Some recent progress on lattice QCD for Kaon physics

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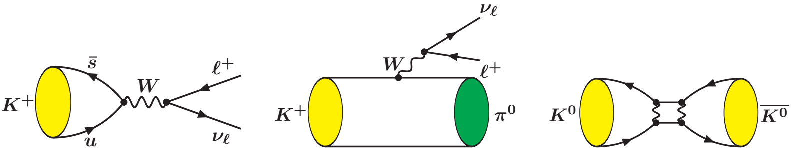
Introduction

- Why we need lattice QCD ⇒
  to evaluate hadronic effects non-perturbatively from first principles

- How it works for flavor physics ⇒
  powerful for standard hadronic matrix elements
  $\langle h_2(p_2)|O(0)|h_1(p_1)\rangle$ or $\langle 0|O(0)|h(p)\rangle$



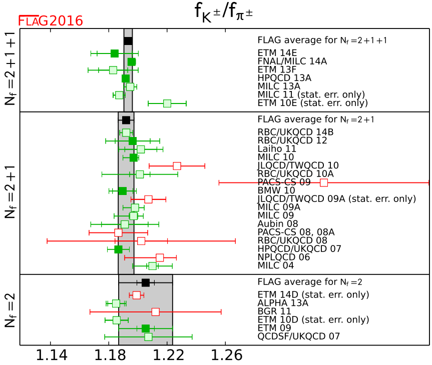
  - single local operator insertion: $O(0)$
  - only single stable hadron $|h\rangle$ or vacuum $|0\rangle$ in the initial/final state
  - spatial momenta p_1, p_2 need to be small compared to $1/a$
    (not a problem for Kaon physics, but essential for B decays)

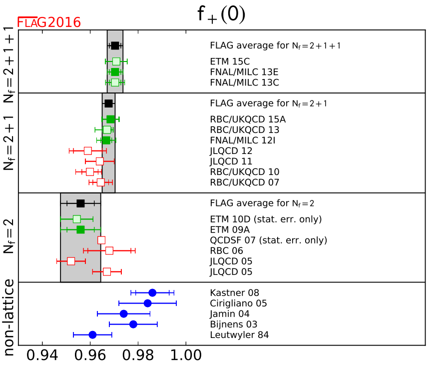
“standard” quantities in Kaon physics: $f_{K^\pm}/f_{\pi^\pm}$ and $f_+(0)$

Flavor Lattice Averaging Group (FLAG): arXiv:1607.00299

$$f_{K^\pm}/f_{\pi^\pm} = 1.1933(29) \quad \Rightarrow \quad 0.25\% \text{ error}$$

$$f_+^{K\pi}(0) = 0.9704(33) \quad \Rightarrow \quad 0.34\% \text{ error}$$





Experimental information: arXiv:1411.5252, 1509.02220

$$K_{\ell 3} \quad \Rightarrow \quad |V_{us}| f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2231(9)$$

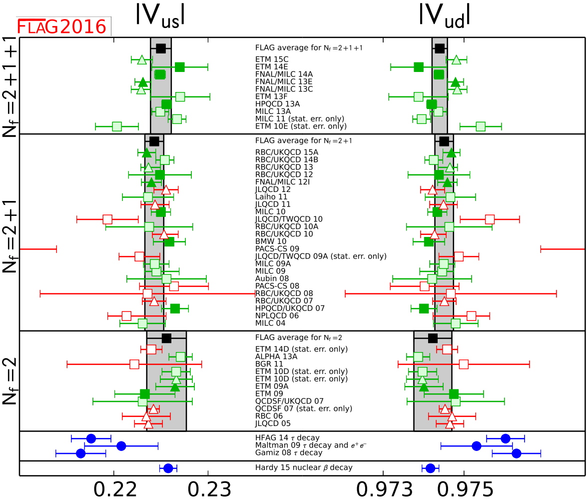
$$K_{\mu 2}/\pi_{\mu 2} \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2313(7)$$

V_{us} from lattice QCD

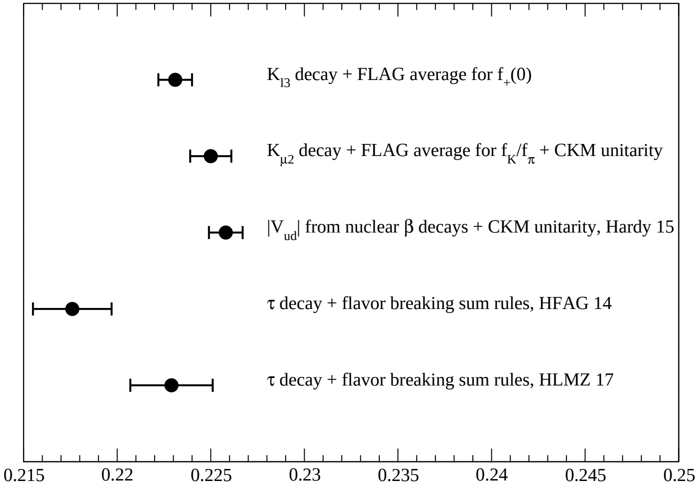
- Test the CKM unitarity

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.980(9) \quad \Rightarrow \quad 2\sigma \text{ deviation from } 1$$

- Experiment + CKM unitarity + $f_{K^\pm}/f_{\pi^\pm}$ or $f_+(0)$ $\Rightarrow$ V_{us} and V_{ud}

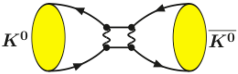


V_{us}: comparison with other determinations



- $> 3\sigma$ deviation from semi-inclusive τ decay (HFAG 14) seems resolved by a new implementation of sum rules (HLMZ 17, arXiv:1702.01767)
- $< 0.5\%$ uncertainty requires the inclusion of $O(\alpha_e)$ EM corrections

Short distance dominance ⇒ OPE ⇒ Wilson coeff. $C(\mu)$ × operator $Q^{\Delta S=2}(\mu)$



$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} C(\mu) Q^{\Delta S=2}(\mu)$$

- Serve as a dominant contribution to the indirect CP violation ϵ_K

$$\epsilon_K = \exp(i\phi_\epsilon)\sin(\phi_\epsilon)\left[\frac{\mathrm{Im}[\langle\overline{K^0}|\mathcal{H}_{\text{eff}}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} + \frac{\mathrm{Im}[M_{0\bar{0}}^{\text{LD}}]}{\Delta M_K} + \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right]$$

- Within Standard Model, only single operator with $V - A$ structure

$$Q^{\Delta S=2} = [\bar{s}_a\gamma_\mu(1-\gamma_5)d_a][\bar{s}_b\gamma_\mu(1-\gamma_5)d_b]$$

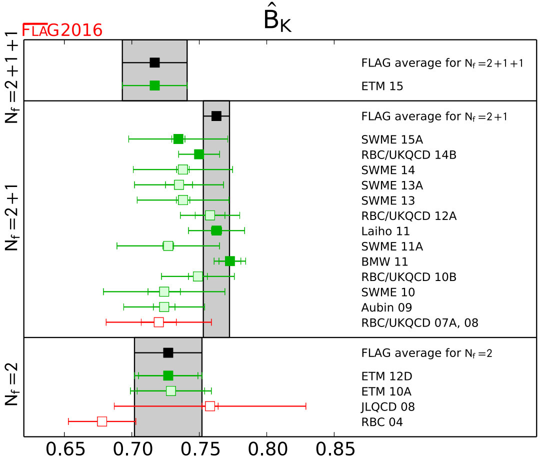
- Beyond SM, 4 other operator possible

$$Q_2^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_a][\bar{s}_b(1-\gamma_5)d_b]$$
$$Q_3^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_b][\bar{s}_b(1-\gamma_5)d_a]$$
$$Q_4^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_a][\bar{s}_b(1+\gamma_5)d_b]$$
$$Q_5^{\Delta S=2} = [\bar{s}_a(1-\gamma_5)d_b][\bar{s}_b(1+\gamma_5)d_a]$$

FLAG average for Standard Model B_K

- B_K in NDR-$\overline{\mathrm{MS}}$ scheme: $B_K(\mu) = \frac{\langle \overline{K^0} | Q^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$
- Renormalization group independent B parameter $\hat{B}_K$:

$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \exp \left\{ \int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g} \right) \right\} B_K(\mu)$$

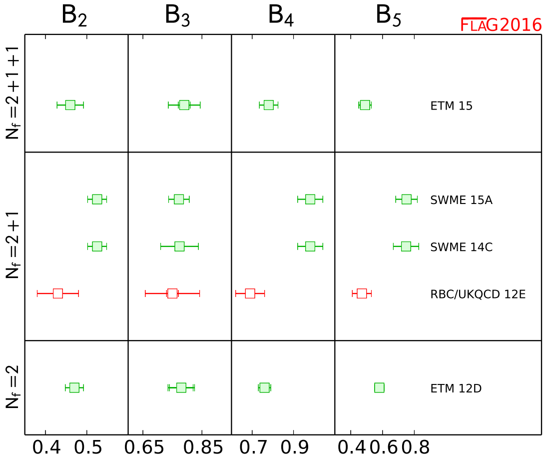


- $N_f = 2+1+1$: $\hat{B}_K = 0.717(24)$
- $N_f = 2+1$: $\hat{B}_K = 0.763(10)$
- $N_f = 2$: $\hat{B}_K = 0.727(25)$

Status for BSM B_i

$$B_i(\mu) = \frac{\langle \overline{K^0} | Q_i(\mu) | K^0 \rangle}{N_i \langle \overline{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}, \quad \{N_2, \cdots, N_5\} = \{-5/3, 1/3, 2, 2/3\}$$

$B_i(\mu)$ at $\mu_{\overline{\mathrm{MS}}} = 3$ GeV



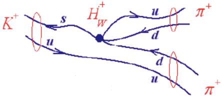
Uncontrolled systematic effects may cause the observed deviations:

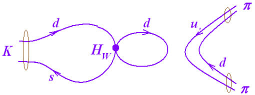
- NPR renormalization
- Matching procedure
- Continuum limit

No FLAG average yet

Go beyond "standard" quantities in lattice Kaon physics

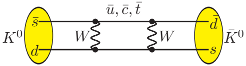
- $K \to \pi\pi$ decays and direct CP violation



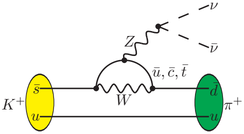


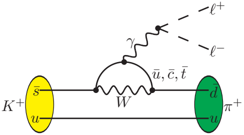
Final state involve $\pi\pi$ (multi-hadron system)

- Long-distance contributions to flavor changing processes
  - ΔM_K and ϵ_K



  - Rare kaon decays: $K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$





Hadronic matrix element for bilocal operators

$$\int d^4x \langle f|T[Q_1(x)Q_2(0)]|i\rangle$$

$K \to \pi\pi$ decays and direct CP violation

- Kaon decays into the isospin $I = 2$ and 0 $\pi\pi$ states

$$\Delta I = 3/2 \text{ transition:} \quad \langle \pi\pi(I=2)|H_W|K^0\rangle = A_2 e^{i\delta_2}$$
$$\Delta I = 1/2 \text{ transition:} \quad \langle \pi\pi(I=0)|H_W|K^0\rangle = A_0 e^{i\delta_0}$$

- If CP symmetry were protected $\Rightarrow$ A_2 and A_0 are real amplitudes

- Direct CP violation is described by the parameter ϵ'

$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}[A_2]}{\mathrm{Re}[A_0]} \left(\frac{\mathrm{Im}[A_2]}{\mathrm{Re}[A_2]} - \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]} \right)$$

- ϵ' is 1000 times smaller than the indirect CP violation ϵ_K

$$\text{PDG}: \quad |\epsilon'| = 3.70(53) \times 10^{-6}, \quad |\epsilon| = 2.228(11) \times 10^{-3}$$

Thus direct CP violation ϵ' is very sensitive to New Physics

Recent results for $K \to \pi\pi(I = 2)$

Results for A_2 [RBC-UKQCD, PRD91 (2015) 074502]

- Use two ensembles (both at $m_\pi = 135$ MeV) for continuum extrapolation

$$48^3 \times 96, \quad a = 0.11 \text{ fm}, \quad L = 5.4 \text{ fm}$$
$$64^3 \times 128, \quad a = 0.084 \text{ fm}, \quad L = 5.4 \text{ fm}$$

- After continuum extrapolation:

$$\mathrm{Re}[A_2] = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8} \text{ GeV}$$
$$\mathrm{Im}[A_2] = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13} \text{ GeV}$$

- Experimental measurement:

$$\mathrm{Re}[A_2] = 1.479(3) \times 10^{-8} \text{ GeV}$$
$$\mathrm{Im}[A_2] \text{ is unknown}$$

- Scattering phase at $E_{\pi\pi} = M_K$

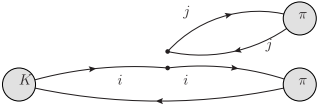
$$\delta_2 = -11.6(2.5)(1.2)^\circ$$

consistent with phenomenological analysis [Schenk, '91]

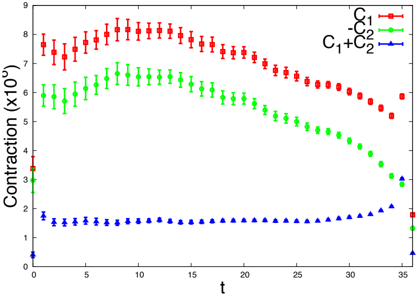
Resolve the puzzle of $\Delta I = 1/2$ rule

$\Delta I = 1/2$ rule: $A_0 = 22.5 \times A_2$

- $\mathrm{Re}[A_2]$ is dominated by diagrams C_1 (left) and C_2 (right)



- Opposite sign in C_1 and C_2 leads to large cancellation in $\mathrm{Re}[A_2] \propto C_1 + C_2$
  - Such cancellation is first observed in an earlier calculation [RBC-UKQCD, PRL110 (2013) 152001]
  - It is further confirmed in the latest calculation of A_2 [RBC-UKQCD, PRD91 (2015) 074502]



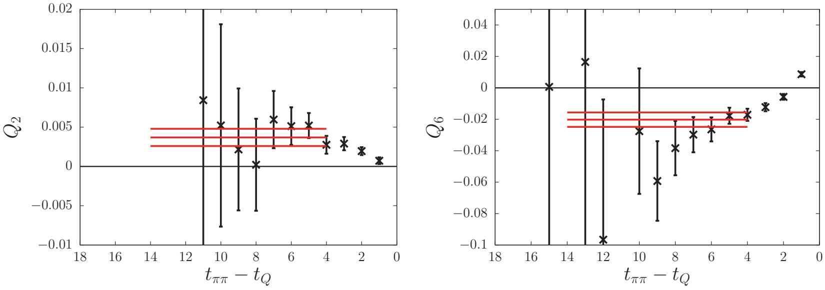
Puzzle of $\Delta I = 1/2$ rule is resolved from first principles

Results for A_0 [RBC-UKQCD, PRL115 (2015) 212001]

- Use a $32^3 \times 64$ ensemble, $a = 0.14$ fm, $L = 4.53$ fm

  $M_\pi = 143.1(2.0)$ MeV, $M_K = 490(2.2)$ MeV, $E_{\pi\pi} = 498(11)$ MeV

- G-boundary condition is used: non-trivial to tune the volume $\Rightarrow M_K = E_{\pi\pi}$
- The largest contributions to $\mathrm{Re}[A_0]$ and $\mathrm{Im}[A_0]$ come from Q_2 (current-current) and Q_6 (QCD penguin) operator



Results for $\mathrm{Re}[A_0]$, $\mathrm{Im}[A_0]$ and $\mathrm{Re}[\epsilon'/\epsilon]$

- Determine the $K \to \pi\pi(I=0)$ amplitude A_0
  - Lattice results

$$\mathrm{Re}[A_0] = 4.66(1.00)_{\mathrm{stat}}(1.26)_{\mathrm{syst}} \times 10^{-7}\ \mathrm{GeV}$$
$$\mathrm{Im}[A_0] = -1.90(1.23)_{\mathrm{stat}}(1.08)_{\mathrm{syst}} \times 10^{-11}\ \mathrm{GeV}$$

  - Experimental measurement

$$\mathrm{Re}[A_0] = 3.3201(18) \times 10^{-7}\ \mathrm{GeV}$$
$$\mathrm{Im}[A_0]\ \text{is unknown}$$

- Determine the direct CP violation $\mathrm{Re}[\epsilon'/\epsilon]$

$$\mathrm{Re}[\epsilon'/\epsilon] = 0.14(52)_{\mathrm{stat}}(46)_{\mathrm{syst}} \times 10^{-3} \quad \text{Lattice}$$
$$\mathrm{Re}[\epsilon'/\epsilon] = 1.66(23) \times 10^{-3} \quad \text{Experiment}$$

2.1 σ deviation $\Rightarrow$ require more accurate lattice results

Long-distance contributions to flavor changing processes

ΔM_K and ϵ_K

K^0-$\overline{K^0}$ mixing: time evolution

$$i\frac{d}{dt}\begin{pmatrix}K^0\\ \overline{K}^0\end{pmatrix} = \left[\begin{pmatrix}M_{00} & M_{0\bar{0}}\\ M_{\bar{0}0} & M_{\bar{0}\bar{0}}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{00} & \Gamma_{0\bar{0}}\\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}}\end{pmatrix}\right]\begin{pmatrix}K^0\\ \overline{K}^0\end{pmatrix}$$

- To 2_{nd}-order in H_W

$$M_{ij} = M_K\delta_{ij} + \langle i|H_W|j\rangle + \mathcal{P}\sum\!\!\!\!\!\!\int_\alpha \frac{\langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle}{M_K - E_\alpha}$$

$$\Gamma_{ij} = 2\pi\sum\!\!\!\!\!\!\int_\alpha \langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle\,\delta(E_\alpha - M_K)$$

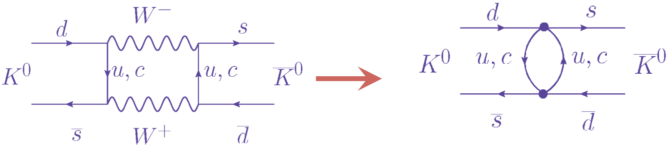
- ΔM_K and ϵ_K is related to $\mathrm{Re}[M_{0\bar{0}}]$ and $\mathrm{Im}[M_{0\bar{0}}]$, respectively

$$\Delta M_K = M_{K_S} - M_{K_L} = 2\mathrm{Re}[M_{0\bar{0}}]$$

$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left[\frac{\mathrm{Im}[M_{0\bar{0}}]}{\Delta M_K} + \frac{\mathrm{Im}[A_0]}{\mathrm{Re}[A_0]}\right], \quad \phi_\epsilon = \arctan\frac{\Delta M_K}{\Delta\Gamma_K/2}$$

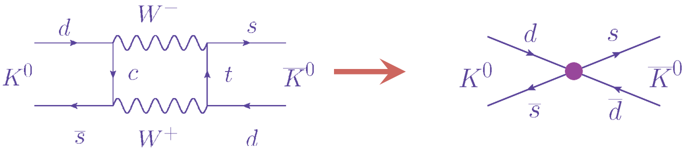
Long-distance contribution: ΔM_K and ϵ_K

- $\Delta M_K \Rightarrow \mathrm{Re}[M_{0\bar{0}}] \Rightarrow$ CP conserving part of K^0-$\overline{K^0}$ mixing



Dominant contribution from charm-charm loop: $\lambda_c^2 \frac{m_c^2}{M_W^2} > \lambda_t^2 \frac{m_t^2}{M_W^2}$
$\Rightarrow$ historically led to the predication of the mass of charm quark

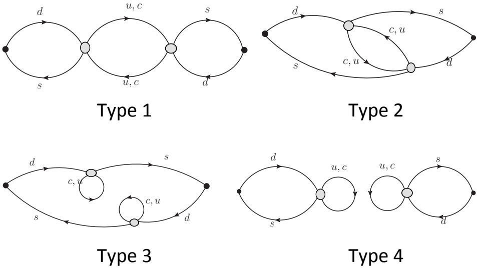
- $\epsilon_K \Rightarrow \mathrm{Im}[M_{0\bar{0}}] \Rightarrow$ CP violating part of K^0-$\overline{K^0}$ mixing



Top-top, top-charm and charm-charm loops compete in size
$\Rightarrow$ important top-top loop, thus ϵ_K is sensitive to the CKM input V_{cb}

Results for ΔM_K

- Use $32^3 \times 64$ ensemble: $1/a = 1.37$ GeV, $m_\pi = 170$ MeV and $m_c = 750$ MeV
  [Preliminary results, from Z. Bai, N. Christ]

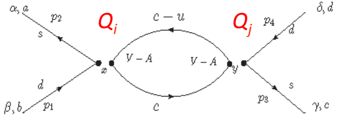


	ΔM_K x 10^{+12} MeV
Types 1-4	3.26(63)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, I=0	-0.097(49)
$\pi\pi$, I=2	-6.56(6) x 10^{-4}
D_{FV}	0.029(19)
Expt.	3.483(6)

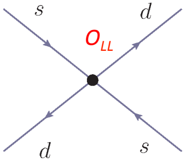
- New project: $64^3 \times 128$, $1/a = 2.38$ GeV, $m_c = 1.2$ GeV, $m_\pi = 140$ MeV
  - Based on 60 configurations: $\Delta M_K = 4.0(2.4) \times 10^{-12}$ MeV

$\lambda_t\lambda_u$ contribution to ϵ_K [calculated by Z. Bai, RBC-UKQCD]

- Without top quark in the lattice QCD calculation, logarithmic divergence



$-X_{ij}(\mu)$ x



  - Subtract $X_{ij}(\mu)\,[(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}]$ to remove the lattice cutoff effects
  - We thus define the bilocal operator in the RI/SMOM scheme

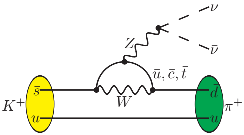
- Preliminary results at $m_\pi = 340$ MeV and $m_c = 970$ MeV

μ_{RI}	$\mathrm{Im}\,M_{\bar{0}0}{}^{ut,RI}$	$\mathrm{Im}\,M_{\bar{0}0}{}^{ut,RI\to\overline{MS}}$	$\mathrm{Im}\,M_{\bar{0}0}{}^{ut,ld\,corr}$	contribution to ε_K
1.54	-1.30(69)	0.352	-0.95(69)	$0.186(135)\times10^{-3}$
1.92	-1.49(69)	0.476	-1.01(69)	$0.199(135)\times10^{-3}$
2.11	-1.58(69)	0.537	-1.04(69)	$0.205(135)\times10^{-3}$
2.31	-1.65(69)	0.599	-1.05(69)	$0.206(135)\times10^{-3}$
2.56	-1.73(69)	0.674	-1.06(69)	$0.207(135)\times10^{-3}$

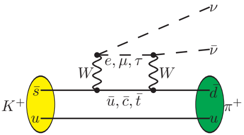
Experimental value for $|\epsilon_K| = 2.228\times10^{-3}$

Rare Kaon decays

$K^+ \to \pi^+ \nu\bar{\nu}$: Experiment vs Standard model



Z-exchange



W-W

$K^+ \to \pi^+ \nu\bar{\nu}$: largest top quark contribution, thus theoretically clean

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\rm EM}}{2\pi \sin^2\theta_W} \lambda_t X_t(x_t)}_{\mathcal{N} \sim 2\times 10^{-5}} \cdot (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Probe the new physics at scales of $\mathcal{N}^{-\frac{1}{2}} M_W = O(10 \text{ TeV})$

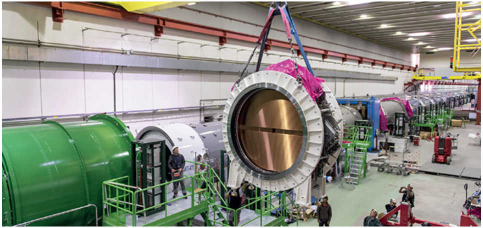
Past experimental measurement is 2 times larger than SM prediction

$$\text{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \qquad \text{[BNL E949, '08]}$$
$$\text{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = 9.11 \pm 0.72 \times 10^{-11} \qquad \text{[Buras et. al., '15]}$$

but still consistent with $> 60\%$ exp. error

New generation of experiment: NA62 at CERN

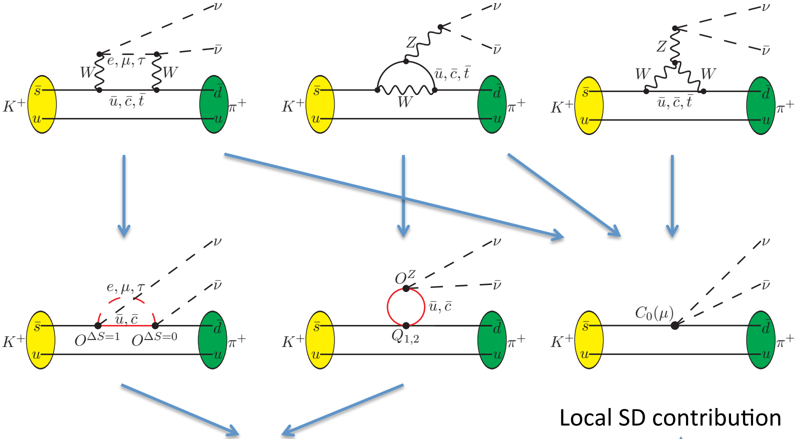
- aims at observation of $O(100)$ events in 2-3 years
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$



$K_L \to \pi^0 \nu\bar{\nu}$

- even more challenging since all the particles involved are neutral
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe K_L decays

OPE: integrate out the heavy fields, Z, W, t, $\cdots$



Local SD contribution

Bilocal LD contribution

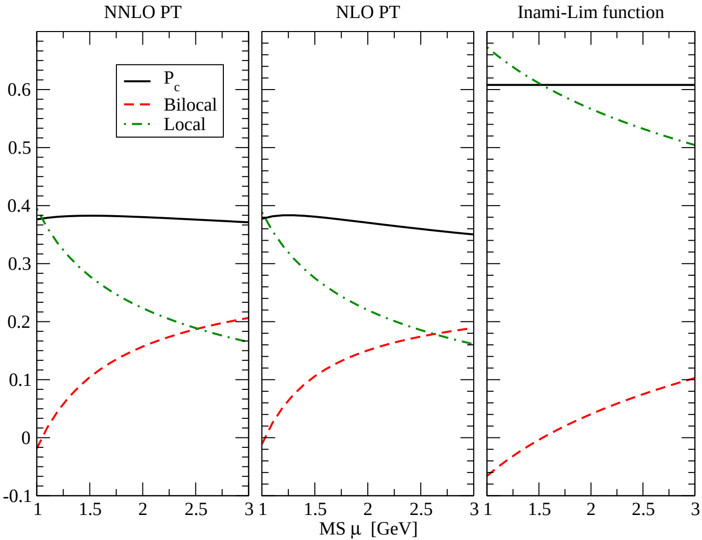
Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu \bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

Bilocal contribution vs local contribution

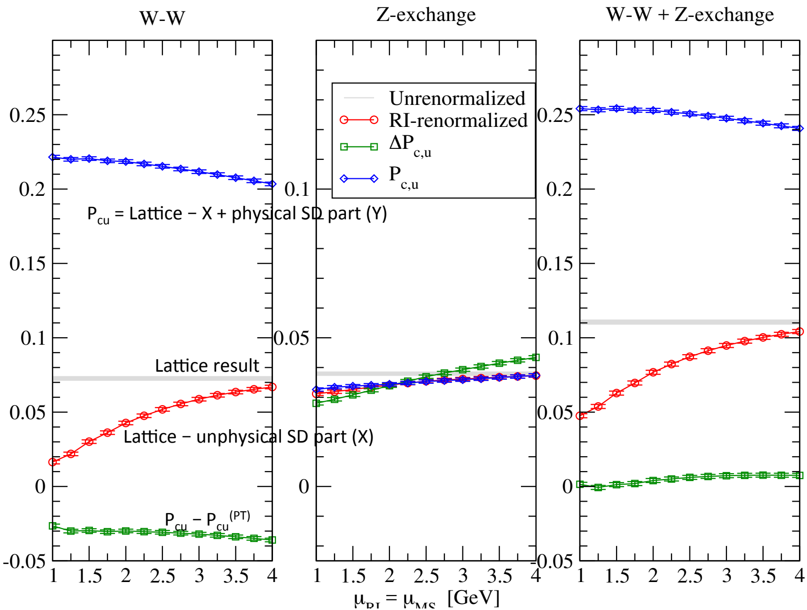
Bilocal $C_A^{\overline{\rm MS}}(\mu)C_B^{\overline{\rm MS}}(\mu)r_{AB}^{\overline{\rm MS}}(\mu)$ vs Local $C_0^{\overline{\rm MS}}(\mu)$

[Buras, Gorbahn, Haisch, Nierste, '06]



At $\mu = 2.5$ GeV, 50% charm quark contribution from bilocal term

First results at m_π = 420 MeV, m_c = 860 MeV [RBC-UKQCD, arXiv:1701.02858]



Results for charm quark contribution

Charm quark contribution P_c

$$P_c = P_c^{\mathrm{SD}} + \delta P_{c,u}$$

NNLO QCD [Buras, Gorbahn, Haisch, Nierste, ‘06]:

$$P_c^{\mathrm{SD}} = 0.365(12)$$

Phenomenological ansatz [Isidori, Mescia, Smith, ‘05]

$$\delta P_{c,u} = 0.040(20)$$

Lattice results

$$\Delta P_{c,u} = 0.0040(\pm 13)_{\mathrm{stat}}(\pm 32)_{\mathrm{syst}}(-45)_{\mathrm{FV}}$$

- Cancellation in W-W and Z-exchange diagrams leads to small $\Delta P_{c,u}$
- Important to check whether such large cancellation also occurs for physical quark masses

$K \to \pi\ell^+\ell^-$ decay: motivation

- CP conserving decays $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ are dominated by long-distance contribution induced by photon exchange

$$
\begin{aligned}
T^{\mu}_{+,S} &= \int d^4x\, e^{iqx} \langle \pi(p) | T\{J^{\mu}_{em}(x)\mathcal{H}^{\Delta S=1}(0)\} | K_{+,S}(k) \rangle \\
&= \frac{G_F M_K^2}{(4\pi)^2} V_{+,S}(z) \left[z(k+p)^{\mu} - (1-r_{\pi}^2) q^{\mu} \right]
\end{aligned}
$$

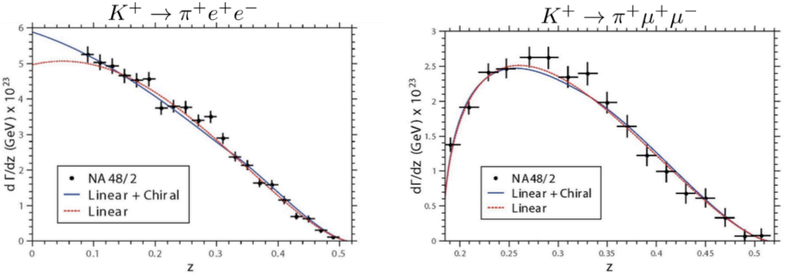
with $q = k - p$, $z = q^2/M_K^2$, $r_\pi = M_\pi/M_K$

- Calculation of $K^+ \to \pi^+\ell^+\ell^-$ can be compared with Exp. + ChPT analysis
- $V_+(z)$ useful for test of lepton flavor universality violation in rare K decays
- Results for $K_S \to \pi^0\ell^+\ell^-$ can be used for the evaluation of the significant interference between direct and indirect CP violation
    - Even the sign of $a_S = V_S(0)$ is useful

summarized by Jorge Portoles at Kaon 2016

Analysis

$$V_j(z) = a_j + b_j z + \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}_{K \to \pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2}\right]}_{F_V(z)} \underbrace{\left[\Phi\left(z/r_\pi^2\right) + \frac{1}{6}\right]}_{\text{loop}}$$



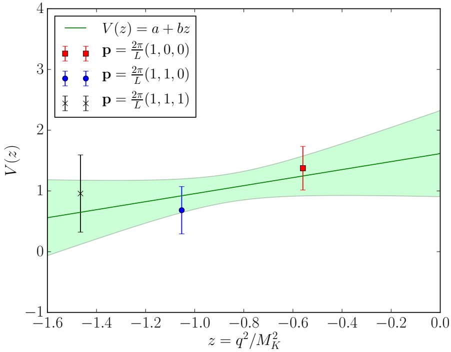
Process	Br x 10^8	*a*	*b*	*b/a*
$K^+ \to \pi^+ e^+ e^-$	31.4 ± 1.0	-0.578 ± 0.016	-0.779 ± 0.066	~ 1.35
$K^+ \to \pi^+ \mu^+ \mu^-$	9.62 ± 0.25	-0.575 ± 0.039	-0.813 ± 0.145	~ 1.41

Use $24^3 \times 64$ ensemble [RBC-UKQCD, PRD94 (2016) 114516]

$$1/a = 1.78\ \text{GeV},\ m_\pi = 430\ \text{MeV},\ m_K = 625\ \text{MeV},\ m_c = 530\ \text{MeV}$$

Momentum dependence of $V_+(z)$

$$V_+(z) = a_+ + b_+ z, \quad \Rightarrow \quad a_+ = 1.6(7),\ b_+ = 0.7(8)$$



Compare with experimental data + phenomenological analysis

Process	Br x 10^8	*a*	*b*	*b/a*
$K^+ \to \pi^+ e^+ e^-$	31.4 ± 1.0	-0.578 ± 0.016	-0.779 ± 0.066	~ 1.35
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Conclusion

- For "standard" quantities such as f_K/f_π, $f_+(0)$ B_K, lattice calculation reach the precision of $O(1\%)$ or better
- It's time to go beyond "standard"
    - $K \to \pi\pi$ and ϵ'
    - ΔM_K and ϵ_K
    - rare kaon decays: $K^+ \to \pi^+\nu\bar{\nu}$ and $K^+ \to \pi^+\ell^+\ell^-$
- Lattice QCD is now capable of first-principles calculation of the above "beyond-standard" quantities
- Realistic calculation await for the next generation of super-computers