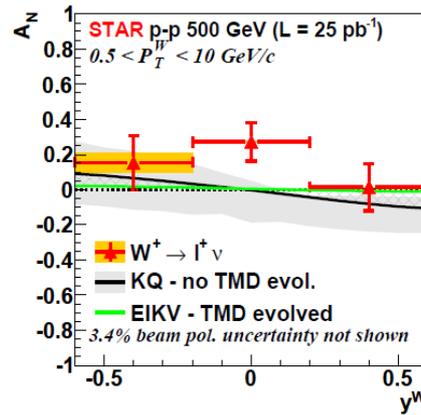
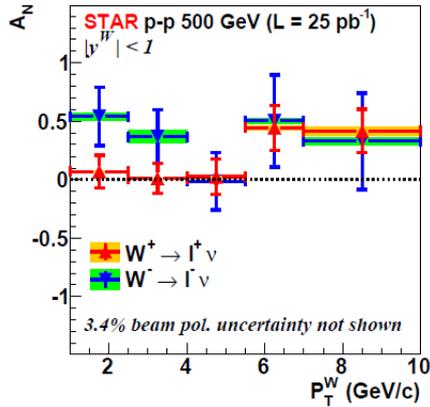
$$[f_{1T}^{q\perp}]_{\text{SIDIS}} = -[f_{1T}^{q\perp}]_{\text{DY}}$$

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***Testing the Sivers  
sign change in  
Drell-Yan processes***

# Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

STAR Collaboration, Phys. Rev. Lett. 116 132301 (2016)



$$A_N^W = \frac{d\sigma^{p \rightarrow W X} - d\sigma^{p \rightarrow W X}}{d\sigma^{p \rightarrow W X} + d\sigma^{p \rightarrow W X}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$= \frac{\sum_{q_1, q_2} |V_{q_1, q_2}|^2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) \mathbf{S} \cdot (\hat{\mathbf{p}}_1 \times \hat{\mathbf{k}}_{\perp 1}) \Delta^N f_{q_1/p^\uparrow}(x_1, \mathbf{k}_{\perp 1}) f_{q_2/p}(x_2, \mathbf{k}_{\perp 2})}{2 \sum_{q_1, q_2} |V_{q_1, q_2}|^2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) f_{q_1/p}(x_1, \mathbf{k}_{\perp 1}) f_{q_2/p}(x_2, \mathbf{k}_{\perp 2})}$$

# Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

- The quark flavours involved in W production include anti-quarks
- In order to estimate  $A_N^W$ , it is important to have a reliable extraction of both quark and anti-quark Sivers functions.

$$\mathbf{W^+} : |V_{u,d}|^2 \left( \Delta^N f_{u/} \otimes f_{\bar{d}/p} + \Delta^N f_{\bar{d}/} \otimes f_{u/p} \right) + |V_{u,s}|^2 \left( \Delta^N f_{u/} \otimes f_{\bar{s}/p} + \Delta^N f_{\bar{s}/} \otimes f_{u/p} \right)$$

$$\mathbf{W^-} : |V_{u,d}|^2 \left( \Delta^N f_{\bar{u}/} \otimes f_{d/p} + \Delta^N f_{d/} \otimes f_{\bar{u}/p} \right) + |V_{u,s}|^2 \left( \Delta^N f_{\bar{u}/} \otimes f_{s/p} + \Delta^N f_{s/} \otimes f_{\bar{u}/p} \right)$$

dominant

suppressed

This single spin asymmetry is very sensitive to  $\bar{u}$  and  $\bar{d}$  as well as  $u_v$  and  $d_v$

# Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

Unpolarized TMD PDF

$$f_{q/p}(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

Unpolarized TMD FF

$$D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}$$

Sivers function

$$\Delta^N f_{q/}(x, k_{\perp}) = 2 \mathcal{N}_q(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$$

$$h(k_{\perp}) = \sqrt{2} e \frac{k_{\perp}}{M_1} e^{-k_{\perp}^2 / M_1^2}$$

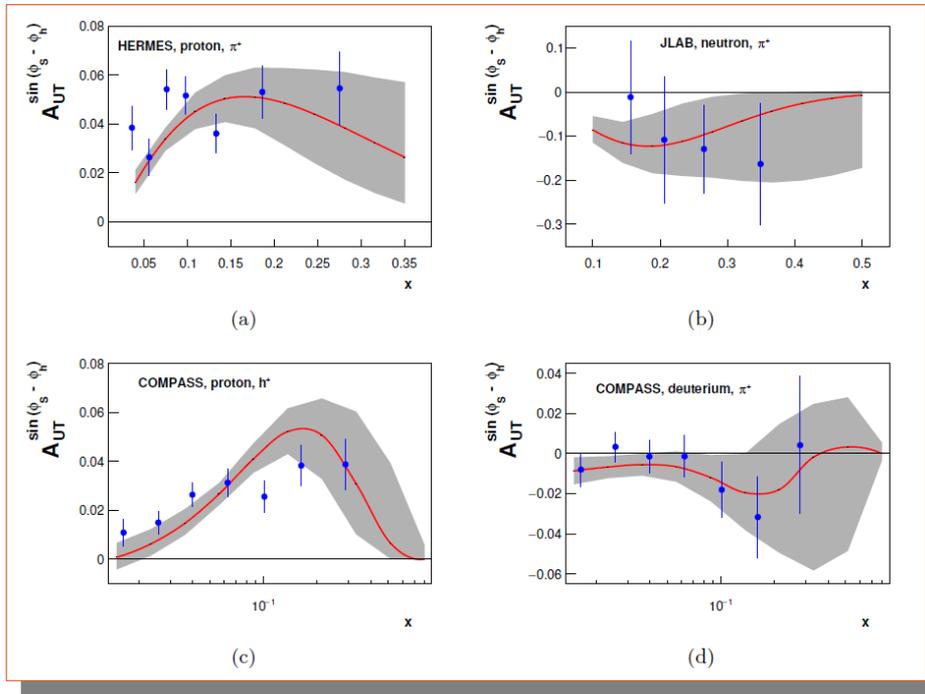
$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$

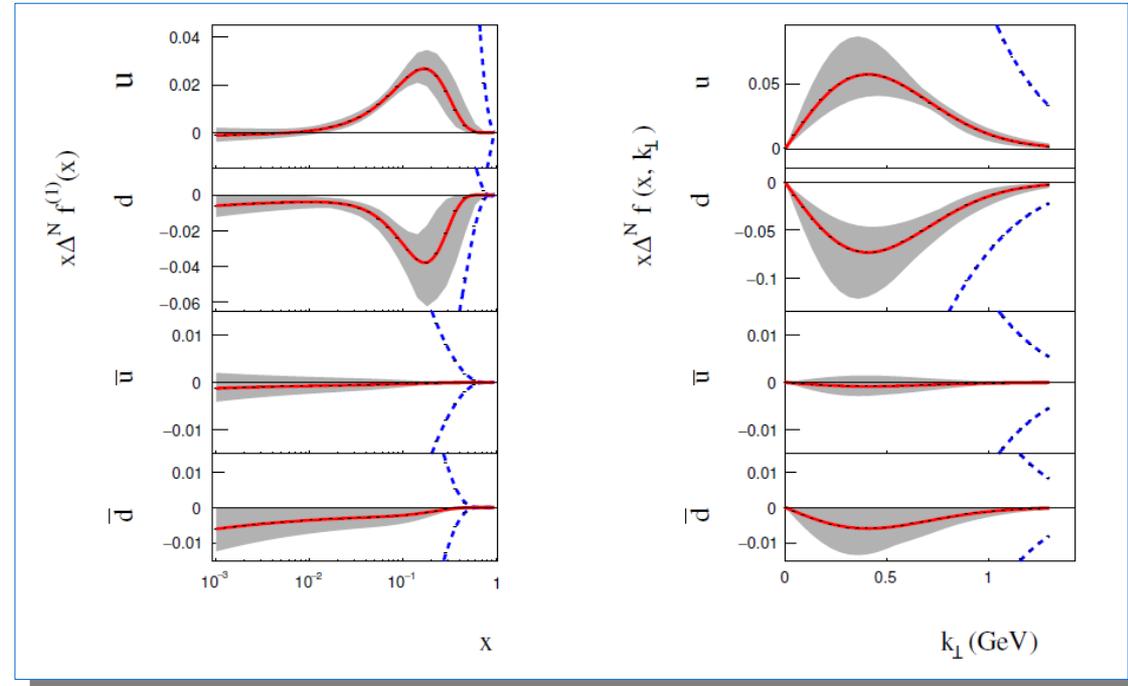
Sivers function parametrized starting from unpolarized PDF

# Extraction of Sivers functions from SIDIS data

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046



$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, P_T) = \frac{[z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_{\perp}^2 \rangle]^2 \langle k_{\perp}^2 \rangle} \exp \left[ -\frac{P_T^2 z^2 (\langle k_{\perp}^2 \rangle - \langle k_S^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_{\perp}^2 \rangle) (z^2 \langle k_{\perp}^2 \rangle + \langle p_{\perp}^2 \rangle)} \right] \times \frac{\sqrt{2} e z P_T \sum_q e_q^2 N_q(x) f_q(x) D_{h/q}(z)}{M_1 \sum_q e_q^2 f_q(x) D_{h/q}(z)}$$



$\langle k_{\perp}^2 \rangle = 0.57 \text{ GeV}^2$   
 $\langle p_{\perp}^2 \rangle = 0.12 \text{ GeV}^2$

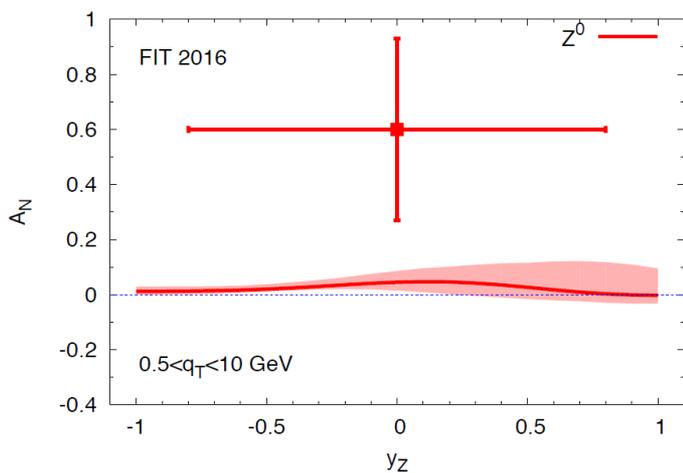
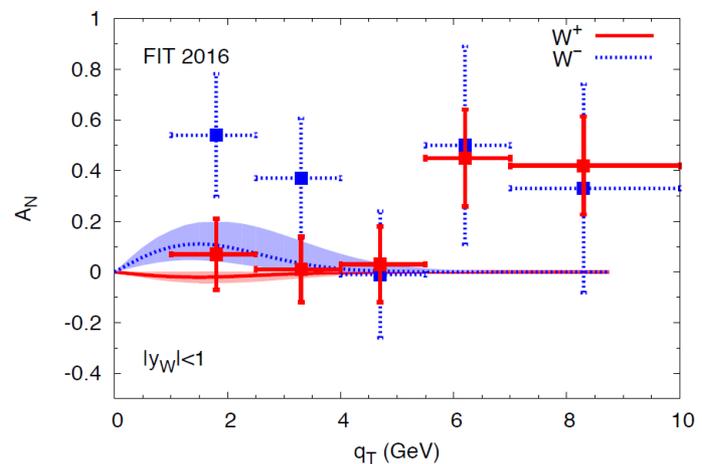
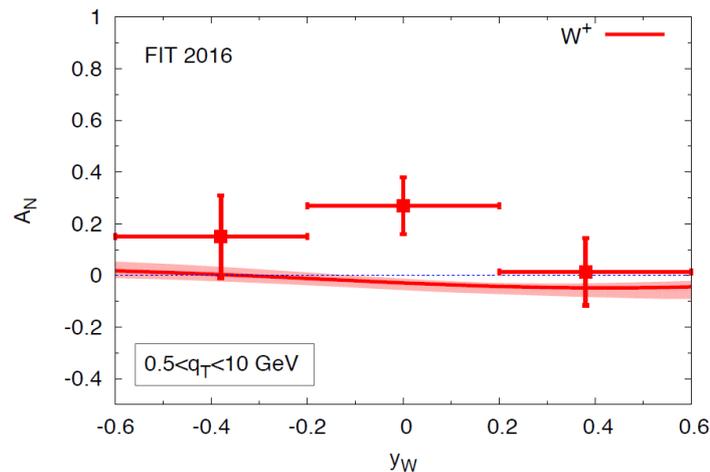
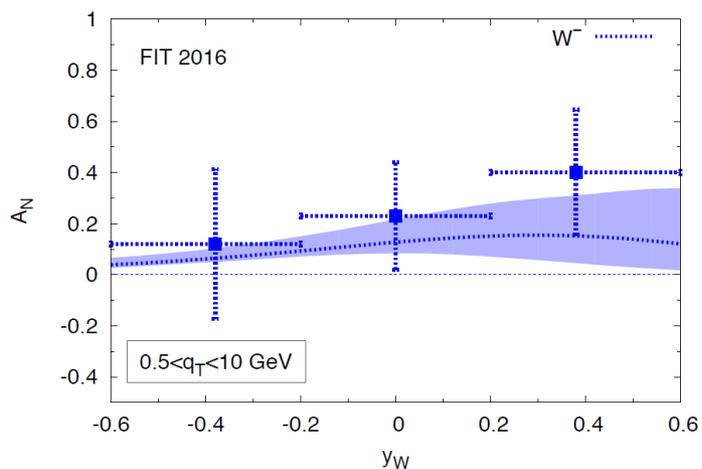
Extracted from HERMES multiplicities

$\chi_{\min}^2/\text{dof} = 1.29$

$u_v$  well determined  
 $d_v$  and  $\bar{d}$  poorly determined  
 $u$  : no information (not even sign)

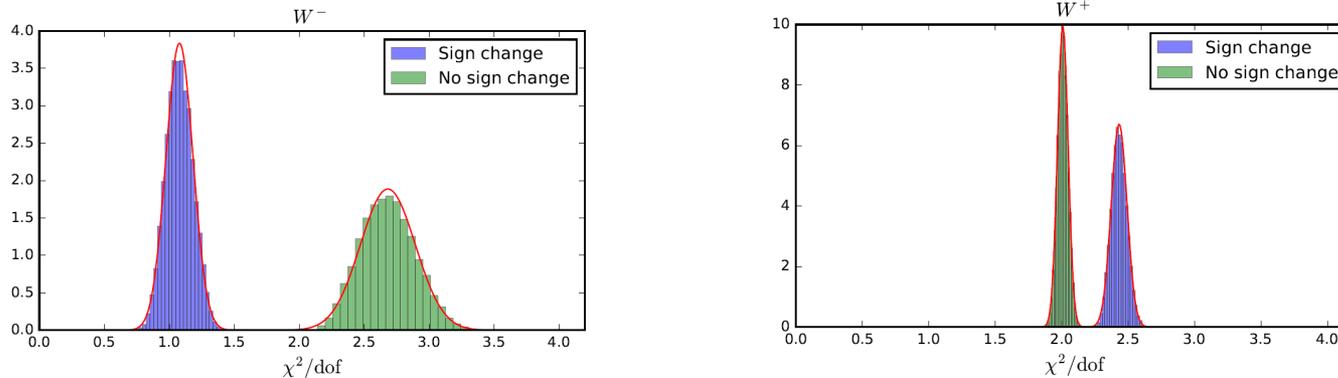
# Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046

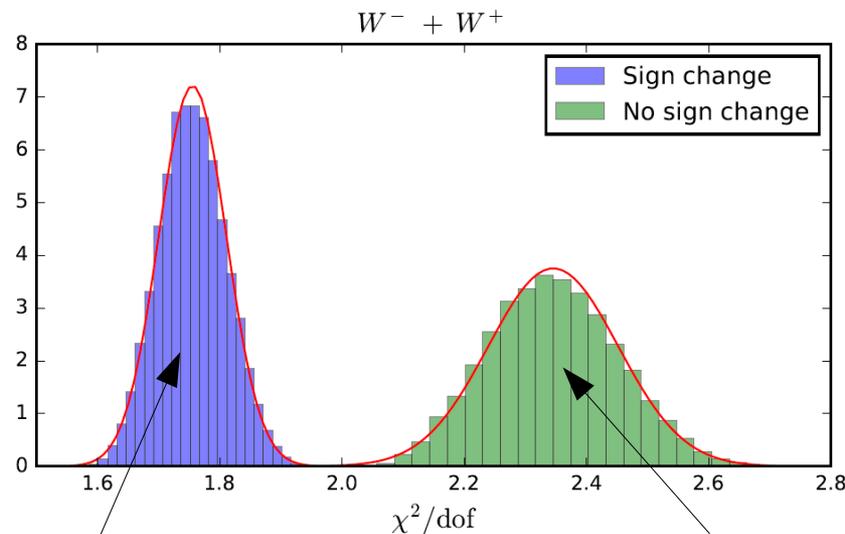


# Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC

Anselmino, Boglione, D'Alesio, Murgia, Prokudin, JHEP 1704 (2017) 046



$W^-$  data are compatible with the sign change, while  $W^+$  data may be compatible with either sign of the Sivers function



$\langle \chi^2/\text{dof} \rangle = 1.75$  and  $\sigma(\chi^2/\text{dof}) = 0.05$

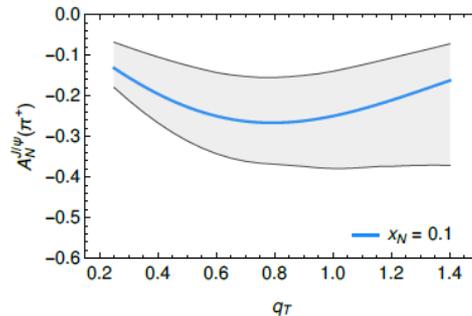
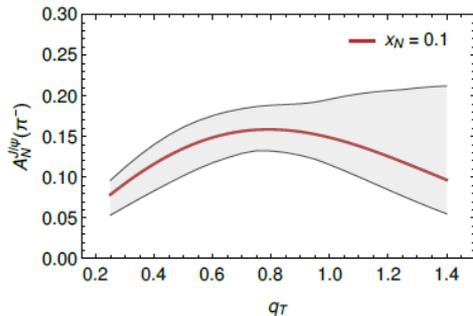
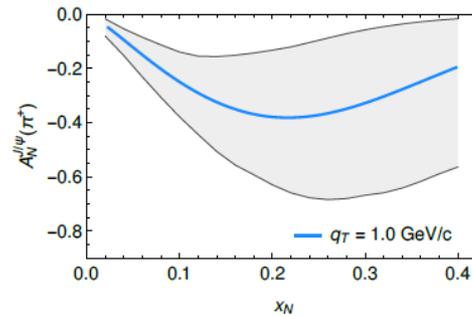
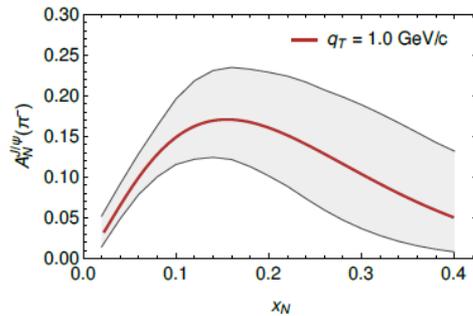
$\langle \chi^2/\text{dof} \rangle = 2.35$  and  $\sigma(\chi^2/\text{dof}) = 0.1$

# Sivers function in Drell-Yan at the $J/\Psi$ peak

M. Anselmino, V. Barone, M. Boglione, Phys. Lett. B (2017)

$$A_N^{J/\Psi}(\pi^-; x_1, x_2, q_T) \simeq \frac{\int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) S \cdot (\hat{\mathbf{p}}_2 \times \hat{\mathbf{k}}_{\perp 2}) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) \Delta^N f_{u/p^+}(x_2, k_{\perp 2})}{2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) f_{\bar{u}/\pi^-}(x_1, k_{\perp 1}) f_{u/p}(x_2, k_{\perp 2})}$$

$$A_N^{J/\Psi}(\pi^+; x_1, x_2, q_T) \simeq \frac{\int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) S \cdot (\hat{\mathbf{p}}_2 \times \hat{\mathbf{k}}_{\perp 2}) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) \Delta^N f_{d/p^+}(x_2, k_{\perp 2})}{2 \int d^2\mathbf{k}_{\perp 1} d^2\mathbf{k}_{\perp 2} \delta^2(\mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2} - \mathbf{q}_T) f_{\bar{d}/\pi^+}(x_1, k_{\perp 1}) f_{d/p}(x_2, k_{\perp 2})}$$



Usual DY elementary cross section

$$e_q^2 \hat{\sigma}_0 = e_q^2 \frac{4\pi\alpha^2}{9M^2}$$

With the replacements

$$\left\{ \begin{array}{l} 16\pi^2\alpha^2 e_q^2 \rightarrow (g_q^V)^2 (g_\ell^V)^2 \\ \frac{1}{M^4} \rightarrow \frac{1}{(M^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \end{array} \right.$$

Measurements from COMPASS  
will soon be available  
See talk by B. Parsamyan

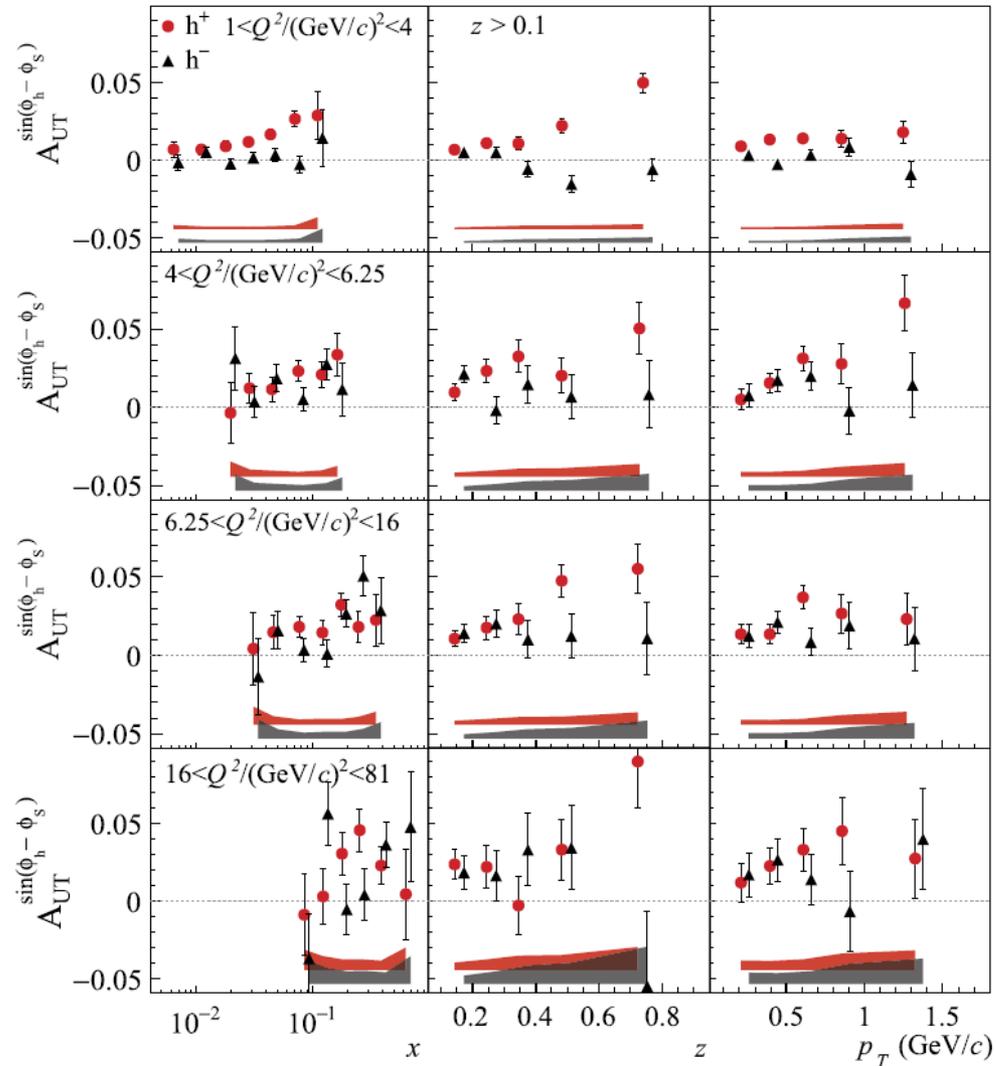


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# ***2017 new data***

# Sivers single spin asymmetry in SIDIS at the hard scales of Drell Yan @ COMPASS

COMPASS Collaboration, *Phys. Lett. B* 770, 138 (2017)

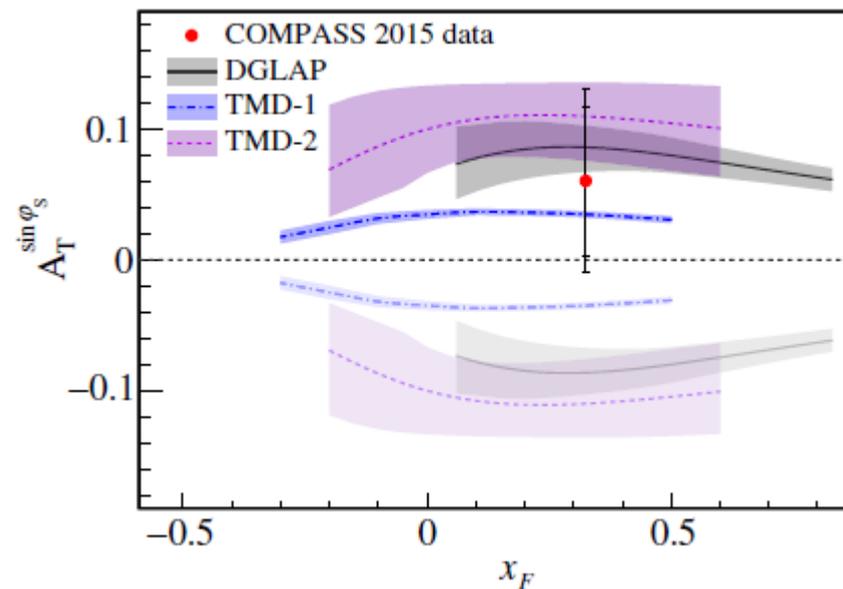


See talk by B. Parsamyan

# Sivers single spin asymmetry in pion induced Drell Yan @ COMPASS

COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

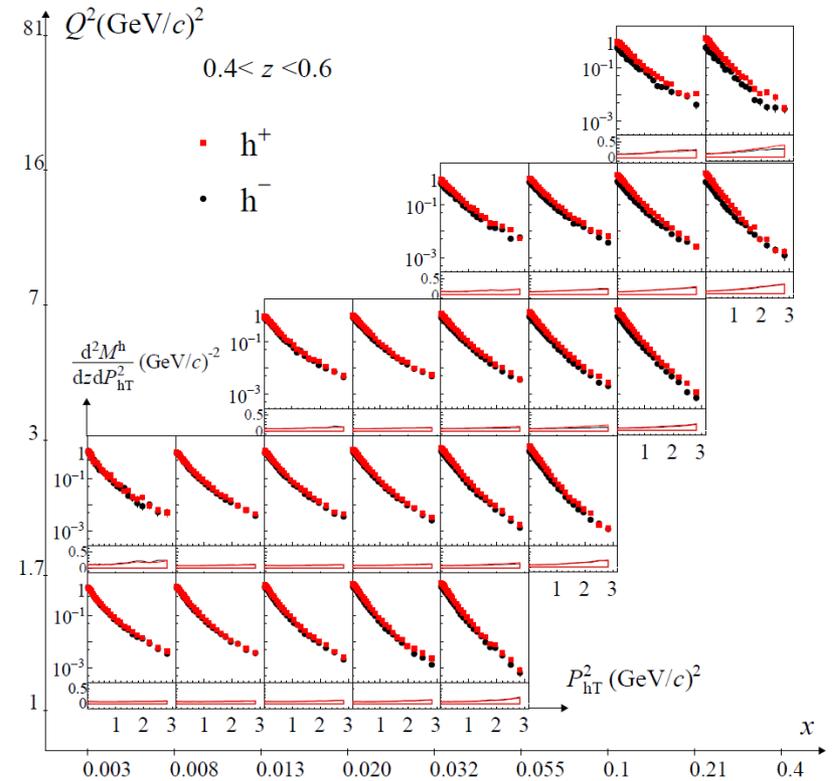
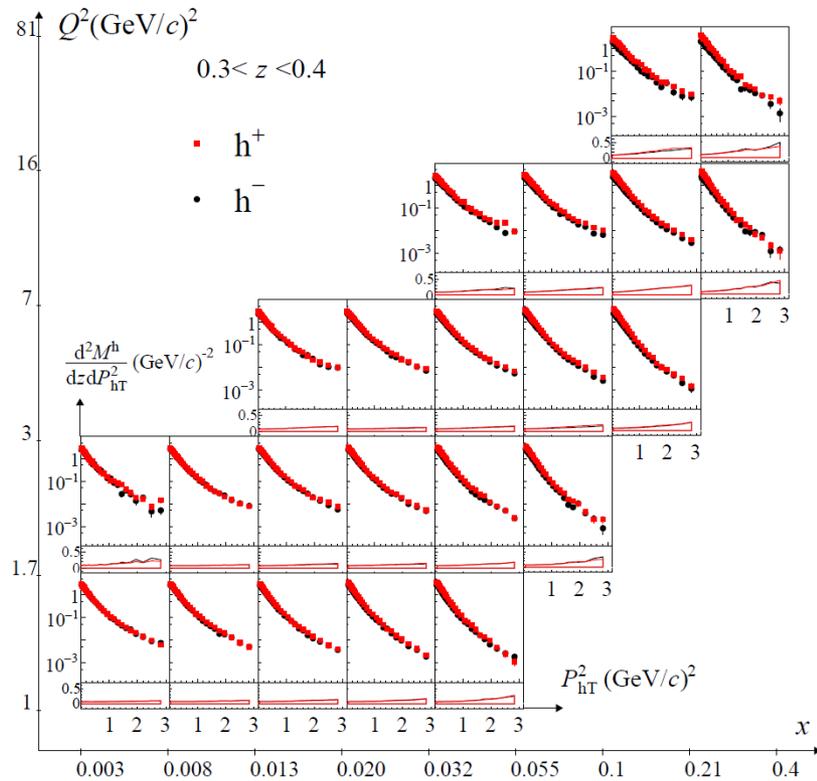
*190 GeV/c  $\pi^-$  beam scattered off a transversely polarized NH<sub>3</sub> target (polarized proton)*



See talk by M. Chiosso

# Multidimensional TMD multiplicities @ COMPASS

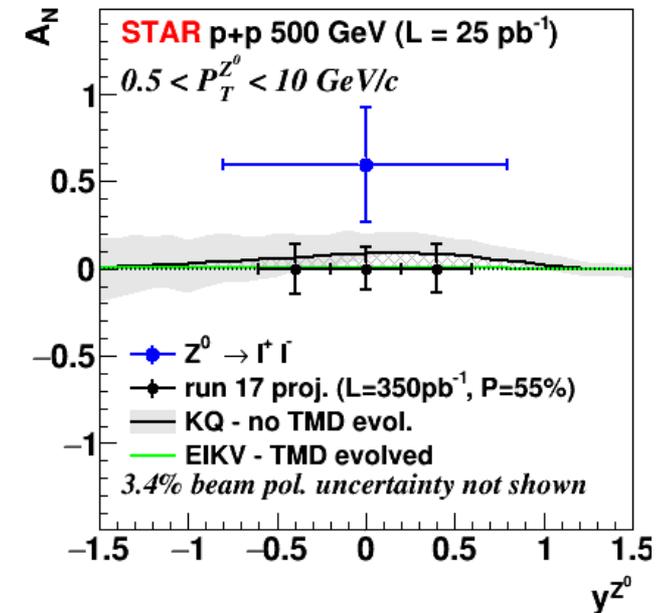
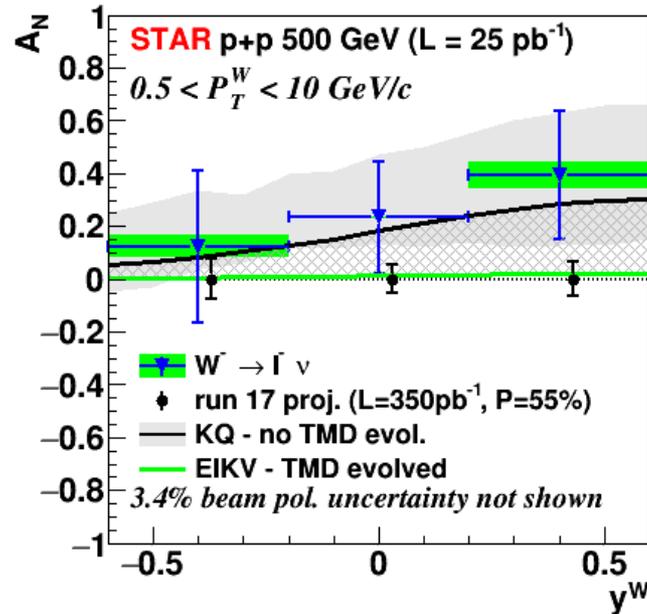
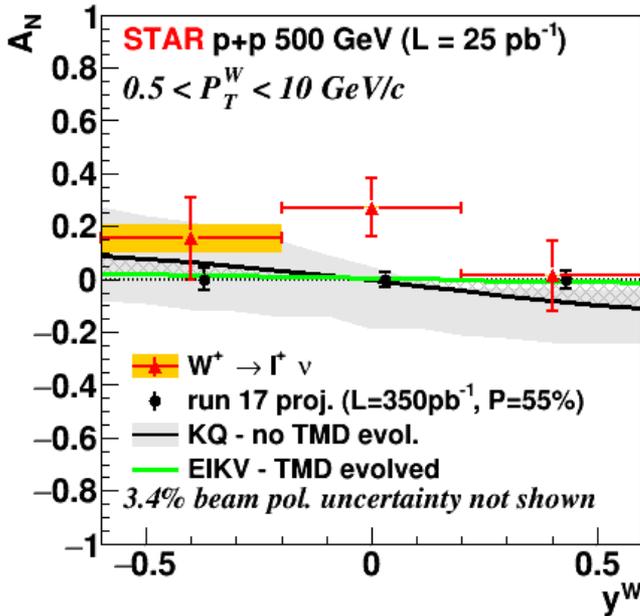
COMPASS Collaboration, arXiv:1709.07374 [hep-ex]



See talk by A. Bressan

# Sivers function in $p^\uparrow + p \rightarrow W^\pm/Z$ @ RHIC RUN 2017

STAR Collaboration, Phys. Rev. Lett. 116 132301 (2016)



See talk by E.C. Ashenauer and S. Fazio

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***Need for a new,  
comprehensive  
study of the  
Sivers effect***

# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

## New parametrization of the Sivers function

$$\Delta^N f_{q/p\uparrow} = \underbrace{N_q x^{\alpha_q} (1-x)^{\beta_q}}_{\text{First moment of the Sivers fn.}} \underbrace{\frac{M_p}{\langle k_{\perp}^2 \rangle_S} k_{\perp} \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle_S}}{\pi \langle k_{\perp}^2 \rangle_S}}_{\text{k}_{\perp} \text{ dependence of the Sivers fn.}}$$

First moment of the Sivers fn.  
Flavour dependent ( $u_v, d_v$ )

$k_{\perp}$  dependence of the Sivers fn.  
Flavour independent

Sivers functions  
not proportional  
to TMD PDFs

No direct control on  
the positivity bound

$M_p$  is a fixed parameter to  
give the right dimensions.  
It is fixed to 1 GeV

- In perspective: parametrization in terms of momentum better suited for the study of TMD evolution
- It makes the expression of the actual Sivers asymmetry as simple as possible (within this model)

Sivers Asymmetry (numerator)

$$F_{UT}^{\sin(\phi_S - \phi_h)} = \frac{1}{2} \frac{z P_T}{\langle P_T^2 \rangle} \frac{e^{-P_T^2 / \langle P_T^2 \rangle_S}}{\pi \langle P_T^2 \rangle_S} \sum_q e_q^2 \left( N_q x^{\alpha_q} (1-x)^{\beta_q} \right) D_{h/q}(z)$$

# Work in progress ...

*Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin*

## Consistent data selection:

- Exclude negative kaons for valence dominance assumption
- $u_v$  seems well constrained
- $d_v$  is not constrained: it can be replaced by sea contributions with equally good fits:  
hard to distinguish where this contribution comes from
- Siverts sea is totally unconstrained

**It is of vital importance to gain information  
on the  $d$  content of the Siverts function**

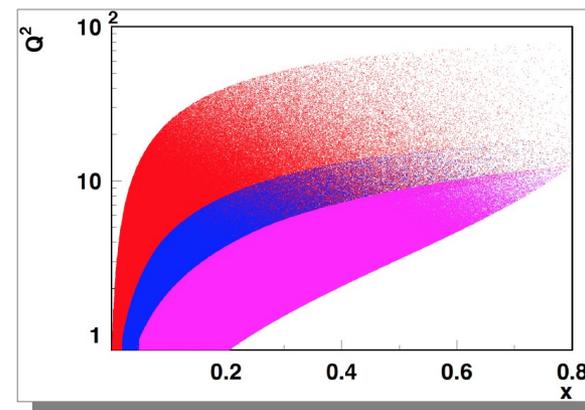
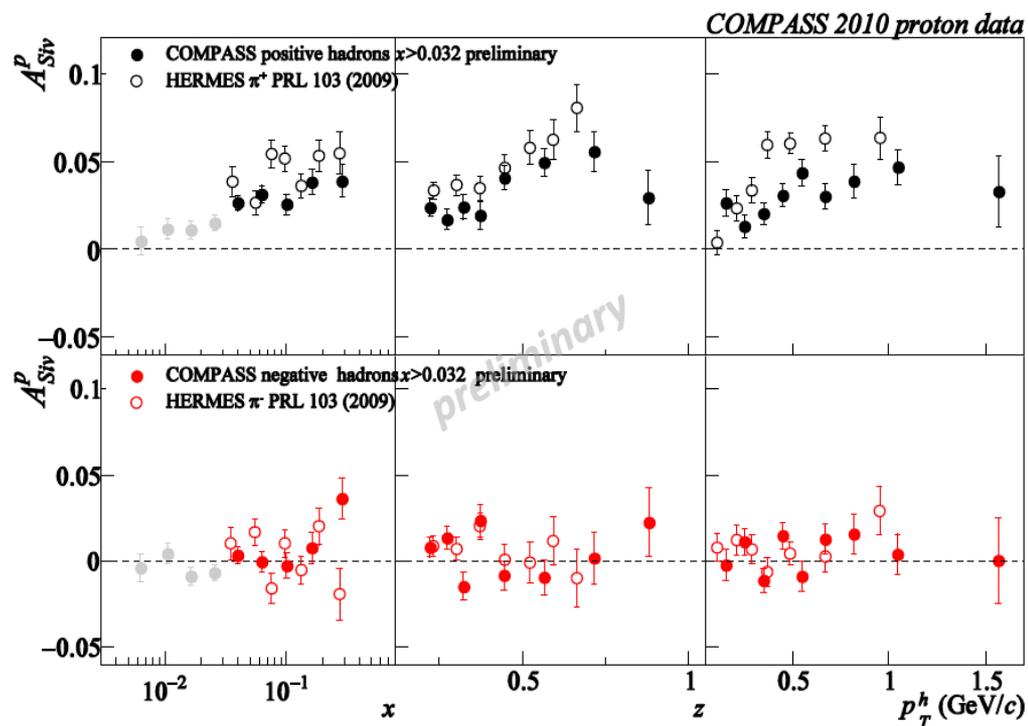
**We strongly rely on SIDIS  
measurements of the Siverts  
asymmetry on deuterium target  
@ COMPASS !**

# Sivers effect: COMPASS vs. HERMES

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Apparently ... signal of some tension between COMPASS and HERMES data

However, COMPASS and HERMES span different ranges in  $Q^2$  and have different  $\langle Q^2 \rangle$ .



Possible signal of TMD evolution?

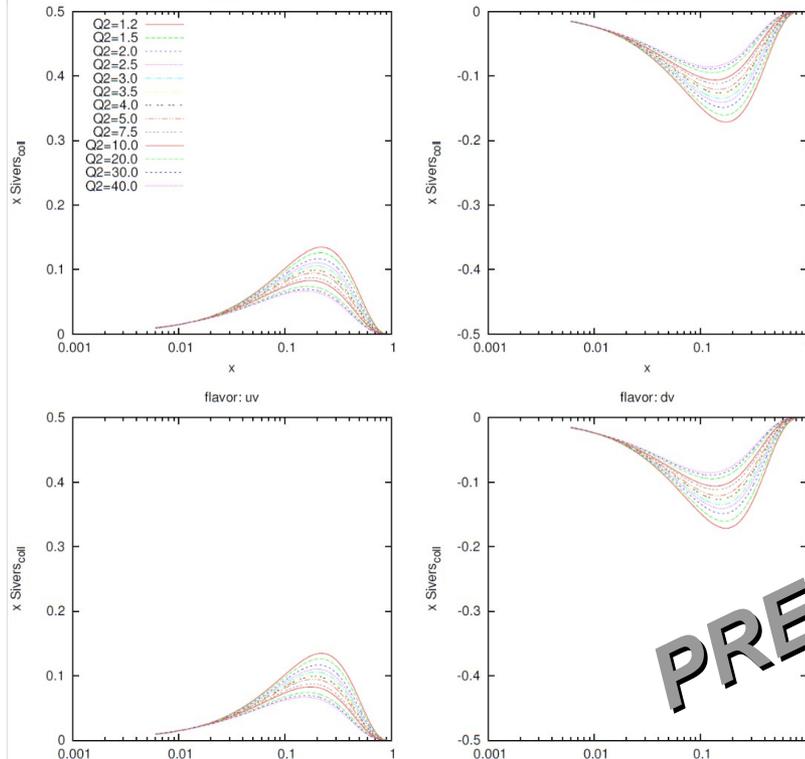
# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

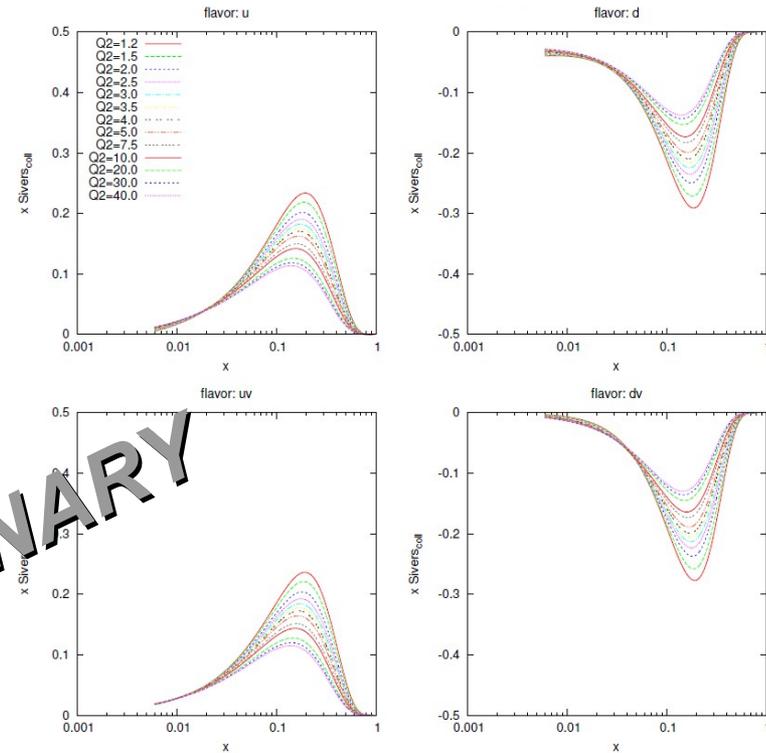
- HERMES 2009
- COMPASS 2009 (deuteron)
- COMPASS 2017 (h+,h-, Q<sup>2</sup> bins)
- JLAB 2011

HOPPET evolution of the whole Sivers function compared to DGLAP evolution of the collinear part only

Sivers function with DGLAP evolution of the collinear part



Sivers function with HOPPET evolution of the whole function

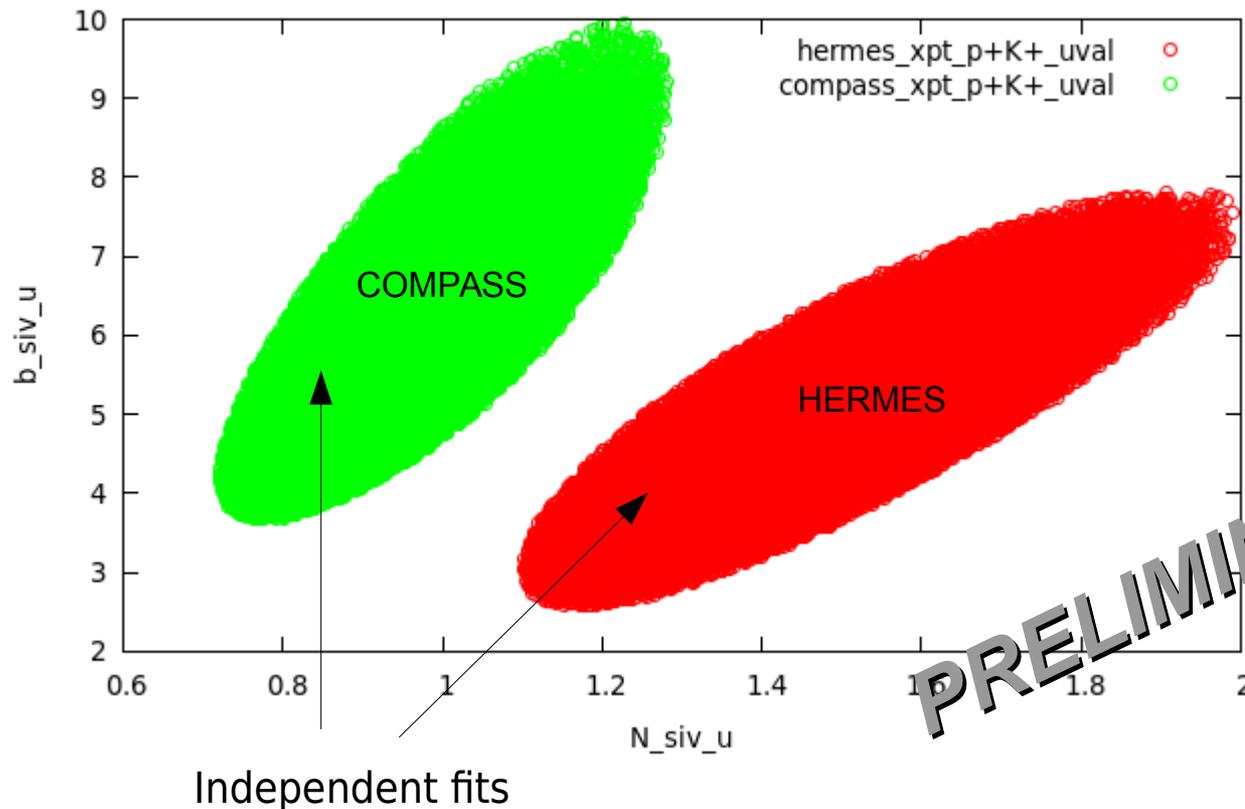


PRELIMINARY

# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Signal of some tension between independent fit solutions for COMPASS and HERMES data



# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

To calculate any spin asymmetry it is crucial to use the appropriate denominator, i.e. the appropriate unpolarized cross section

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\text{with } \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle$$

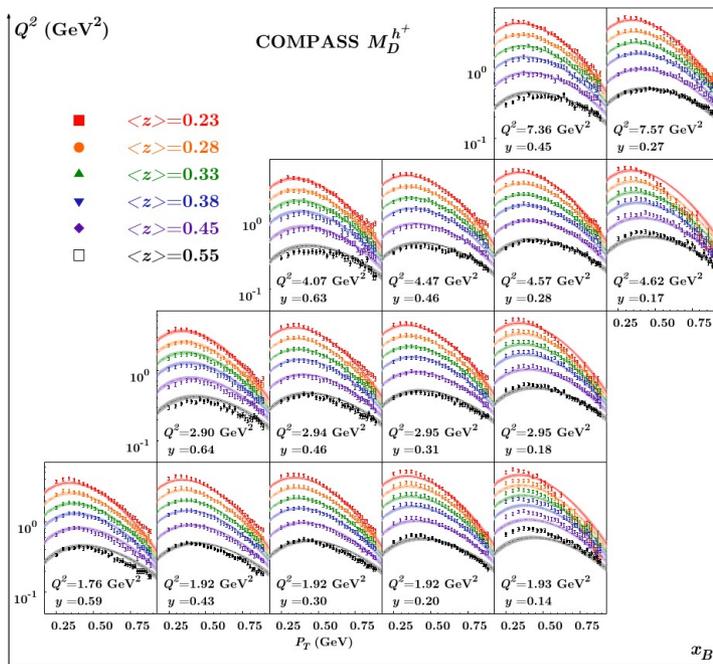
First item on the wish-list:

- Measure  $p_{\perp}$  distributions of unpolarized cross sections in SIDIS, Drell-Yan,  $e+e-$  processes, please

# Naive TMD approach

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, *JHEP* 1404 (2014) 005

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \quad \text{with} \quad \langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

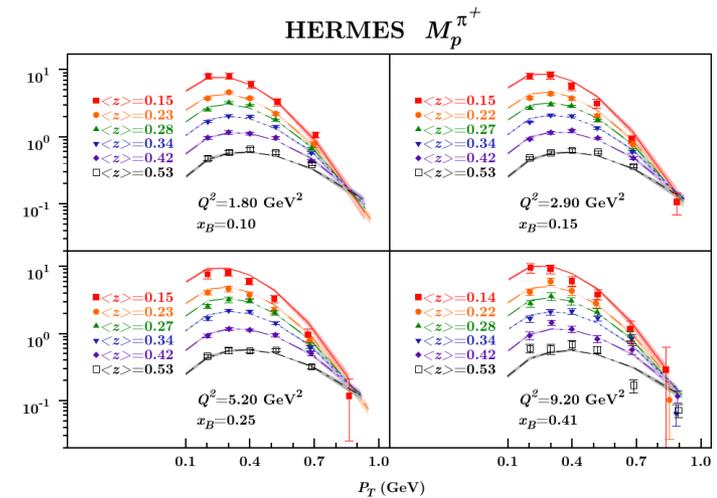


COMPASS, Adolph et al., *Eur. Phys. J. C* 73 (2013) 2531

$$\langle k_\perp^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 3.42$$



Airapetian et al, *Phys. Rev. D* 87 (2013) 074029

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

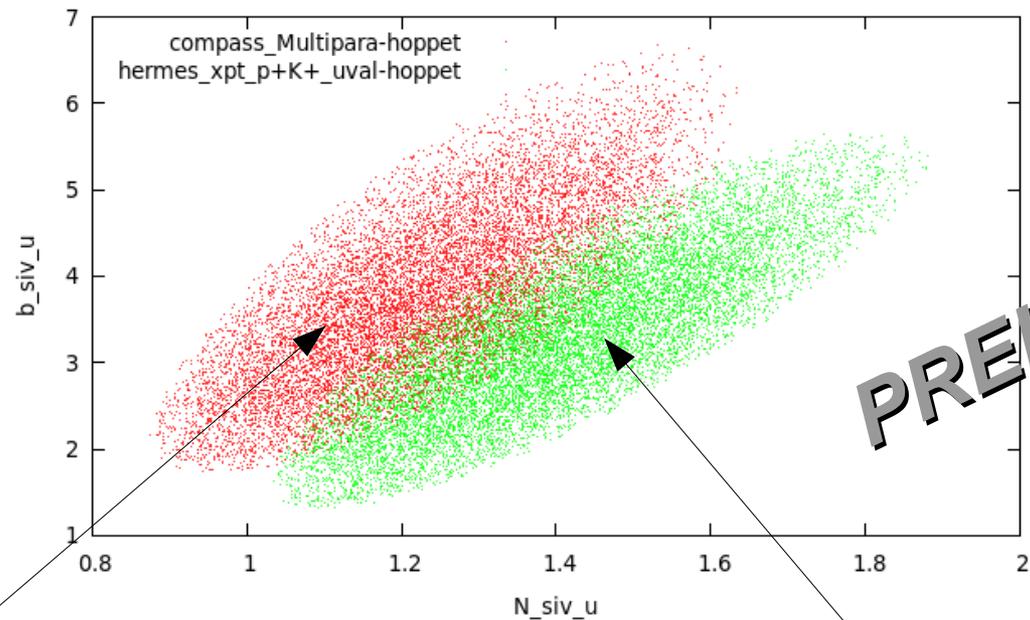
$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set



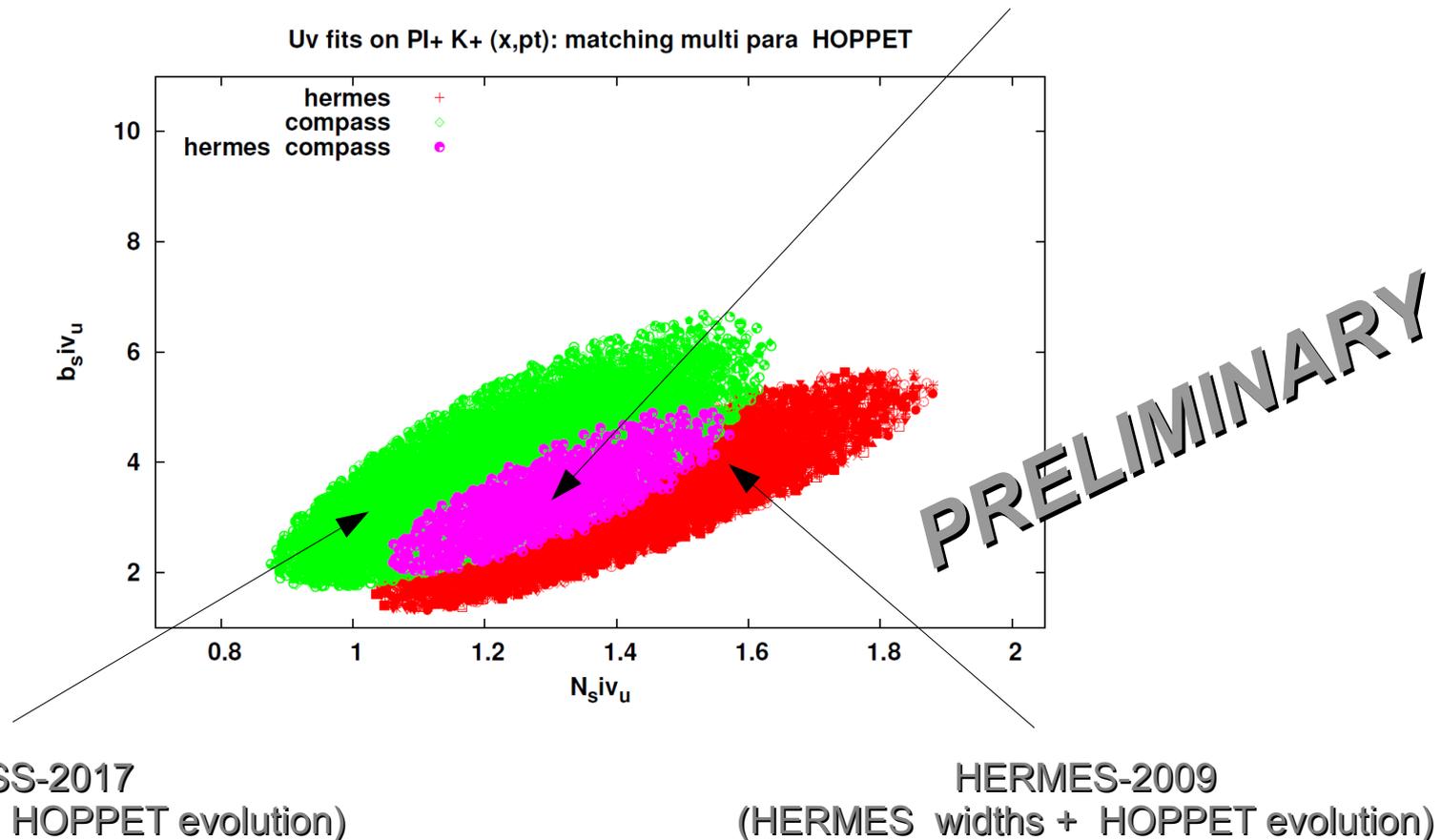
COMPASS-2017  
(COMPASS widths + HOPPET evolution)

HERMES-2009  
(HERMES widths + HOPPET evolution)

# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

Simultaneous fit of  
HERMES-2009 (HERMES widths + HOPPET)  
COMPASS-2017 (COMPASS widths + HOPPET)

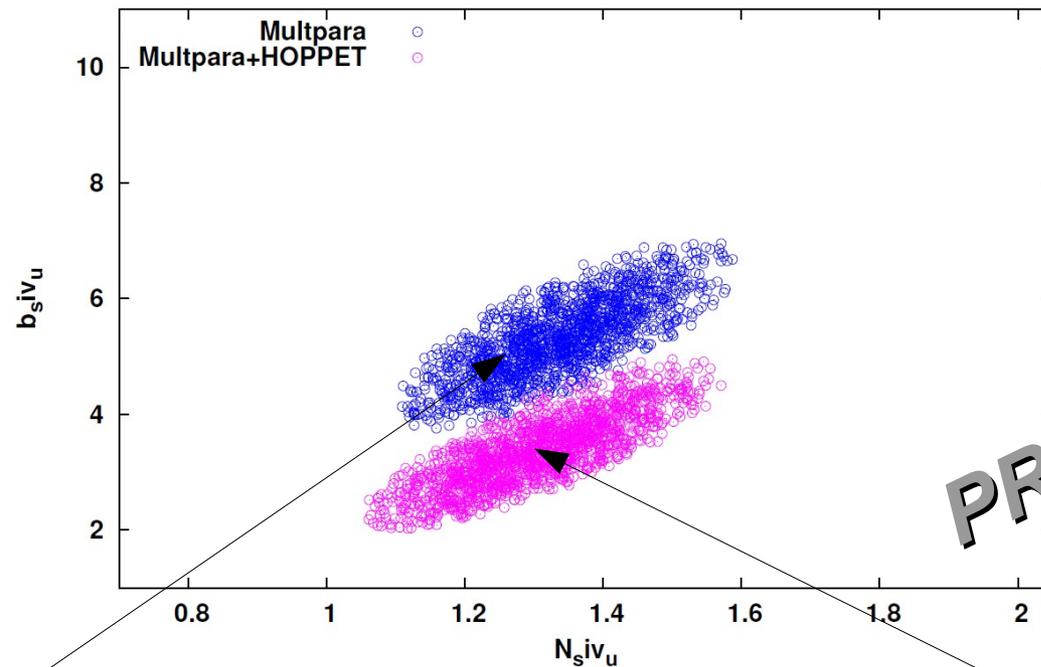


# Work in progress ...

Anselmino, Boglione, D'Alesio, Flore, Gonzalez, Murgia, Prokudin

## Simultaneous fit of HERMES-2009 (HERMES widths) COMPASS-2017 (COMPASS widths)

Combined hermes compass  $U_v$  fits on  $PI+K+(x,pt)$



No evolution

HOPPET evolution)

# Outlooks and perspectives

- Phenomenological studies of TMDs, TMD factorization and TMD extraction have come a long way.
- Some issues remain open and need further investigation
- $P_T$  distributions of SIDIS cross sections need to be measured (over the largest possible  $P_T$  range) and further investigated on the phenomenological point of view.
- Simultaneous fits of SIDIS, Drell-Yan and  $e^+e^-$  annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.
- Data selection is crucial in global fitting:
  - not too many  
(only data within the ranges where the TMD evolution schemes work should be considered)
  - not too few  
(too strict a selection can bias the fit results and neglect important information from experimental data)