

Azimuthal distributions in the Drell-Yan process

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Outline:

- Drell-Yan angular coefficients
- Fixed-order pQCD & phenomenology
- Resummation & phenomenology

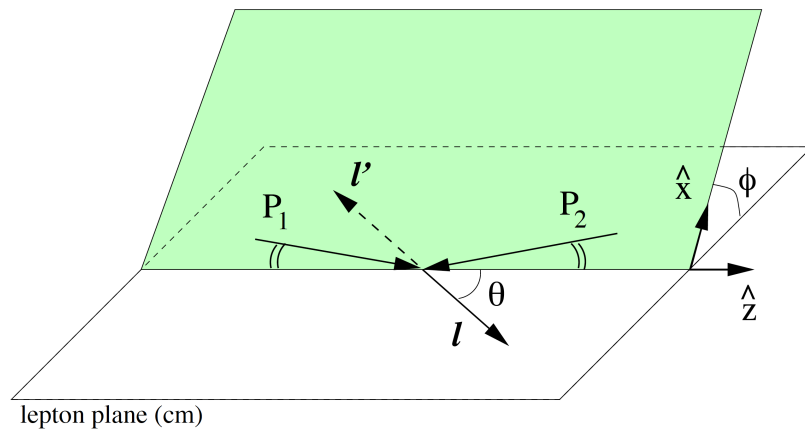
Will focus on “collinear pQCD perspective”
Drell-Yan extremely well explored

Collab. / discussions with M. Lambertsen, J. Steiglechner,
A. Bacchetta, G. Bozzi, F. Piacenza

Drell-Yan angular coefficients

Lepton angular distribution in Drell-Yan (photon exch.):

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha^2}{2\pi N_c Q^2 s^2} \left(W_T (1 + \cos^2 \theta) + W_L (1 - \cos^2 \theta) \right. \\ \left. + W_{\Delta} \sin 2\theta \cos \phi + W_{\Delta\Delta} \sin^2 \theta \cos 2\phi \right)$$



(Collins-Soper frame)

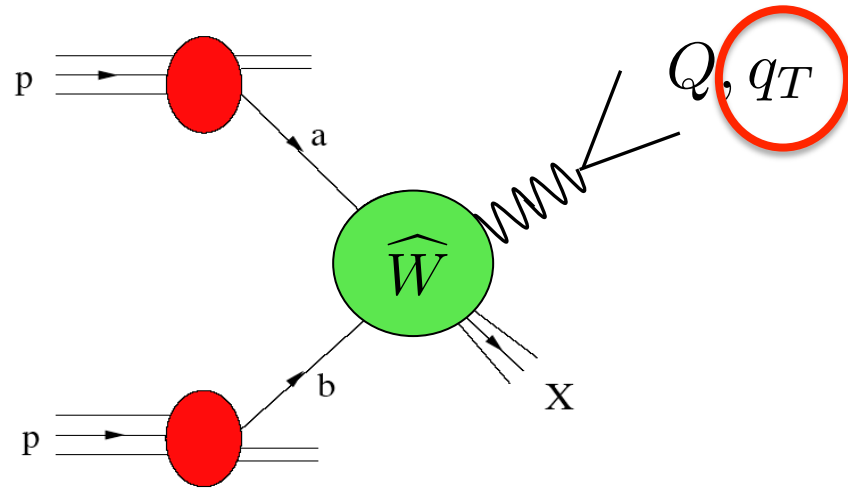
$$\begin{aligned}
\frac{d\sigma}{d^4q d\Omega} &= \frac{\alpha^2}{2\pi N_c Q^2 s^2} \left(W_T (1 + \cos^2 \theta) + W_L (1 - \cos^2 \theta) \right. \\
&\quad \left. + W_\Delta \sin 2\theta \cos \phi + W_{\Delta\Delta} \sin^2 \theta \cos 2\phi \right) \\
&= \frac{3\sigma_0}{4\pi} \frac{1}{\lambda + 3} \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right] \\
&= \frac{3\sigma_0}{16\pi} \left[1 + \cos^2 \theta + \frac{A_0}{2} (1 - 3\cos^2 \theta) + A_1 \sin 2\theta \cos \phi + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \right]
\end{aligned}$$

where:

$$\begin{aligned}
\lambda &= \frac{W_T - W_L}{W_T + W_L}, \quad \mu = \frac{W_\Delta}{W_T + W_L}, \quad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} \\
A_0 &= \frac{2W_L}{2W_T + W_L}, \quad A_1 = \frac{2W_\Delta}{2W_T + W_L}, \quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L}
\end{aligned}$$

Fixed-order pQCD

Collinear factorization:



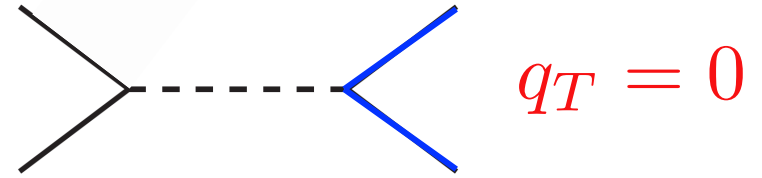
$$W_P = \sum_{a,b} \int dx_a dx_b f_a(x_a, \mu) f_b(x_b, \mu) \widehat{W}_{P,ab}(x_a P_a, x_b P_b, q, \alpha_s(\mu), \mu)$$

($P = T, L, \Delta, \Delta\Delta$)

- \widehat{W}_P partonic structure fcts.: **perturbative**

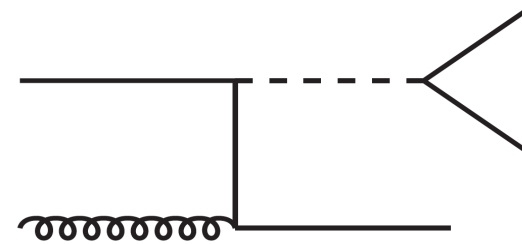
$$\widehat{W}_P = \frac{\alpha_s}{\pi} \widehat{W}_P^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \widehat{W}_P^{(2)} + \dots \quad \text{(fixed-order)}$$

- zeroth order $q\bar{q} \rightarrow V \rightarrow \ell^+\ell^-$:



- $q_T \neq 0$: first non-trivial order (= LO)

$\mathcal{O}(\alpha_s)$

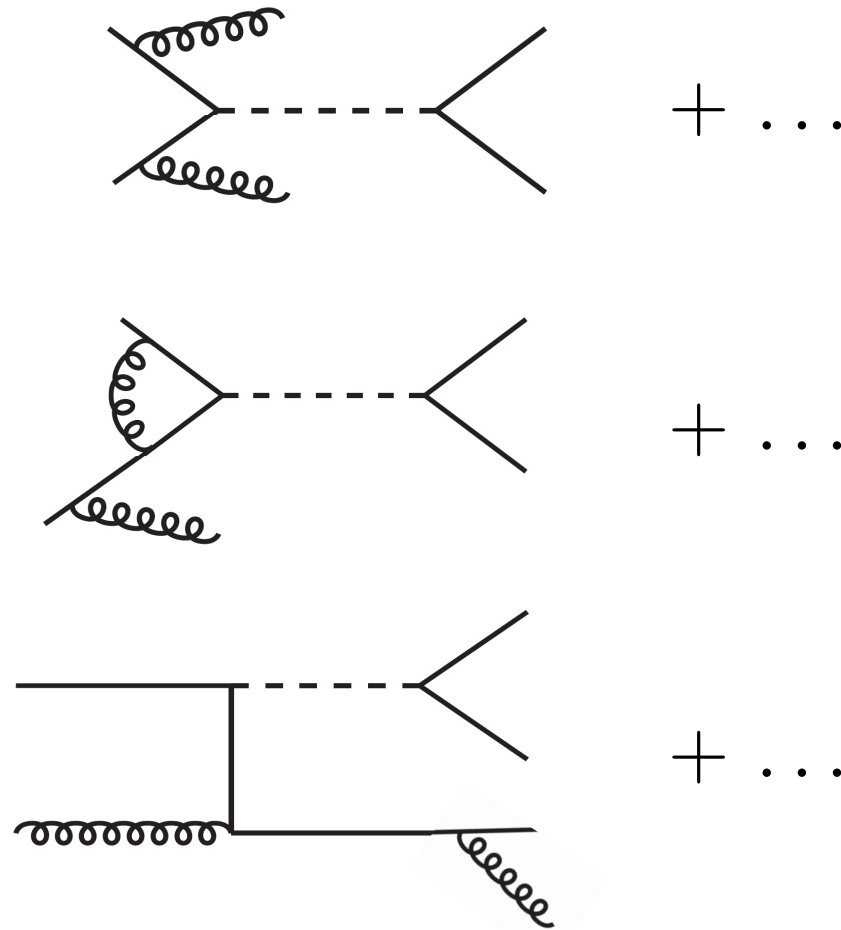


$$\lambda \neq 1, \mu \neq 0, \nu \neq 0$$

- but: $1 - \lambda - 2\nu = 0$ (Lam-Tung relation)

$$A_0 = A_2$$

NLO:



$\mathcal{O}(\alpha_s^2)$

first computed by Mirkes '92; Mirkes, Ohnemus '95

- a lot of work in recent 2 decades on $\mathcal{O}(\alpha_s^2)$ corrections for Drell-Yan process

Hamberg, van Neerven, Matsuura; Harlander, Kilgore;
Anastasiou, Dixon, Melnikov, Petriello; Melnikov, Petriello;
Li, Petriello, Quackenbusch; Catani, Cieri, Ferrera, de Florian, Grazzini;
Karlberg, Re, Zanderighi; ...

- especially: $\mathcal{O}(\alpha_s^2)$ Monte-Carlo codes

FEWZ: Melnikov, Petriello; Melnikov, Petriello;
Li, Petriello, Quackenbusch

DYNNLO: Catani, Cieri, Ferrera, de Florian, Grazzini

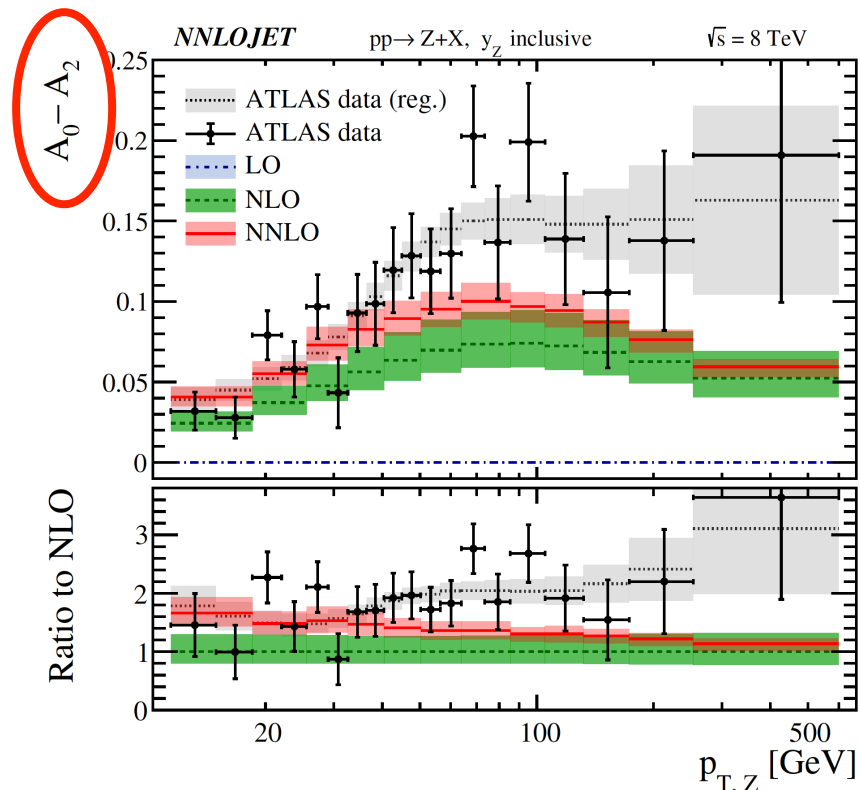
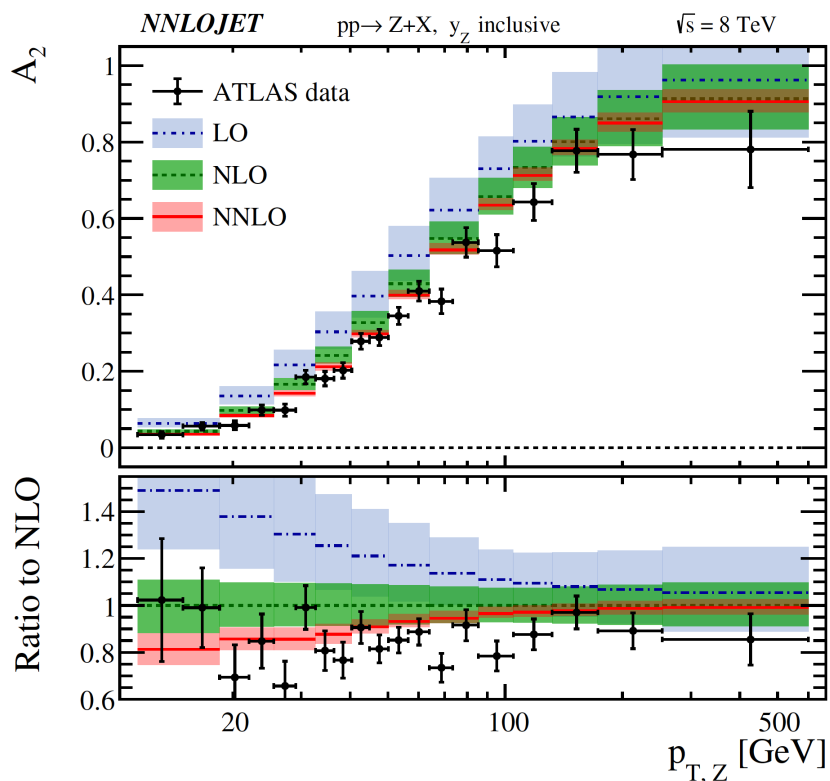
- most recently: $\mathcal{O}(\alpha_s^3)$

Li, von Manteuffel, Schabinger, Zhu;
Anastasiou, Duhr, Dulat, Herzog, Mistlberger;

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Fixed-order phenomenology

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17



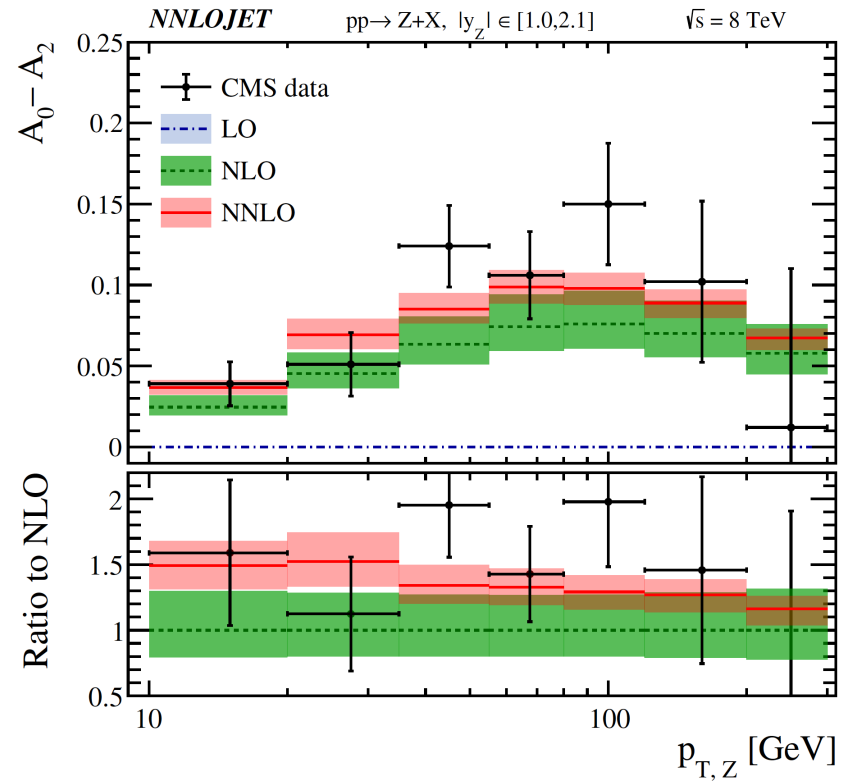
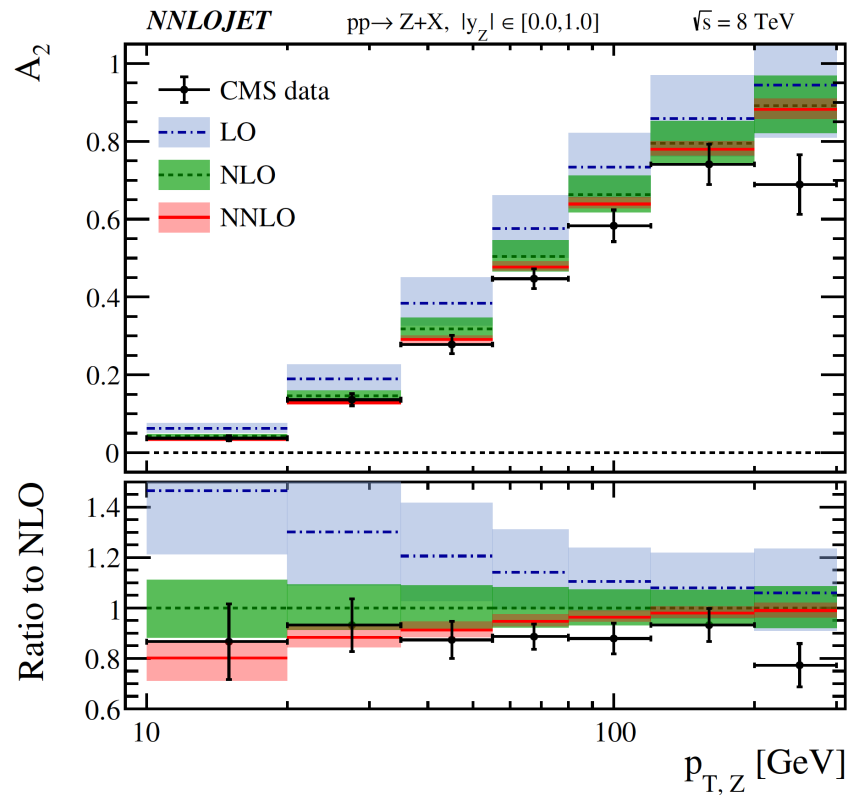
$\mathcal{O}(\alpha_s^2)$ NLO (ATLAS):

$$\chi^2/N_{\text{data}} = 185.8/38 = 4.89$$

$\mathcal{O}(\alpha_s^3)$ NNLO (ATLAS):

$$\chi^2/N_{\text{data}} = 68.3/38 = 1.80$$

Gauld, Gehrman-De Ridder, Gehrman, Glover, Huss '17



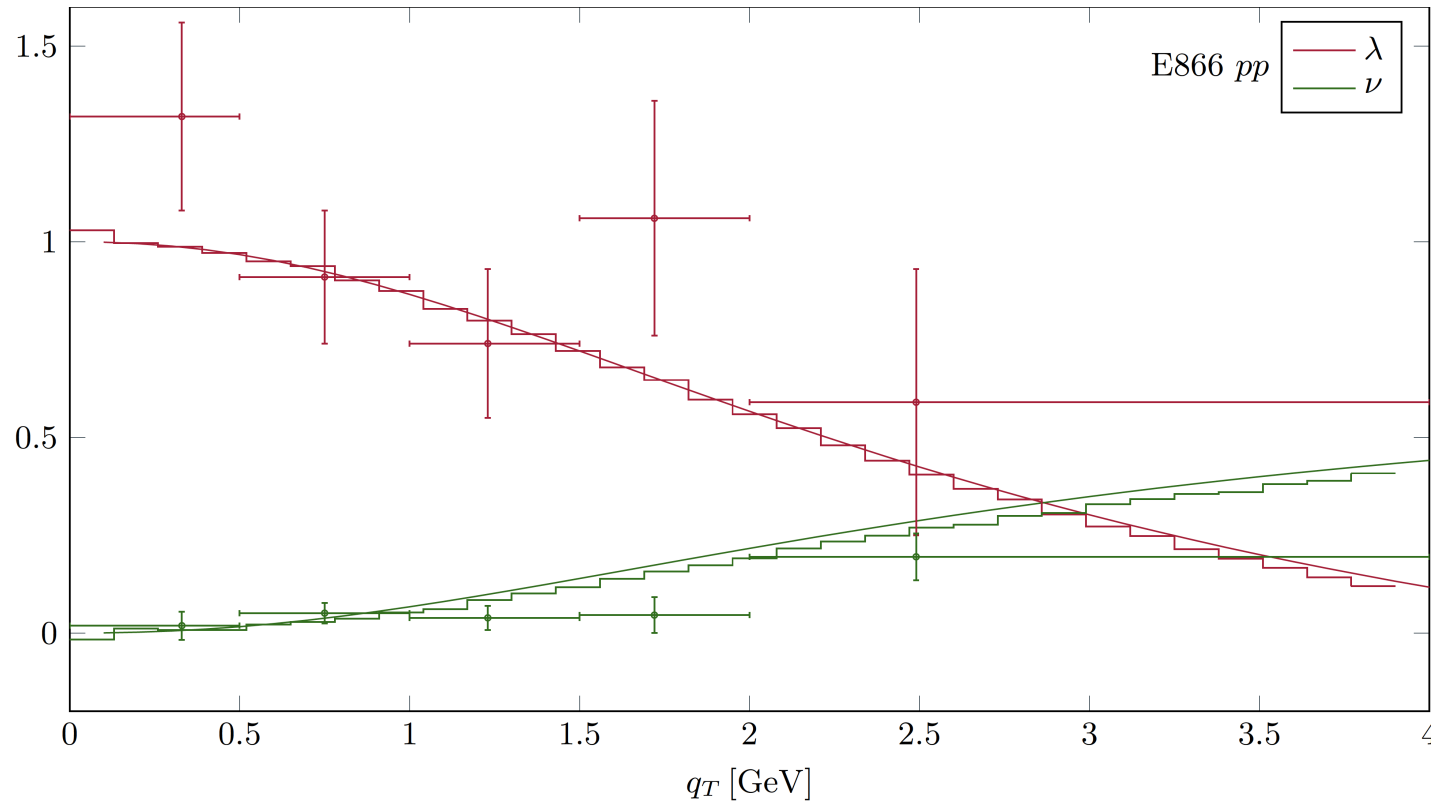
NLO (CMS): $\chi^2 / N_{\text{data}} = 24.5 / 14 = 1.75$

NNLO (CMS): $\chi^2 / N_{\text{data}} = 14.2 / 14 = 1.01$

see also: [Lambertsen, WV '16](#)
[Peng, Chang, McClellan, Teryaev '15](#)

$pp, E = 800 \text{ GeV}$

E866

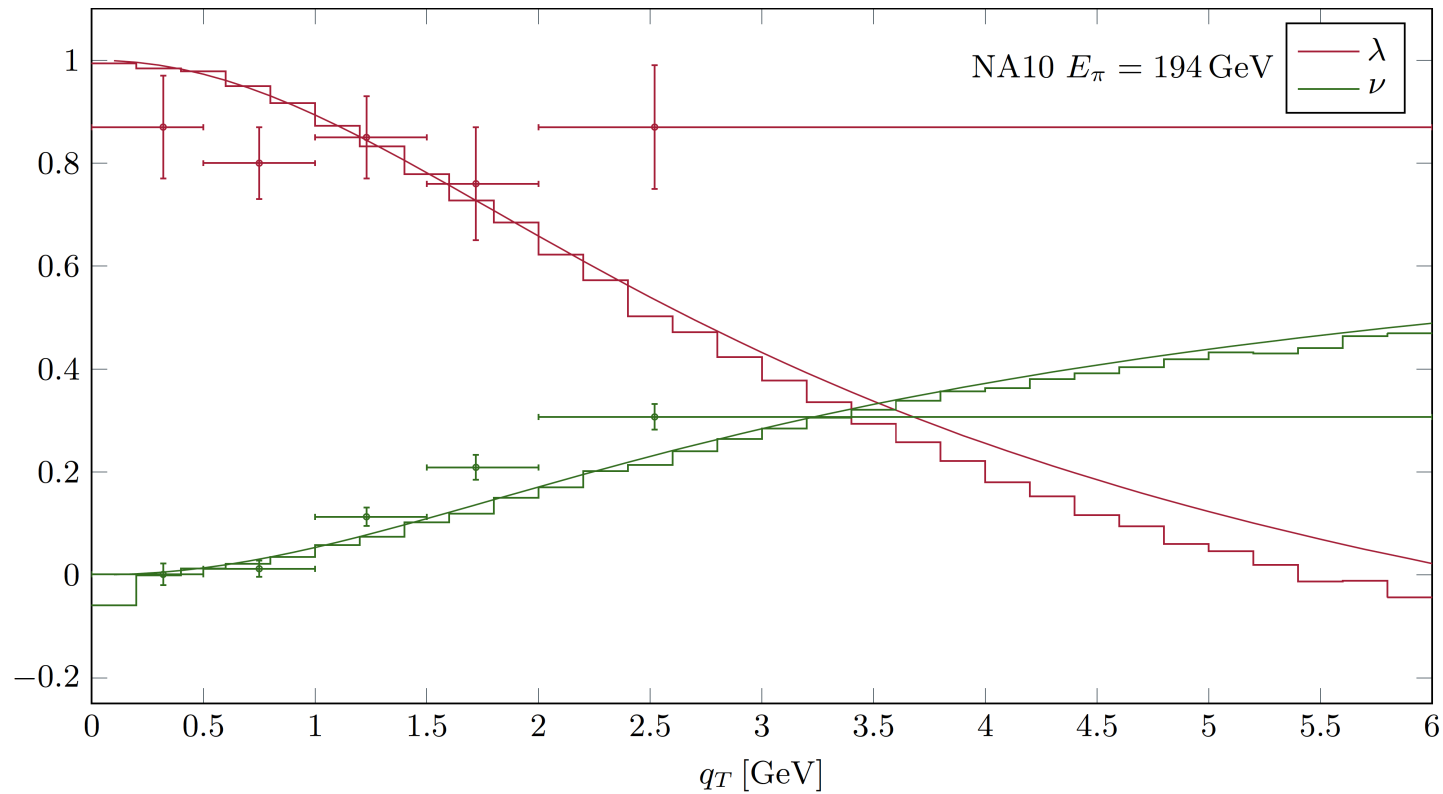


lines: LO $\mathcal{O}(\alpha_s)$

histograms: NLO $\mathcal{O}(\alpha_s^2)$

$\pi W, E_\pi = 194 \text{ GeV}$

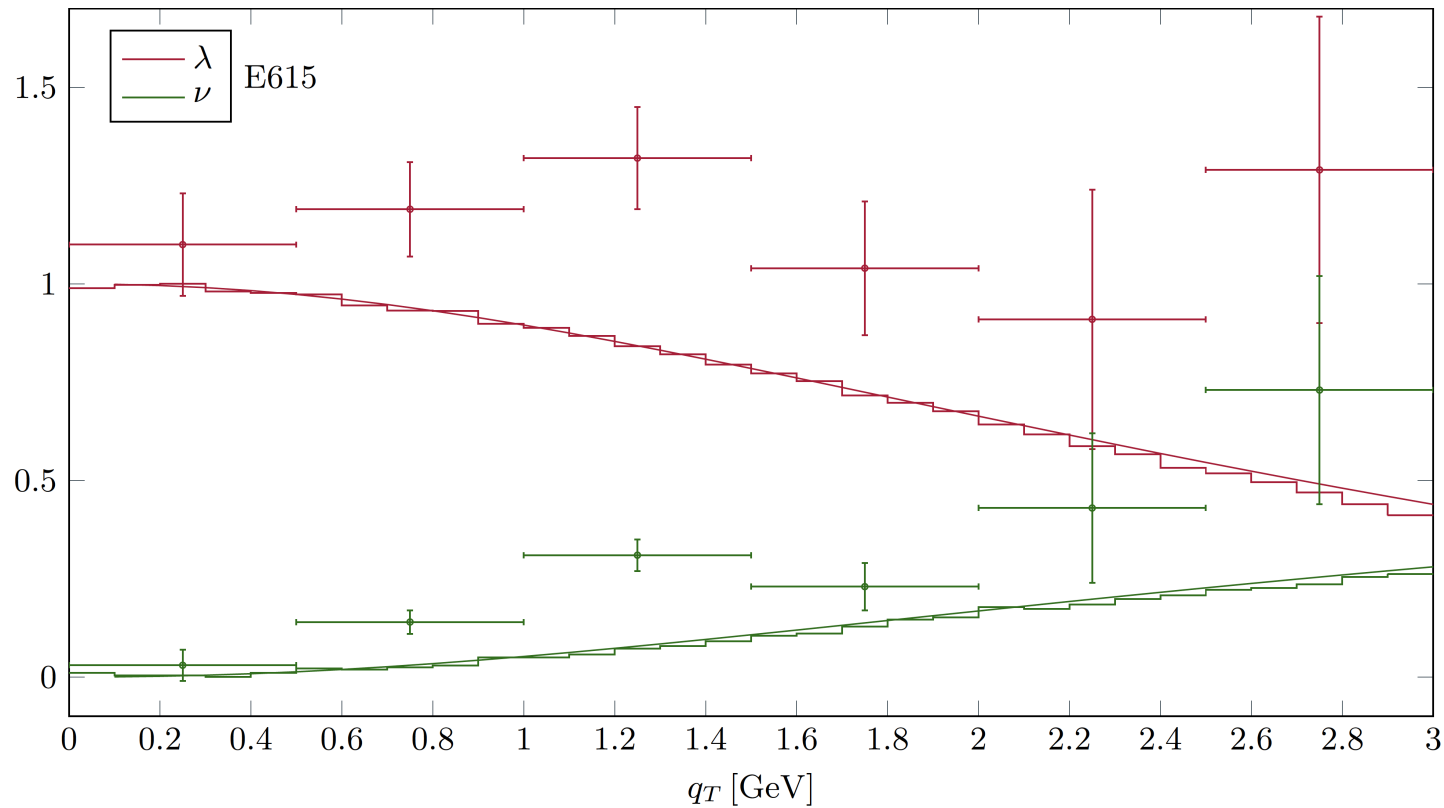
NA10



- “dispel myth” that pQCD cannot describe data:
overall reasonable description

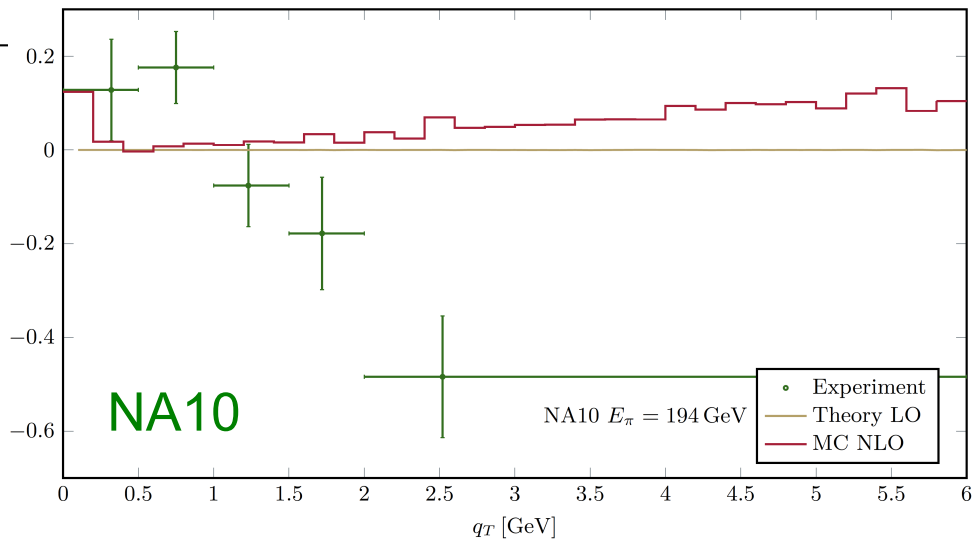
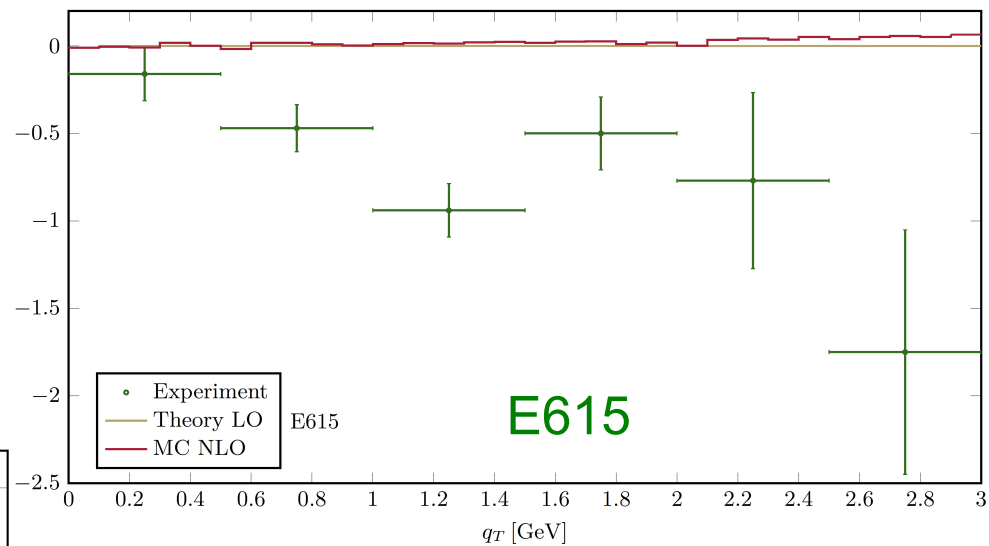
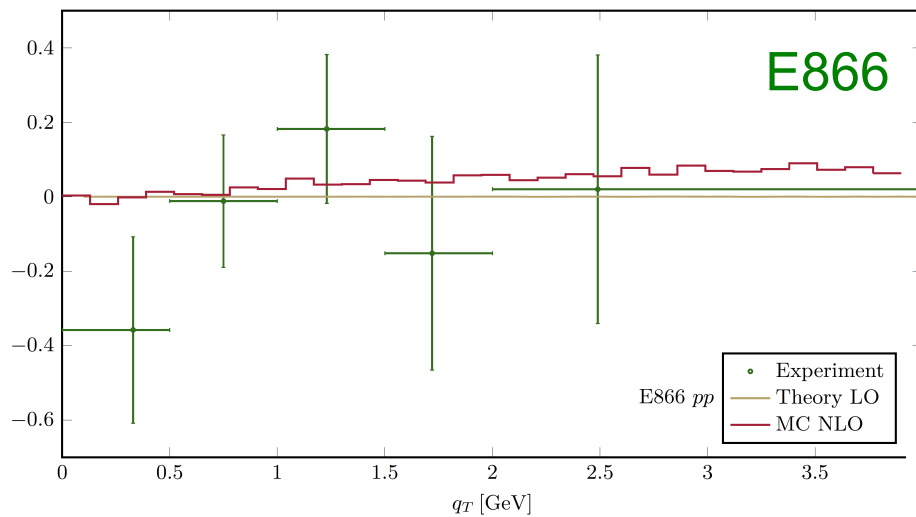
$\pi W, E_\pi = 252 \text{ GeV}$

E615



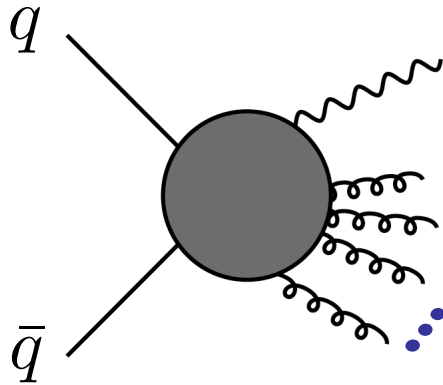
- note: positivity constraint $\lambda \leq 1$ Lam, Tung '78

Lam-Tung $1 - \lambda - 2\nu$



Resummation

- region $q_T \ll Q$:



$$\widehat{W}_T^{(k)} \propto \alpha_s^k \left(\frac{\log^{2k-1}(q_T^2/Q^2)}{q_T^2} \right)_+ + \dots$$

“ q_T logarithms”

- all-order resummation very well understood for W_T
Collins-Soper-Sterman formalism
- 1-1 correspondence to TMD evolution

Collins, Mert Aybat, Rogers, Qiu; Echevarria, Melis, Scimemi, d’Alesio;
Kang, Prokudin, Sun, Yuan;...

- specifically structure functions at LO (CS frame):

$$\widehat{W}_T^{(1)} = -C_F \frac{\alpha_s}{2\pi} \frac{2 \log(q_T^2/Q^2) + 3}{q_T^2} + \dots$$

$$\widehat{W}_L^{(1)} = 2 \widehat{W}_{\Delta\Delta}^{(1)} = -C_F \frac{\alpha_s}{2\pi} (2 \log(q_T^2/Q^2) + 3) + \dots$$

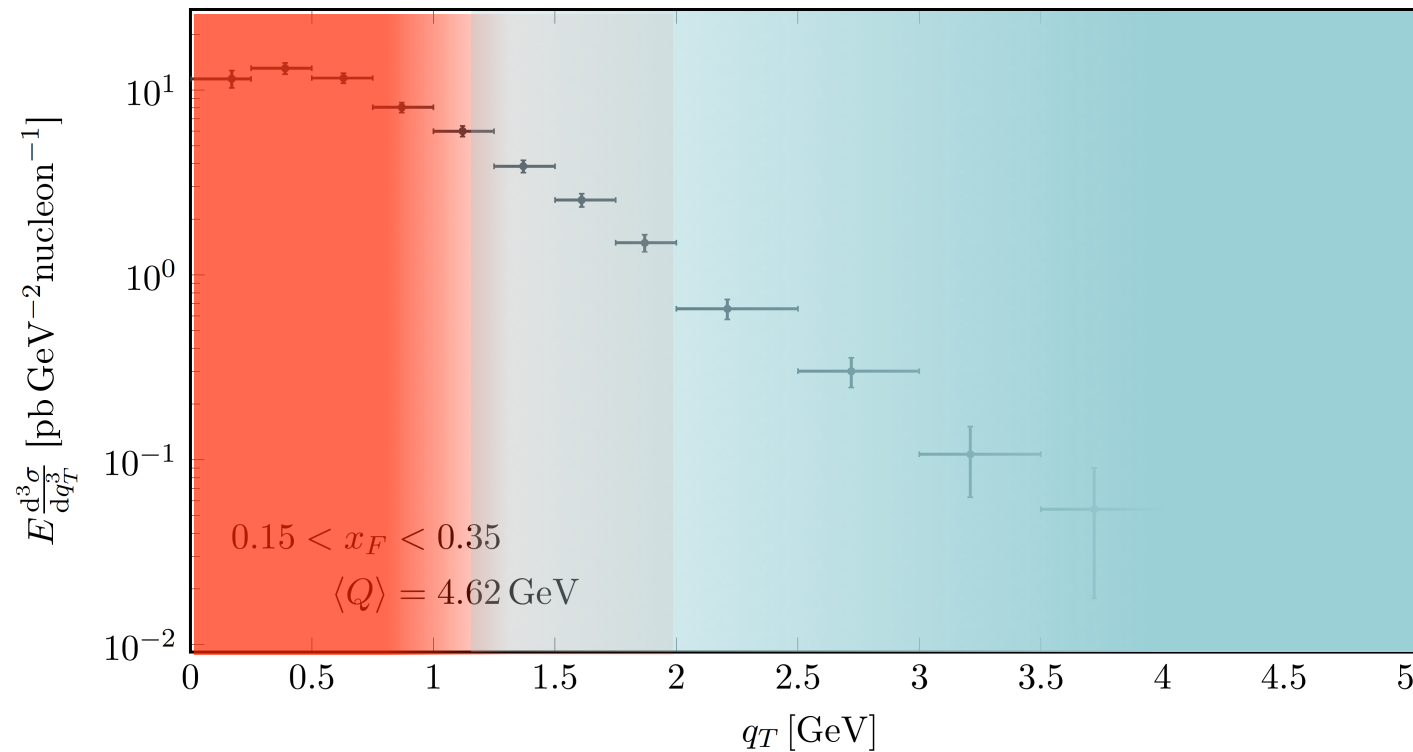
Boer, WV

- leading logs same to all orders in $\widehat{W}_T, \widehat{W}_L, \widehat{W}_{\Delta\Delta}$

Berger, Qiu, Rodriguez

- differences at next-to-leading log Boer, WV
- at present, resummation for angular coeff. **not fully understood**
vital for TMD phenomenology!
- insights from TMD evolution might help Prokudin @ INT2017

- however, need to understand full q_T spectrum:

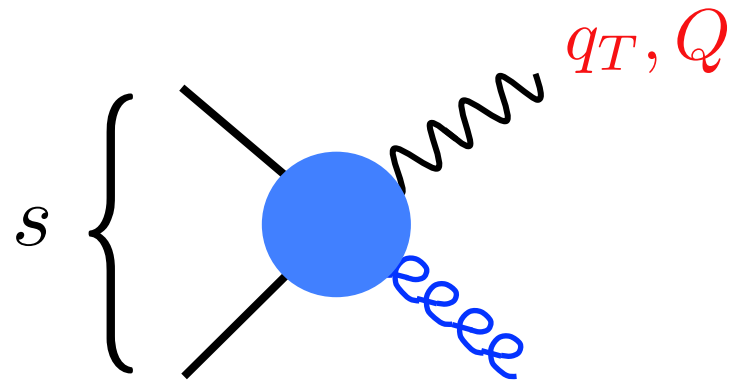


TMD regime

collinear regime

- crucial for matching (“Y term”)
- develops different set of large logs

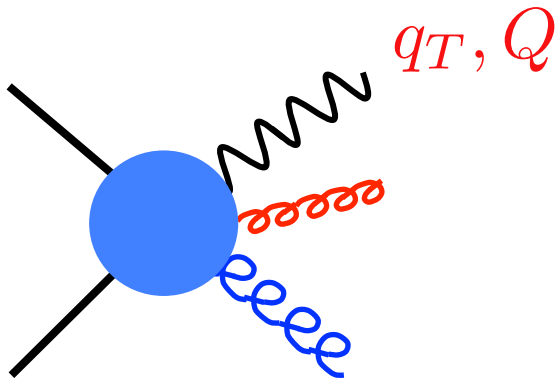
- LO :



$$\sqrt{s} \geq q_T + \sqrt{Q^2 + q_T^2}$$

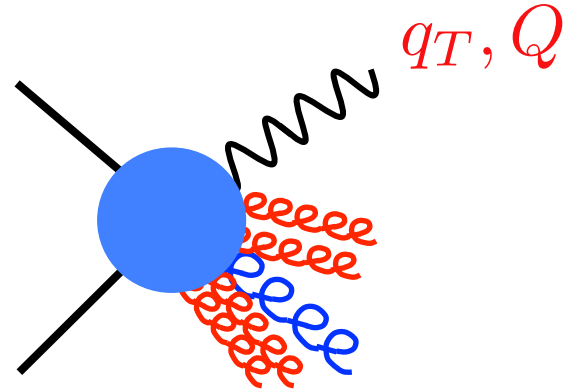
$$y_T \equiv \frac{q_T + \sqrt{q_T^2 + Q^2}}{\sqrt{s}} \leq 1$$

- NLO :



$$\frac{d\hat{\sigma}^{\text{NLO}}}{dq_T} \propto \alpha_s \left[\mathcal{A} \log^2(1 - y_T^2) + \mathcal{B} \log(1 - y_T^2) + \mathcal{C} \right]$$

- $N^k\text{LO}$:



$$\frac{d\hat{\sigma}^{N^k\text{LO}}}{dq_T} \propto \alpha_s^k \log^{2k}(1 - y_T^2) + \dots$$

- threshold logarithms

- threshold resummation:

$$\hat{\sigma}_{q\bar{q}\rightarrow\gamma^*g}(N) \equiv \int_0^1 dy_T^2 (y_T^2)^{N-1} \frac{d\hat{\sigma}_{q\bar{q}\rightarrow\gamma^*g}}{dp_T}$$

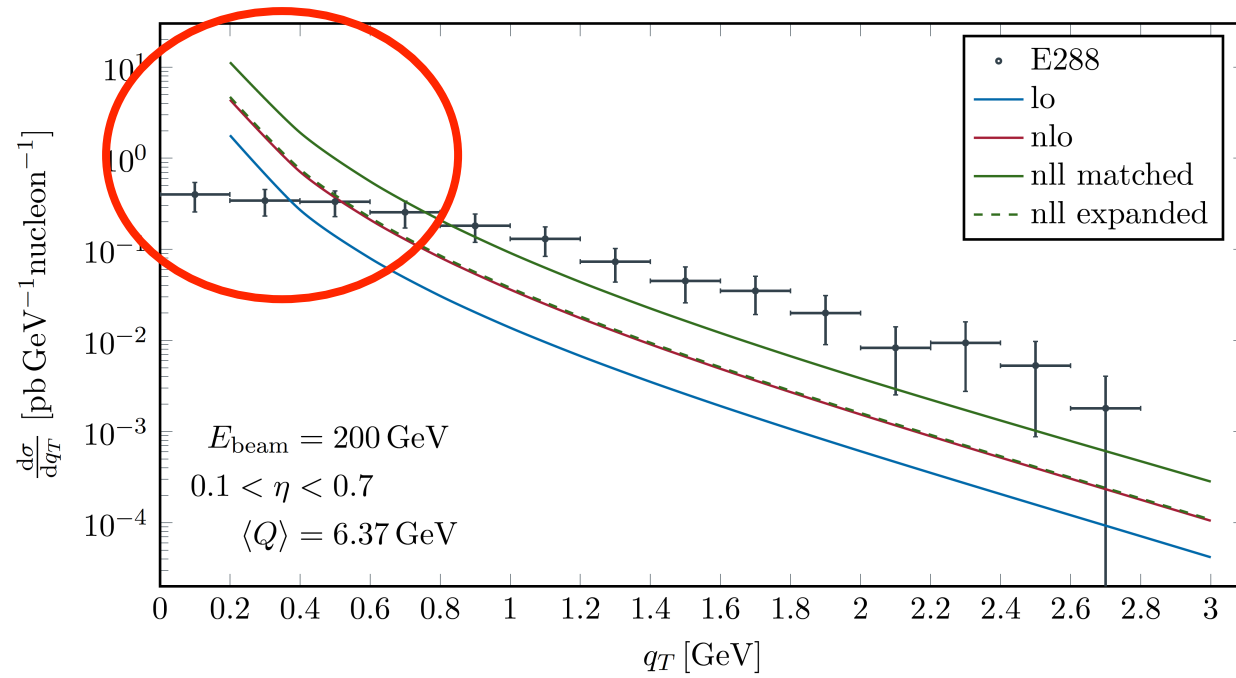
$$\hat{\sigma}_{q\bar{q}\rightarrow\gamma^*g}^{(\text{res})}(N) = C_{q\bar{q}\rightarrow\gamma^*g} \underbrace{\Delta_N^q \Delta_N^{\bar{q}} J_N^g \Delta_N^{(\text{int})}}_{\text{soft/coll. gluons}} \hat{\sigma}_{q\bar{q}\rightarrow\gamma^*g}^{\text{LO}}(N)$$

↑
hard virtual
corrections

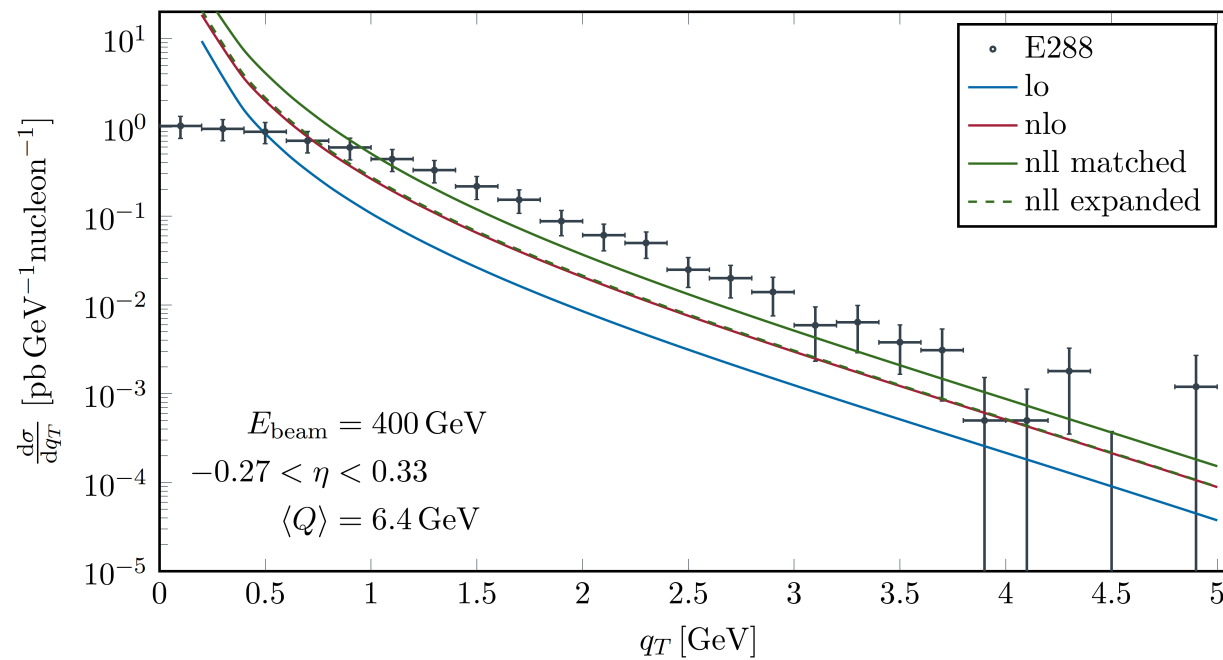
↑
LO cross sec.

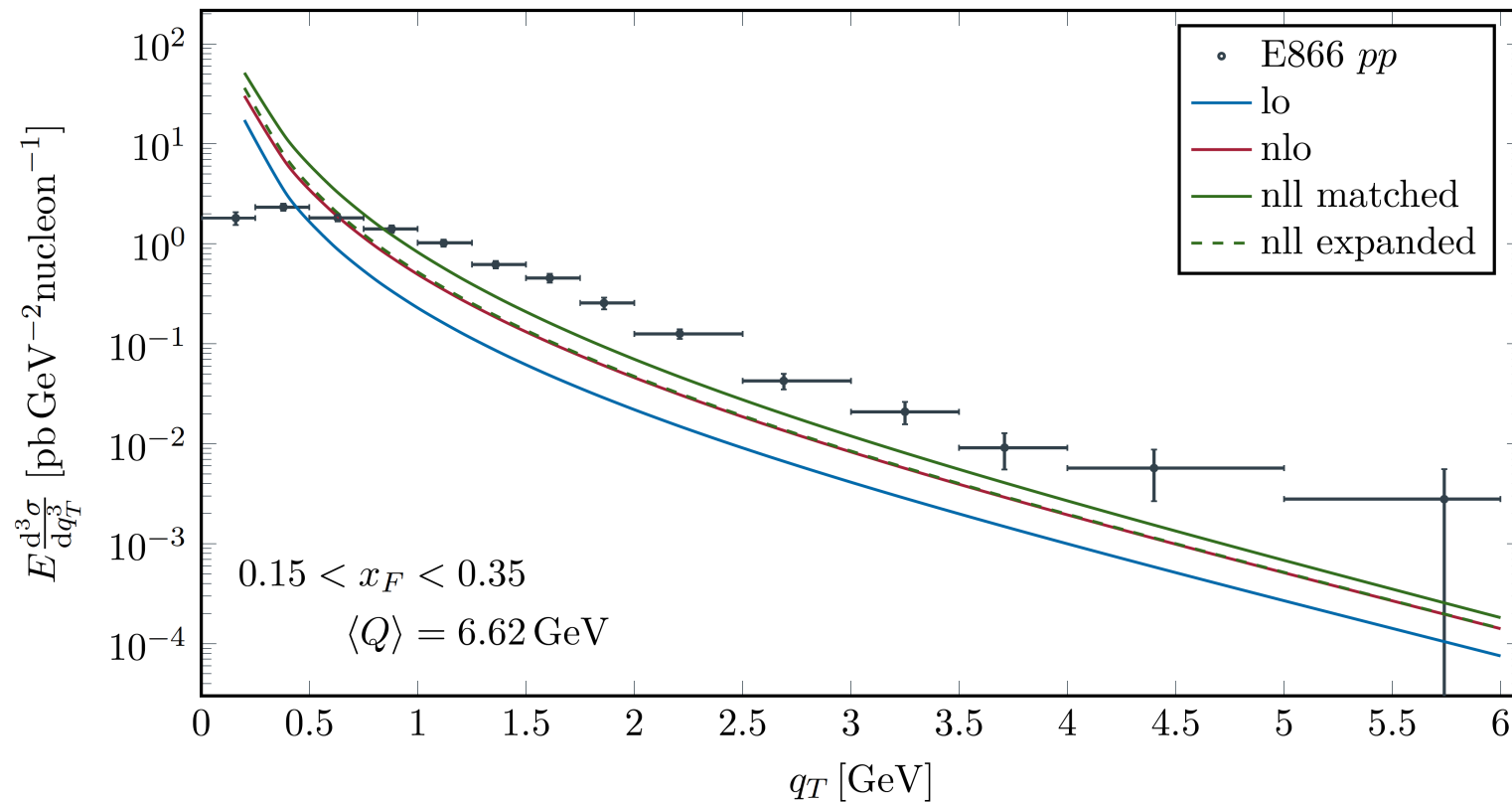
↙ ↘
soft/coll. gluons

$$\ln \Delta_N^q = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_q(\alpha_s(q^2))$$

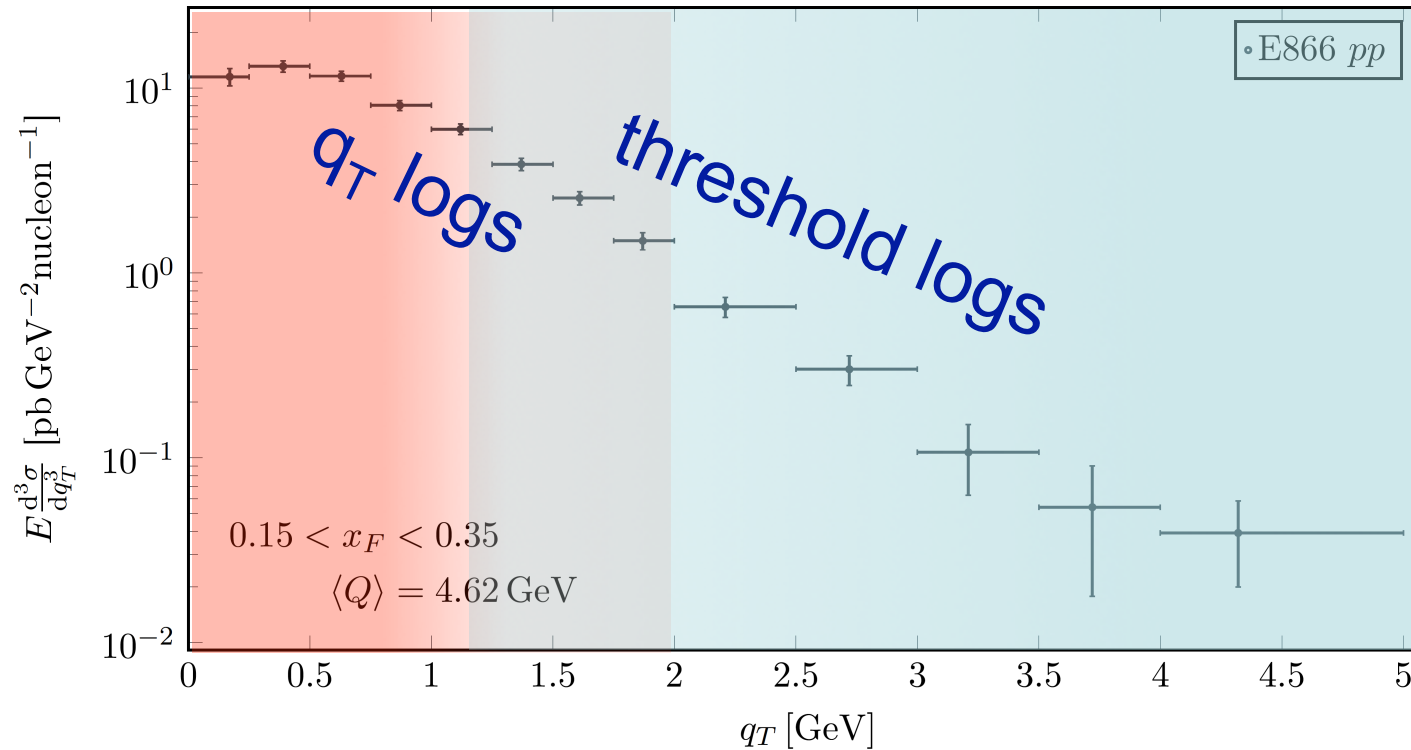


Lambertsen,
Steiglechner,
WV





- note, threshold resummation for angular dependences not known



- ultimately, will need **joint resummation** of both types of logs
- framework exists:
 - Laenen, Sterman, WV '00
 - Lustermans, Waalewijn, Zeune '16
 - Muselli, Forte, Ridolfi '17

- e.g. inclusive Drell-Yan:

$$\hat{\sigma}^{(\text{res})} \propto \exp \left[2 \int_0^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp})) \left[J_0(bk_{\perp}) K_0 \left(\frac{2Nk_{\perp}}{Q} \right) + \ln \left(\frac{\bar{N}k_{\perp}}{Q} \right) \right] \right]$$

- “jointly resummed” cross section:

Laenen, Sterman, WV
Kulesza, Sterman, WV

$N \gg bQ$: threshold logs (e.g. $b=0$)

$bQ \gg N$: q_{T} logs

- Drell-Yan at high q_{T} : Muselli, Forte, Ridolfi

- for pure q_{T} resummation case:

$$\sigma^{\text{eik}}(N, b) \approx \exp \left[-2 \int_0^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A_q(\alpha_s(k_{\perp})) \left(J_0(bk_{\perp}) - 1 \right) \ln \left(\frac{\bar{N}k_{\perp}}{Q} \right) \right]$$

- btw: shows how to match TMD and collinear framework

Conclusions and outlook:

- “dispel myth” that pQCD cannot describe λ, μ, ν
overall reasonable description
- *not* meant to argue that there are no effects beyond fixed-order pQCD
- presently not a really good understanding of Drell-Yan cross section at high q_T
- serious studies of Boer-Mulders etc. should include pQCD radiative effects
(including resummation, probably **joint resummation**)
- **clear call for better data in fixed-target regime**