Quenching of g_A in β and $\beta\beta$ Decays: A Review

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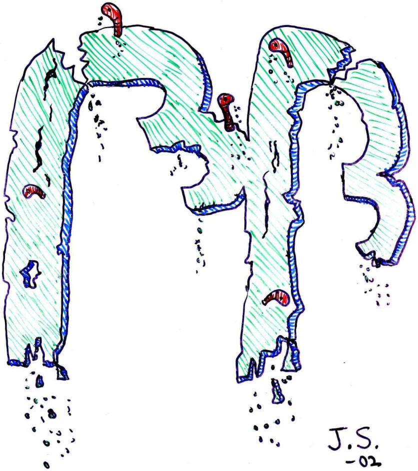
Conference on Neutrino and Nuclear Physics (CNNP2017)
Catania, Italy, Octoberber 15 - 21, 2017

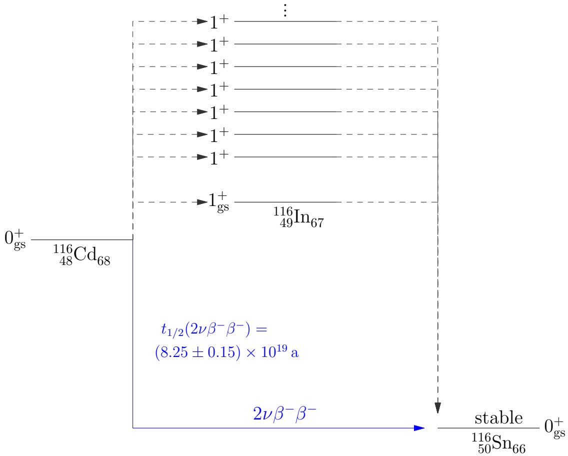
Contents:

- Incentive for g_A studies
- GT and SD β Decays
- Unique Spin-Multipole Decays
- Nonunique forbidden β Decays (spectrum-shape method)

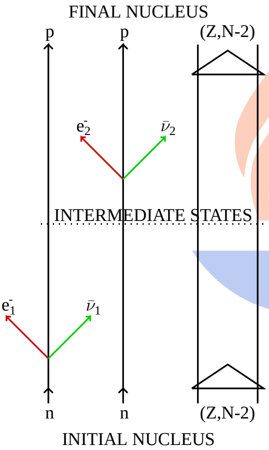
Motivation for the Work: Double Beta Decay



Two-Neutrino Double Beta Decay of ^{116}Cd

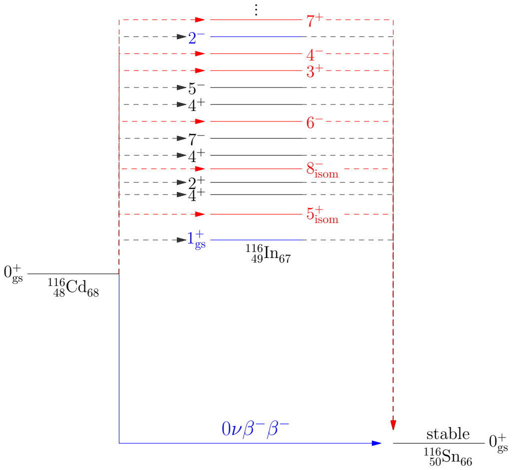


$t_{1/2}(2\nu\beta^-\beta^-) =$
$(8.25 \pm 0.15) \times 10^{19}$ a

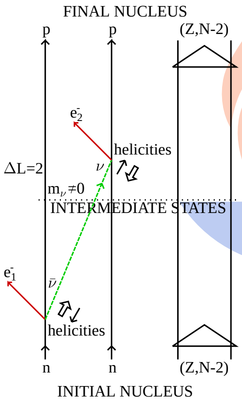


$$\boxed{2\nu\beta\beta - \text{rate} \sim \left|M^{(2\nu)}_{\mathrm{GTGT}}\right|^2 = (g_{\mathrm{A}})^4 \left|\sum\nolimits_{m,n} \frac{M_{\mathrm{L}}(1^+_m)M_{\mathrm{R}}(1^+_n)}{D_m}\right|^2}$$

Neutrinoless Double Beta Decay of ^{116}Cd



MASS MODE



$$\boxed{0\nu\beta\beta - \text{rate} \sim \left| M^{(0\nu)}_{\rm GTGT} \right|^2 = (g_{{\rm A},0\nu})^4 \left| \sum_{J^\pi} (0^+_f || \mathcal{O}^{(0\nu)}_{\rm GTGT}(J^\pi) || 0^+_i) \right|^2}$$

Definitions

The talk is based on "Value of the axial-vector coupling strength in β and $\beta\beta$ decays: A review" to appear in **Frontiers in Physics**.

Quenching:

$$q = g_{\mathrm{A}}/g_{\mathrm{A}}^{\mathrm{free}}$$

Free value of g_{A} (Particle Data Group 2016):

$$g_{\mathrm{A}}^{\mathrm{free}} = 1.2723(23)$$

Effective value of g_{A}:

$$g_{\mathrm{A}}^{\mathrm{eff}} = q g_{\mathrm{A}}^{\mathrm{free}}$$

Gamow-Teller β decays

There are data on:

Gamow-Teller β

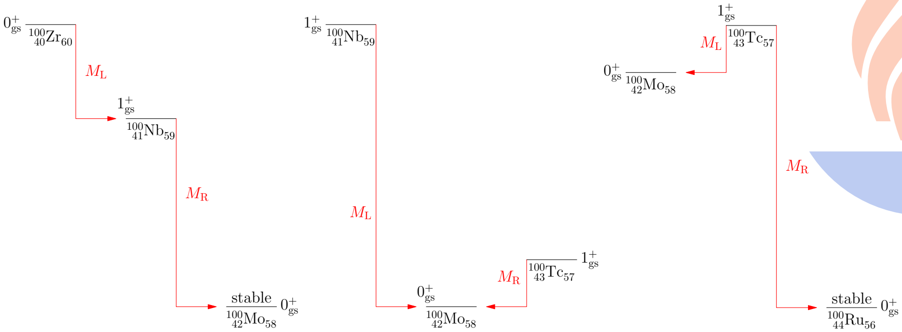
TRANSITIONS

Theoretical approaches:

ISM (Interacting Shell Model)

pnQRPA (proton-neutron QRPA)

Typical Gamow-Teller β transitions



Interacting Shell Model (ISM)

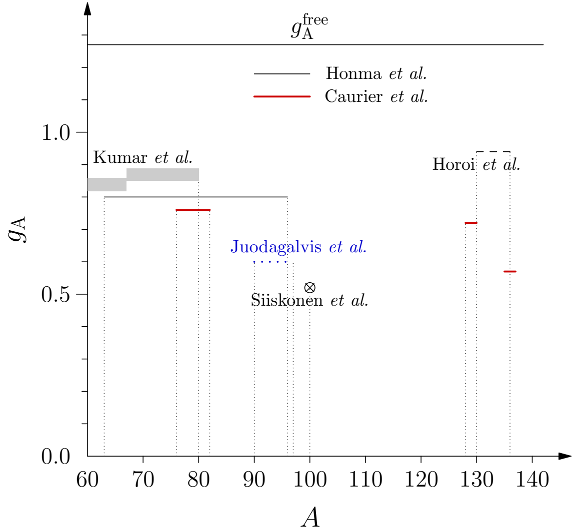
Results from:

Quenching of g_A in the ISM calculations

Results from the ISM

Mass range	$g_{\rm A}^{\rm eff}$	Reference
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$	W. T. Chou *et al.* 1993
$0p$ – low$1s0d$ shell	$1.12^{+0.05}_{-0.04}$	D. H. Wilkinson *et al.* 1974
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$	B. H. Wildenthal *et al.* 1983
	1.0	T. Siiskonen *et al.* 2001
$A = 41 - 50$ ($1p0f$ shell)	$0.937^{+0.019}_{-0.018}$	G. Martínez-Pinedo *et al.* 1996
$1p0f$ shell	0.98	T. Siiskonen *et al.* 2001
^{56}Ni	0.71	T. Siiskonen *et al.* 2001
$A = 52 - 67$ ($1p0f$ shell)	$0.838^{+0.021}_{-0.020}$	V. Kumar *et al.* 2016
$A = 67 - 80$ ($0f_{5/2}1p0g_{9/2}$ shell)	0.869 ± 0.019	V. Kumar *et al.* 2016
$A = 63 - 96$ ($1p0f0g1d2s$ shell)	0.8	M. Honma *et al.* 2006
$A = 76 - 82$ ($1p0f0g_{9/2}$ shell)	0.76	E. Caurier *et al.* 2012
$A = 90 - 97$ ($1p0f0g1d2s$ shell)	0.60	A. Juodagalvis *et al.* 2005
^{100}Sn	0.52	T. Siiskonen *et al.* 2001
$A = 128 - 130$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.72	E. Caurier *et al.* 2012
$A = 130 - 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.94	M. Horoi *et al.* 2016
$A = 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.57	E. Caurier *et al.* 2012

Results from the ISM (illustration)



- Kumar *et al.*: J. Phys. G 43 (2016) 105104
- Honma *et al.*: J. Phys. Conf. Ser. 49 (2006) 45
- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
- Horoi *et al.*: Phys. Rev. C 93 (2016) 024308
- Juodagalvis *et al.*: Phys. Rev. C 72 (2005) 024306
- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

Proton-neutron Quasiparticle Random-Phase Approximation (pnQRPA)

Results from:

Quenching of g_A

in the pnQRPA calculations

Results from the pnQRPA analyses

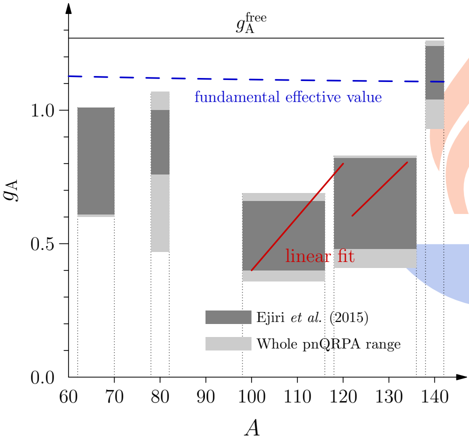
A	pn Conf.	$\bar{g}_{\rm A}^{\rm eff}$ [1]
$62-70$	$1p_{3/2}-1p_{1/2}$	0.81 ± 0.20
$78-82$	$0g_{9/2}-0g_{9/2}$	0.88 ± 0.12
$98-116$	$0g_{9/2}-0g_{7/2}$	0.53 ± 0.13
$118-136$	$1d_{5/2}-1d_{5/2}$	0.65 ± 0.17
$138-142$	$1d_{5/2}-1d_{3/2}$	1.14 ± 0.10

[1] H. Ejiri, J. Suhonen, J. Phys. G 42 (2015) 055201

Other analyses in the whole range:

[2] P. Pirinen, J. Suhonen, Phys. Rev. C 91 (2015) 054309

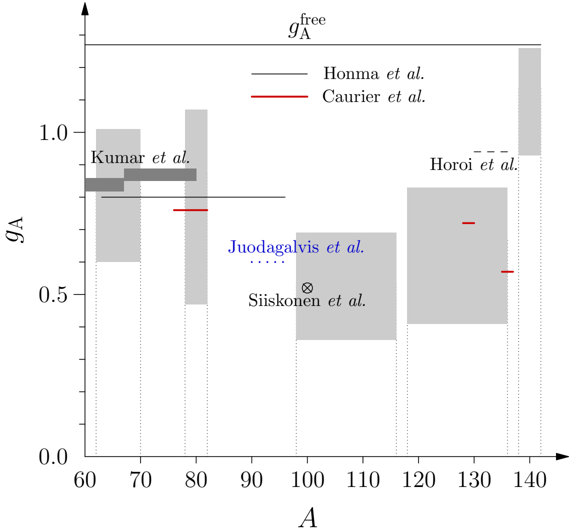
[3] F. Deppisch, J. Suhonen, Phys. Rev. C 94 (2016) 055501



Fundamental quenching: M. Ericson (1971); M. Ericson *et al.* (1973); M. Rho (1974); D. H. Wilkinson (1974)

(Meson-exchange currents → effective two-body operators)

Results from the ISM on top of the pnQRPA ranges



- Kumar *et al.*: J. Phys. G 43 (2016) 105104
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- Caurier *et al.*: Phys. Lett. B 711 (2012) 62
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- Siiskonen *et al.*: Phys. Rev. C 63 (2001) 055501

Calculations for the β decays and $\beta\beta$ decays

Results from:

Quenching of g_{A}

in the pnQRPA-based,

ISM-based and

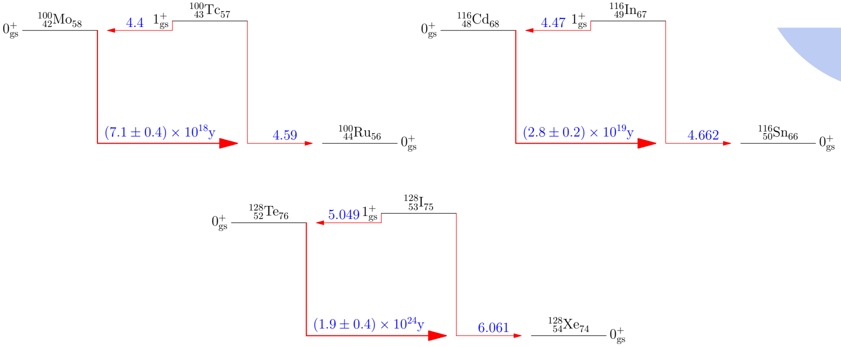
IBM-based calculations

of β decays and $\beta\beta$ decays

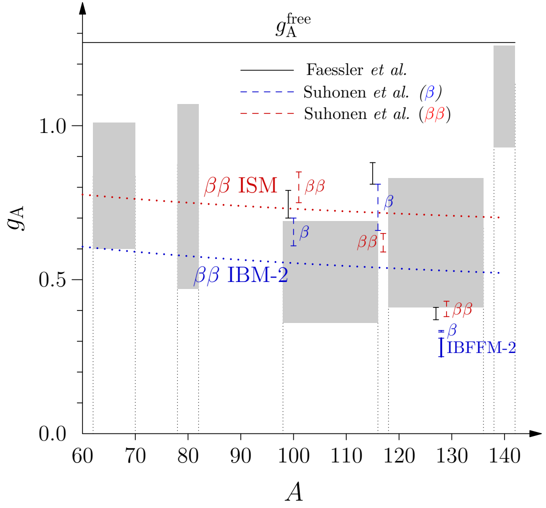
Results from the pnQRPA, IBM-2, and IBFFM-2

A	pnQRPA			IBFFM-2 [1]		IBM-2 [2]
	$g_A(\beta+\beta\beta)$ [3]	$g_A(\beta)$ [4]	$g_A(\beta\beta)$ [4]	$g_A(\beta)$	$g_A(\beta\beta)$	$g_A(\beta\beta)$
100	0.70 – 0.79	0.61 – 0.70	0.75 – 0.85	-	-	0.46(1) [SSD]
116	0.81 – 0.88	0.66 – 0.81	0.59 – 0.65	-	-	0.41(1) [SSD]
128	0.37 – 0.41	0.330 – 0.335	0.38 – 0.43	0.25 – 0.31	0.293	0.55(3) [CA]

[1] N. Yoshida, F. Iachello, Prog. Theor. Exp. Phys. 2013 (2013) 043D01 ; [2] J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315 ; [3] A. Faessler *et al.*, arXiv 0711.3996v1 [Nucl-th] ; [4] J. Suhonen, O. Civitarese, Nucl. Phys. A 924 (2014) 1



Results from the β+$\beta\beta$ calculations against the pnQRPA ranges from Gamow-Teller β decays



- Faessler *et al.*: A. Faessler, G. L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno, F. Šimkovic, arXiv 0711.3996v1 [Nucl-th]
- Suhonen *et al.*: J. Suhonen, O. Civitarese, Nucl. Phys. A 924 (2014) 1
- $\beta\beta$ ISM and IBM-2: J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315

Spin-multipole NMEs

Results from:

Quenching of g_A

as derived from

spin-multipole NMEs

of forbidden unique β decays

Spin-multipole (SM) nuclear matrix elements

General half-life formula for the allowed and unique-forbidden beta decays

$$t_{1/2}^{K}(0_{\rm gs}^{+} \leftrightarrow J^{\pi}) = \frac{\rm Constant}{\frac{g_{\rm A}{}^{2}}{2J_i+1}\left(M^{K}({\rm SM}J^{\pi})\right)^{2} f_K},$$

where

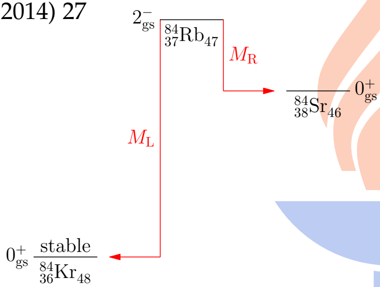
- f_K is the phase-space factor for the K^{th} forbidden (allowed $\equiv 0^{th}$ forbidden) unique β-decay transition,
- $g_{\rm A}$ is the axial-vector coupling constant,
- $J_i = J$ or $J_i = 0$ ($J = K + 1$) is the angular momentum of the decaying state, and
- $M^{K}({\rm SM}J^{\pi})$ is the spin-multipole NME for the K^{th} forbidden unique transition.

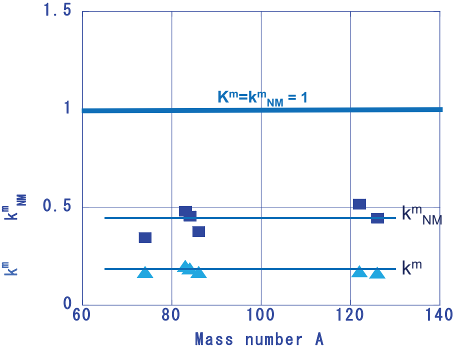
The unique decays are classified as:

K	0 (allowed)	1	2	3	4	5	6	7
J^{π}	1^{+}	2^{-}	3^{+}	4^{-}	5^{+}	6^{-}	7^{+}	8^{-}

Global study for the first-forbidden ($K = 1$) spin-dipole $2^-_{\mathrm{gs}} \to 0^+_{\mathrm{gs}}$ decays

H. Ejiri, N. Soukouti and J. Suhonen, Spin-dipole nuclear matrix elements for double beta decays and astro-neutrinos, Phys. Lett. B 729 (2014) 27





$$\bar{M}(\mathrm{SD2}^-) = \sqrt{M_{\mathrm{L}} M_{\mathrm{R}}}$$

$$\langle k \rangle = \left\langle \frac{\bar{M}_{\mathrm{exp}}(\mathrm{SD2}^-)}{\bar{M}_{\mathrm{qp}}(\mathrm{SD2}^-)} \right\rangle \approx 0.18$$

$$\langle k_{\mathrm{NM}} \rangle = \left\langle \frac{\bar{M}_{\mathrm{exp}}(\mathrm{SD2}^-)}{\bar{M}_{\mathrm{pnQRPA}}(\mathrm{SD2}^-)} \right\rangle \approx 0.45$$

$$\Rightarrow \bar{g}_{\mathrm{A}}^{\mathrm{eff}} \approx 0.57$$

Decays through higher spin-multipole ($K \geq 2$) operators

Question:

WHAT CAN WE LEARN

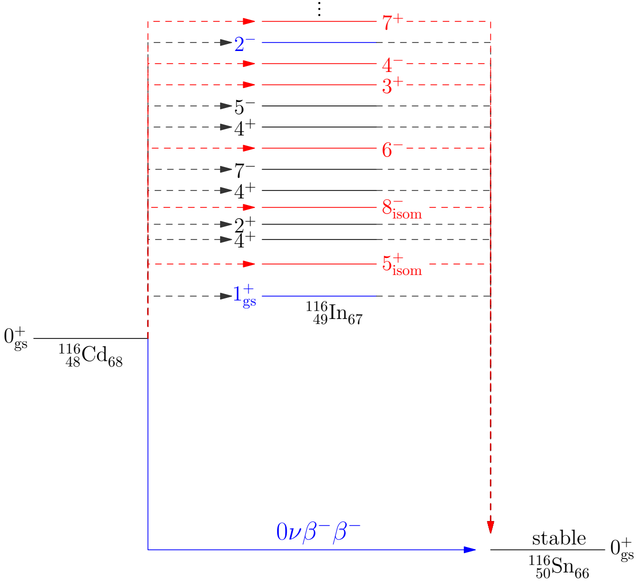
FROM THE UNIQUE HIGHER-FORBIDDEN

β DECAYS?

Answer:

A LOT!

INCENTIVE: $0\nu\beta\beta$ decay through the higher spin-multipole states



Decays through higher spin-multipole ($K \geq 2$) operators

Task:

STUDY 148 UNIQUE HIGHER-FORBIDDEN

β DECAYS IN ISOTOPIC CHAINS

Problem:

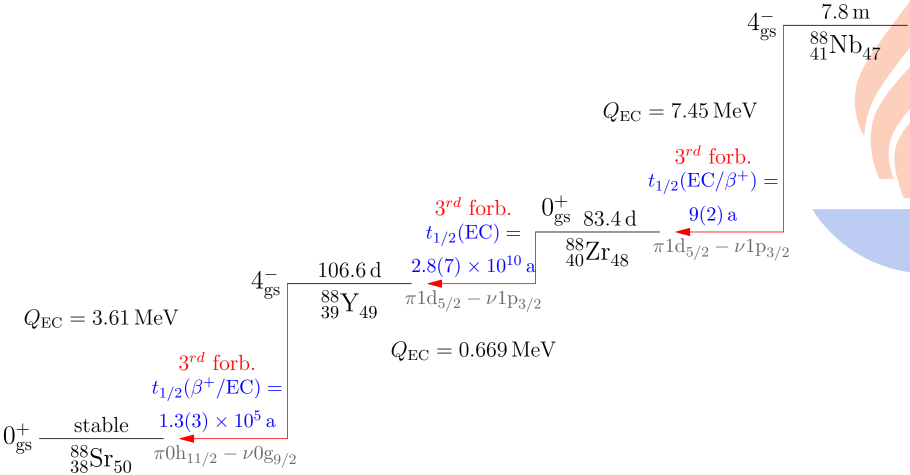
NO EXP. DATA AVAILABLE

Study:

$$k = \frac{M^K_{\rm pnQRPA}(\mathrm{SM}J^\pi)}{M^K_{\rm qp}(\mathrm{SM}J^\pi)} = ?$$

Dependence on K and mass number A?

Example: Decays in the $A = 88$ chain



4^-_{gs} 7.8 m $^{88}_{41}\mathrm{Nb}_{47}$

$Q_{\mathrm{EC}} = 7.45\,\mathrm{MeV}$

3^{rd} forb.
$t_{1/2}(\mathrm{EC}/\beta^+) =$
9(2) a

$\pi 1\mathrm{d}_{5/2} - \nu 1\mathrm{p}_{3/2}$

0^+_{gs} 83.4 d $^{88}_{40}\mathrm{Zr}_{48}$

3^{rd} forb.
$t_{1/2}(\mathrm{EC}) =$
$2.8(7) \times 10^{10}$ a

$\pi 1\mathrm{d}_{5/2} - \nu 1\mathrm{p}_{3/2}$

$Q_{\mathrm{EC}} = 0.669\,\mathrm{MeV}$

4^-_{gs} 106.6 d $^{88}_{39}\mathrm{Y}_{49}$

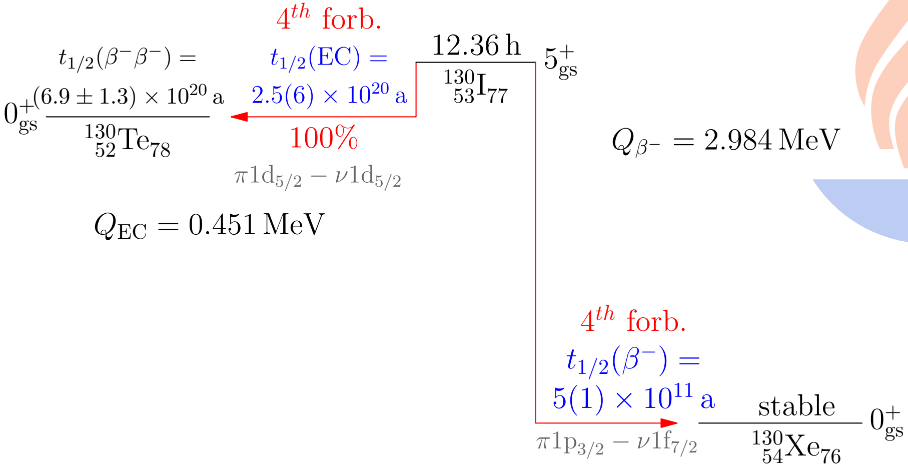
$Q_{\mathrm{EC}} = 3.61\,\mathrm{MeV}$

3^{rd} forb.
$t_{1/2}(\beta^+/\mathrm{EC}) =$
$1.3(3) \times 10^5$ a

$\pi 0\mathrm{h}_{11/2} - \nu 0\mathrm{g}_{9/2}$

0^+_{gs} stable $^{88}_{38}\mathrm{Sr}_{50}$

Example: Decays in the $A = 130$ chain (including a $\beta\beta$ decay)



$t_{1/2}(\beta^-\beta^-) =$

$(6.9 \pm 1.3) \times 10^{20}$ a

0^+_{gs} ${}^{130}_{52}\mathrm{Te}_{78}$

4^{th} forb.

$t_{1/2}(\mathrm{EC}) =$

$2.5(6) \times 10^{20}$ a

100%

$\pi 1\mathrm{d}_{5/2} - \nu 1\mathrm{d}_{5/2}$

12.36 h

${}^{130}_{53}\mathrm{I}_{77}$ 5^+_{gs}

$Q_{\beta^-} = 2.984$ MeV

$Q_{\mathrm{EC}} = 0.451$ MeV

4^{th} forb.

$t_{1/2}(\beta^-) =$

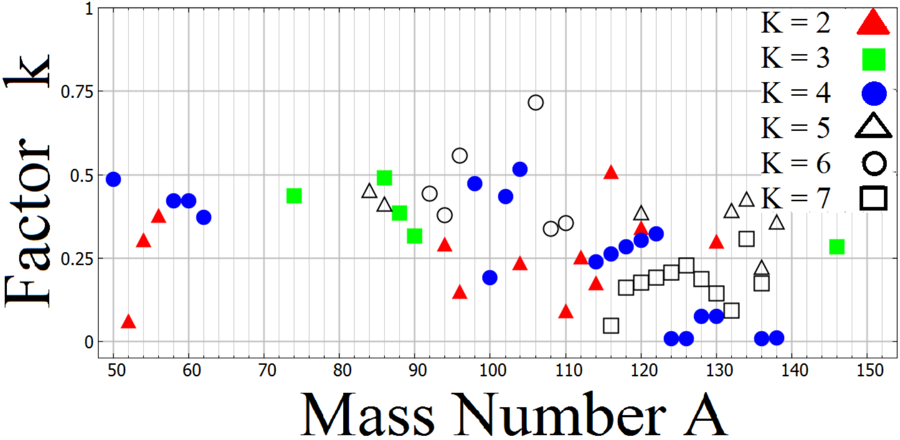
$5(1) \times 10^{11}$ a

$\pi 1\mathrm{p}_{3/2} - \nu 1\mathrm{f}_{7/2}$

stable

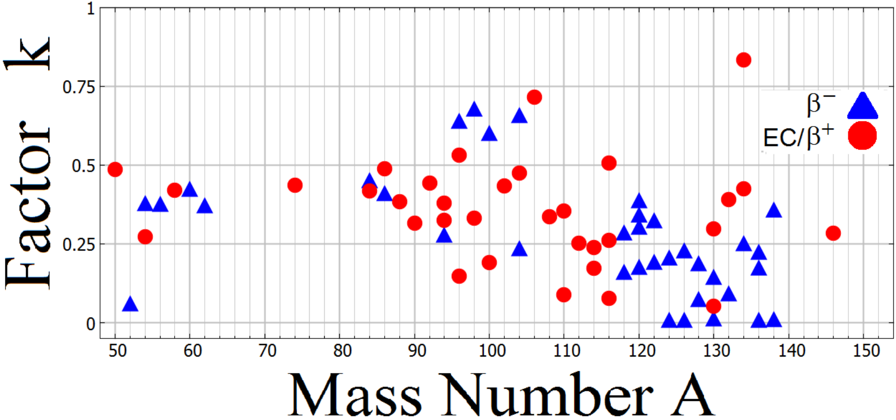
${}^{130}_{54}\mathrm{Xe}_{76}$ 0^+_{gs}

Ratio k for 74 β decays involving non-magic nuclei



k extracted using the geometric mean of the full set of K^{th} ($K = 2 - 7$) forbidden β-decay transitions in an isobaric chain (J. Kostensalo, J. Suhonen, Phys. Rev. C 95 (2017) 014322)

Separation to β^- and β^+/EC decays



Results for the Ratio $k = M^K_{\text{pnQRPA}}(\text{SM}J^\pi)/M^K_{\text{qp}}(\text{SM}J^\pi)$

A	GT [1]	$K=1$ [2]	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=7$	Avg.
$50-88$	0.35	0.40	0.25	0.46	0.43	0.43	-	-	0.39
$90-122$	0.52	0.40	0.25	0.35	0.34	0.38	0.41	0.13	0.31
$122-146$	0.40	0.40	0.30	0.28	0.07	0.35	-	0.19	0.24
Average	0.42	0.40	0.27	0.36	0.28	0.39	0.41	0.16	0.31

[1] H. Ejiri, J. Suhonen, J. Phys. G: Nucl. Part. Phys. 42 (2015) 055201

[2] H. Ejiri, N. Soukouti, J. Suhonen, Phys. Lett. B 729 (2014) 27

Conclusion: k is roughly independent of K ⇒ Low-energy quenching of g_{A} derivable from the hatched regions of the Gamow-Teller studies in the pnQRPA framework:

Mass range	$A = 76-82$	$A = 100-116$	$A = 122-136$
$g^{\text{eff}}_{\text{A},0\nu}$	$0.7-0.9$	0.5	$0.5-0.7$

Assumption: Also the forbidden non-unique virtual transitions behave like the forbidden unique virtual transitions.

Spectrum-shape method (SSM)

Results from:

Quenching of g_{A}

as derived from

electron spectra of

forbidden non-unique β decays

Spectrum shape of forbidden non-unique β decays

Half-life:

$$t_{1/2} = \kappa/\tilde{C}\,.$$

Dimensionless integrated shape function:

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_\mathrm{e} (w_0 - w_e)^2 F_0(Z_f, w_e) \mathrm{d}w_e\,.$$

Shape factor:

$$C(w_e) = \sum_{k_e, k_\nu, K} \lambda_{k_e} \left[M_K(k_e, k_\nu)^2 + m_K(k_e, k_\nu)^2 - \frac{2\gamma_{k_e}}{k_e w_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right] ,$$

where

$$\lambda_{k_e} = \frac{F_{k_e-1}(Z, w_e)}{F_0(Z, w_e)}\;; \quad \gamma_{k_e} = \sqrt{k_e^2 - (\alpha Z_f)^2}\,,$$

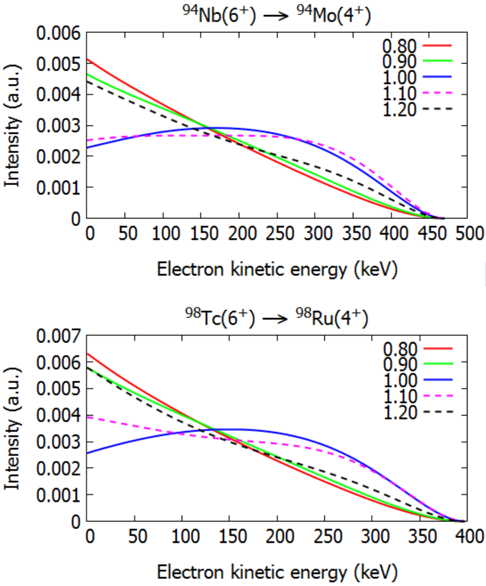
$F_{k-1}(Z, w_e)$ being the generalized Fermi function.

Decomposition of the shape factor:

$$\boxed{C(w_e) = g_\mathrm{V}^2 C_\mathrm{V}(w_e) + {g_\mathrm{A}}^2 C_\mathrm{A}(w_e) + g_\mathrm{V} g_\mathrm{A} C_\mathrm{VA}(w_e).}$$

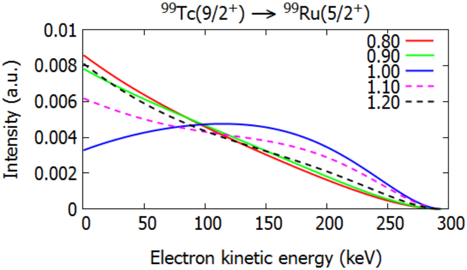
ISM-computed β spectra for different values of g_A

Normalized ISM-computed electron spectra for the 2*nd*-forbidden nonunique β^- decays of ^{94}Nb and ^{98}Tc ($g_V = 1.0$).

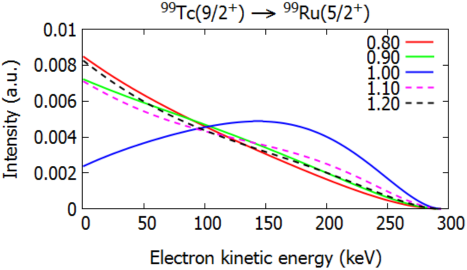


Example: ISM- and MQPM-computed electron spectra

Normalized ISM- and MQPM-computed electron spectra for the 2*nd*-forbidden nonunique β^- decay of ^{99}Tc ($g_V = 1.0$) using different values of g_A.



(ISM)



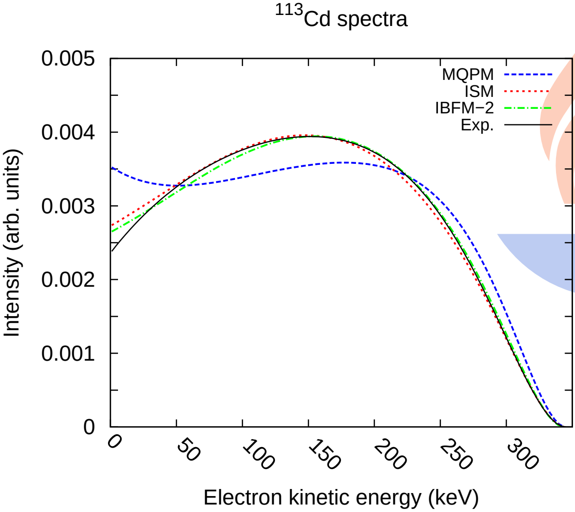
(MQPM)

Example: Decay of ^{113}Cd – Comparison with data

Normalized electron spectra for the 4*th*-forbidden nonunique β^- decay $^{113}\mathrm{Cd}(1/2^+) \rightarrow {}^{113}\mathrm{In}(9/2^+)$ $(g_\mathrm{V} = 1.0)$.

Experimental data from:

P. Belli *et al.*, Phys. Rev. C 76 (2007) 064603



All three nuclear models give:

$g_\mathrm{A} \approx 0.92$!

Summary of the exploratory work on β spectra

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ (n_f)	Branching	K	Sensitivity	Nucl. model
$^{36}\mathrm{Cl} \rightarrow {}^{36}\mathrm{Ar}$	2^+	0^+ (gs)	**98%**	2	None	ISM
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Sc}$	0^+	4^+ (2)	$\sim$0%	4	None	ISM
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Sc}$	0^+	6^+ (gs)	$\sim$0%	6	None	ISM
$^{50}\mathrm{V} \rightarrow {}^{50}\mathrm{Cr}$	6^+	2^+ (1)	$\sim$0%	4	Weak	ISM
$^{60}\mathrm{Fe} \rightarrow {}^{60}\mathrm{Co}$	0^+	2^+ (1)	**100%**	2	None	ISM
$^{85}\mathrm{Br} \rightarrow {}^{85}\mathrm{Kr}$	$3/2^-$	$9/2^+$ (gs)	$\sim$0%	3	Moderate	MQPM
$^{87}\mathrm{Rb} \rightarrow {}^{87}\mathrm{Sr}$	$3/2^-$	$9/2^+$ (gs)	**100%**	3	Moderate	MQPM, ISM
$^{93}\mathrm{Zr} \rightarrow {}^{93}\mathrm{Nb}$	$5/2^+$	$9/2^+$ (gs)	5 $\leq$%	2	Weak	MQPM
$^{94}\mathrm{Nb} \rightarrow {}^{94}\mathrm{Mo}$	6^+	4^+ (2)	**100%**	2	**Strong**	NSM
$^{96}\mathrm{Zr} \rightarrow {}^{96}\mathrm{Nb}$	0^+	4^+ (2)	$\sim$0%	4	None	ISM
$^{96}\mathrm{Zr} \rightarrow {}^{96}\mathrm{Nb}$	0^+	6^+ (gs)	$\sim$0%	6	**Strong**	ISM
$^{97}\mathrm{Zr} \rightarrow {}^{97}\mathrm{Nb}$	$1/2^+$	$9/2^+$ (gs)	$\sim$0%	4	**Strong**	MQPM
$^{98}\mathrm{Tc} \rightarrow {}^{98}\mathrm{Ru}$	6^+	4^+ (3)	**100%**	2	**Strong**	ISM
$^{99}\mathrm{Tc} \rightarrow {}^{99}\mathrm{Ru}$	$9/2^+$	$5/2^+$ (gs)	**100%**	2	**Strong**	MQPM, ISM
$^{101}\mathrm{Mo} \rightarrow {}^{101}\mathrm{Tc}$	$1/2^+$	$9/2^+$ (gs)	$\sim$0%	4	**Strong**	MQPM
$^{113}\mathrm{Cd} \rightarrow {}^{113}\mathrm{In}$	$1/2^+$	$9/2^+$ (gs)	**100%**	4	**Strong**	MQPM, ISM, IBFM-2
$^{115}\mathrm{Cd} \rightarrow {}^{115}\mathrm{In}$	$1/2^+$	$9/2^+$ (gs)	$\sim$0%	4	**Strong**	MQPM
$^{115}\mathrm{In} \rightarrow {}^{115}\mathrm{Sn}$	$9/2^+$	$1/2^+$ (gs)	**100%**	4	**Strong**	MQPM, ISM, IBFM-2

Summary on β spectra continues

Transition	$J_i^{\pi_i}$ (gs)	$J_f^{\pi_f}$ (n_f)	Branching	K	Sensitivity	Nucl. model
^{117}Cd → ^{117}In	$1/2^+$	$9/2^+$ (gs)	∼0%	4	**Strong**	MQPM
^{119}In → ^{119}Sn	$9/2^+$	$1/2^+$ (gs)	∼0%	4	**Strong**	MQPM
^{123}Sn → ^{123}Sb	$11/2^-$	$1/2^+$ (4)	∼0%	5	Weak	MQPM
^{126}Sn → ^{126}Sb	0^+	2^+ (5)	**100%**	2	None	ISM
^{135}Cs → ^{135}Ba	$7/2^+$	$3/2^+$ (gs)	**100%**	2	None	MQPM
^{137}Cs → ^{137}Ba	$7/2^+$	$3/2^+$ (gs)	5.4%	2	None	MQPM, ISM
^{125}Sb → ^{125}Te	$7/2^+$	$9/2^-$ (3)	7.2%	1	None	MQPM
^{141}Ce → ^{141}Pr	$7/2^-$	$5/2^+$ (gs)	31%	1	Weak	MQPM
^{159}Gd → ^{159}Tb	$3/2^-$	$5/2^+$ (1)	26%	1	None	MQPM
^{161}Tb → ^{161}Dy	$3/2^+$	$5/2^-$ (1)	∼0%	1	None	MQPM
^{169}Er → ^{169}Tm	$1/2^-$	$3/2^+$ (1)	45%	1	None	MQPM

Conclusions and Outlook

Conclusions:

- The long chain of ISM calculations and the recent pnQRPA and IBM-2 calculations of Gamow-Teller β decays and $2\nu\beta\beta$ decays are (surprisingly!) consistent with each other and clearly point to a A-dependent quenched g_{A}
- Previous studies on GT 1^+ and SD 2^- β decays shed light on the suppression chain: quasiparticle NME → pnQRPA NME → experimental NME
- Studies of unique high-forbidden β decays ($K \geq 2$) give the suppression chain: quasiparticle NME → pnQRPA NME → Previous GT and SD studies → one can speculate about modifications in the pnQRPA-computed $0\nu\beta\beta$-decay half-lives (About the impact on the sensitivity of $0\nu\beta\beta$ experiments, see also arXiv:1708.09604 [nucl-th])
- The spectrum-shape method (SSM) for forbidden non-unique β decays is a robust tool (largely independent of the nuclear model, the assumed Hamiltonian and mean field) to seach for the effective value of g_{A}

Outlook:

- Urge measurements of the β spectra for the (5) interesting decays amenable to the SSM
- Find ways to use the present studies in a more reliable prediction of the pnQRPA-based $0\nu\beta\beta$ NMEs