Investigation on the spiral inflector and central region of the IsoDAR test-bench cyclotron

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for IsoDAR / DAEδALUS collaboration
IsoDAR project
(ISOtope Decay-At-Rest)

- Goal
- Experimental set-up
- Requirements
- Injection test and test-bench cyclotron
IsoDAR (ISOtope Decay-At-Rest)

**Goal**: look for the existence of *sterile neutrinos*

New physics beyond the Standard Model?

- Proton beam into neutron-producing target: \( p + ^9\text{Be} \rightarrow ^8\text{Li} + 2p + n \)
- Secondary neutrons into \( \sim 50 \) kg pure \(^7\text{Li}\) blanket: \(^7\text{Li} + n \rightarrow ^8\text{Li}\)
- \(^8\text{Li}\) decay produces \( \bar{\nu}_e \) with \( \langle E_{\bar{\nu}_e} \rangle = 6.4 \text{ MeV} \): \( ^8\text{Li} \rightarrow ^8\text{Be} + e^- + \bar{\nu}_e \)
- \( \bar{\nu}_e \) detected by inverse beta decay (IBD) in liquid scintillator-based detector: \( \bar{\nu}_e + p \rightarrow e^+ + n \)
IsoDAR requirements

Experiment decisive in 5 years ($\approx 8 \cdot 10^5$ IBD events) \rightarrow High electronic antineutrinos flux \rightarrow High proton beam intensity

IsoDAR requires 10 mA proton beam at 60 MeV

How to obtain the beam?

Cyclotron able to deliver a 5 mA $H_2^+$ beam at 60 AMeV:

- 4 sectors
- Harmonic four
- Extraction through electrostatic deflector
- Stripping of the $H_2^+$ ions in protons outside the machine

Courtesy of Daniela Campo
**IsoDAR test-bench cyclotron $H_2^+$ 1 AMeV**

**Main critical issue:** beam injection from the ion source into the cyclotron due to the **high beam current** and correlated serious **space charge effects**

![IsoDAR cyclotron diagram]

**Main parts:**
- Ion source (*Multicusp source*)
- Focusing lense
- Vacuum pump
- RFQ
- Spiral inflector
- Central region

First design of the test-bench cyclotron already realized at INFN-LNS

Study of the spiral inflector and central region in progress in collaboration with IBA
Possible injection test at AIMA

IsoDAR test-bench cyclotron with the AIMA return yoke and coils

The use of the AIMA cyclotron would allow to reduce the costs!
Design of the spiral inflector and central region of the IsoDAR test-bench cyclotron

- Goals
- Constraints
- Calculation code used
- Method used
Goals & constraints

Goals:

• Good horizontal and vertical beam centering
• Vertical focusing
• Good longitudinal acceptance
• Minimization of the losses
• Orbit turn separation

Very important to avoid losses on the electrostatic deflector!

Input parameters considered during the study: see table

<table>
<thead>
<tr>
<th>Ion</th>
<th>( H_2^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current</td>
<td>5 mA</td>
</tr>
<tr>
<td>Injection energy</td>
<td>35 AkeV</td>
</tr>
<tr>
<td>RF cavities</td>
<td>4</td>
</tr>
<tr>
<td>RF frequency</td>
<td>32.8 MHz (Harmonic 4)</td>
</tr>
<tr>
<td>Dee voltage</td>
<td>70 kV</td>
</tr>
<tr>
<td>Dee angle</td>
<td>36°</td>
</tr>
</tbody>
</table>

Constraints related to the magnetic structure (LNS cyclotron design):

• Magnetic field level and shape in the cyclotron centre
• Geometrical space available for the central region and spiral inflector
Calculation code AOC

“AOC” (Advanced Orbit Calculations), IBA code for the particle orbit tracking

We used OPERA3D to realize the 3D design and to obtain the field maps to import in AOC.

AOC tracks accelerator orbits under the combined action of static magnetic and RF electric fields, using time as independent variable.

Space charge effects have been included in our simulations. A particle-to-particle method is used (self-field acting on one particle is obtained as the sum of contributions of all other particles).
Stepwise approach used to design the central region and the spiral inflector

**Step 1**: Initially decouple between central region and spiral inflector to find the matching point.

The **matching point** is the particle starting position that allow to achieve the best orbit centering looking at the forward tracked particle orbit in the central region.

**Step 2**: Design the centroid curve of the spiral inflector intercepting well the matching point.

**Step 3**: Make a full 3D electric/magnetic model (inflector + c.r.)

**Step 4**: Fine tuning of the 3D geometry of the spiral inflector

**Step 5**: Verify the beam dynamics, also considering space charge calculations.
Results of the simulations
Central region

The smallest vertical gap has been chosen taking into account the presence of the transit time factor effects on the beam dynamics especially in the first turn.
The spiral inflector

- **Inflector parameters:**
  - Bending radius $A_0$: 50 mm
  - $L=79.74$ mm
  - Tilt-parameter $k'$: -1
  - Rotation angle $\alpha$: -150.86°
  - Constant gap: 14 mm
  - **Voltage:** ± 18.88 kV
  - Aspect ratio: 2.5

- **Collimator parameters:**
  - Circular shape
  - Internal radius: 6 mm
  - Thickness: 16 mm
  - Distance from SI: 10 mm

1) Is this value high? 2) Could it produce electrical discharge problems?

The collimator reduces the fringing field effects at the inflector entrance.
Features of the central orbit in the full model

At the inflector exit, the central orbit is onto the median plane and its vertical momentum is near to zero. The centroid curve is well centered with respect to the inflector electrodes.

The injection system model (spiral inflector + central region) allows to obtain a good orbit centering and a small vertical excursion of the reference trajectory in the central region.

**Red dots**: particle position when $V_{\text{dee}}=0$

**Green dots**: particle position when $V_{\text{dee}}=V_{\text{max}}$
Features of the central orbit in the full model

At the inflector exit, the central orbit is onto the median plane and its vertical momentum is near to zero. The centroid curve is well centered with respect to the inflector electrodes.

The injection system model (spiral inflector + central region) allows to obtain a good orbit centering and a small vertical excursion of the reference trajectory in the central region.
What happens when we inject a beam in the designed injection system?
Verify the beam dynamics

Track an emittance

Round beam => same for x and y

Uniform distribution in the spaces x-x' and y-y'

Starting position: 100 mm from the median plane

Space charge effect included in the simulations (<I>=5 mA)

Different bunch lengths: 30°, 20°, 10°, 5°

<table>
<thead>
<tr>
<th>Number of particles</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting energy</td>
<td>70 keV</td>
</tr>
<tr>
<td>Energy spread</td>
<td>±2 %</td>
</tr>
<tr>
<td>x, y</td>
<td>5 mm</td>
</tr>
<tr>
<td>Norm. Emittance in the planes x-x' and y-y'</td>
<td>1 π mm mrad</td>
</tr>
<tr>
<td>Geometrical emittance</td>
<td>115 π mm mrad</td>
</tr>
<tr>
<td>βx, βy</td>
<td>0.217 mm</td>
</tr>
<tr>
<td>αx, αy</td>
<td>2.065</td>
</tr>
</tbody>
</table>

Spatial distribution x-y

Emittance in the x-x' space

<table>
<thead>
<tr>
<th>Bunch length (deg)</th>
<th>Bunch length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6.6</td>
</tr>
<tr>
<td>20</td>
<td>4.4</td>
</tr>
<tr>
<td>10</td>
<td>2.2</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Bunch during its motion through the spiral inflector and in the central region

For simplicity, not all particles of the bunch are showed in the figure
Verify the beam dynamics

Track an emittance

Losses on the exit housing and in the first turn

No orbit turn separation

This occurs in presence of the space charge effects and for all bunch lengths considered and also in absence of the space charge effects.

Work in progress
### Transmission and losses evaluation

Starting position: 100 mm from the median plane

<table>
<thead>
<tr>
<th>Current (mA), Angle (deg)</th>
<th>Losses (#particles and %)</th>
<th>Transmission (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=0 mA, 30 deg</td>
<td>645 (12.9)</td>
<td>87.1</td>
</tr>
<tr>
<td>I=5 mA, 30 deg</td>
<td>1667 (33.3)</td>
<td>66.7</td>
</tr>
<tr>
<td>I=5 mA, 20 deg</td>
<td>1891 (37.8)</td>
<td>62.2</td>
</tr>
<tr>
<td>I=5 mA, 10 deg</td>
<td>2216 (43.3)</td>
<td>55.7</td>
</tr>
<tr>
<td>I=5 mA, 5 deg</td>
<td>2414 (48.2)</td>
<td>51.8</td>
</tr>
</tbody>
</table>

#### Diagram 1

- **Bunch length**
- **Space charge forces**
- **Beam transverse defocusing**

#### Diagram 2

- **% particles hitting the radial probe**

#### Diagram 3

- **# particles hitting the diff radial probe**

#### Legend

- I=5 mA b. length 5 deg
- I=5 mA b. length 10 deg
- I=5 mA b. length 20 deg
- I=5 mA b. length 30 deg
- I=0 mA b. length 30 deg
RF phase dispersion - spiral inflector

The spiral inflector produces beam RF phase dispersion. It appears due to the different length paths of the ions inside the inflector with respect to the reference particle.

The beam phase dispersion depends on the position of the particles (x and y coordinates) but also on direction (x' and y') (The spiral inflector is a linear device).

No space charge effects
No bunch length (unique RF phase for all particles)
No energy spread
Starting position and beam features equal to that considered previously

Does the tilt parameter influence the beam phase dispersion??
**Conclusion and perspectives**

The design of the IsoDAR injection system poses serious problems due to the high beam current and the space charge effects non negligible.

It will be needed to improve the injection system design, looking in particular at the orbit turn separation, transmission, beam size, energy spread.

**How to do this?**

- Look at the beam motion along the vertical axis and spiral inflector to reduce the RF debunching, one of the causes of the absence of orbit turn separation
- Look at the beam motion in the central region (could the pillars help the beam dynamics?)
- Look at the beam motion in the full injection system

Space charge effects have to be included in the simulations. Simulations considering different bunch lengths, different average currents and different beam size are mandatory to understand their effect on the beam dynamics.

- Investigation on the maximum voltage applicable on the inflector electrodes to avoid electrical discharges.
Thanks for your attention
IsoDAR (ISOtope Decay-At-Rest)

$\bar{\nu}_e$ disappearance due to oscillations $\rightarrow \Delta m^2$ anomalies (1 eV)

Anomalies: short baseline accelerator neutrino oscillation experiment, short baseline reactor experiments, radioactive source experiments

Sterile neutrinos: (3+1) and (3+2) models can solve the problem

Oscillation L/E waves in IsoDAR

High statistic and good L/E resolution
The $\text{H}_2^+$ cyclotron

Critical point concerning the injection \textbf{Space charge effects}

Acceleration of $\text{H}_2^+$ molecules to produce high intensity proton beam

Generalized Perveance (Reiser’s formula)

$$K = \frac{qI}{2 \cdot \pi \cdot \varepsilon_0 \cdot m \cdot \gamma^3 \beta^3}$$

2 mA protons @ 30 keV/amu \quad Perveance \quad 5 mA $\text{H}_2^+$ @ 35 keV/amu

The perveance measures the strength of the space charge effects
The $\text{H}_2^+$ cyclotron

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{inj}}$</td>
<td>35 keV/n</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>61.7 MeV/n</td>
</tr>
<tr>
<td>$B_0$</td>
<td>1.075 T</td>
</tr>
<tr>
<td>$&lt;B&gt;$ at $R_{\text{ext}}$</td>
<td>1.166 T</td>
</tr>
<tr>
<td>$&lt;R_{\text{inj}}&gt;$</td>
<td>51.58 mm</td>
</tr>
<tr>
<td>$&lt;R_{\text{ext}}&gt;$</td>
<td>2000 mm</td>
</tr>
<tr>
<td>N. Sectors</td>
<td>4</td>
</tr>
<tr>
<td>Hill width</td>
<td>$25.5^\circ \pm 36.5^\circ$</td>
</tr>
<tr>
<td>Valley gap</td>
<td>1800 mm</td>
</tr>
<tr>
<td>Hill gap</td>
<td>100 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>6240 mm</td>
</tr>
<tr>
<td>Full height</td>
<td>2700 mm</td>
</tr>
<tr>
<td>N. Cavities</td>
<td>4</td>
</tr>
<tr>
<td>Cavities $\lambda/2$</td>
<td>Double gap</td>
</tr>
<tr>
<td>RF Harmonic</td>
<td>4$^{\text{th}}$</td>
</tr>
<tr>
<td>RF frequency</td>
<td>32.8 MHz</td>
</tr>
<tr>
<td>Acc. Voltage</td>
<td>70 ÷ 250 kV</td>
</tr>
<tr>
<td>Power cavity</td>
<td>&lt;160 kW</td>
</tr>
<tr>
<td>Coil size</td>
<td>200x250 mm$^2$</td>
</tr>
<tr>
<td>Current density</td>
<td>3.17 A/mm$^2$</td>
</tr>
</tbody>
</table>

Courtesy of Daniela Campo
MIST-1 Ion Source

- Confinement in multicusp field generated by permanent magnets
- Electrons are created through thermionic emission from filament
- Electrons are accelerated towards front plate
- Electron impact collision with molecules, atoms and ions leads to ionization
- Ions drift through extraction hole and are accelerated in extraction system → beam

Courtesy of Daniel Winklehner
RFQ direct injection

Why an RFQ?

Accelerate
- Lower energy required from ion source
- Smaller HV platform and peripherals

Separate
- Early and efficient separation of $p^+$ and $H_2^+$
- No need for additional dipole magnet

Focus
- Strong focusing, 99% transmission efficiency

Bunch
- Very high bunching efficiency (>60%)
- Better phase acceptance in cyclotron

Compact for Underground

Improved $H_2^+$ Current

Courtesy of Daniel Winklehner
Some data concerning the ion source and the RFQ

- Ion Currents (worst case): $H_2^+\colon 12\ mA$, $p$, $H_3^+\colon 5\ mA$ each

- RFQ:
  - Alignment: Vertical
  - Cyclotron Bore hole: $< 30\ cm$
  - Frequency: $32.8\ MHz$ (harmonic 4)
  - Length: $120-130\ cm$
  - Injection Energy: $15\ keV$
  - RFQ final Energy: $70\ keV$
  - Maximum Energy spread: $\pm 2\%$
  - Distance of RFQ exit to Cyclotron mid-plane: $20\ cm$
  - Bunching efficiency: $50\%$
  - Exit ion current: $7-7.5\ mA$
## IsoDAR test-bench cyclotron - Info

<table>
<thead>
<tr>
<th><strong>what</strong></th>
<th><strong>value</strong></th>
<th><strong>units</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>external radius of the yoke</td>
<td>75</td>
<td>cm</td>
</tr>
<tr>
<td>semi-height of the cyclo</td>
<td>37</td>
<td>cm</td>
</tr>
<tr>
<td>total weight</td>
<td>6.321</td>
<td>Tons</td>
</tr>
<tr>
<td>coil cross-section</td>
<td>r16xz18</td>
<td>cm²</td>
</tr>
<tr>
<td>current density</td>
<td>224</td>
<td>Amp/cm²</td>
</tr>
<tr>
<td>electrical power per coil w/out and with cooling (to get the total: this number×2)</td>
<td>5.62-6.8</td>
<td>kW</td>
</tr>
<tr>
<td>pole radius</td>
<td>37</td>
<td>cm</td>
</tr>
<tr>
<td>1 MeV radius at the center of the hill</td>
<td>27</td>
<td>cm</td>
</tr>
<tr>
<td>energy of the last closed orbit found w/ GENSPE</td>
<td>1.4</td>
<td>AMeV</td>
</tr>
<tr>
<td>internal radius of the skirt valley</td>
<td>36</td>
<td>cm</td>
</tr>
<tr>
<td>external radius of the skirt valley</td>
<td>37</td>
<td>cm</td>
</tr>
<tr>
<td>internal radius of coil</td>
<td>40</td>
<td>cm</td>
</tr>
<tr>
<td>external radius of the coil</td>
<td>56</td>
<td>cm</td>
</tr>
<tr>
<td>internal radius of the yoke</td>
<td>58</td>
<td>cm</td>
</tr>
<tr>
<td>coil distance in z</td>
<td>3.6*2</td>
<td>cm</td>
</tr>
<tr>
<td>distance between coil and iron in the z direction</td>
<td>0.8</td>
<td>cm</td>
</tr>
<tr>
<td>valley height</td>
<td>22.5x2</td>
<td>cm</td>
</tr>
<tr>
<td>volume of the semi-sector (1/16 of the model)</td>
<td>50083</td>
<td>cm³</td>
</tr>
<tr>
<td>iron specific weight</td>
<td>77300</td>
<td>N/m³</td>
</tr>
<tr>
<td>weight x volume</td>
<td>3871.42</td>
<td>N</td>
</tr>
<tr>
<td>weight of the semi-sector</td>
<td>395.04</td>
<td>kg</td>
</tr>
<tr>
<td>residual field at (0,0,57) cm</td>
<td>145</td>
<td>Gauss</td>
</tr>
<tr>
<td>residual field at (80,0,0) cm</td>
<td>&lt;5</td>
<td>Gauss</td>
</tr>
</tbody>
</table>
The cyclotron has **good vertical focusing properties**.
Phase slip per turn as radius function

Integrated phase slip as radius function

In addition, it presents a good isochronism (the integrated phase slip is < 10 deg)
IsoDAR test-bench cyclotron - Properties

Energy and magnetic field along the vertical axis

Energy gain per turn=532 keV

Energy as radius function

Energy gain per turn=532 keV
Possible injection test at AIMA - Costs

- Coil construction (30 k€)
- Power supply for the coil (30 k€)
- Hydraulic piston and crane (100 k€)
- Return yoke iron (60-80 k€)
- Support pillars, cables (15 k€)
- Pumping system (30 k€)
- Cooling system for the cyclotron (20 k€)
- Mapping system (15 k€)

Total 300-320 k€ (Saved)

- Cyclotron poles machining (30 k€)
- Axial hole drilling (15 k€)
- Vacuum chamber (20-30 k€)
- Labor cost for field mapping and shimming (5 k€)
- 4 RF cavities (160 k€)
- RF amplifier for the cavities (160 k€)
- Ancillary costs (30 k€)

Total 435 k€
Possible injection test at AIMA - Images

Distance between the median plane and the bottom/top return yoke is 26.5 mm in AIMA cyclotron, in the LNS design is 22.5 mm. New poles design

Cyclotron support and bottom space to install the RFQ and ion source

The 12 T crane of the accelerator vault
Calculations code AOC

“AOC” (Advanced Orbit Calculations), the IBA code for the particle orbit tracking

AOC tracks accelerator orbits under the combined action of static magnetic and RF electric fields, using time as independent variable.
OPERA3D can be used for the 3D design of the electrodes and to obtain the 3D field maps to import in AOC

AOC allows to calculate the design orbit of a cyclotron spiral inflector considering the electric fringing field at the entrance and exit and 3D dependence of the magnetic field in the volume occupied by the inflector.

AOC is able to monitor beam losses on RF electrodes or magnetic iron. In addition, there are several tools in AOC, to process the calculated beam, as radial probes or patches.

REFERENCE: AOC, A beam dynamics design code for medical and industrial accelerators at IBA, W. Kleeven et al., Proceedings of IPAC2016, Busan, Korea
**Calculations code AOC**

<table>
<thead>
<tr>
<th>SCOPE OF APPLICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclotrons (isochronous, synchronous)</td>
</tr>
<tr>
<td>FFAG’s, Rhodotrons, PT-gantries</td>
</tr>
<tr>
<td>Static magnetic fields, RF and/or static electric fields</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EQUATIONS OF MOTION SOLVED</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-trajectories, Twiss-functions, inflector design orbits</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INTEGRATION METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time domain, Runge-Kutta 5th order, adaptive step</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPACE CHARGE CALCULATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully relativistic, fully 3D centroids</td>
</tr>
<tr>
<td>E- and B-self fields, iterative or single-step solution</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OTHER IMPORTANT FEATURES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclotron injection (axial, internal source)</td>
</tr>
<tr>
<td>Cyclotron extraction (stripping, ESD), Multi-threading</td>
</tr>
<tr>
<td>Detection of beam-losses, orbit back-tracking</td>
</tr>
<tr>
<td>Choice between cartesian and polar coordinate systems</td>
</tr>
<tr>
<td>Application of angular kicks, Dee-voltage ripple</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MAGNETIC AND ELECTRIC FIELD INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear or non-linear expansion from 2D B-maps</td>
</tr>
<tr>
<td>3D B-maps of multiple magnets on multiple grids</td>
</tr>
<tr>
<td>3D E-maps on multiple grids, or simplified dee-gaps</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>POST-PROCESSING OPTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam rms-analysis, radial probe tracks</td>
</tr>
<tr>
<td>patch intersections, cyclotron Smith-Garren analysis</td>
</tr>
</tbody>
</table>

REFERENCE: AOC, A beam dynamics design code for medical and industrial accelerators at IBA, W. Kleeven et al., Proceedings of IPAC2016, Busan, Korea
Stepwise approach used to design the central region and the spiral inflector

**Step 1:** Initially decouple between central region and spiral inflector to find the matching point

The **matching point** is the particle starting position that allows to achieve the best orbit centering looking at the forward tracked particle orbit in the central region.

**Forward tracking** from 70 keV, corresponding to the injection energy, towards the outer radius of the cyclotron

Design of the 3D dees and dummy-dees
Method used to design the central region and the spiral inflector

Step 2: Design the centroid curve of the spiral inflector intercepting well the matching point

AOC simulates the inflector by an electric field that is always perpendicular to the orbit. The matching point is used to optimize the inflector parameters.
The spiral inflector is specified by:
- its electric bend radius $A_0$
- its length L (along the orbit)
- its tilt parameter $k'$
- its entrance height $z_0$
- its rotation $a$ around the z-axis
- width of fringe fields at the inflector entrance and exit.

The code optimizes L and $z_0$, such that the particle is injected exactly onto the median plane.
The code optimizes the angle $a$ such that the injected orbit passes through the matching point. Optimization of the injected orbit angle at the matching point is done by changing $k'$.

✓ Varying 3D magnetic field
✓ Electric fringing field at the inflector entrance and exit
Method used to design the central region and the spiral inflector

**Step 3:** Make a full 3D electric model

The real 3D inflector electrode have been designed around the estimated reference orbit using a tool in OPERA3D.

**Step 4:** Fine tuning of the 3D geometry of the spiral inflector

- Previous step 2 repeated using the more precise width of the fringing field obtained from the 3D model of the spiral inflector;
- Optimization of the inflector voltage
- Movement the spiral inflector along the vertical direction.

**Step 5:** Verify the beam dynamics in the full injection system, also considering space charge calculations
The inflector voltage that deflects the particle on the median plane is $V_{\text{infl}} = \pm 18.88 \text{ kV}$

The theoretical value is $\pm 19.6 \text{ kV}$:

$$V = E_{\text{inf}} \cdot \text{gap} = 2 \cdot \frac{V_{\text{inj}}}{A_0} \cdot \text{gap} = 2 \cdot \frac{70 \text{ kV}}{50 \text{ mm}} \cdot 14 \text{ mm}$$

$$= 39.2 \text{ kV} = 2 \cdot 19.6 \text{ kV}$$

It was needed to decrease the voltage on the electrodes to reduce the vertical momentum of the reference particle at the exit of the spiral inflector.

In addition, the spiral inflector has been moved along the vertical direction of 0.7 mm to put the reference orbit perfectly onto the median plane.
Inflector central orbit features by tracking in the 3D full model

The energy variation in the inflector is 2 keV which means that the particle follows pretty well the central orbit.

The max displacement wrt to the central orbit is $d \approx 0.8 \text{ mm}$:

$$d = gap \cdot \frac{\Delta E}{qV} = 14 \text{ mm} \cdot \frac{2 \text{ keV}}{1e*37.76 \text{ kV}} \approx 0.8 \text{ mm}$$
Accepted particle phases of the central region for the centering (< 2 mm)

The orbit center amplitude is < 2 mm for the phases included in the interval [25° - 70°]
Accepted particle phases of the central region for the CP-phase

For the phases included in the interval [25°-70°] the CP phase is < 20°
Accepted particle phases of the central region for z-stability

A variation of the inflector voltage (±0.5 kV) has been used to individuate the accepted particle phases for the z-stability.

The z oscillation is < 10 mm for the particle phases included in the interval [40°-80°].
Accepted particle phases for centering, CP phase and z-stability

Accepted particle phases by central region $\rightarrow$ [10°-80°]

Accepted particles phases for centering (< 2 mm) $\rightarrow$ [25°-70°]

Accepted particles phases for CP-phase $\rightarrow$ [25°-70°]

Accepted particles phases for z stability $\rightarrow$ [40°-80°]

Phase acceptance in degrees for centering, CP-phase and z-stability

[40°-70°]
Beam phase dispersion - spiral inflector
Beam phase dispersion – spiral inflector

The spiral inflector is a linear device

RF phase dispersion vs x coordinate

RF phase dispersion vs y coordinate