

Scalar mesons - peculiarities and normality. What can it be?

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Genova X 2016

$$I^G J^{PC} = 0^+ 0^{++}$$

q̄

$q\bar{q}$

$q\bar{q}q\bar{q}$

gg

$q\bar{q}$

$q\bar{q}q\bar{q}$

$q\bar{q}gg$

gg

$q\bar{q}$

$q\bar{q}q\bar{q}$

MM

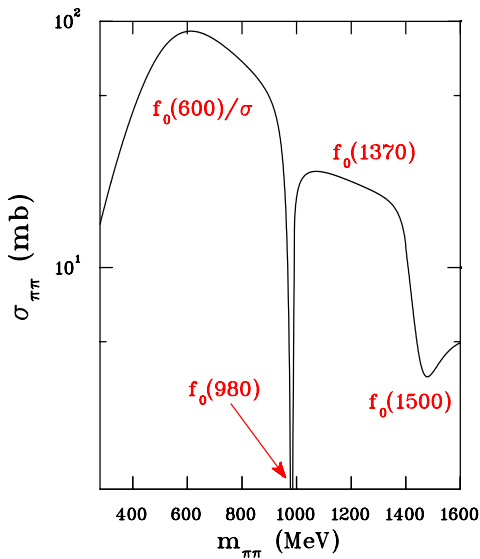
q̄qgg

gg

q̄q

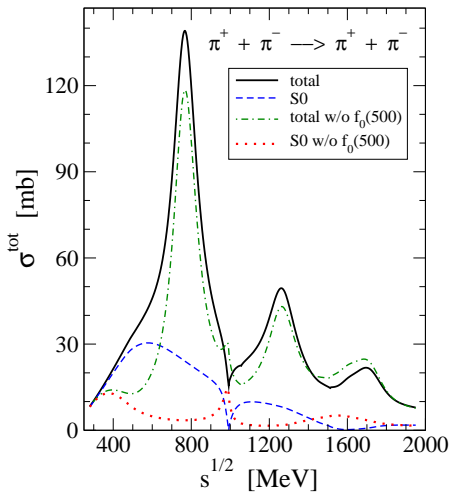
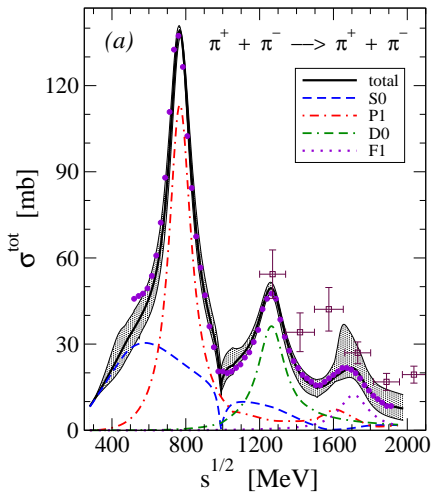
q̄qq̄

Puzzling ($J=0$) S_0 wave $\pi\pi$ cross section

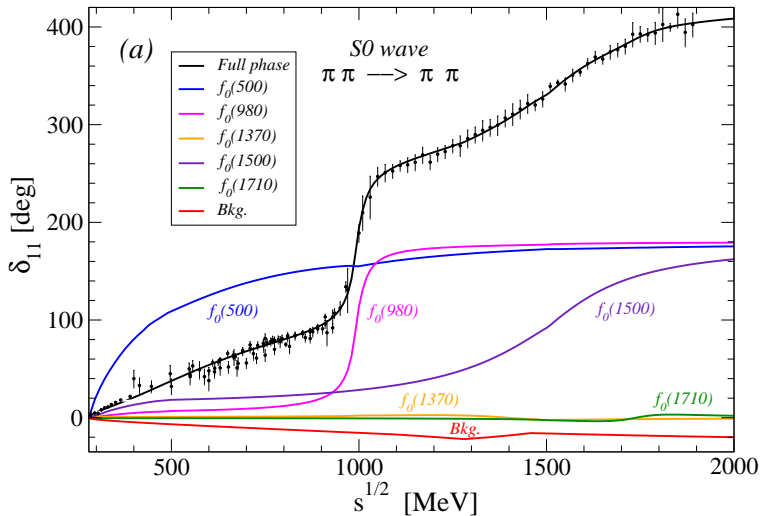


Total cross section and components

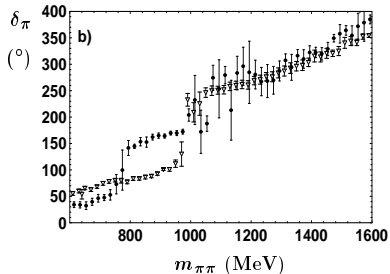
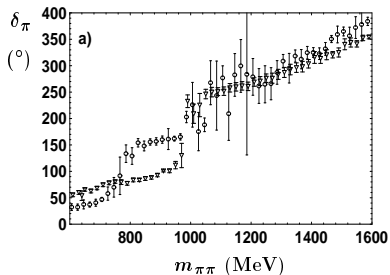
In elastic region: $\sigma \sim \text{Sin}(\delta)^2$



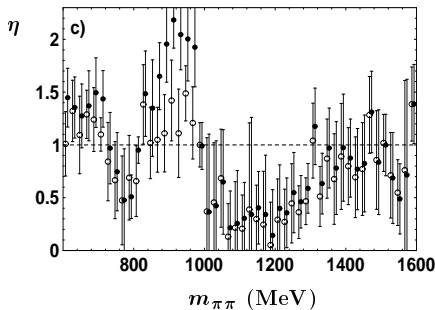
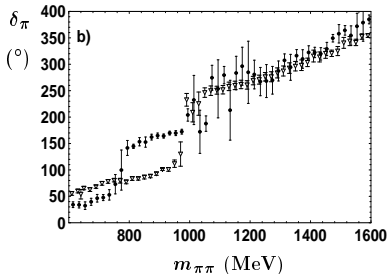
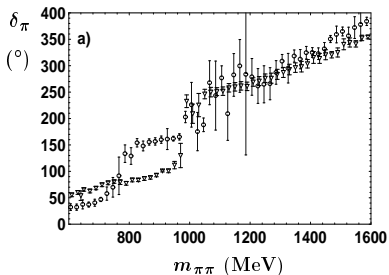
phase shifts of components in the S_0 wave



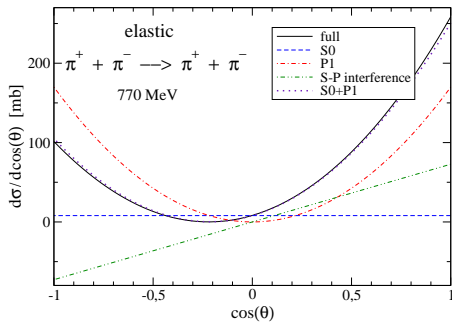
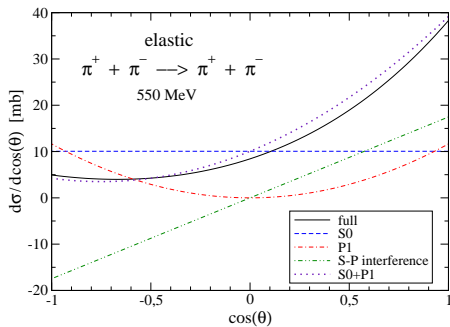
Experimental "data" for the $\pi\pi$ in the $S0$ wave



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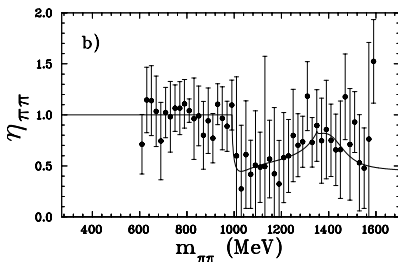
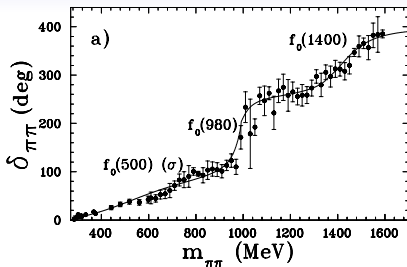


Differential cross section and components



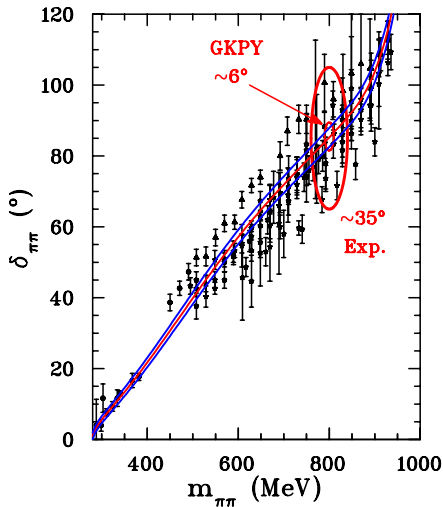
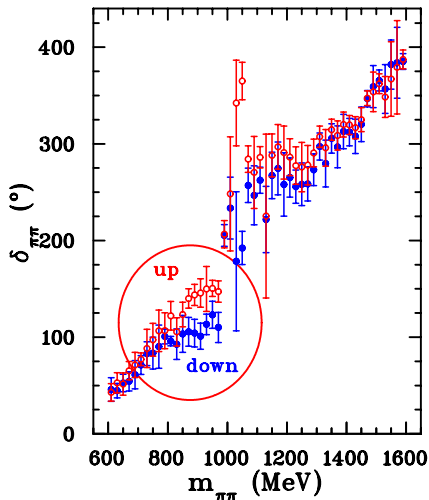
what is the meson $f_0(500)$?

- formally $f_0(500)$ (informally σ) with mass (before 2012): M : 400 – 1200 MeV and width Γ : 500 – 1000 MeV,
- the lightest scalar-isoscalar meson with $J^{G} J^{PC} = 0^{+} 0^{++}$, decays into $\pi\pi$,
- had a rich but difficult life,
- very important for e.g.
 - calculation of quark condensate mass,
 - determination of $q\bar{q} - gg$ couplings,
 - parameterization of $\pi\pi$ S wave amplitudes in e.g. many heavy meson decays (FSI)
- difficult to study



Experimental data for the $\pi\pi$ in the S_0 wave (JI)

In PWA (CERN-Munich group'74) $A(s, t) \sim \text{Cos}(\theta_S - \theta_P)$

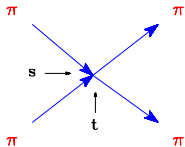


The pion-pion scattering amplitude'2008 (Roy eqs) ...



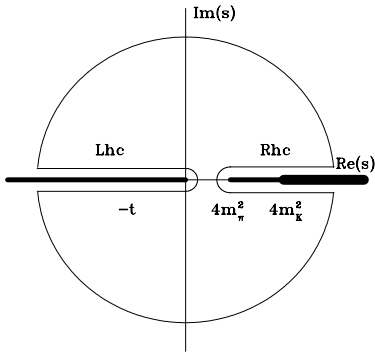
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory \longleftrightarrow experiment

crossing symmetry:



$$\rightarrow \vec{T}_s(s, t) = \hat{C}_{st} \vec{T}_t(t, s)$$

$\vec{T}(s, t)$ + crossing symmetry \rightarrow dispersion relations for $4m_\pi^2 < s < \sim (1150 \text{ MeV})^2$

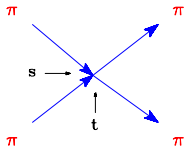


Once subtracted DR:

$$\begin{aligned} \text{Re } \vec{F}(s, t) &= \text{Re } \vec{F}(s_0, t) + \frac{s - s_0}{\pi} \\ &\times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im } \vec{F}(s', t)}{(s' - s_0)(s' - s)} \right. \\ &\left. + \int_{-t}^{-\infty} ds' \frac{\text{Im } \vec{F}(s', t)}{(s' - s_0)(s' - s)} \right] \end{aligned}$$

Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory \longleftrightarrow experiment

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$\vec{T}(s, t)$ + crossing symmetry \rightarrow dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150 \text{ MeV})^2}$

Once subtracted dispersion relations ("GKPY" for the S and P waves):

$$\text{Re } t_\ell^{l(OUT)}(s) = \sum_{l'=0}^2 C_{st}^{ll'} a_0^{l'} + \sum_{l'=0}^2 \sum_{\ell'=0}^4 \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{ll'}(s, s') \text{Im } t_{\ell'}^{l'(IN)}(s')$$

$a_0^{l'}$ - subtraction constant = $\vec{T}_s(s = 4m_\pi^2, t = 0)$ - scattering lengths from only S wave

due to $\text{Re } t_\ell^l(k) = k^{2\ell}(a_\ell^l + b_\ell^l k^2 + O(k^4))$

$$\underline{\text{Re } t_\ell^{l(OUT)}(s) - \text{Re } t_\ell^{l(IN)}(s) \rightarrow 0}$$

GKPY equations and $\pi\pi$ amplitudes

partial waves: Jl

experiment

F1

D2

S0

D0

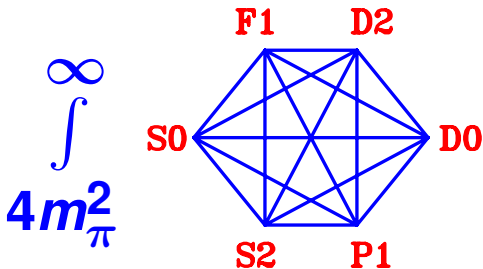
S2

P1

GKPY equations and $\pi\pi$ amplitudes

partial waves: Jl

experiment + theory (GKPY)



$\pi\pi$ amplitudes - 6 partial waves: $S_0, S_2, P_1, D_0, D_2, F_1$

Spin-isospin configuration: $J + I$ always even (Bose symmetry)

Used for GPKY equations: Madrid-Kraków group for the S_0 wave (PRD'2011):

$$\cot \delta_0^{(0)}(s) = \frac{s^{1/2}}{2k} \frac{M_\pi^2}{s - \frac{1}{2}z_0^2} \times \left\{ \frac{z_0^2}{M_\pi \sqrt{s}} + B_0 + B_1 w(s) + B_2 w(s)^2 + B_3 w(s)^3 \right\}.$$

$$w(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}, \quad s_0 = 4M_K^2 \sim (992 \text{ MeV})^2.$$

$$\delta_0^{(0)}(s) = \begin{cases} d_0 \left(1 - \frac{|k_2|}{k_M}\right)^2 + \delta_M \frac{|k_2|}{k_M} \left(2 - \frac{|k_2|}{k_M}\right) + |k_2|(k_M - |k_2|) \left(8\delta'_M + c \frac{(k_M - |k_2|)}{M_K^3}\right), \\ d_0 + B \frac{k_2^2}{M_K^2} + C \frac{k_2^4}{M_K^4} + D \theta(s - 4M_\eta^2) \frac{k_2^2}{M_\eta^2}, \end{cases} \quad (1)$$

Below and above 850 MeV (up to 1420 MeV - above - Regge)

Totally for all 6 waves: 52 free parameters

$\pi\pi$ amplitudes - 6 partial waves: $S_0, S_2, P_1, D_0, D_2, F_1$

Used in Dubna-Kraków-Prague group (PRD'2014):

$$S_{\pi\pi} = \frac{d_{res}(-k_\pi, k_2, \dots)}{d_{res}(k_\pi, k_2, \dots)}, \quad d_{res} = \prod_r \left[M_r^2 - s - i \sum_{j=1}^N \rho_{rj}^{2J+1} R_{rj} f_{rj}^2 \theta(s - s_j) \right], \quad (2)$$

Totally \sim 60 parameters

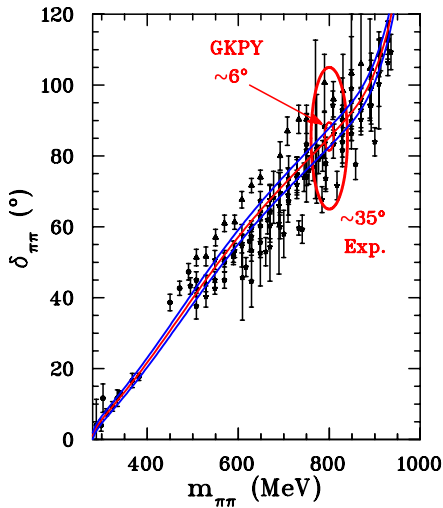
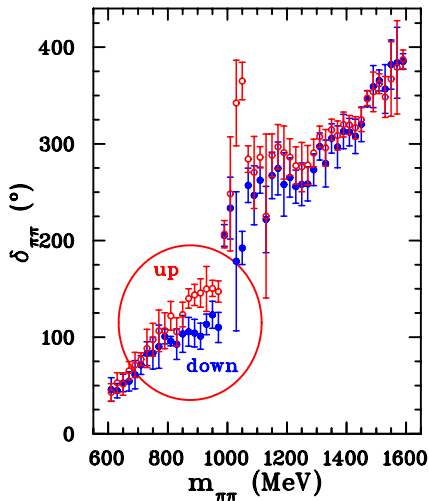
$$\chi_{Data}^2(k) = \sum_{i=1}^{N_\delta^k} \frac{(\delta_i^{exp} - \delta_i^{th})^2}{(\Delta\delta_i^{exp})^2} + \sum_{i=1}^{N_\eta^k} \frac{(\eta_i^{exp} - \eta_i^{th})^2}{(\Delta\eta_i^{exp})^2} \quad (3)$$

and

$$\chi_{DR}^2(k) = \sum_{i=1}^{N_{DR}} \frac{[\text{Re } t_\ell^{(OUT)}(s_i) - \text{Re } t_\ell^{(IN)}(s_i)]^2}{[\Delta\text{Re } t_\ell^{(OUT)}(s_i)]^2}, \quad (4)$$

Experimental data for the $\pi\pi$ in the S_0 wave (JI)

In PWA (CERN-Munich group'74) $A(s, t) \sim \text{Cos}(\theta_S - \theta_P)$



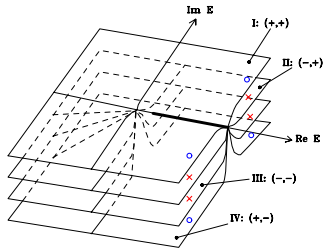
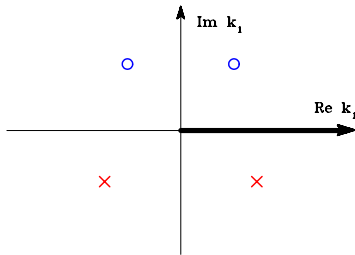
GKPY equations:

$$\operatorname{Re} t_{\ell}^{I(OUT)}(s) = \sum_{\ell'=0}^2 C^{\ell\ell'} t_0^{I(IN)}(4m_{\pi}^2) + \sum_{\ell'=0}^2 \sum_{\ell''=0}^4 \int_{4m_{\pi}^2}^{\infty} ds' K_{\ell\ell''}^{\ell\ell'}(s, s') \operatorname{Im} t_{\ell'}^{I(IN)}(s')$$

$$\operatorname{Re} t_{\ell}^{I(OUT)}(s) = \operatorname{Re} t_{\ell}^{I(IN)}(s)$$

and poles of the $\pi\pi$ amplitudes: 2^n Riemann sheets,

$$k_j = \pm \sqrt{(k_i^2 + m_i^2 - m_j^2)}$$



$\pi\pi$ amplitudes and the σ pole

Another group - "Bern" group:

H. Leytwyller, J. Gasser, G. Colangelo, I. Caprini: 2001-2011

FAQ: ... for sure your solution is not unique

The Role of the input in Roy's equations for pi pi scattering G. Wanders, Eur. Phys. J. C17 (2000) 323-336

In the abstract:

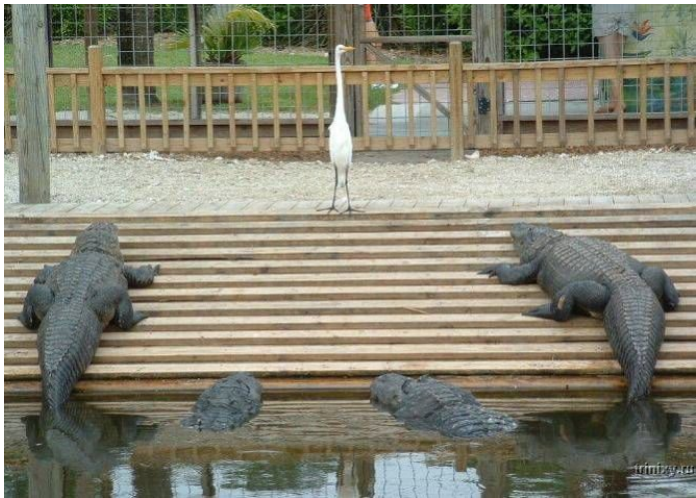
An updated survey of known results on the dimension of the manifold of solutions is presented. The solution is unique for a low energy interval with upper end at 800 MeV. We determine its response to small variations of the input: S-wave scattering lengths and absorptive parts above 800 MeV.

I.e.:

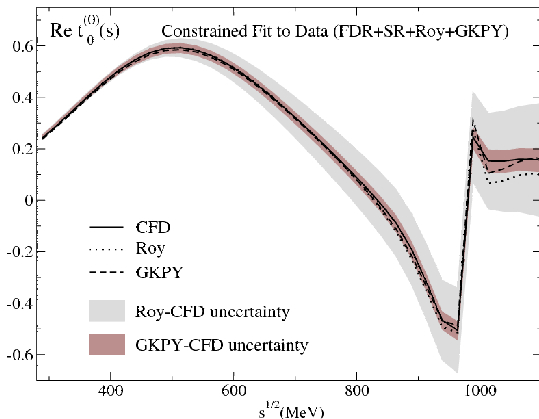
Fixed two boundary conditions for the $\pi\pi$ amplitude:

- at the threshold (S0 wave scattering length) and
- at 800 MeV

tiny error bands: common target



precision of the Roy and GKPY equations



"Precise determination of the $f_0(500)$ and $f_0(980)$ pole parameters from a dispersive data analysis",

R. Garcia-Martin, R. Kamiński, J.R. Pelaez, J. Ruiz de Elvira,

Phys. Rev. Lett. **107** (2011) 072001

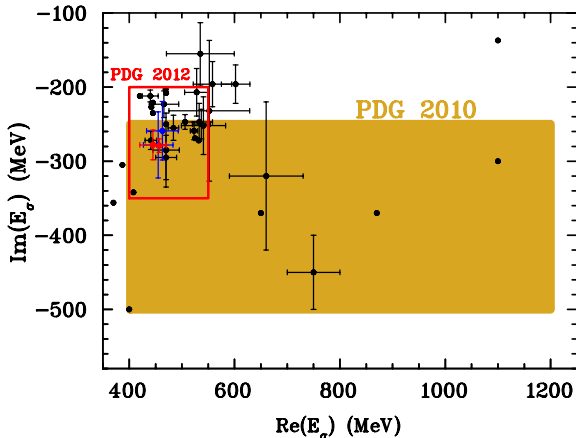
precise determination of $f_0(500)$ (σ) meson and threshold parameters

$f_0(500)$ (σ)

- PDG 2010:
 $M = 400 - 1200$ MeV
 $\Gamma = 2 \times (250 - 500)$ MeV
- PDG 2012:
 $M = 400 - 550$ MeV
 $\Gamma = 2 \times (200 - 350)$ MeV
- GKPY:
 $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV

threshold parameters, e.g. a_0^0 :

- ChPT + Roy eqs (Bern group):
 $0.220 \pm 0.005 m_\pi^{-1}$
- GKPY:
 $0.220 \pm 0.008 m_\pi^{-1}$



Before 2012

Since year 2012

Citation: C. Amsler et al. (Particle Data Group), PL **B667**, 1 (2008) and 2009 partial update for the 2010 edition (URL: <http://pdg.lbl.gov>) OR **D86**, 010001 (2012) and 2013 partial update for the 2014 edition (URL: <http://pdg.lbl.gov>) OR **D86**, 010001 (2012) and 2013 partial update for the 2014 edition (URL: <http://pdg.lbl.gov>)

$f_0(600)$
or σ

$$I^{G(J^{PC})} = 0^{++}$$

A REVIEW GOES HERE – Check our WWW

$f_0(600)$ T-MATRIX POLE \sqrt{s}

Note that $\Gamma \approx 2 \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN
(400–1200)–i(250–500) OUR ESTIMATE		
• • • We do not use the following data for averages, fits, limits, et		
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	1 CAPRINI	08 RVUE
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	2 CAPRINI	08 RVUE
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	3 ABLIKIM	07A BES2
$(466 \pm 18) - i(223 \pm 28)$	4 BONVICINI	07 CLEO
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..07	RVUE
$(441^{+16}_{-8}) - i(272^{+9}_{-12.5})$	5 CAPRINI	06 RVUE
$(470 \pm 50) - i(285 \pm 25)$	6 ZHOU	05 RVUE
$(541 \pm 39) - i(252 \pm 42)$	7 ABLIKIM	04A BES2
$(528 \pm 32) - i(207 \pm 23)$	8 GALLEGOS	04 RVUE
$(440 \pm 8) - i(212 \pm 15)$	9 PELAEZ	04A RVUE
$(533 \pm 25) - i(247 \pm 25)$	10 BUGG	03 RVUE
532 - i272	BLACK	01 RVUE
$(470 \pm 30) - i(295 \pm 20)$	5 COLANGELO	01 RVUE

• • • We do not use the following data for averages, fits, limits, et

VALUE (MeV)	DOCUMENT ID	TECN
(400–550)–i(200–350) OUR ESTIMATE		
• • • We do not use the following data for averages, fits, limits, et		
$(440 \pm 10) - i(238 \pm 10)$	1 ALBALADEJO	12
$(445 \pm 25) - i(278^{+22}_{-18})$	2,3 GARCIA-MAR..11	
$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	2,4 GARCIA-MAR..11	
$(442^{+5}_{-8}) - i(274^{+6}_{-5})$	5 MOUSSALLAM11	
$(452 \pm 13) - i(259 \pm 16)$	6 MENNESSIER	10
$(448 \pm 43) - i(266 \pm 43)$	7 MENNESSIER	10
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	8 CAPRINI	08
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	9 CAPRINI	08
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	10 ABLIKIM	07
$(466 \pm 18) - i(223 \pm 28)$	11 BONVICINI	07
$(472 \pm 30) - i(271 \pm 30)$	12 BUGG	07
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..07	

$f_0(500)$ or σ
was $f_0(600)$

$$I^{G(J^{PC})}$$

A REVIEW GOES HERE – Check our

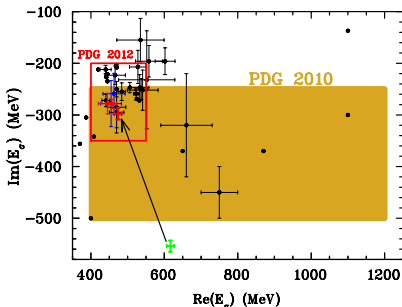
$f_0(500)$ T-MATRIX POLE

Note that $\Gamma \approx 2 \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID
(400–550)–i(200–350) OUR ESTIMATE	
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$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	9 CAPRINI 08
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	10 ABLIKIM 07
$(466 \pm 18) - i(223 \pm 28)$	11 BONVICINI 07
$(472 \pm 30) - i(271 \pm 30)$	12 BUGG 07
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..07

what forces GKPY eqs to pull up-left the sigma pole?



Two things: **trigonometry** and **crossing symmetry algebra** lead to narrower and lighter σ .

Modified $\pi\pi$ amplitude with σ pole PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

What really σ can be?

JRP printed on May 8, 2014

1

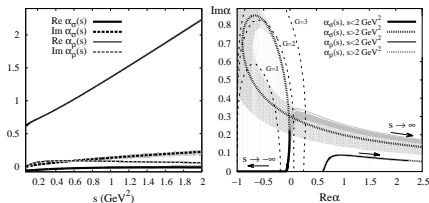


Fig. 1. (Left) $\alpha_\rho(s)$ and $\alpha_\sigma(s)$ Regge trajectories, from our constrained Regge-pole amplitudes. (Right) $\alpha_\sigma(s)$ and $\alpha_\rho(s)$ in the complex plane. At low and intermediate energies (σ thick continuous lines), the trajectory of the σ is similar to those of Yukawa potentials $V(r) = -Ga \exp(-r/a)/r$ [8] (thin dashed lines). Beyond 2 GeV^2 we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

Furthermore, in Fig. 1 we show the striking similarities between the $f_0(500)$ trajectory and those of Yukawa potentials in non-relativistic scattering [8]. From the Yukawa $G=2$ curve in that plot, which lies closest to our result for the $f_0(500)$, we can estimate $a \simeq 0.5 \text{ GeV}^{-1}$, following [8]. This could be compared, for instance, to the S-wave $\pi\pi$ scattering length $\simeq 1.6 \text{ GeV}^{-1}$. Thus it seems that the range of a Yukawa potential that would mimic our low energy results is comparable but smaller than the $\pi\pi$ scattering length in the scalar isoscalar channel. Of course, our results are most reliable at low energies (thick continuous line) and the extrapolation should be interpreted cautiously. Nevertheless, our results suggest that the $f_0(500)$ looks more like a low-energy resonance of a short range potential, *e.g.* between pions, than a bound state of a confining force between a quark and an antiquark.

In summary, our formalism and the results for the $f_0(500)$ explains why the lightest scalar meson has to be excluded from the ordinary linear Regge fits of ordinary mesons.

"Identification of non-ordinary mesons from the dispersive connection between their poles and their Regge trajectories: The $f_0(500)$ resonance"

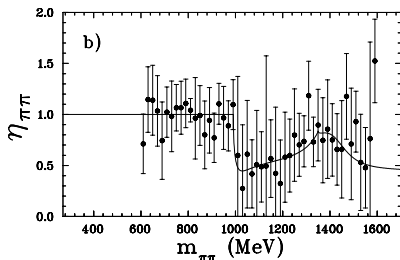
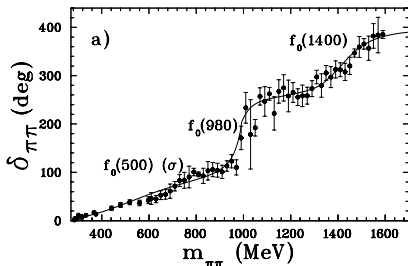
by

J. R. Pelaez, J. T. Londergan,
J. Nebreda and A. Szczepaniak

Phys.Lett. B729 (2014) 9-14

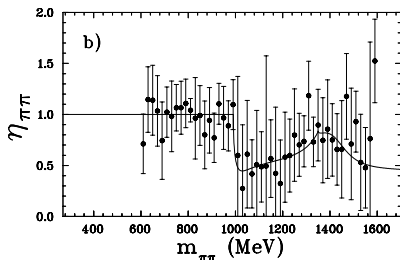
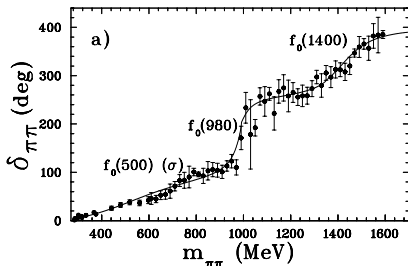
All $0^{+}0^{++}$ resonances

- $f_0(500)$ (σ) - now well known and for sure not pure $q\bar{q}$ state, most likely mix with $2q2\bar{q}$ (or loosely bound $\pi\pi$) state but why not gg ?
- $f_0(980)$ (mass: 990 ± 20 MeV, width: 40-100 MeV) - $K\bar{K}$ threshold: 992 MeV, so possible components: $K\bar{K}$ bound state ($Br_{KK} \approx 25\%$!), $q\bar{q}$, $2q2\bar{q}$, $2q2g$, let's look at new papers by Francesco Giacosa on a_0 states,
- $f_0(1370)$ (mass: 1200-1500 MeV, width: 200-500 MeV) - most probably ordinary $q\bar{q}$ state decaying mostly into $\pi\pi$ and 4π ,
- $f_0(1500)$ (mass: 1505 ± 6 MeV, width: 109 ± 7 MeV) - most probably ordinary $q\bar{q}$ state decaying into $\pi\pi$ (35%), 4π (50%), $K\bar{K}$ (9%) and $\eta\eta$ (5%),
- $f_0(1720)$ - possible glueball - lattice calculations and series of works by FG



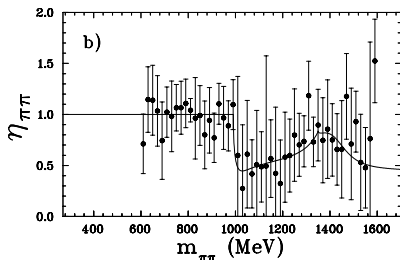
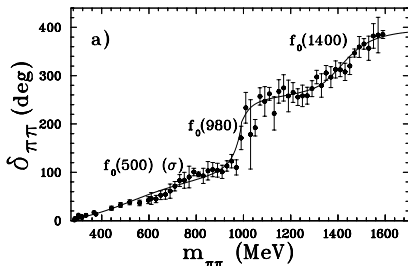
All $0^{+}0^{++}$ resonances

- $f_0(500)$ (σ) - now well known and for sure not pure $q\bar{q}$ state, most likely mix with $2q2\bar{q}$ (or loosely bound $\pi\pi$) state but why not gg ?
- $f_0(980)$ (mass: 990 ± 20 MeV, width: 40-100 MeV) - $K\bar{K}$ threshold: 992 MeV, so possible components: $K\bar{K}$ bound state ($Br_{KK} \approx 25\%$!), $q\bar{q}$, $2q2\bar{q}$, $2q2g$, let's look at new papers by Francesco Giacosa on a_0 states,
- $f_0(1370)$ (mass: 1200-1500 MeV, width: 200-500 MeV) - most probably ordinary $q\bar{q}$ state decaying mostly into $\pi\pi$ and 4π ,
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- $f_0(1720)$ - possible glueball - lattice calculations and series of works by FG



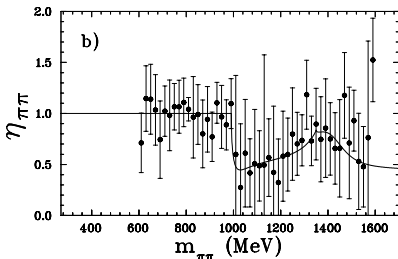
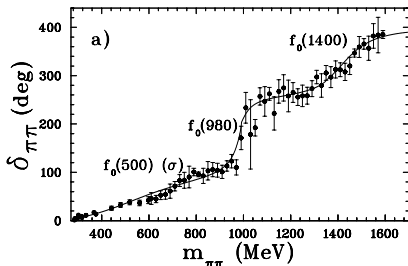
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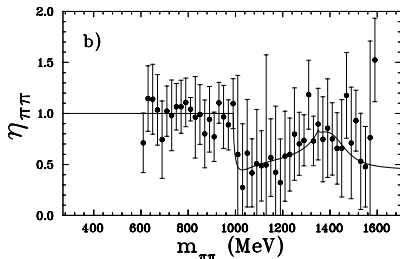
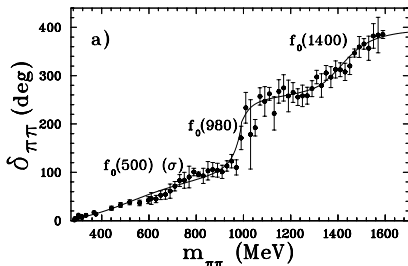
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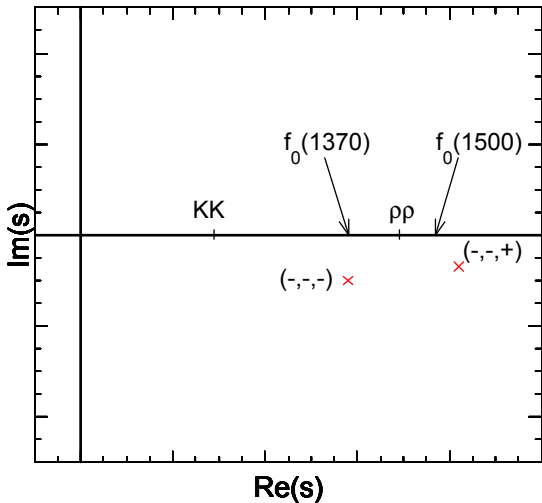


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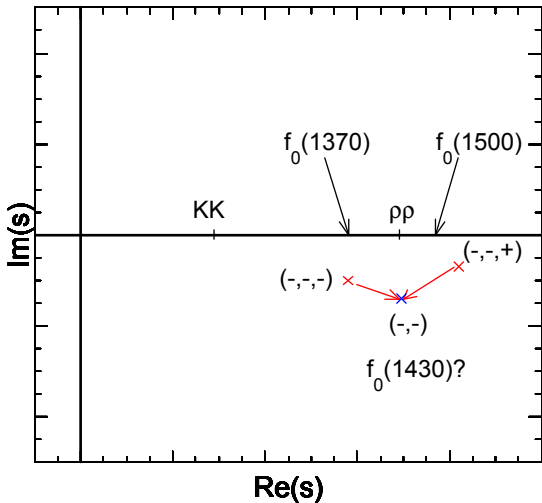
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$f_0(1370)$ and $f_0(1500)$: positions of poles, $\lambda = 1$



$f_0(1370)$ and $f_0(1500)$: positions of poles, $\lambda = 0$



$f_0(980)$: scalar - isovector 1^-0^{++} section - a_0 mesons

Works by Francesco Giacosa (now Kielce University)

" $a_0(980)$ revisited", Phys. Rev. D93 (2016), 014002

and proceedings:

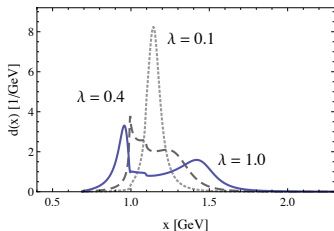
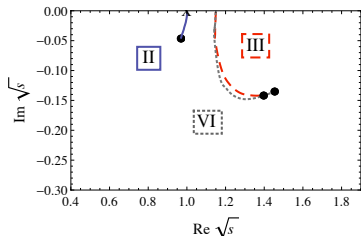
" $a_0(980)$ as a companion pole of $a_0(1450)$ "

issue:

coupling of the heavy scalar-isovector **seed $q\bar{q}$ state $a_0(1450)$** to $\pi\eta$, $K\bar{K}$ and $\pi\eta'$ is capable to **dynamically generate** the light state $a_0(980)$.

$$\Delta^{-1}(\mathbf{s}) = \mathbf{s} - m_0 - \Pi(\mathbf{s}),$$

where m_0 : "bare mass" of the $a_0(1450)$ and $\Pi(\mathbf{s}) = \sum_i \Pi_i(\mathbf{s})$ - "self energy"

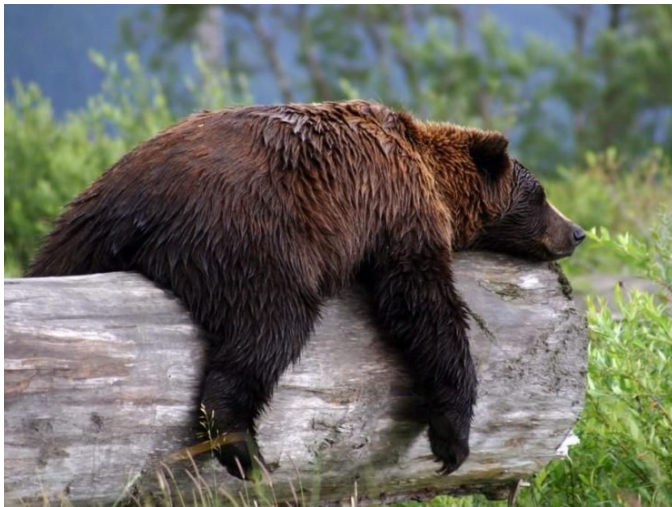


Spectral
function

We were checking all details and ...



... we have gotten tired so ...



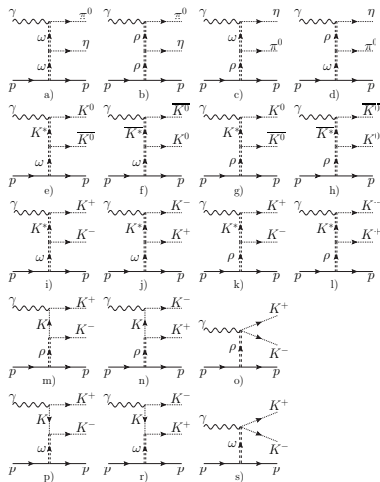
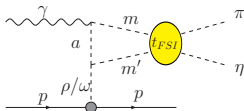
some photo-processes

with, for example, $\pi\eta$ and $K\bar{K}$ coupled channels in FS (exotic $\pi_1(1400) 1^{-+}$)

Let's a , m and m' correspond to mesons exchanged in the intermediate state

FSI - final state interactions taken from various analyses. The best are - with one type of amplitude and with wide energy range (threshold - about 2 GeV).

Amplitudes must be analytic and unitary (crossing symmetry too!)



Conclusions

Experimental situation in the $0^{+}0^{++}$ sector is finally clear (although without new experiments!),

- $f_0(500)$ (σ) - loosely bound $\pi\pi$ system or/and $2q2\bar{q}$ state,
- $f_0(980)$ - dynamically generated $2q2\bar{q}$ state or/and $K\bar{K}$ quasi bound state,
- $f_0(1370)$ - ordinary $q\bar{q}$ state,
- $f_0(1500)$ - ordinary $q\bar{q}$ state + glueball,
- $f_0(1720)$ - glueball

Probably $K_0^*(800)$ (κ) is also dynamical effect of coupling of $K_0^*(1430)$