Double-Beta Decay of Medium-Mass Nuclei within the Realistic Shell Model

Luigi Coraggio

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

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- L. De Angelis (INFN-NA)
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Outline

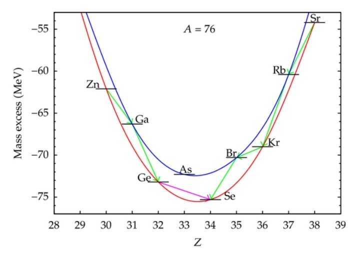
- The neutrinoless double-β decay
- The problematics of the calculation of the nuclear matrix element (NME) of $0\nu\beta\beta$ decay
- The realistic nuclear shell model
- Testing the theoretical framework: calculation of the GT strengths and the nuclear matrix element of $2\nu\beta\beta$ decay
- Outlook

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

- Its detection
  - would correspond to a violation of the conservation of the leptonic number,
  - may provide more informations on the nature of the neutrinos (the neutrino as a Majorana particle, determination of its effective mass, ..).

The double β-decay

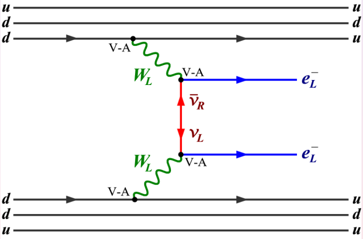
The semiempirical mass formula provides two different parabolas for even-mass isobars:



- Maria Goeppert-Mayer (1935) suggested the possibility to detect $(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e$
- Historically, G. Racah (1937) and W. Furry (1939) were the first ones, to suggest to test the neutrino as a Majorana particle, considering the process: $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$

The neutrinoless double β-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME



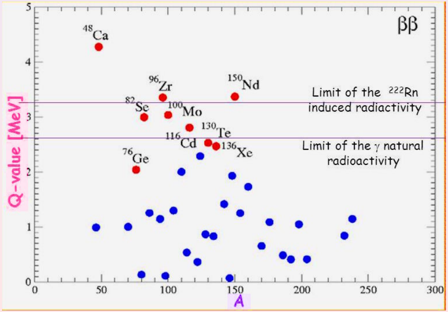
$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} \left| M^{0\nu} \right|^2 \langle m_\nu \rangle^2 \quad ,$$

- $G^{0\nu}$ is the so-called phase-space factor, obtained by integrating over she single electron energies and angles, and summing over the final-state spins;
- $\langle m_\nu \rangle = \mid \sum_k m_k U_{ek}^2 \mid$ effective mass of the Majorana neutrino, U_{ek} being the lepton mixing matrix.

The detection of the $0\nu\beta\beta$-decay

It is necessary to locate the nuclei that are the best candidates to detect the $0\nu\beta\beta$-decay

- The main factors to be taken into account are:
  - the Q-value of the reaction;
  - the phase-space factor $G^{0\nu}$;
  - the isotopic abundance



- First group: ^{76}Ge, ^{130}Te, and ^{136}Xe.
- Second group: ^{82}Se, ^{100}Mo, and ^{116}Cd.
- Third group: ^{48}Ca, ^{96}Zr, and ^{150}Nd.

Our aim is to compute the $0\nu\beta\beta$-decay NME for ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe.

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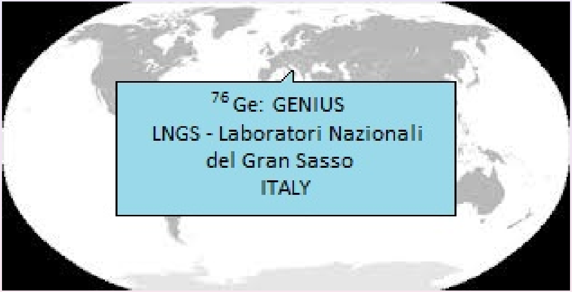

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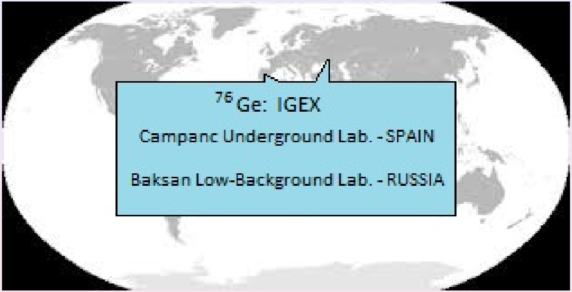

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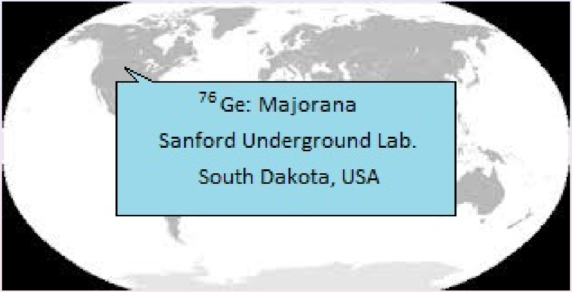

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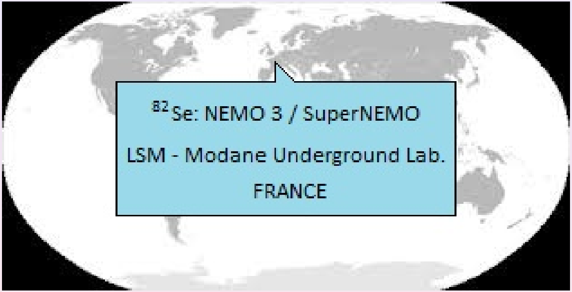

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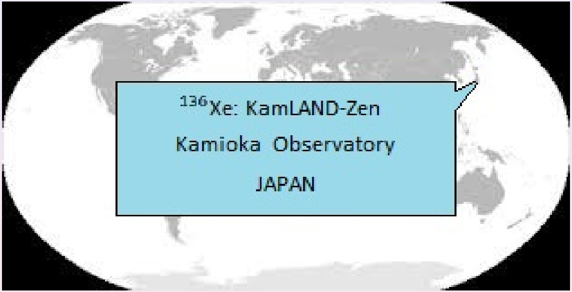


The calculation of the NME

The NME is given by

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} - M^{0\nu}_{T} \quad ,$$

where the matrix elements are defined as follows:

$$M^{0\nu}_{\alpha} = \sum_{m,n} \left\langle 0^+_f \mid \tau^-_m \tau^-_n O^{\alpha}_{mn} \mid 0^+_i \right\rangle \quad ,$$

with $\alpha = (GT,\ F,\ T)$.
Since the transition operator is a two-body one, we may write it as:

$$M^{0\nu}_{\alpha} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\left(j_p j_{p'}, j_n j_{n'}; J_\pi\right) \left\langle j_p j_{p'}; J^\pi T \mid \tau^-_1 \tau^-_2 O^{\alpha}_{12} \mid j_n j_{n'}; J^\pi T \right\rangle \quad ,$$

where the two-body transition-density matrix elements are defined as

$$TBTD\left(j_p j_{p'}, j_n j_{n'}; J_\pi\right) = \langle 0^+_f \mid (a^\dagger_{j_p} a^\dagger_{j'_p})^{J^\pi} (a_{j'_n} a_{j_n})^{J^\pi} \mid 0^+_i \rangle$$

and the Gamow-Teller (GT), Fermi (F), and tensor (T) operators as

$$\begin{aligned} O^{GT}_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \ , \\ O^{F}_{12} &= H_F(r) \ , \\ O^{T}_{12} &= \left[3\left(\vec{\sigma}_1 \cdot \hat{r}\right)\left(\vec{\sigma}_1 \cdot \hat{r}\right) - \vec{\sigma}_1 \cdot \vec{\sigma}_2\right] H_T(r) \ . \end{aligned}$$

These operators should be regularized consistently with the two-body NN potential

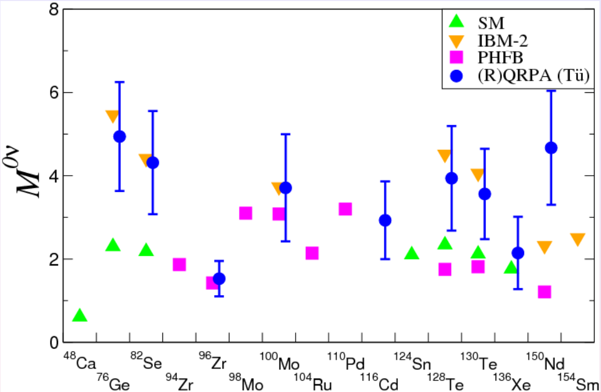
The calculation of the NME

To describe the nuclear properties detected in the experiments, one needs to resort to nuclear structure models.

- Every model is characterized by a certain number of parameters.
- The calculated value of the NME may depend upon the chosen nuclear structure model.

All models may present advantages and/or shortcomings to calculate the NME

Nuclear structure calculations



- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

The realistic shell model

- The derivation of the shell-model hamiltonian, starting from a realistic nuclear potential V_{NN} and using the many-body theory, may provide a reliable approach to the study of the $0\nu\beta\beta$ decay
- The model space may be "shaped" according to the computational needs of the diagonalization of the shell-model hamiltonian
- In such a case, the effects of the neglected degrees of freedom are taken into account by the effective hamiltonian $H_{\rm eff}$ theoretically

Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
2. Renormalize its short range correlations
3. Determine the model space better tailored to study the system under investigation
4. Derive the effective shell-model hamiltonian by way of the many-body perturbation theory
5. Calculate the physical observables (energies, e.m. transition probabilities, ...)

Parameter-free shell model?

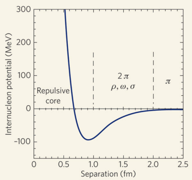
Three issues to be addressed:

1. The “realistic” nuclear potential
2. The renormalization parameters (cutoff choice)
3. The order where we arrest the perturbative expansion

Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- EFT inspired approaches
  - $V_{\rm low-k}$
  - SRG
  - chiral potentials

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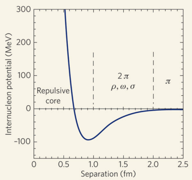

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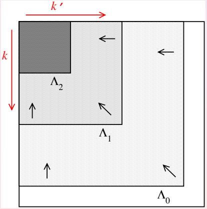

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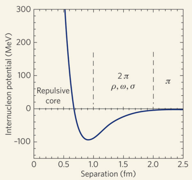

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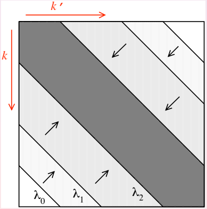

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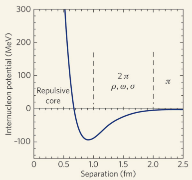

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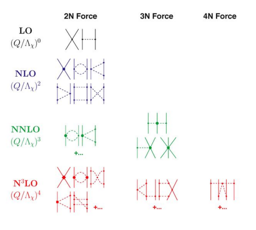


The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \underset{Q\mathcal{H}P=0}{\overset{\mathcal{H}=X^{-1}HX}{\Longrightarrow}} \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$

$$H_1^{\mathrm{eff}}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P -$$
$$-PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\mathrm{eff}}(\omega)$$

The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\rm eff}$ $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

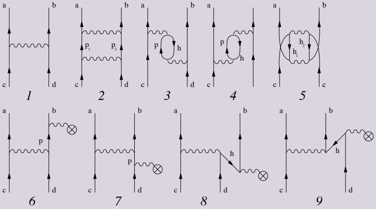
The perturbative approach to the shell-model $H^{\rm eff}$

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box



The shell-model effective operators

Consistently, any shell-model effective operator may be calculated

It has been demonstrated that, for any bare operator Θ, a non-Hermitian effective operator $\Theta_{\rm eff}$ can be written in the following form:

$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q} + \hat{Q} \hat{Q}_2 + \cdots)(\chi_0 + \\ + \chi_1 + \chi_2 + \cdots) ~,$$

where

$$\hat{Q}_m = \frac{1}{m!} \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \Big|_{\epsilon=\epsilon_0} ~,$$

ϵ_0 being the model-space eigenvalue of the unperturbed hamiltonian H_0

K. Suzuki and R. Okamoto, Prog. Theor. Phys. ***93*** *, 905 (1995)*

The shell-model effective operators

The χ_n operators are defined as follows:

$$\begin{aligned}
\chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} \quad , \\
\chi_1 &= (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.) \quad , \\
\chi_2 &= (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q} \hat{Q} + h.c.) + \\
&\quad (\hat{\Theta}_{02} \hat{Q} \hat{Q} + h.c.) + \hat{Q} \hat{\Theta}_{11} \hat{Q} \quad , \\
&\quad \cdots
\end{aligned}$$

and

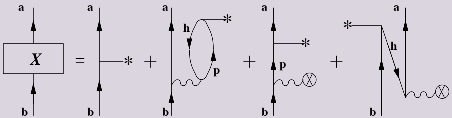
$$\begin{aligned}
\hat{\Theta}(\epsilon) &= P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P \quad , \\
\hat{\Theta}(\epsilon_1; \epsilon_2) &= P\Theta P + PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times \\
&\quad Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P \quad ,
\end{aligned}$$

$$\hat{\Theta}_m = \frac{1}{m!} \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m} \bigg|_{\epsilon=\epsilon_0} \quad , \quad \hat{\Theta}_{nm} = \frac{1}{n!\,m!} \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1; \epsilon_2) \bigg|_{\epsilon_1=\epsilon_0, \epsilon_2=\epsilon_0}$$

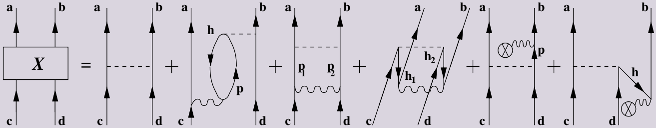
The shell-model effective operators

We arrest the χ series at the leading term χ_0, and then expand it perturbatively:

One-body operator

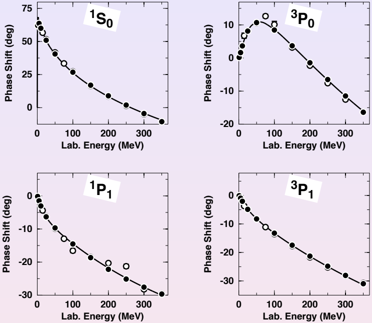


Two-body operator



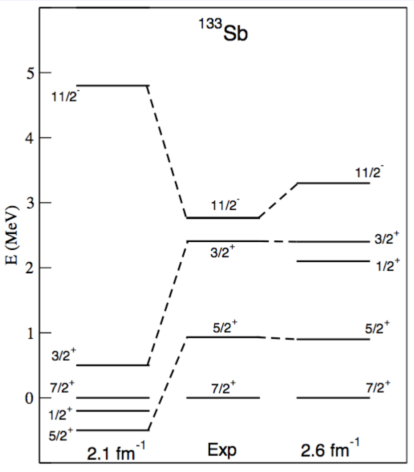
Our recipe for realistic shell model

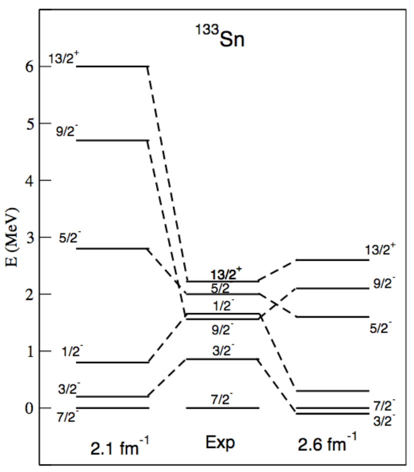
- First issue: input V_{NN}, the high-precision NN CD-Bonn potential:



- Second issue: we renormalize the input V_{NN} deriving a $V_{\rm low-k}$ with a cutoff: $\Lambda = 2.6$ fm^{-1}.
- Third issue: $H_{\rm eff}$ obtained calculating the Q-box up to the 3rd order in perturbation theory.
- Single-particle energies and two-body matrix elements are obtained from the one- and two-body components of $H_{\rm eff}$ to diagonalize the shell-model hamiltonian
- Effective operators are consistently derived by way of the MBPT

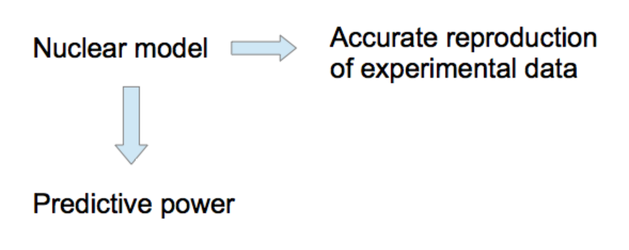
Second issue: the choice of the cutoff Λ





L. C., A. Gargano, and N. Itaco, JPS Conf. Proc. ***6****, 020046 (2015)*

Nuclear models and predictive power



Realistic shell-model calculations for ^{130}Te and ^{136}Xe

$\Downarrow$

Check this approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and compare the results with data.

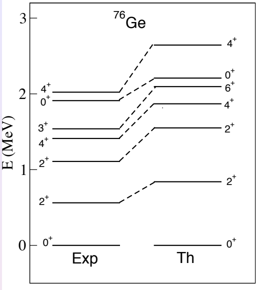
$$\left[T^{2\nu}_{1/2}\right]^{-1} = G^{2\nu}\left|M^{\mathrm{GT}}_{2\nu}\right|^2$$

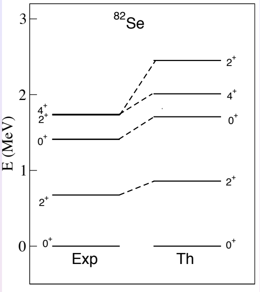
Shell-model calculations

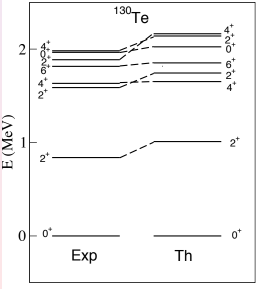
- ^{76}Ge,^{82}Se: four proton and neutron orbitals outside double-closed ^{56}Ni
  $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- ^{130}Te,^{136}Xe: five proton and neutron orbitals outside double-closed ^{100}Sn *
  $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

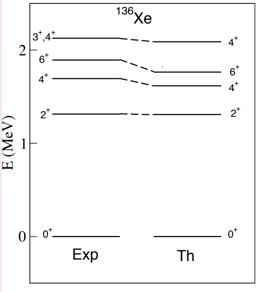
* *L.C., L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* ***95****, 064324 (2017)*

Energy spectra







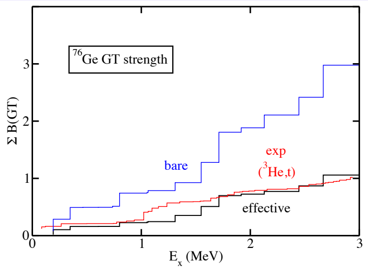


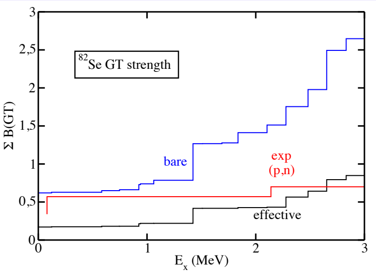
Electric quadrupole transition strengths

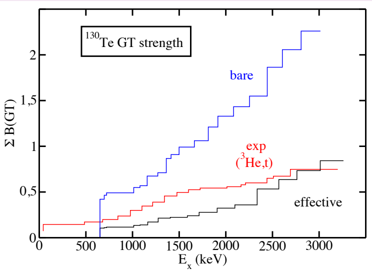
Nucleus	$J_i \rightarrow J_f$	$B(E2)_{Expt}$	$B(E2)_{Theor}$	Nucleus	$J_i \rightarrow J_f$	$B(E2)_{Expt}$	$B(E2)_{Theor}$
^{76}Ge				^{82}Se			
	$2_1^+ \rightarrow 0_1^+$	560 ± 20	540		$2_1^+ \rightarrow 0_1^+$	360 ± 6	390
	$2_2^+ \rightarrow 2_1^+$	750 ± 100	750		$0_2^+ \rightarrow 2_1^+$	78 ± 2	218
	$2_2^+ \rightarrow 0_1^+$	17.2 ± 0.6	3.6		$2_2^+ \rightarrow 2_1^+$	106 ± 3	113
	$4_1^+ \rightarrow 2_1^+$	730 ± 170	730		$2_2^+ \rightarrow 0_1^+$	30 ± 3	32
	$0_2^+ \rightarrow 2_1^+$	100 ± 40	40		$4_1^+ \rightarrow 2_1^+$	400 ± 60	550

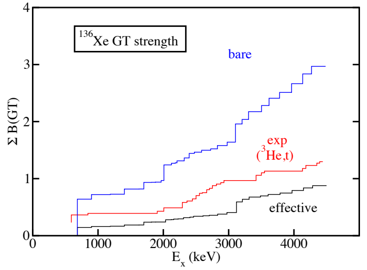
Nucleus	$J_i \rightarrow J_f$	$B(E2)_{Expt}$	$B(E2)_{Theor}$	Nucleus	$J_i \rightarrow J_f$	$B(E2)_{Expt}$	$B(E2)_{Theor}$
^{130}Te				^{136}Xe			
	$2_1^+ \rightarrow 0_1^+$	580 ± 20	430		$2_1^+ \rightarrow 0_1^+$	420 ± 20	300
	$6_1^+ \rightarrow 4_1^+$	240 ± 10	220		$4_1^+ \rightarrow 2_1^+$	53 ± 1	9
					$6_1^+ \rightarrow 4_1^+$	0.55 ± 0.02	1.58

GT$^-$ running sums

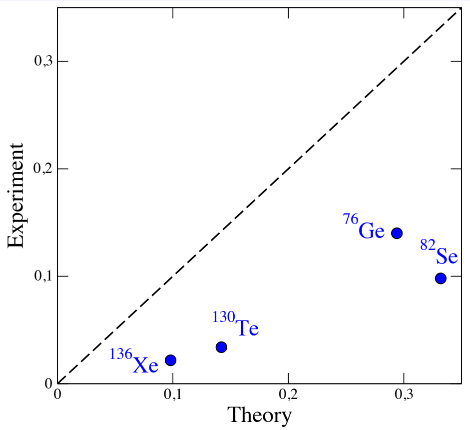








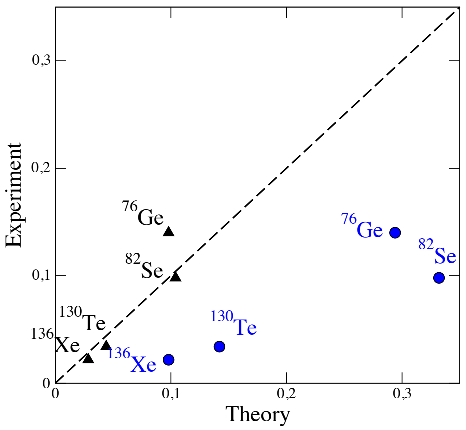
$2\nu\beta\beta$ nuclear matrix elements



Blue dots: bare GT operator

Decay	Expt.	Bare
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.294
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.332
$^{130}Te \rightarrow ^{130}Xe$	0.034 ± 0.003	0.142
$^{136}Xe \rightarrow ^{136}Ba$	0.0218 ± 0.0003	0.0975

$2\nu\beta\beta$ nuclear matrix elements



Blue dots: bare GT operator
Black triangles: effective GT operator

Decay	Expt.	Eff.
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.098
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.104
$^{130}Te \rightarrow ^{130}Xe$	0.034 ± 0.003	0.044
$^{136}Xe \rightarrow ^{136}Ba$	0.0218 ± 0.0003	0.0285

Matrix elements of the neutron-proton effective GT$^-$ operator

$n_a l_a j_a$ $n_b l_b j_b$	3rd order GT$^-_{\rm eff}$	quenching	$n_a l_a j_a$ $n_b l_b j_b$	3rd order GT$^-_{\rm eff}$	quenching
$0f_{5/2}$ $0f_{5/2}$	-0.977	0.37	$0g_{7/2}$ $0g_{7/2}$	-1.239	0.50
$0f_{5/2}$ $1p_{3/2}$	-0.143		$0g_{7/2}$ $1d_{5/2}$	-0.019	
$1p_{3/2}$ $0f_{5/2}$	0.046		$1d_{5/2}$ $0g_{7/2}$	0.131	
$1p_{3/2}$ $1p_{3/2}$	2.030	0.62	$1d_{5/2}$ $1d_{5/2}$	1.864	0.64
$1p_{3/2}$ $1p_{1/2}$	-1.621	0.55	$1d_{5/2}$ $1d_{3/2}$	-1.891	0.61
$1p_{1/2}$ $1p_{3/2}$	1.713	0.58	$1d_{3/2}$ $1d_{5/2}$	1.794	0.58
$1p_{1/2}$ $1p_{1/2}$	-0.697	0.67	$1d_{3/2}$ $1d_{3/2}$	-1.023	0.66
$0g_{9/2}$ $0g_{9/2}$	3.121	0.70	$1d_{3/2}$ $2s_{1/2}$	-0.093	
			$2s_{1/2}$ $1d_{3/2}$	0.117	
			$2s_{1/2}$ $2s_{1/2}$	1.598	0.65
			$0h_{11/2}$ $0h_{11/2}$	2.597	0.69

Third issue: perturbative properties of the effective operator

Decay	1st ord $M^{\rm GT}_{2\nu}$	2nd ord $M^{\rm GT}_{2\nu}$	3rd ord $M^{\rm GT}_{2\nu}$	Expt.
$^{130}\mathrm{Te} \rightarrow ^{130}\mathrm{Xe}$	0.142	0.042	0.044	0.034 ± 0.003
$^{136}\mathrm{Xe} \rightarrow ^{136}\mathrm{Ba}$	0.0975	0.0272	0.0285	0.0218 ± 0.0003

More than 70% from 1st $\rightarrow$ 2nd order
Less than 5% from 2nd $\rightarrow$ 3rd order

Outlook

- $2\nu\beta\beta$
  - Role of real three-body forces and two-body currents (present collaboration with Pisa group)
  - Evaluation of the contribution of many-body correlations (blocking effect)
- $0\nu\beta\beta$
  - Derivation of the two-body effective operator
  - Calculation of the two-body transition-density matrix elements (in collaboration with Frédéric Nowacki)
  - SRC calculated consistently with $V_{\mathrm{low-k}}$

$^{130}Te \rightarrow ^{130}Xe$ nuclear matrix element