Physics of Gravitational Waves

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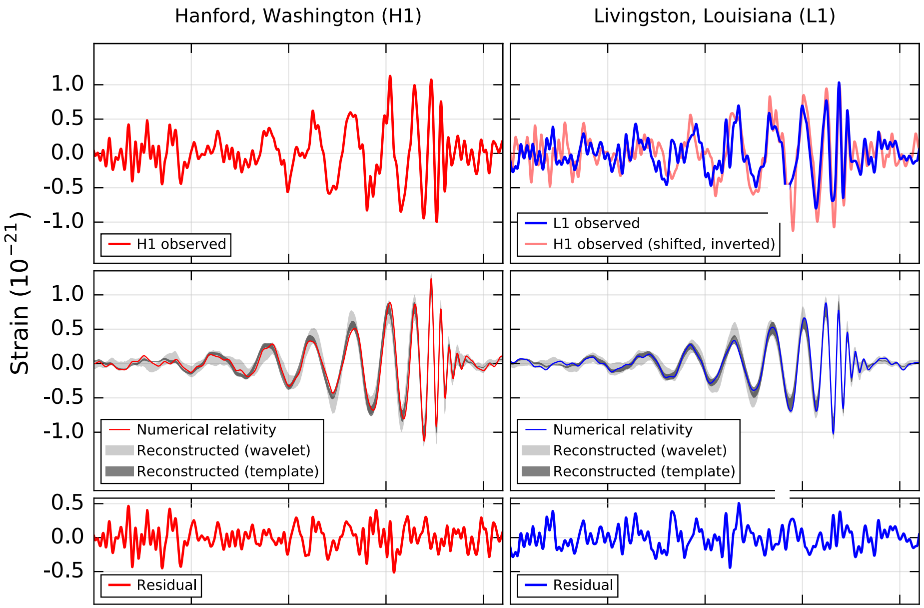
September 15th, 2015: Gravitational Waves detection



first detection:GW150914

$m_1 = 29.1^{+3.7}_{-4.4}\ M_\odot$

$m_2 = 36.2^{+5.2}_{-3.8}\ M_\odot$ $\quad a_1, a_2 = -0.06^{+0.14}_{-0.14}$

final BH

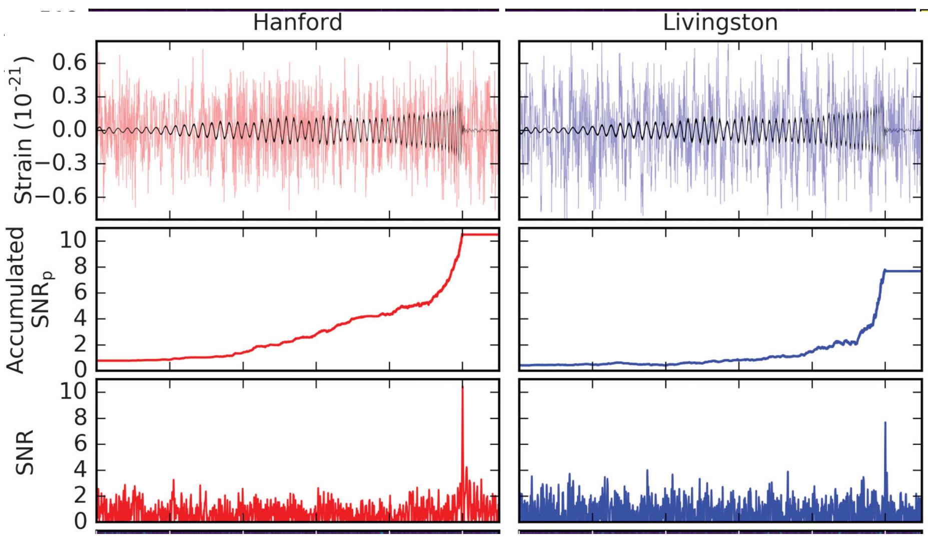
$M = 62.3^{+3.7}_{-3.1}\ M_\odot$

$a = J/M = 0.68^{+0.05}_{-0.06}$

$D_L = 420^{+150}_{-180} Mpc$ $\quad z = 0.09^{+0.03}_{-0.04}$

SNR=23.7

radiated $E_{GW} = 3M_\odot c^2$



second detection:GW151226

$m_1 = 7.5^{+2.3}_{-2.3}\ M_\odot$

$m_2 = 14.2^{+8.3}_{-3.7}\ M_\odot$ $\quad a_1, a_2 = 0.21^{+0.20}_{-0.10}$

final BH

$M = 20.8^{+6.1}_{-1.7}\ M_\odot$

$a = J/M = 0.74^{+0.06}_{-0.06}$

$D_L = 440^{+180}_{-190} Mpc$ $\quad z = 0.09^{+0.03}_{-0.04}$

SNR=13.0

radiated $E_{GW} = 1M_\odot c^2$

for both detections the significance is > 5 σ

third candidate: LVT151012

$m_1 = 13^{+4}_{-5}\ M_\odot$

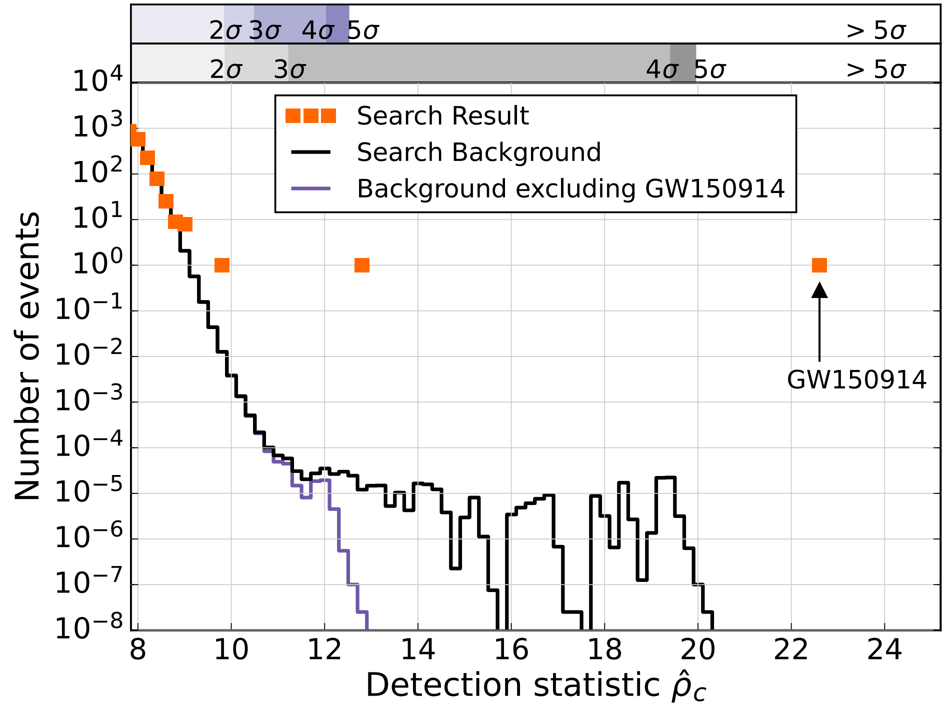
$m_2 = 23^{+18}_{-6}\ M_\odot$ $\qquad a_1, a_2 = 0.0^{+0.3}_{-0.2}$

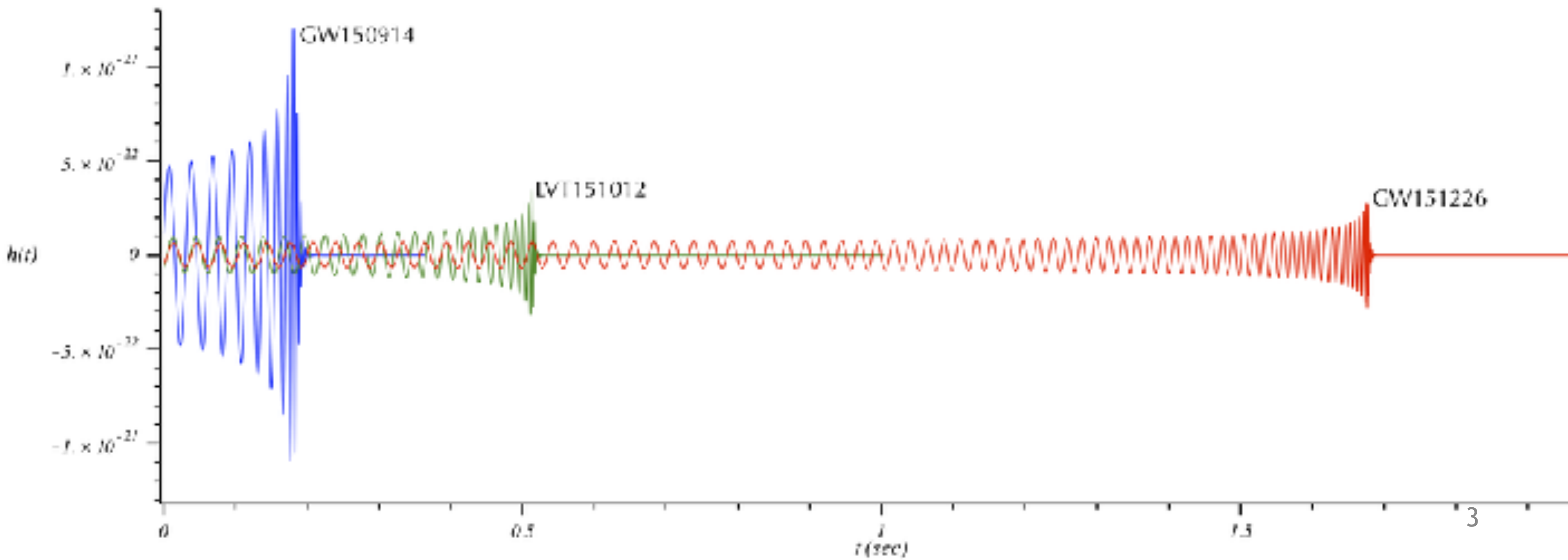
$M = 35^{+14}_{-4}\ M_\odot$

$a = J/M = 0.66^{+0.09}_{-0.10}$

$D_L = 1^{+0.5}_{-0.5} Gpc$ $\qquad z = 0.20^{+0.09}_{-0.09}$

the significance is ≤ 2 σ





How did the LIGO-Virgo collaboration reach the conclusion that the observed gravitational signal is due to the coalescence of two black holes?

What did LIGO detectors see on September 15th 2015?

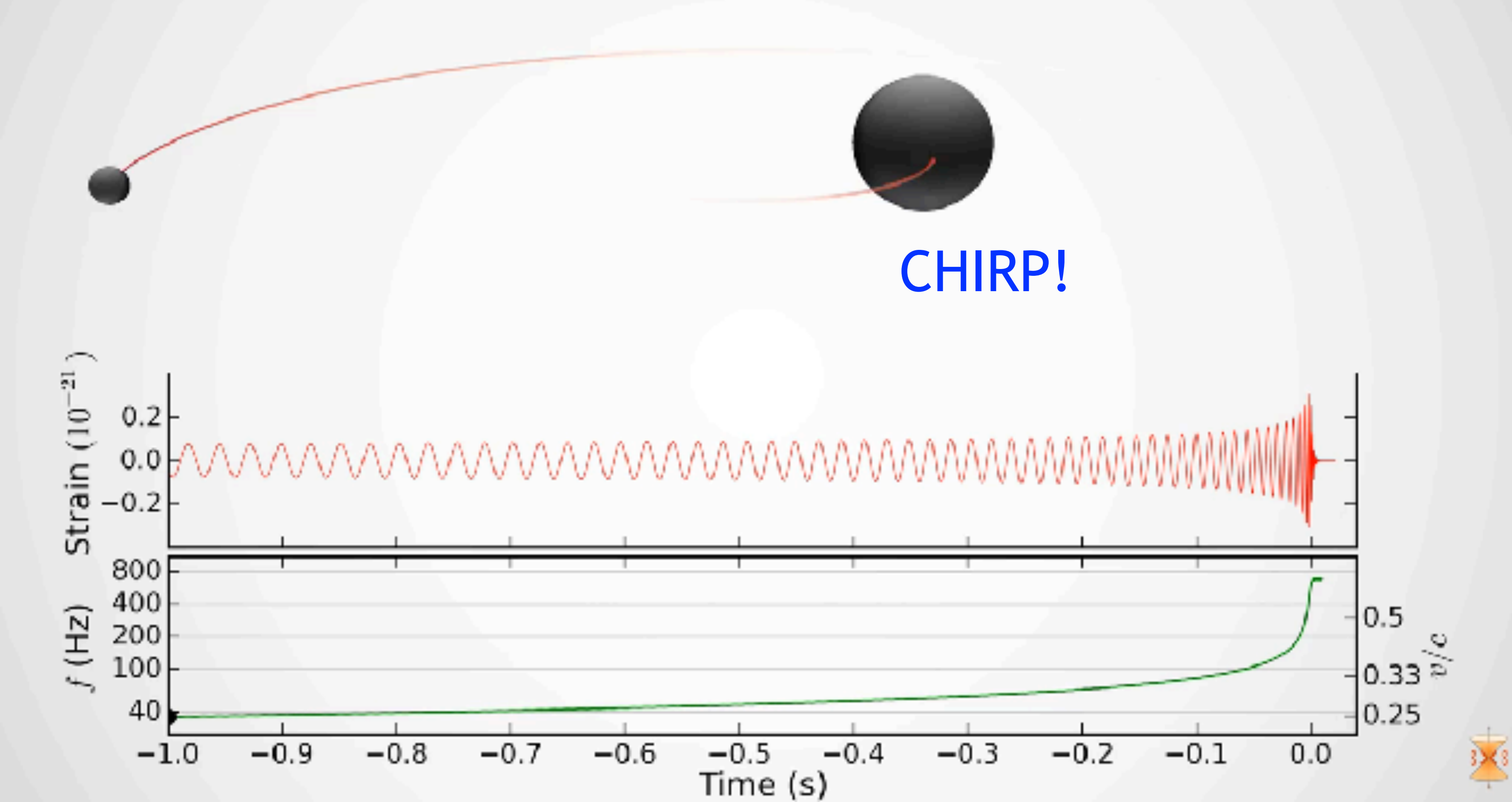
during the inspiralling the orbit shrinks due to GW emission:

the orbital period decreases

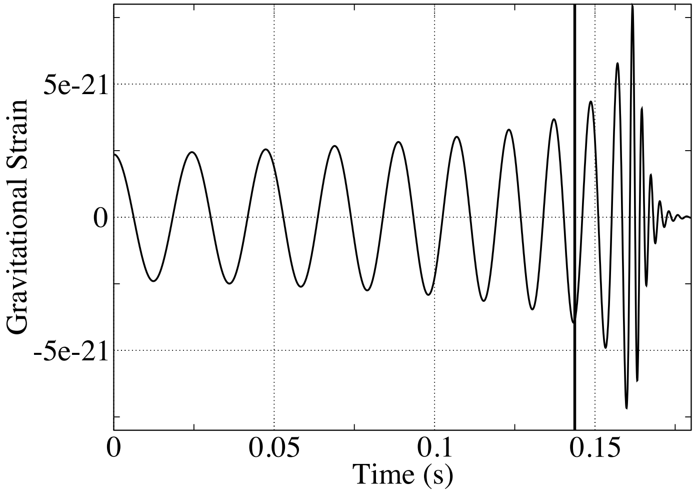
$\nu_{GW} = 2/P_{orb}$
the frequency increases

$h_0 \propto \nu_{GW}^{2/3}$
the amplitude increases

The inspiralling part of the signal is computed by a post-Newtonian expansion of the equations of motion in GR, assuming two point masses in circular orbit



$$h(\nu) = h_0 e^{i\Psi(\nu)}$$



Information from the wave phase

$$\Psi = -2\left[\frac{c^3(t_{coal} - t)}{5G\,\mathcal{M}}\right]^{5/8} + \Psi^{in}$$

"Chirp mass"

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3}\,\nu^{-11/3}\,\dot{\nu}\right]^{3/5}$$

measuring the wave frequency and its time derivative, we measure the chirp mass

PROBLEM: in the chirp mass formula, frequency and its time derivative are evaluated in the source frame, but the wave frequency is measured in the detector frame:

$$\nu_{obs} = \frac{\nu}{1+z} \quad \Rightarrow \quad [\nu_{obs}^{-11/3}\dot{\nu}_{obs}]^{3/5} = \frac{[\nu^{-11/3}\dot{\nu}]^{3/5}}{(1+z)}$$

This means that since we do not know the source redshift, what we measure is the "redshifted mass", i.e.

$$\mathcal{M}' = \mathcal{M}(1+z)$$

The same scaling remains true even if we include further terms in the Post-Newtonian expansion

FROM THE WAVE AMPLITUDE WE GAIN INFORMATION ON THE SOURCE DISTANCE

In the detector frame the wave amplitude is

$$h_0(t) = \frac{4\pi^{2/3} G^{5/3}}{c^4} \times \frac{\mathcal{M}'}{D} \times [\mathcal{M}'\nu_{obs}]^{2/3}$$

where $\mathcal{M}' = \mathcal{M}(1+z)$

D is the luminosity distance

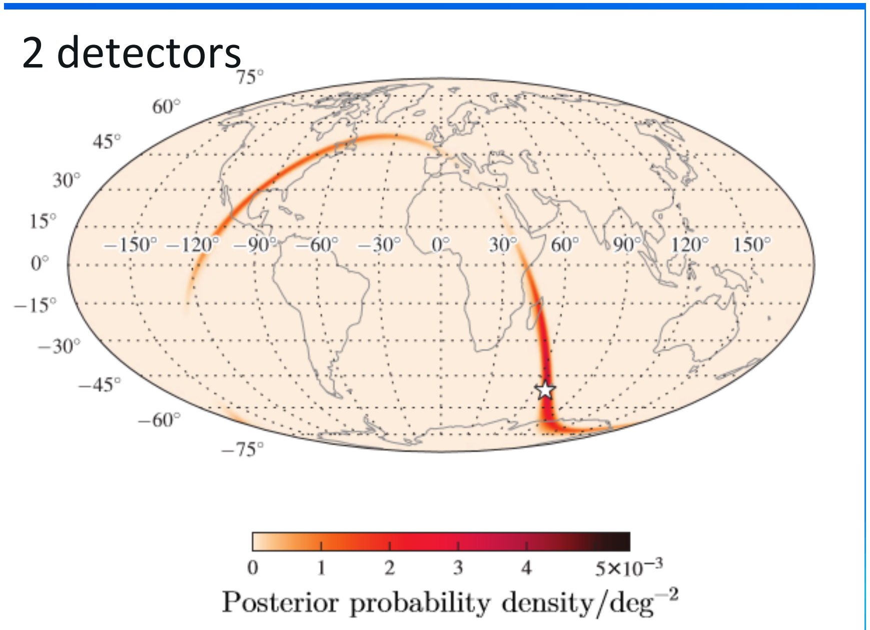
$$D(z) = \frac{2}{H_0 \Omega_0^2}[\Omega_0 z - (2-\Omega_0)(\sqrt{1+\Omega_0 z} - 1)$$

H_0 is the Hubble constant $\Omega_0 = \frac{8\pi}{3}\frac{\rho_{m_0}}{H_0^2}$, $\rho_{m_0} =$ present matter density

from the wave amplitude we can infer the source luminosity distance up to a factor (1+z), i.e.

$$d_{eff} = \frac{D(z)}{1+z}$$

But in the case of the detected signals we do not know the redshift z!



to measure the redshift of the galaxy hosting the source, this must me localized: more detectors are needed

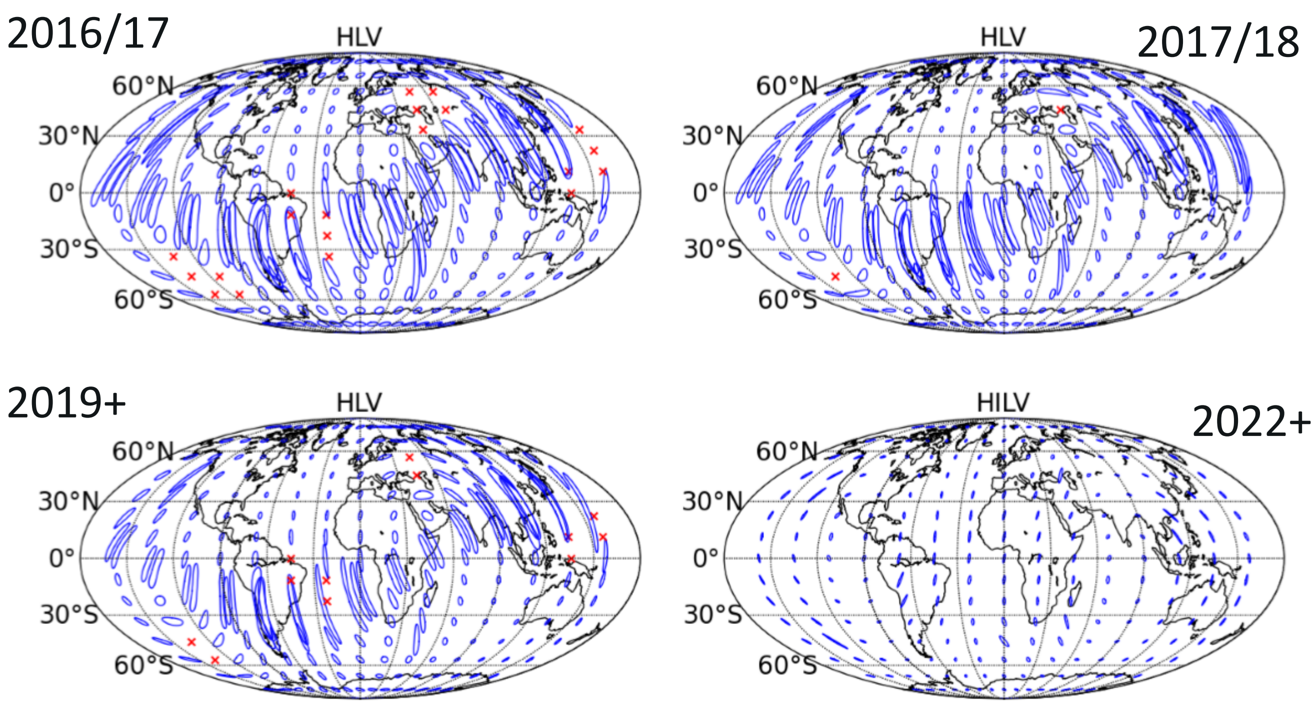
Abbott et al. *"Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO and Advanced Virgo"*
Living Rev. Relativity, 19, (2016), 1

GW150914 is localized in a sky area of 230 deg^2

850 deg^2 for GW151226

1600 deg^2 for LVT151012

The sky area is expected to scale inversely with the square of the SNR



$$d_{eff} = \frac{D(z)}{1+z}$$

$$D(z) = \frac{2}{H_0 \Omega_0^2} [\Omega_0 z - (2 - \Omega_0)(\sqrt{1 + \Omega_0 z} - 1)$$

$$d_{eff} = \left[\frac{2}{H_0 \Omega_0^2} [\Omega_0 z - (2 - \Omega_0)(\sqrt{1 + \Omega_0 z} - 1) \right] / (1+z)$$

ASSUMING A COSMOLOGICAL MODEL

ΛCDM cosmology

$H_0 = 67.9 \ km \ s^{-1} Mpc^{-1}$

$\Omega_0 = 0.306, \quad \text{Planck 2015}$

from the measured **d_{eff}**, we can infer the source redshift **z**

$$GW150914 : \quad z = 0.09^{+0.03}_{-0.04}$$

$$GW151226 : \quad z = 0.09^{+0.03}_{-0.04}$$

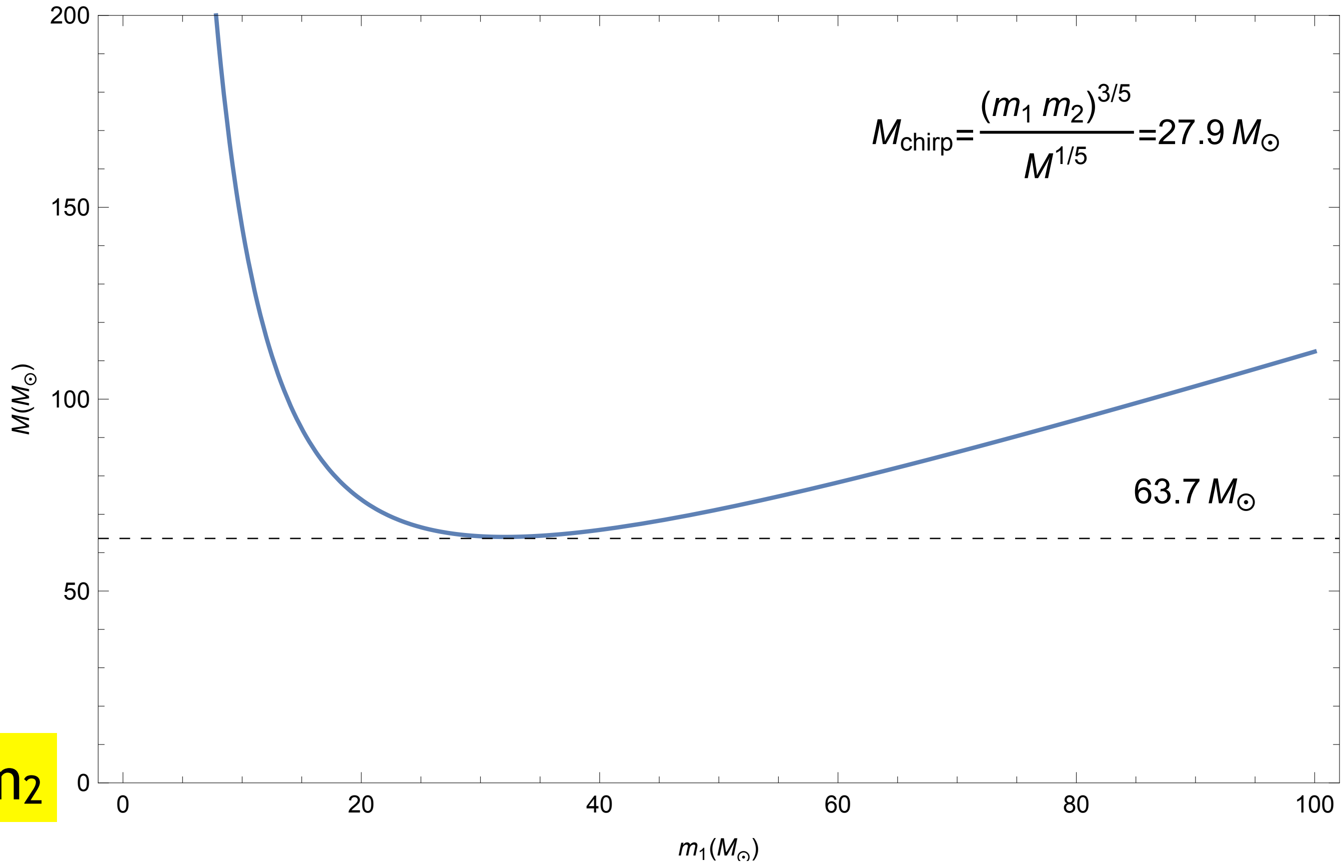
$$LVT151012 : \quad z = 0.2^{+0.09}_{-0.09}$$

given the redshift we find the "true" chirp mass

$$\mathcal{M} = \frac{\mathcal{M}'}{1+z}$$

where $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$

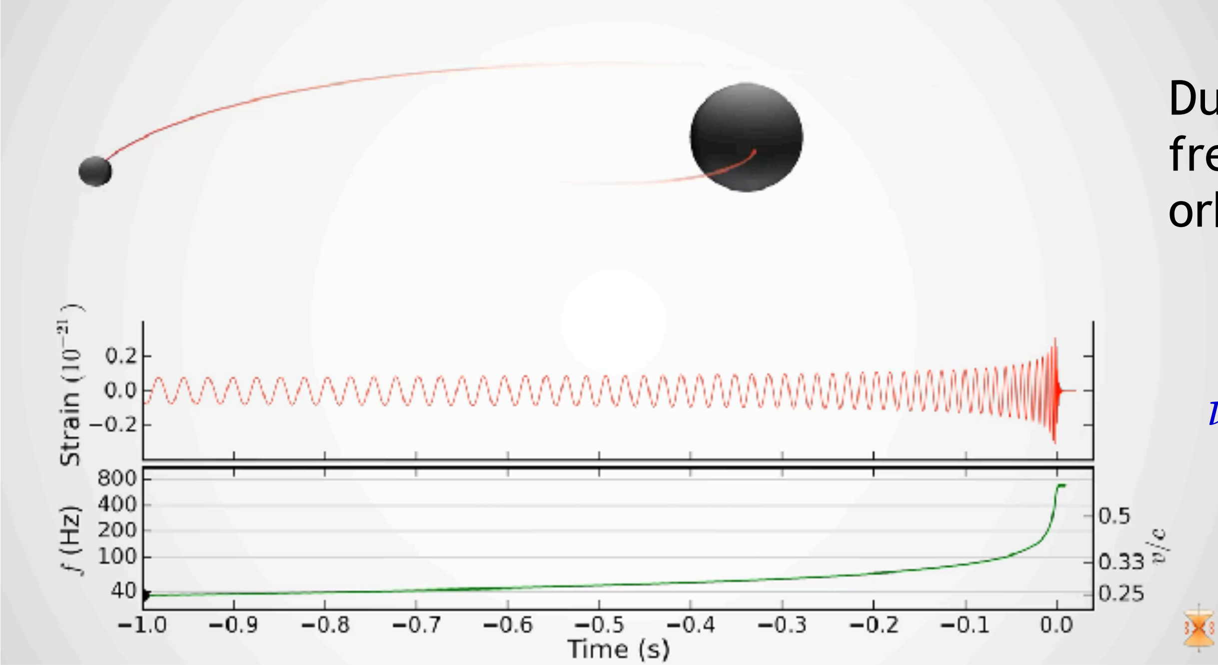
GW150914



$M = m_1 + m_2$

$$\mathcal{M} \simeq 28 M_\odot \to (m_1 + m_2) \gtrsim 63.7 M_\odot$$

Too large to be two neutron stars



During the inspiralling the wave frequency is related to the orbital distance by

$$\nu_{GW}(t) = \frac{1}{\pi}\sqrt{\frac{G(m_1+m_2)}{d^3_{orb}(t)}}$$

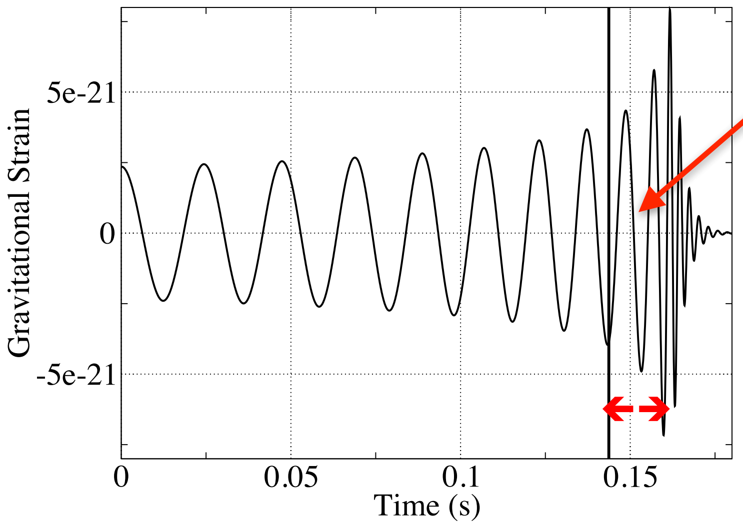
For GW150914 the total mass is ≳ 63.7 M⊙

over 0.2 s the wave frequency increases from 35 to 150 Hz, from which we infer that, just before merging, the distance bewteen the two masses was

$$d_{orb}(150\ Hz) \simeq 339\ km$$

The two objects must be extremely compact!

Are they Black Holes?



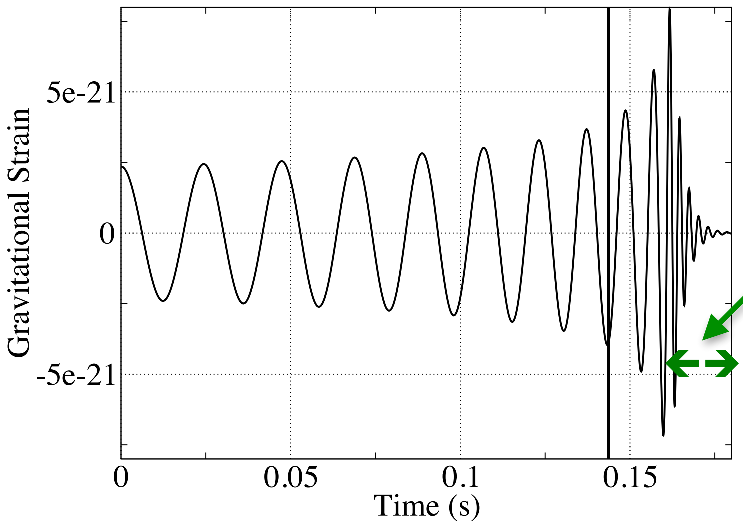
signal emitted during the merging: to be found by solving numerically Einstein's equations in the non linear regime

These studies started in the late 1990s with the Grand Challenge project to simulate head-on binary black hole collision

After decades of numerical studies on BH coalescence, a bank of templates has been set up

Fitting formulae based on numerical simulations of BH merging have been found, which compared to the merging part of the signal allow to estimate:

individual masses and spins
mass and angular momentum of the final black hole



Ringdown: part of the signal emitted by the final black hole, which oscillates in its proper modes: the Quasi-Normal-Modes (QNM)

the ringdown is a superposition of damped sinusoids at the frequencies and with the damping times of the QNMs

In General Relativity the QNM frequencies depends only on the black hole mass and the angular momentum **(no hair theorem)**

$$\mathbf{M} = \mathbf{n}\mathbf{M}_{\odot} \qquad \nu_{\mathbf{0}} \sim (\mathbf{12}/\mathbf{n})\mathbf{kHz} \qquad \tau \sim \mathbf{n} \cdot \mathbf{5.5} \times \mathbf{10}^{-\mathbf{5}}\ \mathbf{s}$$

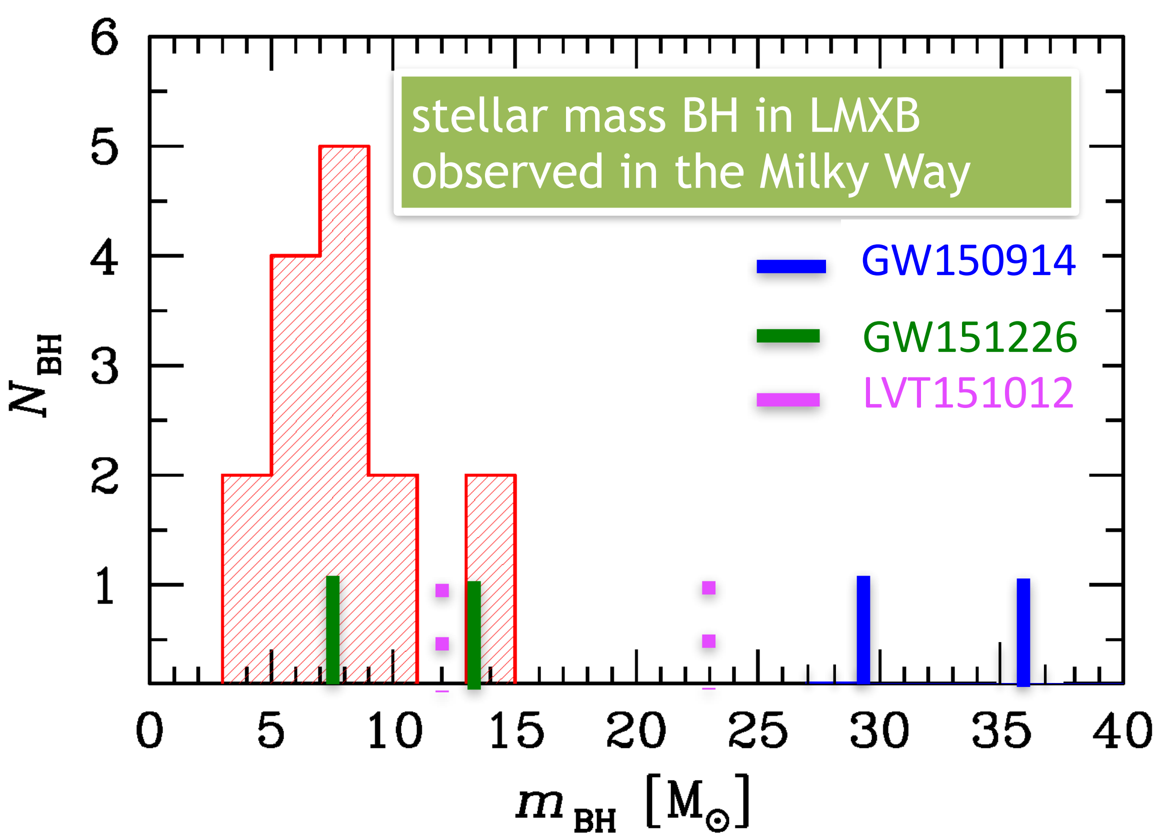
frequency increases up to 30% if the BH rotates

The frequency of the lowest quasi-normal mode has been extracted from the detected ringdown of the firts event GW150914. The black hole **mass and angular momentum** agree with the values found from the merging

WHAT DID WE KNOW ABOUT BLACK HOLES BEFORE GW DETECTION

- supermassive black holes $10^6\ M_\odot \lesssim M \lesssim 10^{11}\ M_\odot$

- stellar mass black holes $5\ M_\odot \lesssim M \lesssim 15 - 20\ M_\odot$



GW150914

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LVT151012

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Black holes exist in a mass range much broader than previously observed

Orosz etal 2003
Ozel etal 2013

How did the “heavy” BHs and BH binaries form?

- “heavy” BHs as in GW150914 ~ (29,36) $M_\odot$ are most likely formed in the direct collapse of low metallicity stars (below $Z \approx 0.5\, Z_\odot$, where $Z_\odot \approx 1{,}6\,\%$ of the total mass)

B.P. Abbott et al., Observation of gravitational waves from a binary black hole merger, Physical review Letters 116, 2016.

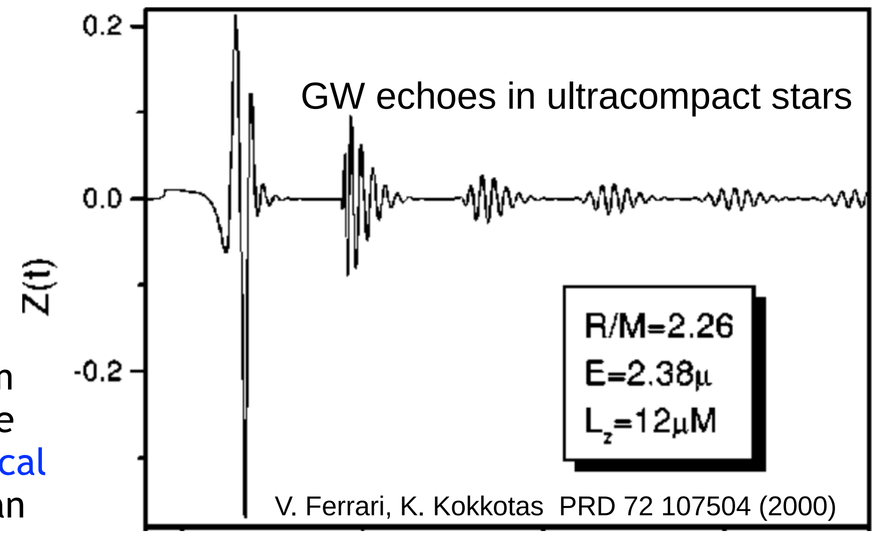
- the observed BH binaries have probably been formed dinamically, by close encounters in three-body systems possible in dense clusters

Ziosi et al MNRAS 441, 2014
Kimpson et al MNRAS 463, 2016

The formation channel depends not only on the mass ratio, but also on the BH spins: these are not measured with sufficient accuracy in the detected signals. More events and larger signal-to-noise ratios will be needed

The coalescing compact objects were two black holes or … something else?

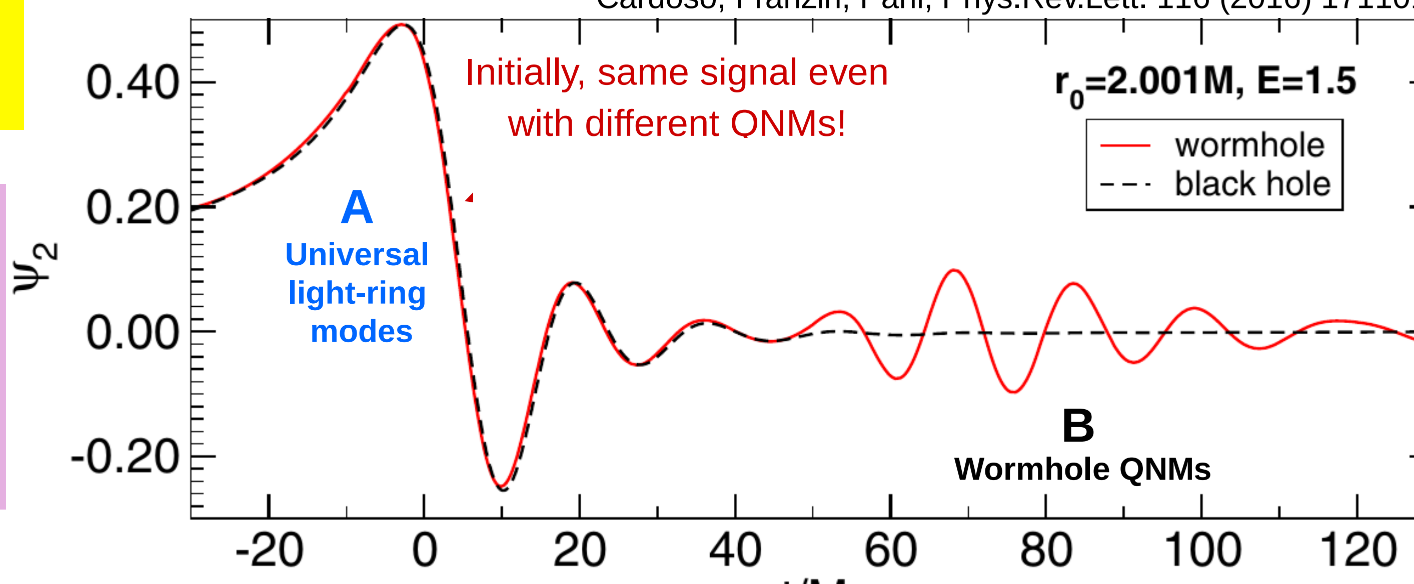
- ✭ We are sure that the coalescing objects are extremely compact
- ✭ the mass and spin of the final BH estimated from the merging part of the signal agrees with those extracted from the ringing tail, in the frame of General Relativity
- ✭ However, the quality of the data is such that some room is left for alternative interpretations that do not involve black holes, but other objects that, either within classical General Relativity, or in modified theories of gravity, can be equally massive and compact, i.e. gravastars, boson stars, whormholes etc



More signature to be considered: tidal heating, tidal deformability, etc

Future detections with larger SNR will shed light on this important question

Cardoso, Franzin, Pani; Phys.Rev.Lett. 116 (2016) 17110



There are other sources we are looking for and unsolved questions we want to answer using gravitational waves

Neutron stars (NS) in different phases of their life:

- Coalescing binaries composed of two neutron stars, or of a neutron star and a black hole
- gravitational collapse to a neutron stars
- spinning neutron stars
- oscillating of neutron stars

Astrophysics:

are coalescing NS-NS or NS-BH sourcing Gamma Ray Bursts?

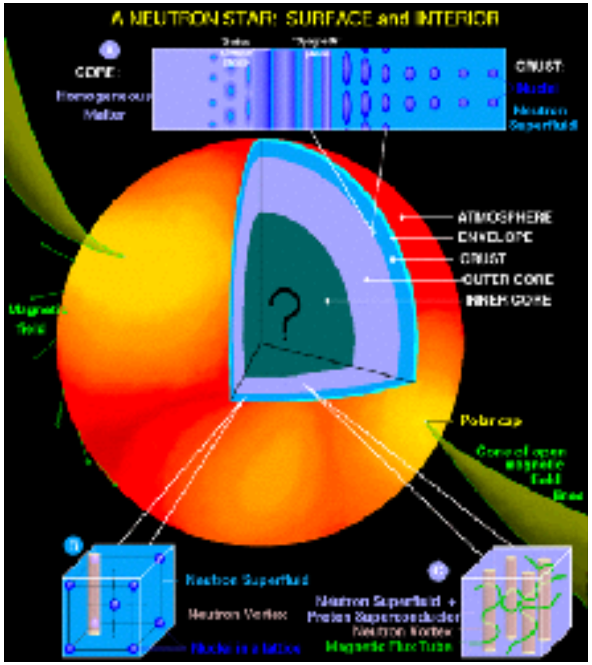
gravitational collapse: how is it ignited?

what is the shape of a neutron star?

are there sources which we do not know?

Fundamental physics:

how does matter behave at the extreme densities of a neutron star core?



credits D. Page

NEUTRON STARS:

observed mass: [1-2]$M_{\odot}$

radius: difficult to measure (about 13-15 % accuracy)
[10-15] km (teoretical)

In the inner part of the core of a neutron star, the density can be larger than the equilibrium density of nuclear matter

$$\rho_0 = 2.67 \times 10^{14} \text{ g /cm3}$$

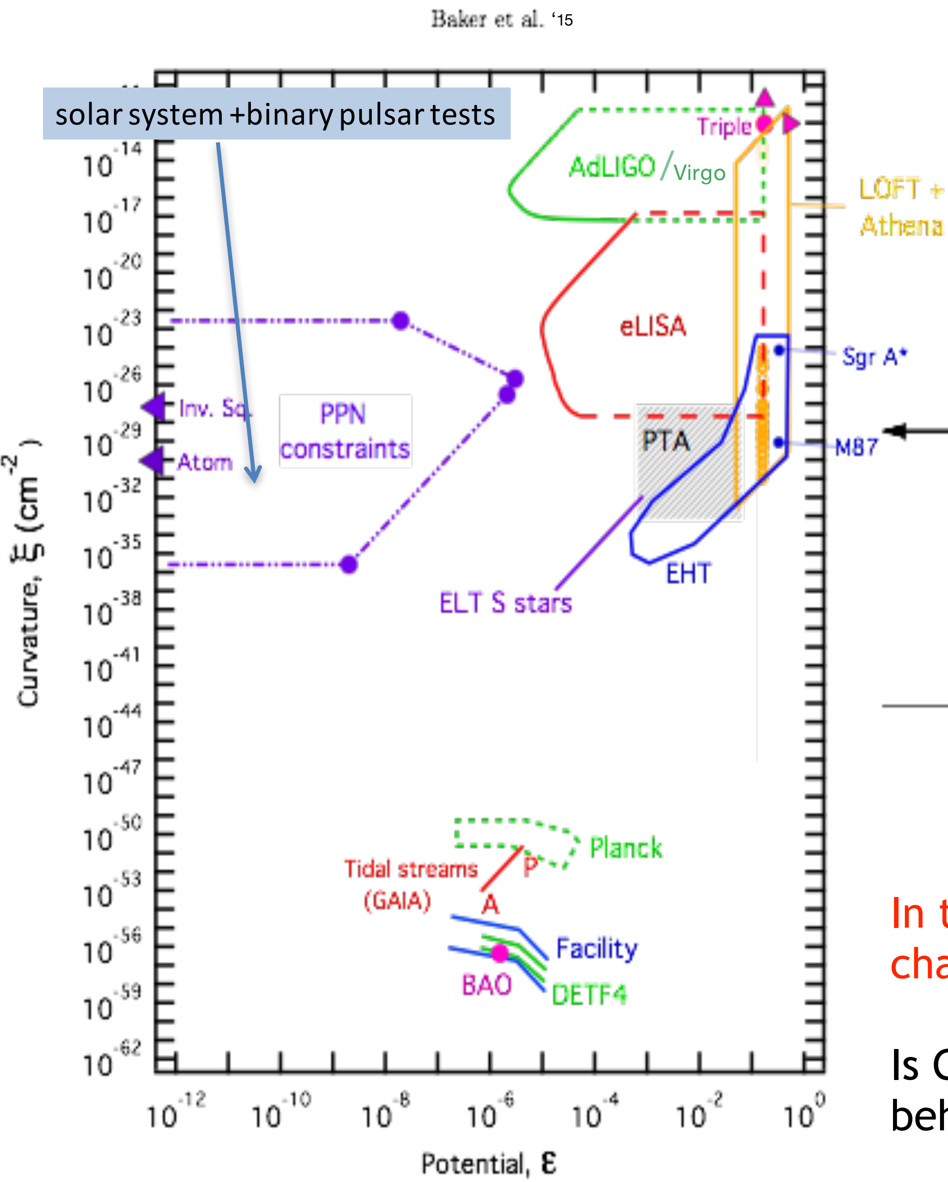
typical densities ≈ 2-5 ρ_0 or more

At these densities (unreachable in a laboratory) hadrons interactions cannot be neglected, and have to be treated in the framework of the theory of Quantum Cromo Dynamics

even the particle content is unknown: Hadrons? Hyperons? Meson condensates? Deconfined quark matter?

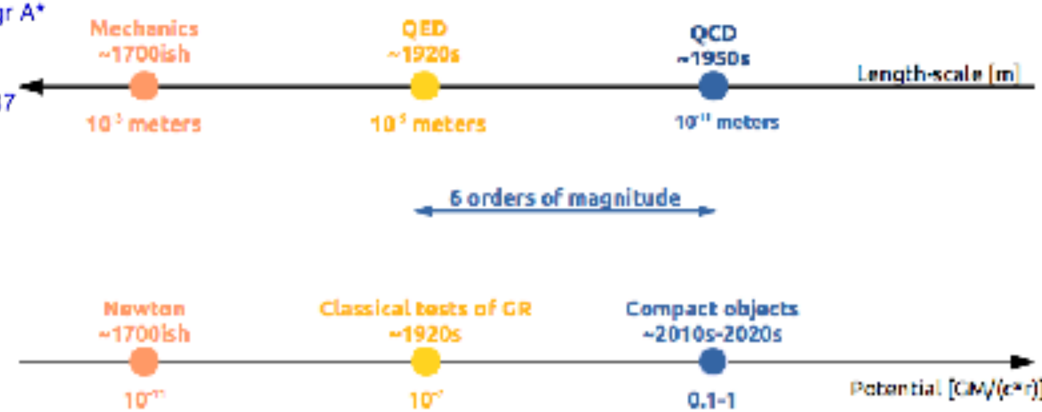
Several different models have been proposed which have to be tested

- **Before GW150914, we had tested only the weak-field regime of gravity (solar system tests, binary pulsars) Now, the realm of strong gravity is open to exploration!**



$$\epsilon = \frac{GM}{r} \quad \text{gravitational potential}$$

$$\zeta = \frac{GM}{r^3} \quad \text{spacetime curvature}$$



In the past, when changing scale interaction also changed!

Is General Relativity appropriate to describe the behaviour of gravity at the horizon scale?

Gravitational waves will be the probe through which we will be able to explore this mysterious and fascinating region of the spacetime