

Measurements of top-quark production and properties at ATLAS

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- Latest cross-section measurements (5 analyses)
- 2) Measurements of spin-related observables (3 analyses)



Top-quark production







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Aims of analyses

ATLAS EXPERIMENT

- Large cross-sections O (100 pb) and data sets
 ➡ top-quark factory
- Test pQCD at NNLO precision (fixed-order)
- Tune MC generators
 - NLO ME generators
 - (New) parton shower generators
- Constrain parton distribution functions (PDFs)
- Determine SM parameters (m_t, |V_{tb}|)
- Measure rare processes: tt+W, tt+Z, tt+H; tZ, tH (not in this talk)
- Constrain new physics models
 - Anomalous couplings
 - Effective Field Theory operators
- Direct searches: $t\overline{t}$ resonances, W' $\rightarrow t\overline{b}$ (not in this talk)



Predictions made at NNLO+NNLL:

	7 TeV	8 TeV	13 TeV
$\sigma(t\bar{t})$	182 pb	259 pb	842 pb
$\Delta\sigma / \sigma$	±6%	±6%	±6%



Part 1: Latest cross-section measurements

$t\bar{t}$ production

- 1) Differential cross-sections in the eµ channel at \sqrt{s} = 13 TeV
- 2) Ratios of $t\bar{t}$ to Z-boson cross-sections at \sqrt{s} = 13, 8, 7 TeV
- 3) Differential cross-sections in the all-hadronic channel using **highly boosted** top quarks at $\sqrt{s} = 13$ TeV

Single top-quarks

- 1) t-channel production at \sqrt{s} = 8 TeV fiducial, total, differential cross-sections
- 2) tW production at $\sqrt{s} = 13$ TeV

Single top-quark production in the t-channel





- Separate tq and t
 *t*q production (lepton charge)
- Two jets, charged lepton, E_T^{miss}
- Neural network to separate signal and background
- Binned maximum-likelihood fit

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arXiv: 1702.02859 [hep-ex]

Measured cross-sections:

- Fiducial
- Total (cross-section ratio, f_{LV} |V_{tb}| extraction)
- Differential (particle-level and parton-level)



Fiducial and total tq and $\bar{t}q$ cross-sections

$$\sigma_{\rm fid} = \frac{N_{\rm fid}}{N_{\rm sel}} \cdot \frac{\hat{\nu}}{L_{\rm int}}$$

- Total uncertainties: 5.8 % (top-quark) and 7.8 (top-antiquark)
- Comparison to predictions with NLO ME generators + different parton-shower programs



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- Total uncertainties: +7.6 % / -6.7 % (top-quark) and +9.1 % / -8.4 % (top-antiquark)
- Comparison to fixed-order calculations.



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Cross-section ratio R_t of tq to $\bar{t}q$ production



$$R_t = \frac{\sigma_{\text{tot}}(tq)}{\sigma_{\text{tot}}(\bar{t}q)}$$

- Total uncertainty: ± 5.0 %
- Statistically limited
- Comparison to predictions with different PDF sets.





Differential tq and $\bar{t}q$ cross-sections



Enrich tq events by cut on NN discriminant

Events / 10 GeV

<u>Data</u> Pred.

1500

000

500

50



Observe good agreement with predictions by NLO generators.

arXiv: 1702.02859 [hep-ex]

100

200

150

300

250

m(lvb) [GeV]

Differential cross-sections: untagged jet

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- Unfold p_T and |y| of the untagged (forward) jet at particle level
- Train specific network without |η| to avoid depletion at high rapidity after cutting on the network





Good agreement with predictions by NLO generators, with a slight trend to a softer p_T spectrum.

arXiv: 1702.02859 [hep-ex]

Determination of $|f_{LV} \cdot V_{tb}|$





tW production at 13 TeV





Measured cross-section:

$$\sigma_{Wt} = 94 \pm 10 \,(\text{stat.})^{+28}_{-22} \,(\text{syst.}) \pm 2 \,(\text{lumi.}) \,\text{pb}$$

 $\sigma_{\text{theory}} = 71.7 \pm 1.8 \,(\text{scale}) \pm 3.4 \,(\text{PDF}) \,\text{pb}$

- Total uncertainty: ± 31 %
- Total systematic unc.: ± 30 %
- Main components:
 - Jet energy scale: ± 21 %
 - NLO ME generator: ± 18 %



- Use BDTs to separate signal and background
- Constrain $t\bar{t}$ background with event yield in 2j2b channel





Differential $t\bar{t}$ cross-sections in the e μ channel



- 5 kinematic variables:
 - > Top quark: $p_T(t)$, |y(t)|
 - > $t\bar{t}$ system: $p_T(t\bar{t})$, $|y(t\bar{t})|$, m($t\bar{t}$)
- Normalised and absolute fiducial differential cross-sections
- Use neutrino-weighting method for reconstruction to solve under-constrained system (weight based on agreement of reconstructed and measured E_T^{miss})
- POWHEG + HERWIG++(7) / PYTHIA8 have been retuned based on these results



Cross-section ratio: $\sigma^{\rm tot}(t\bar{t}) \, / \, \sigma^{\rm fid}(Z)$

<u>arXiv:1612.03636 [hep-ex]</u> JHEP 1702 (2017) 117

- Use previously published ATLAS measurements and new $Z \rightarrow \ell^+ \ell^-$ measurement at 13 TeV.
- Correct for common phase space where required.
- Account for correlations of systematic uncertainties.
- Compare to predictions at NNLO(+NNLL) accuracy (DYNNLO 1.5 and Top++v2.0) made with six different PDF sets.



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- Common pattern observed.
- > ABM12 set yields lowest values.
- CT14, NNPDF3.0, MMHT14 largest values, ATLAS-epWZ12 and HERAPDF2.0 in the middle.
- \succ Reason: differences in the gluon density and α_s values used in the sets.
- \blacktriangleright ABM12, ATLAS-epWZ12 and HERAPDF2.0 do not include collider data, ABM12 uses lower value of α_s





JHEP 1702 (2017) 117

Boosted $t\bar{t}$ differential cross-section



- All-hadronic channel with boosted top-quarks.
- Use 14.7 fb⁻¹ of 2015 and 2016 data at \sqrt{s} = 13 TeV.
- Form top-quark candidates with large-radius jets (DR = 1.0) using Top-Tagging algorithm.



ATLAS-CONF-2016-100

Boosted $t\bar{t}$ differential cross-section

• Use 12 observables of top-quarks and the $t\bar{t}$ system.



• Most important uncertainties: large-R jet calibration and reconstruction, $t\bar{t}$ MC modelling, b-tagging





production angle



Part 2: Spin-related observables



Top-quark decay





- Large decay width due to large mass: $\Gamma \propto m_t^{-3}$
- Life time ≪ formation time of hadrons, spin de-correlation time
- Polarisation and spin correlations in production are transferred to decay products.
- Source of on-shell polarised *W* bosons.

Top-quark spin observables in $t\bar{t}$ production

- Use $t\bar{t}$ di-lepton events (ee, eµ, µµ)
- Aim: determine coefficients of spin density matrix
- Measure 15 polarisation and spin correlation observables (follow Bernreuther, Heisler, Si, JHEP 12 (2015) 026)
- Use 3 orthogonal spin quantisation axes:
 - helicity axis \vec{k} : top-quark direction in the $t\bar{t}$ rest frame
 - transverse axis \vec{n} : \perp production plane
 - r-axis \vec{r} : $\perp \vec{k}$ and $\perp \vec{n}$
- Define 6 angles: 3 axes × 2 charged leptons

$$\begin{array}{cccc} \theta_{+}^{k} & \theta_{+}^{n} & \theta_{+}^{r} \\ \\ \theta_{-}^{k} & \theta_{-}^{n} & \theta_{-}^{r} \end{array}$$





arXiv:1612.07004 [hep-ex]

- Unfold distributions of
 - Polarisation observables: $\cos\theta_{+}^{k}$, $\cos\theta_{-}^{k}$, $\cos\theta_{-}^{n}$, ...
 - → 6 distributions
 - Spin correlation observables: $\cos\theta_{+}^{k} \cdot \cos\theta_{-}^{k}$, $\cos\theta_{+}^{n} \cdot \cos\theta_{-}^{n}$, ...
 - → 3 distributions
 - Cross correlations observables: $\cos \theta^{n_{+}} \cdot \cos \theta^{k_{-}} + \cos \theta^{k_{+}} \cdot \cos \theta^{n_{-}},$ $\cos \theta^{n_{+}} \cdot \cos \theta^{k_{-}} - \cos \theta^{k_{+}} \cdot \cos \theta^{n_{-}}, ...$ $\rightarrow 6$ distributions
- Relate mean values of observables to polarisation and spin correlation coefficients:

 $B_{+}^{k} = 3 < \cos \theta_{+}^{k} >, ...$

 $C(k, n) = 9 < \cos \theta^{k_{+}} \cdot \cos \theta^{n_{-}} >, \dots$





arXiv:1612.07004 [hep-ex]

Measured polarisations and spin correlations





- Measurements at parton-level (and particle-level, not shown)
- Use Bayesian unfolding with marginalisation of systematic uncertainties
- Good agreement to SM.
- Spin polarisation along transverse axis C(n,n) differs from zero by 5.1 standard deviations.

arXiv:1612.07004 [hep-ex]

Measurement of the W polarisation in top-quark decays



- W bosons from top-quark decays are polarised due to the V-A structure of the *Wtb* vertex.
- The angle θ^{*} between the spin analyser (charged lepton or down-type quark) and the reversed direction of flight of the b-quark from the top-quark decay in the W-boson rest frame.



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Hadronic analyser

Leptonic analyser



arXiv:1612.02577 [hep-ex]

Events / 0.13

Data/Fit

7000

6000

5000

4000

3000

2000

1000

Limits on anomalous couplings



Generalized Wtb Lagrangian:

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}} \bar{b} \gamma^{\mu} (V_{\rm L} P_{\rm L} + V_{\rm R} P_{\rm R}) t W_{\mu}^{-} - \frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_{\nu}}{m_W} (g_{\rm L} P_{\rm L} + g_{\rm R} P_{\rm R}) t W_{\mu}^{-} + \text{h.c.}$$



Wtb vertex structure in production and decay



- Measure 8 asymmetries sensitive to polarisation effects in production and decay.
- Good agreement with SM predictions.

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Conclusions



- Splendid performance of LHC → large data sets of top-quark candidate events.
- Precision measurements: ratios, couplings, differential measurements; for example:
 - \succ f_{LV} · |V_{tb}| = 1.029 ± 0.048
 - ightarrow R_t = 1.72 ± 0.09 @ 8 TeV (5 % precision)
 - → $\sigma^{\text{tot}}(t\bar{t}) / \sigma^{\text{fid}}(Z) = 0.480 \pm 0.012$ @ 8 TeV (2.6 % precision)

> $t\bar{t}$ spin correlation: C(n,n) = 0.304 ± 0.060 (5.1 σ)

- Input for PDF fits, tuning of MC generators, limits on BSM modells
- New opportunities to explore phase space: boosted $t\bar{t}$ differential cross-section
- Looking forward to analyse full Run 2 data set with 100 fb⁻¹.
 - > Use high statistics to improve systematic uncertainties (e.g. profiling)
 - > Use new PS generators (HERWIG7 and PYTHIA8) for more consistent treatment of modelling uncertainties.
 - > Intensive use of Effective Field Theory to investigate BSM physics (foe example: UFO models in MadGraph)





Backup slides

Single top-quark cross-sections





	7 TeV	8 TeV	13 TeV
$\sigma (tq + t\bar{q})$	65 pb	85 pb	217 pb
$\Delta\sigma / \sigma$	±4%	±5%	±4%

Single top-antiquark *t*-channel (post fit)





Uncertainties of the fiducial tq cross-sections





Source	$\frac{\Delta\sigma_{\rm fid}(tq) / \sigma_{\rm fid}(tq)}{[\%]}$	$\begin{array}{c} \Delta\sigma_{\rm fid}(\bar{t}q) \ / \ \sigma_{\rm fid}(\bar{t}q) \\ [\%] \end{array}$
Data statistics	± 1.7	± 2.5
Monte Carlo statistics	± 1.0	± 1.4
Background normalisation	< 0.5	< 0.5
Background modelling	± 1.0	± 1.6
Lepton reconstruction	± 2.1	± 2.5
Jet reconstruction	± 1.2	± 1.5
Jet energy scale	\pm 3.1	\pm 3.6
Flavour tagging	± 1.5	± 1.8
$E_{\rm T}^{\rm miss}$ modelling	± 1.1	± 1.6
b/\bar{b} tagging efficiency	± 0.9	± 0.9
PDF	± 1.3	± 2.2
$tq~(\bar{t}q)$ NLO matching	± 0.5	< 0.5
$tq~(\bar{t}q)$ parton shower	± 1.1	± 0.8
$tq~(\bar{t}q)$ scale variations	± 2.0	± 1.7
$t\bar{t}$ NLO matching	± 2.1	± 4.3
$t\bar{t}$ parton shower	± 0.8	± 2.5
$t\bar{t}$ scale variations	< 0.5	< 0.5
Luminosity	± 1.9	± 1.9
Total systematic	± 5.6	± 7.3
Total (stat. $+$ syst.)	\pm 5.8	\pm 7.8



Source	$\Delta R_t/R_t$ [%]
Data statistics	\pm 3.0
Monte Carlo statistics	± 1.8
Background modelling	± 0.7
Jet reconstruction	± 0.5
$E_{\rm T}^{\rm miss}$ modelling	± 0.6
$tq(\bar{t}q)$ NLO matching	± 0.5
$tq \ (\bar{t}q)$ scale variations	± 0.7
$t\bar{t}$ NLO matching	± 2.3
$t\bar{t}$ parton shower	± 1.7
PDF	± 0.7
Total systematic	± 3.9
Total (stat. $+$ syst.)	\pm 5.0



At least one jet with $p_{\rm T} > 25 \,{\rm GeV}, \, |\eta| < 2.5$

Exactly two leptons of opposite charge with $p_{\rm T} > 20 \,{\rm GeV}$,

 $|\eta| < 2.5$ for muons and $|\eta| < 2.47$ excluding $1.37 < |\eta| < 1.52$ for electrons

At least one lepton with $p_{\rm T} > 25 \,{\rm GeV}$, veto if third lepton with $p_{\rm T} > 20 \,{\rm GeV}$

At least one lepton matched to the trigger object

Different flavour	$E_{\mathrm{T}}^{\mathrm{miss}} > 50 \mathrm{GeV},$ $E_{\mathrm{T}}^{\mathrm{miss}} > 20 \mathrm{GeV},$	$ \label{eq:metric} \begin{split} & \text{if } m_{\ell\ell} < 80 \text{GeV} \\ & \text{if } m_{\ell\ell} > 80 \text{GeV} \end{split} $
	$E_{\rm T}^{\rm miss} > 40 {\rm GeV},$	always
	veto,	if $m_{\ell\ell} < 40 \mathrm{GeV}$
Same flavour	$4E_{\rm T}^{\rm miss} > 5m_{\ell\ell},$	if $40 \mathrm{GeV} < m_{\ell\ell} < 81 \mathrm{GeV}$
	veto,	if $81 \mathrm{GeV} < m_{\ell\ell} < 101 \mathrm{GeV}$
	$2m_{\ell\ell} + E_{\rm T}^{\rm miss} > 300 {\rm GeV},$	if $m_{\ell\ell} > 101 \mathrm{GeV}$

tW cross-section uncertainties



Δμ

			-0.6 -0.4 -0.2 0 0.2 0.4 0.6
Source	$\Delta \sigma_{Wt} / \sigma_{Wt} [\%]$		
Jet energy scale	21	Parton Shower generator	
Jet energy resolution	8.6	° °	
$E_{\rm T}^{\rm miss}$ soft terms	5.3	JES: flavour composition	
b-tagging	4.3	JES: Eff1	· · · · · · · · · · · · · · · · · · ·
Luminosity	2.3	4. L/COD	
Lepton efficiency, energy scale and resolution	1.3	u //FSR	
NLO matrix element generator	18	Luminosity	
Parton shower and hadronisation	7.1	Wt ME generator	
Initial-/final-state radiation	6.4	JES: η intercal. model	
Diagram removal/subtraction	5.3		
Parton distribution function	2.7	PDF central value	
Non- $t\bar{t}$ background normalisation	3.7	JES: pileup ρ	
Total systematic uncertainty	30	b-jet efficiency scale fac. 0	
Data statistics	10	ATLAS	Pull
Total uncertainty	31		Vs = 13 TeV, 3.2 fb ⁻⁺ Pre-fit Impact on μ Post-fit Impact on μ

3

2

0

1

-3

-2

-1

Differential $t\bar{t}$ cross-sections (absolute)





Differential $t\bar{t}$ cross-sections: rapidity distributions



10 <u>ਰ</u> · <u>do</u> / Unit Rapidity do / Unit Rapidity 0.9⊨ ATLAS Fiducial phase-space ATLAS Fiducial phase-space √s = 13 TeV, 3.2 fb⁻¹ √s = 13 TeV, 3.2 fb⁻¹ 0.8⊟ Data Data Stat. Stat. 0.7 Stat ⊕ Syst. POWHEG Box + PYTHIA 6 - POWHEG Box + PYTHIA 6 0.6 POWHEG Box + HERWIG++ POWHEG Box + HERWIG++ POWHEG Box + PYTHIA 8 POWHEG Box + PYTHIA 8 0.5⊟ Sherpa Sherpa 0.4⊨ MG5 aMC@NLO + HERWIG++ MG5 aMC@NLO + HERWIG++ 0 MG5_aMC@NLO + PYTHIA 8 MG5_aMC@NLO + PYTHIA 8 0.3 -10 0.2⊨ 10⁻² . 0.1⊨ Pred. / Data Pred. / Data 1.1 1.1 0.9 0.9 Pred. / Data Pred. / Data 1.1 1.1 0.9 0.9 2.5 3.5 2.5 3.5 0.5 1.5 2 3 0.5 1.5 2 3 0 1 0 |y(t)|

|y(tt̄)|



	p_{T}	(t)	y(t)	t)	$p_{\mathrm{T}}(\cdot)$	$t\bar{t})$	y(t)	$\overline{t}) $	m(t	(\bar{t})
Predictions	χ^2/NDF	<i>p</i> -value	χ^2/NDF	<i>p</i> -value	χ^2/NDF	<i>p</i> -value	χ^2/NDF	<i>p</i> -value	χ^2/NDF	<i>p</i> -value
Powheg + Pythia 6	5.2/4	0.27	0.5/3	0.92	5.5/6	0.48	0.6/2	0.74	3.9/4	0.42
Powheg $+$ Pythia 8	4.6/4	0.33	1.3/3	0.73	5.1/6	0.53	0.0/2	1.00	5.7/4	0.22
Powheg + Herwig++	14.6/4	0.01	1.4/3	0.71	4.1/6	0.66	1.0/2	0.61	12.0/4	0.02
$MG5_aMC@NLO + HERWIG++$	2.0/4	0.74	1.3/3	0.73	0.6/6	1.00	0.2/2	0.90	0.9/4	0.92
$MG5_aMC@NLO + Pythia 8$	3.6/4	0.46	0.6/3	0.90	10.7/6	0.10	0.1/2	0.95	2.7/4	0.61
Sherpa	3.8/4	0.43	0.8/3	0.85	0.7/6	0.99	0.0/2	1.00	2.3/4	0.68
Powheg + Pythia 6 (radHi)	7.8/4	0.10	0.6/3	0.90	0.9/6	0.99	0.4/2	0.82	3.8/4	0.43
POWHEG + PYTHIA 6 (radLow)	5.5/4	0.24	0.8/3	0.85	9.6/6	0.14	0.8/2	0.67	4.5/4	0.34

Overall there is good agreement between the distributions predicted by MC generators and data, except for Powheg+HERWIG++ (low p-value).

Statistical correlation matrices

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		$\sigma \pm \text{ stat} \pm \text{ syst [pb]}$	
$\sqrt{s} [\text{TeV}]$	13	8	7
$\sigma_{Z \to ee}^{\text{fid}}$	$778.3 \pm 0.7 \pm 17.7$	$507.0 \pm 0.2 \pm 11.0$	$451.2 \pm 0.5 \pm 8.7$
$\sigma_{Z \to \mu\mu}^{\rm fid}$	$774.4 \pm 0.6 \pm 18.2$	$504.7 \pm 0.2 \pm 10.8$	$450.0 \pm 0.3 \pm 8.8$
$\sigma_{t\bar{t}\to e\mu+X}^{\rm fid}$	$9.94 \pm 0.09 \pm 0.37$	$3.04 \pm 0.02 \pm 0.10$	$2.30 \pm 0.04 \pm 0.08$
$\sigma_{tar{t}}^{ m tot}$	$818 \pm 8 \pm 35$	$243 \pm 2 \pm 9$	$183 \pm 3 \pm 6$



		$\delta \sigma_Z^{ m fid}$		($\delta \sigma_{t\bar{t}}^{ m tot}$	
Source / \sqrt{s} [TeV]	13	8	7	13	$8^{"}$	7
Luminosity	A	В	С	A	В	С
Beam energy	A	А	А	A	А	Α
Muon (lepton) trigger	A	\mathbf{A}^*	А	A	В	В
Muon reconstruction/ID	A	В	\mathbf{C}	A	D	D
Muon isolation	A	А	А	В	\mathbf{C}	D
Muon momentum scale	A	А	А	A	А	Α
Electron trigger	А	А	А	А		
Electron reconstruction/ID	A	В	\mathbf{C}	A	D	D
Electron isolation	A	А		В	\mathbf{C}	D
Electron energy scale	A	А	А	A	А	А
Jet energy scale				A	В	В
b-tagging				A	В	В
Background	A	А	А	В	В	В
Signal modelling (incl. PDF)	A	А	А	B*	В	В

2D analysis: $\sigma^{\text{tot}}(t\bar{t})$ versus $\sigma^{\text{fid}}(Z)$





Quantitative analysis of agreement between measurements and predictions with different PDF sets:

	ATLAS-epWZ12	CT14	MMHT14	NNPDF3.0	HERAPDF2.0	ABM12
$\chi^2/{ m NDF}$	8.3 / 6	15 / 6	13 / 6	17 / 6	10 / 6	25 / 6
p-value	0.22	0.02	0.05	0.01	0.11	< 0.001

Differential cross-sections in:

- \triangleright p_T^{t,1}, p_T^{t,2}, |y^{t,1}|, |y^{t,2}|, p_T(t \bar{t}),
- \succ |y($t\bar{t}$)|, m($t\bar{t}$), H_T($t\bar{t}$)
- \succ cos θ^* (production angle in the Collins-Soper frame)
- \succ y_B($t\bar{t}$) = $\frac{1}{2}$ (y^{t,1} + y^{t,2}) (longitudinal boost)

 $\succ \Delta \phi(t_1, t_2)$

 $\succ \chi(t\bar{t}) = \exp 2|y^*|$ with $y^* = \frac{1}{2}(y^{t,1} - y^{t,2})$ production angle

Large-R jet reconstruction

- |η| < 2.0
- Trimming algorithm with $R_{sub} = 0.2$ and $f_{cut} =$ 0.05 to suppress QCD radiation and pile-up effects
- Top-tagging mass requirement: $122.5 \text{ GeV} < m_1 < 222.5 \text{ GeV}$
- Use N-subjetiness ratio τ_{32} requirement, p_T dependent
- Top-tagging efficiency is 50 % constant in p_T

Predicted polarisations and spin correlations



Expectation values	NLO predictions	Observables
B^k_+	0.0030 ± 0.0010	$\cos heta_+^k$
B^k	0.0034 ± 0.0010	$\cos heta_{-}^k$
B^n_+	0.0035 ± 0.0004	$\cos heta_+^n$
B_{-}^{n}	0.0035 ± 0.0004	$\cos heta_{-}^n$
B^r_+	0.0013 ± 0.0010	$\cos heta_+^r$
B_{-}^{r}	0.0015 ± 0.0010	$\cos heta_{-}^r$
C(k,k)	0.318 ± 0.003	$\cos heta_+^k \cos heta^k$
C(n,n)	0.332 ± 0.002	$\cos heta_+^n \cos heta^n$
C(r,r)	0.055 ± 0.009	$\cos \theta^r_+ \cos \theta^r$
C(n,k) + C(k,n)	0.0023	$\cos\theta_+^n\cos\theta^k + \cos\theta_+^k\cos\theta^n$
C(n,k) - C(k,n)	0	$\cos\theta_+^n\cos\theta^k - \cos\theta_+^k\cos\theta^n$
C(n,r) + C(r,n)	0.0010	$\cos\theta^n_+\cos\theta^r + \cos\theta^r_+\cos\theta^n$
C(n,r) - C(r,n)	0	$\cos\theta_+^n\cos\theta^r - \cos\theta_+^r\cos\theta^n$
C(r,k) + C(k,r)	-0.226 ± 0.004	$\cos\theta^r_+\cos\theta^k + \cos\theta^k_+\cos\theta^r$
C(r,k) - C(k,r)	0	$\cos\theta_+^r\cos\theta^k - \cos\theta_+^k\cos\theta^r$



Asymmetry	Angular observable	Polarisation observable	SM prediction
$A_{ m FB}^\ell$	$\cos \theta_{\ell}$	$\frac{1}{2}\alpha_{\ell}P$	0.45
$A_{\rm FB}^{tW}$	$\cos \theta_W \cos \theta_\ell^*$	$\frac{3}{8}P(F_{\rm R}+F_{\rm L})$	0.10
$A_{\rm FB}$	$\cos heta^*_\ell$	$\frac{3}{4}\langle S_3\rangle = \frac{3}{4}\left(F_{\rm R}-F_{\rm L}\right)$	-0.23
$A_{\rm EC}$	$\cos heta^*_\ell$	$\frac{3}{8}\sqrt{\frac{3}{2}}\langle T_0\rangle = \frac{3}{16}(1-3F_0)$	-0.20
$A_{\rm FB}^T$	$\cos \theta_{\ell}^{T}$	$\frac{3}{4}\langle S_1 \rangle$	0.34
$A_{\rm FB}^N$	$\cos \theta_{\ell}^{N}$	$-\frac{3}{4}\langle S_2\rangle$	0
$A_{ m FB}^{T,\phi}$	$\cos\theta^*_{\ell}\cos\phi^*_T$	$-\frac{2}{\pi}\langle A_1\rangle$	-0.14
$A_{ m FB}^{N,\phi}$	$\cos\theta^*_\ell\cos\phi^*_N$	$\frac{2}{\pi}\langle A_2\rangle$	0