HADRONIC CONTRIBUTION TO g-2 OF THE MUON AND TO THE FINE STRUCTURE CONSTANT AT THE Z-MASS SCALE

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MOTIVATION

Hadronic (g – 2)μ : signal for Physics beyond SM

Current tension: 2.7 σ

CULPRIT: DATA ON $e^+ e^- \longrightarrow$ hadrons (uds, c, b)

NEW APPROACH: ENTIRELY FROM QCD

PQCD (c,b) & PQCD & LQCD (c,b)

α_{QED} (M^2_Z) : Higgs Mass

MUON

$$a_\mu|_{EXP} = 1\,165\,920\,8.9\ (6.3) \times 10^{-10}$$

$$a_\mu|_{THY} = 1\,165\,918\,1.8\ (7.6) \times 10^{-10}$$

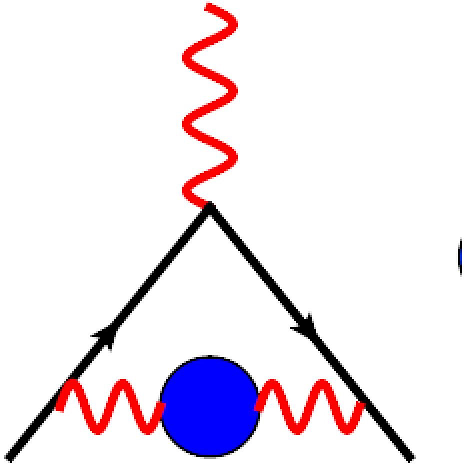
$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10}$$

$$\Delta a_\mu : 2.7\ \sigma$$

CONTRIBUTIONS TO a = (g-2)/2

a_{QED} a_{HAD} a_{EW}

$$a_{HAD} = a|_{HAD}(LO) + a|_{HAD}(HO) + a|_{HAD}(LBL)$$



IN UNITS OF 10^{-10}

$$a_\mu|_{QED} = 11\,658\,471.8853 \pm 0.3650$$

$$a_\mu|_{EW} = 15.4\ (2)$$

$$a_\mu|_{HAD}\ (HO) = -\ 9.84\ (7)$$

$$a_\mu|_{HAD}\ (LBL) = 11.6\ (4.0)$$

$$a_\mu|_{HAD}\ (LO) = 692.7\ (6.5)$$

$$a_\mu|_{THY} = 11\,659\,181.8 \pm 7.6$$

$$a_\mu|_{EXP} - a_\mu|_{THY} = 27.1\ (9.9) \times 10^{-10} \quad [\Delta a_\mu : 2.7\ \sigma]$$

$a_\mu|_{HAD}$ (LO) = 692.7 (6.5)

(in units of 10^{-10})

RELIES ENTIRELY ON (somewhat unreliable) DATA ON

$e^+ e^- \rightarrow$ hadrons

(uds) + (charm) + (bottom)

Need: $a_\mu|_{HAD}$ (LO) = 710 - 730

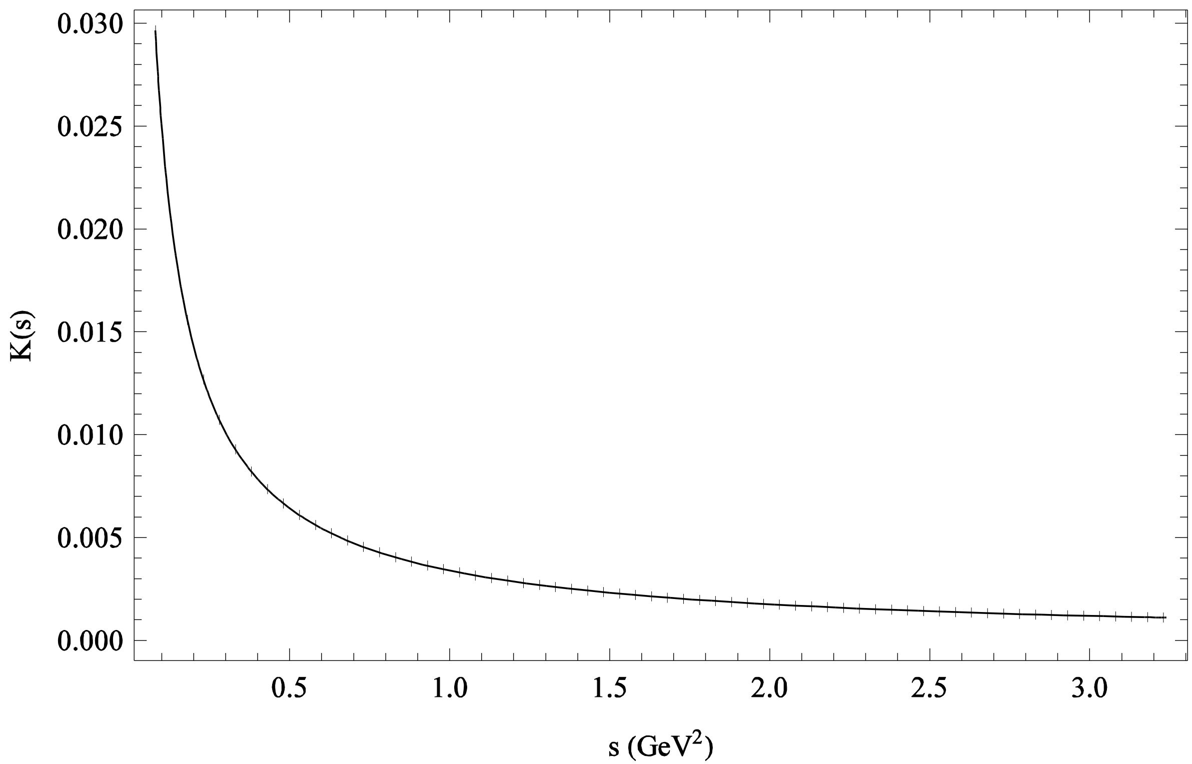
$$a^{HAD} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} \, K(s) \, R(s) \quad (s=E^2)$$

$$R(s) \equiv \frac{\sigma(e+ e- \to hadrons}{\sigma(e+ e- \to leptons} \propto \mathrm{Im}\,\Pi(s)$$

$$\Pi_{\mu\nu}(q^2) = i \int d^4x \, e^{iqx} < 0 | T(J_\mu(x) \, J_\nu^+(0)) | 0 >$$

$$= (-g_{\mu\nu} q^2 + q_\mu q_\nu) \Pi_{EM}(q^2)$$

$$\mathrm{Im}\,\Pi_{EM}(q^2) = \frac{1}{8\pi}\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$


0.030
0.025
0.020
0.015
0.010
0.005
0.000
K(s)
0.5
1.0
1.5
2.0
2.5
3.0
s (GeV^2)

STRATEGY TO DETERMINE $a_\mu|_{HAD}$ (LO) IN QCD

- (1) SUBSTITUTE KERNEL K(s) BY A FIT FUNCTION

  - $K(s) = a_0 + a_1\, s + \frac{a_2}{s^2} + \frac{a_3}{s^3} + \cdots$

- (2) SINGULAR TERMS ⟹ POLES ⟹ RESIDUES

- RESIDUES $\propto \frac{d}{d\,q2} \Pi_{QCD}(q^2)|\, q^2{=}0$

  - RESIDUES ⟹ LATTICE QCD

A THEORETICAL QCD CALCULATION OF

$$a_\mu|_{HAD}\,(LO) = a_\mu|_{uds} + a_\mu|_c + a_\mu|_b$$

USING LQCD FOR $d\Pi(q^2)/d\,q^2$ & $d^2\Pi(q^2)/(d\,q^2)^2$ (u,d,s)

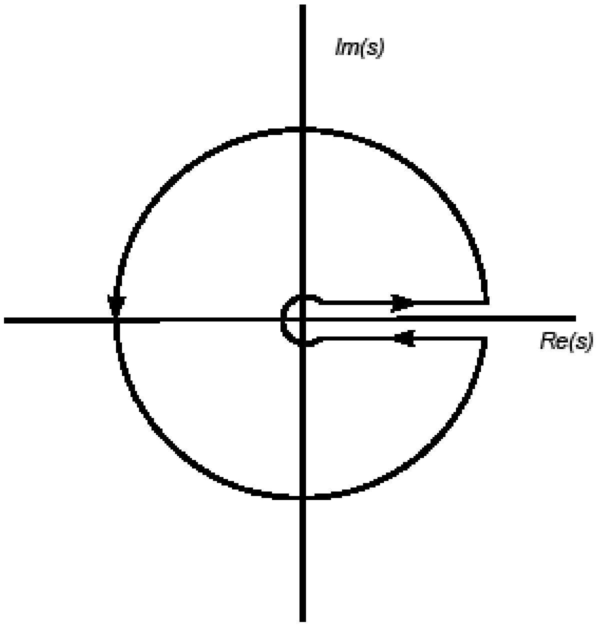
USING ENTIRELY PERTURBATIVE QCD (quark mass) EXPANSION (c,b)

$$\Pi_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx} < 0|T(J_\mu(x)\, J_\nu^{\,+}(0))|0 >$$
$$= (-g_{\mu\nu}\, q^2 + q_\mu\, q_\nu)\,\Pi_{EM}(q^2)$$

$$\mathrm{Im}\,\Pi_{EM}(q^2) = \frac{1}{8\pi}\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

CAUCHY'S THEOREM

$$\oint_C \Pi(s)\, ds = 0$$



$$\oint_C \Pi(s)\, ds = \sum_i (\text{Residue Pole})_i$$

$$\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, \Pi(s) + \int_{s_{th}}^{s_0} ds\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s) = \sum_i \mathrm{Res}_\mathrm{i}$$

HEAVY QUARK SECTOR
c & b

$$\mathrm{K(s)} \rightarrow K_1(s) = \frac{a_1}{s} + \frac{a_2}{s^2}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(\mathrm{s}) = \mathrm{Res}\left[\Pi(\mathrm{s})\, \frac{K_1(s)}{s}\right]_{\mathrm{s}=0} - \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \Pi(\mathrm{s})$$

PERTURBATIVE QCD EXPANSION (HEAVY QUARKS)

$$z = \frac{s}{4{m_Q}^2}$$

$$\Pi(s)_{\text{PQCD}} \propto \sum_{n\geq 0} C_n \, z^n$$

PQCD up to 4-loop level

$a^{HAD}|_c = 14.4 \pm 0.1 \times 10^{-10}$ $a^{HAD}|_b = 0.29 \pm 0.01 \times 10^{-10}$

(S. Bodenstein et al. 2012)

- LQCD
- (2014)

- $a^{HAD}|_c = 14.1 \pm 0.1 \times 10^{-10}$ (ETM Coll.)

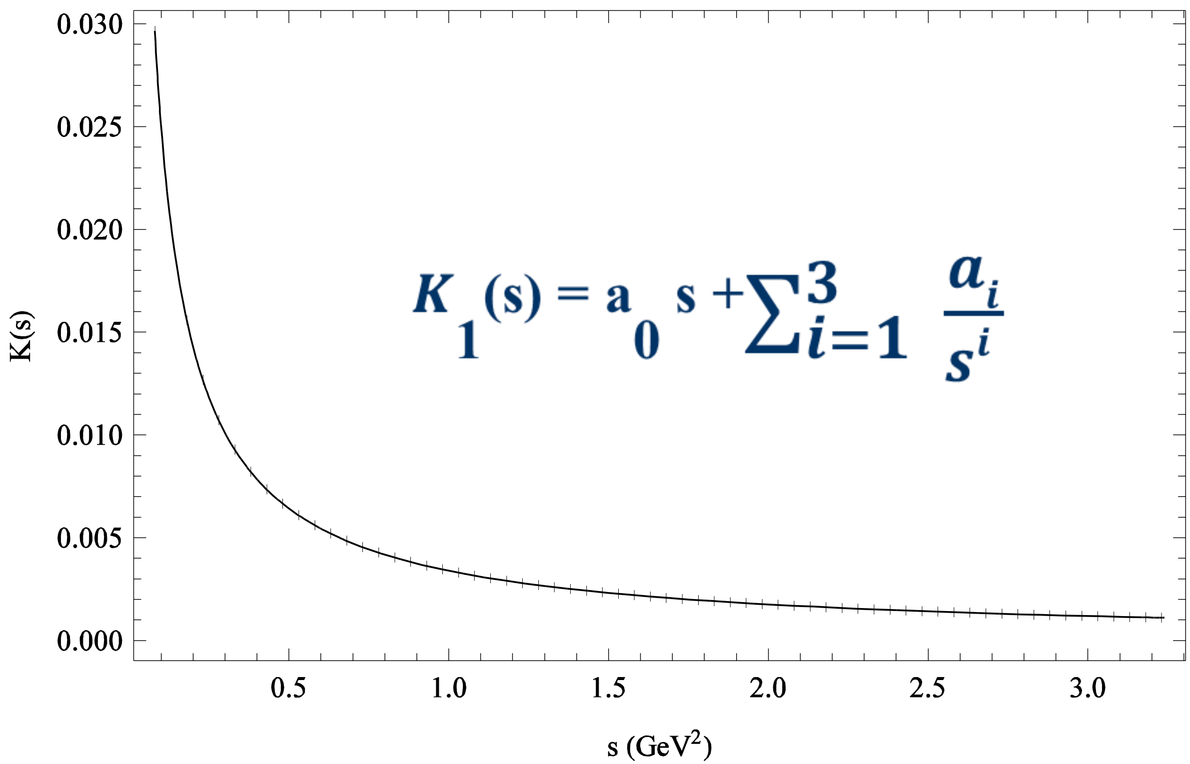
- $a^{HAD}|_c = 14.42 \pm 0.39 \times 10^{-10}$ (HPQCDColl.)

uds

$$\mathbf{K(s)} \rightarrow \boldsymbol{K_1}\mathbf{(s)} = \mathbf{a_0\, s} + \sum_{n=1}^{3} \frac{a_n}{s^n}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s}\; K_1(\mathrm{s})\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(\mathrm{s}) = \mathrm{Res}\, [\Pi(\mathrm{s})\, \frac{K_1(s)}{s}\,]_{\mathrm{s}=0}$$

$$-\, \frac{1}{2\,\pi\, i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(\mathrm{s})\, \Pi(\mathrm{s})$$



0.030
0.025
0.020
0.015
0.010
0.005
0.000
K(s)
0.5
1.0
1.5
2.0
2.5
3.0
s (GeV^2)
$K_1(s) = a_0 s + \sum_{i=1}^{3} \frac{a_i}{s^i}$

$$\text{Res}\left[\Pi(s)\frac{K_1((s)}{s}\right]_{s=0} = \lim_{s\to 0} \Sigma \frac{a_n}{n!}\left(\frac{d}{ds}\right)^n \Pi(s)$$

NO (QCD) LOW ENERGY THEOREM FOR

$\Pi(s)|_{uds}$

Need: $d\,\Pi(s) / ds|_{s=0}$ (dominant residue)

LQCD: available information

Lattice QCD: H.Horsch, B. Jaeger, H. Wittig

$\mathbf{d\,\Pi(s) / ds|_{s=0} = 0.072 - 0.087\ GeV^{-2}}$ (HJW):

$\mathbf{a_\mu|_{HAD} = 729\text{-}871}$ **(710-730)**

Second derivative < 0 **(negligible)**

HADRONIC CONTRIBUTION TO $\alpha_{EM}(M_Z^2)$

- $\alpha(s) = \dfrac{\alpha(0)}{1 - \Delta\alpha(s)L - \Delta\alpha(s)HAD}$

- $\Delta\alpha(M^2{}_Z)HAD = 4\,\pi\alpha(0)\,\{\,\Pi(0) - \mathrm{Re}\,[\Pi_{EM}(M^2{}_Z)]\,\}$

- $\Delta\alpha(M^2{}_Z)HAD \propto \int_{4m_\pi^2}^{\infty} ds\, R(s)\, /[s\,(s - M^2{}_Z)]$

- Charm, bottom, top ⟹ PQCD

- up, down, strange ⟹ LQCD

RESULTS

- $274.2 \pm 1.0 \times 10^{-4}$ (Davier et al.)
- $\Delta\alpha(M^2_Z)_{HAD} =$
- $275.7 \pm 0.8 \times 10^{-4}$ (Bodenstein et al.)
-
- NEW LQCD data on Π(-s) & Π(0) (Wittig et al.) :
- **CONSIDERABLE REDUCED UNCERTAINTY**

THANK YOU