Puzzles in $|V_{ub}|$ and $|V_{cb}|$
Towards a solution?

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Why $|V_{cb}|$ and $|V_{ub}|$?

$|V_{xb}|$: crucial inputs
To indirect search of New Physics

$|V_{cb}|$ and $|V_{ub}|$
discrepancy between different determinations: $3\sigma$ effect

$\Gamma(B \to D(*)\tau\nu_\tau) / \Gamma(B \to D(*)\ell\nu_\ell)$
Enhanced respect to SM Predictions ($\sim 4\sigma$)

Predictions of FCNC processes

$\propto |V_{tb}V_{ts}| \approx |V_{cb}|^2 [1 + O(\lambda^2)]$

Kaon physics $\epsilon_K \approx x|V_{cb}|^2 + ...$

$|V_{ub}|$ opposite to angle $\beta$: compare Tree with loops

Semileptonic measurements provide Form-Factors, crucial for SM predictions on $R(D)-R(D^*)$

Study of $B \to D^{**}$ crucial to constrain backgrounds in $|V_{ub}|$ inclusive $|V_{cb}|$ and $R(D)-R(D^*)$ determinations
Exclusive $B \to \pi \ell \nu$

- For massless leptons only one Form Factor

\[ \frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = |V_{ub}|^2 \frac{G_F^2 \tau_B}{24\pi^3 p_\pi^3} f^{B\pi+}_{+}(q^2) \]

- Lattice QCD (UKQCD, FNAL, …)
  - Works at high $q^2$
  - Unquenched calculations (2+1, 2+1+1)
  - Other mesons ($\rho$, $\omega$, …) difficult on lattice

- Light Cone Sum Rules
  - Reliable at low $q^2$
  - Works for both pseudo-scalars and vector decays
Exclusive $B \to \pi \ell \nu$

- **Lattice QCD** (HPQCD, FNAL,...)
  - Works at high $q^2$
  - Unquenched calculations (2+1, 2+1+1)
  - Other mesons ($\rho, \omega,...$) difficult on lattice
- **Light Cone Sum Rules**
  - Reliable at low $q^2$
  - Works for both pseudo-scalars and vector decays

- For massless leptons only one Form Factor

\[
\frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = |V_{ub}|^2 \frac{G_F^2 \tau_B}{24\pi^3} p_{\pi} |f_+^{B\pi}(q^2)|^2
\]

**Strategies for $|V_{ub}|$ extraction**

- Measure $\Delta Br$ in regions where theory is reliable

\[
|V_{ub}|^2 = \frac{\Delta Br}{\tau_B \Delta \tilde{\Gamma}_{theory}}
\]

- Simultaneous fit to data and theory
  - Measure $\Delta Br$ in bins of $q^2$
  - $|V_{ub}|$ and form factor shape from a fit to data and theory
  - Exploiting the recent lattice calculations in many points at high $q^2$
Untagged $B \to \pi \ell \nu$

- Combined $\pi$ with a lepton $\ell$; the neutrino from the rest of the event

L = 416 fb$^{-1}$
Nsig = 12.5K ± 400

Signal extracted in 12 $q^2$-bins

L = 605 fb$^{-1}$
Nsig = 21.5K ± 500

Signal extracted in 13 $q^2$-bins

\[ m_{ES} = \sqrt{E_{\text{beam}}^* - p_{\pi \ell \nu}^*} \]
\[ \Delta E = E_{\pi \ell \nu}^* - E_{\text{beam}}^* \]

Phys. Rev. D83(2011) 071101
Tagged $B \rightarrow \pi \ell \nu$

- Using the hadronic tag
  
  $L = 711 \text{ fb}^{-1}$
  $N(B \rightarrow \pi^2 \ell \nu) \sim 500$, $N(B \rightarrow \pi^0 \ell \nu) \sim 200$

- Reduce combinatorial backgrounds
- Improve kinematic resolution
  - Signal $B$ direction determined by $B_{\text{tag}}$

\[ B \rightarrow \pi^0 \ell \nu \]

\[ B \rightarrow \pi^+ \ell \nu \]

\[ B \rightarrow \pi^0 \ell \nu \]

\[ B \rightarrow \pi^+ \ell \nu \]
**HFLAV average**

- Include the most precise measurements
- Partial $B_r$ averaged with a likelihood fit

**Theoretical inputs:**
- Lattice QCD at high $q^2$
- HFLAG average of FNAL/MILC + HPQCD
- LCSR ta $q^2=0$
  Bharucha, JHEP 1205 (2012) 092

**Form Factor parameterization BCL**
Bourrely, Caprini, Lellouch, PRD79, 013008 (2009)

\[ f_+(q^2, \bar{b}) = \frac{1}{1 - q^2/m_B^2} \sum_{k=0}^{K} b_k(t_0) z(q^2)^k \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>V_{ub}</td>
</tr>
<tr>
<td>$b_1^+$</td>
<td>$0.421 \pm 0.017$</td>
</tr>
<tr>
<td>$b_2^+$</td>
<td>$-0.390 \pm 0.033$</td>
</tr>
<tr>
<td>$b_3^+$</td>
<td>$-0.650 \pm 0.126$</td>
</tr>
</tbody>
</table>

Tot $\sigma \sim 4\%$!
\(|V_{ub}|\) at LHCb

- B-baryons provide complementary informations to B-mesons
- Copious production of \(\Lambda_b\)
- Kinematic constraints allow the determination of the \(p_{\Lambda_b}\) (modulo 2-fold ambiguity)
- Large background from \(\Lambda_b \rightarrow \Lambda_c \mu\nu\)
- LHCb determines (in the high \(q^2\) region) the ratio

\[
R_{exp} = \frac{\mathcal{B}(\Lambda_b \rightarrow p\mu\nu)}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c\mu\nu)}
\]

\(-\) Detmold et al PRD92(2015)034503

\(\Lambda_b \rightarrow p\mu\nu\)

\(q^2 > 15\text{ GeV}^2\)

\(\Lambda_b \rightarrow \Lambda_c\mu\nu\)

\(q^2 > 7\text{ GeV}^2\)
$\Lambda_b \rightarrow p\mu\nu$ signal & $|V_{ub}|$

$$M_{corr} = \sqrt{p_{\perp}^2 + M_{p\mu}^2} + p_{\perp}$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.080 \pm 0.004_{\text{Exp.}} \pm 0.004_{\text{F.F.}}$$

$\sigma_{\text{tot}} = 7\%$

$N_{\text{sig}} = 17687 \pm 733$

$$R = \frac{B(\Lambda_b \rightarrow p\mu\nu)_{q^2>15 \ GeV^2}}{B(\Lambda_b \rightarrow \Lambda_c\mu\nu)_{q^2>7 \ GeV^2}} = (0.95 \pm 0.04 \pm 0.07) \times 10^{-2}$$

Systematics dominated by $\text{BF}(\Lambda_c \rightarrow pK\pi) = (6.46 \pm 0.24)\%$

HFLAV using BESIII-Belle measurements
$|V_{ub}|$ from inclusive decays

- Large background from $B \to X_c \ell \nu$
- Kinematics to extract the signal: $m_u \ll m_c$
  - Cut limited region of phase space ($f_u$)
    - Non perturbative shape-function needed
    - Universal only at leading order in $\Lambda/m_b$

$E_\ell = \text{lepton energy}$

$q^2 = (P_B - P_X)^2 = (P_\ell - P_\nu)^2$

$M_X = X_u \text{ hadronic mass}$

Experimental resolution leads to “irreducible” $b \to c \ell \nu$ contamination
- partially suppressed
  - With K and D* vetos

$\frac{\Gamma(b \to c\ell\nu)}{\Gamma(b \to u\ell\nu)} \approx 50$
\[ |V_{ub}| \text{ from inclusive decays} \]

- Large background from $B \to X_c \ell \nu$

\[ \frac{\Gamma(b \to c \ell \nu)}{\Gamma(b \to u \ell \nu)} \approx 50 \]

\[
\begin{align*}
\text{Experimental resolution} & \rightarrow \text{"irreducible" } \\
\text{Experimental resolution} & \cdots \text{contamination} \\
& \cdots \text{partially suppressed} \\
& \text{With K and D* vetos}
\end{align*}
\]

- Kinematics to extract the signal: $m_u \ll m_c$

\[
\text{Non perturbative shape-function needed} \\
\text{Universal only at leading order in } \frac{\Lambda}{m_b}
\]

\[
|V_{ub}| = \sqrt{\frac{\Delta \Gamma}{\tau_B \Delta \Gamma_{\text{theory}}}}
\]

\[
M_X = X_u \text{ hadronic mass}
\]

\[
\begin{align*}
& \text{Small } f_u \\
& \text{Large } f_u
\end{align*}
\]

\[
\begin{align*}
\text{Not to scale!}
\end{align*}
\]

\[
\begin{align*}
\text{DN De Fazio, Neubert JHEP9905,017 (1999)} \\
\text{Claimed in BLNP to be superseeded} \\
\text{GGOU Gambino, Giordano, Ossola, Uraltsev, JHEP908 10, 058 (2007)} \\
\text{DGE Andersen, Gardi, JHEP 0601, 097 (2006)}
\end{align*}
\]

\[
\begin{align*}
\text{ADFR Aglietti, Di Ludovico, Ferrara, Ricciardi EPJC, Vol. 59 (2009)}
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\text{Only valid in the } m_X-q^2 \text{ two-dimensions cut}
\end{align*}
\]

\[
\begin{align*}
\text{Only in the } m_X-q^2 \text{ two-dimensions cut}
\end{align*}
\]

\[
\begin{align*}
\text{Not to scale!}
\end{align*}
\]
Fit results in limited regions of phase space

\[ B \rightarrow X_u \ell \nu \]

\[ B \rightarrow X_c \ell \nu + \text{cascades + fake } \ell \]

\[ \frac{\Delta B(X_u \ell \nu)}{B(X \ell \nu)} = \frac{N_{b \rightarrow u}}{N_{X \ell \nu}} \cdot \frac{F}{\epsilon_{sel}} \]
Status of inclusive $|V_{ub}|$

- Consistency between difference acceptance regions
- Calculations agree with each other
- Correlated uncertainties
  - HQE parameters $m_b$, $m_\mu^2$: from Global Fit for inclusive $|V_{cb}|$
  - Common experimental tools: EvtGen, JETSET $X_u$ hadronisation, $b\rightarrow c\ell\nu$
- $|V_{ub}|$ is calculated from partial rates measured with only one signal model
  (Belle multivariate, adjust the signal model to match the GGOU predictions)

### Framework $|V_{ub}|[10^{-3}]$

| Framework                  | $|V_{ub}|[10^{-3}]$ |
|----------------------------|---------------------|
| BLNP                      | $4.44 \pm 0.15^{+0.21}_{-0.22}$ |
| DGE                       | $4.52 \pm 0.16^{+0.15}_{-0.16}$ |
| GGOU                      | $4.52 \pm 0.15^{+0.11}_{-0.14}$ |
| ADFR                      | $4.08 \pm 0.13^{+0.18}_{-0.12}$ |
| BL (m_X/q^2 only)         | $4.62 \pm 0.20^{+0.29}_{-0.29}$ |

Most recent measurements is dated 2012

- CLEO ($E_c$): $4.23 \pm 0.49 + 0.22 - 0.31$
- BELLE sim. ann. (m_X, q^2): $4.52 \pm 0.47 + 0.25 - 0.28$
- BELLE ($E_c$): $4.95 \pm 0.46 + 0.16 - 0.21$
- BABAR ($E_c$): $4.52 \pm 0.26 + 0.17 - 0.24$
- BELLE multivariate ($p^*$): $4.62 \pm 0.28 + 0.09 - 0.10$
- BABAR (m_X < 1.55): $4.30 \pm 0.20 + 0.20 - 0.21$
- BABAR (m_X < 1.7): $4.10 \pm 0.23 + 0.16 - 0.17$
- BABAR (m_X < 1.7, q^2 > 8): $4.33 \pm 0.23 + 0.24 - 0.27$
- BABAR ($P^* < 0.66$): $4.25 \pm 0.26 + 0.26 - 0.27$
- BABAR (m_X, q^2 fit, $p^* > 1$GeV): $4.44 \pm 0.24 + 0.09 - 0.10$
- BABAR ($p^* > 1.3$GeV): $4.43 \pm 0.27 + 0.09 - 0.11$

Average +/- exp + theory - theory: $4.52 \pm 0.15 + 0.11 - 0.14$

F_u ~ 90%

- $|V_{ub}|$ is calculated from partial rates measured with only one signal model
- (Belle multivariate, adjust the signal model to match the GGOU predictions)
New inclusive $|V_{ub}|$

- Inclusive electron spectrum measurement

  **Fit Strategy**
  - Fit simultaneously on-Y(4S) and off-Y(4S)
    - 5 separate $b \rightarrow c$ components
    - Secondary leptons $b \rightarrow c \rightarrow e$
    - $b \rightarrow X_u e \nu$
  - Spectrum range $[p_{\text{min}}, 2.7]$ GeV, $p_{\text{min}}$ from 0.8 GeV

  **Dataset:** 467M Y(4S)

  - Large statistics: $>10^6$ events / 50 MeV bin; statistical uncertainties dominated by continuum subtraction
  - Use 44.4 fb$^{-1}$ taken off resonance

  **Signal model** obtained mixing known existing exclusive final states with calculations for $b \rightarrow X_u e \nu$ (Hybrid model). Four different calculations considered for $b \rightarrow X_u e \nu$ Inclusive spectrum

  B.Kowalewski@CKM16 Phys.Rev.D 95, 072001 (2017)
B → X_{u}e\nu in Y(4S) frame

- B → X_{u}e\nu electron spectra for p_{e} > 0.8 GeV after continuum, B → X_{c}e\nu and cascade subtraction
Results on total rate and $|V_{ub}|$

- Highest sensitivity to $B \to X_u \text{ev}$ in the wide bin $2.1-2.7$ GeV
- Models make different predictions for the fractional rate in this bin
  - The normalization of the $B \to X_u \text{ev}$ is fixed by this bin!
- This dependence on the signal model could impact any measurement that extends in the $B \to X_u \text{ev}$ region

Results are lower than previous measurement (not for BLNP!)

How existing analyses would be affected by the signal model is difficult to predict without re-analysing old data!

The effect could be smaller than the one observed here!
Results on total rate and $|V_{ub}|$

- Highest sensitivity to $B \rightarrow X_u \text{ev}$ in the wide bin 2.1-2.7 GeV
- Models make different predictions for the fractional rate in this bin
- The normalization of the $B \rightarrow X_u \text{ev}$ is fixed by this bin!
- This dependence on the signal model impact any measurement that extends in the $B \rightarrow X_u \text{ev}$ region

In the future it will be crucial to improve
- Knowledge about $B \rightarrow X_c$ composition and kinematics: rates and FFs for $D/D^*/D^{**}$...
- Constrain the signal model measuring exclusive $B \rightarrow n\pi \text{ev}$: up to now resonant and non-resonant contributions are combined in an ad-hoc procedure
- WA, $X_u$ hadronisation...
HQE is the successful tool to include perturbative and non-perturbative QCD corrections that allow to connect measurements of semileptonic B-meson decays to $|V_{cb}|^2$.

No new experimental results since 2010

Latest fits in Kinetic Scheme:

**Gambino, Schwanda**
*PhysRevD* 89,014022 (2014)
Include charm-quark mass from sum-rule results (PRD80,074010 (2009))

**Alberti, Gambino, Healey, Nandi**
- Includes corrections of $O(\alpha_s^2\Lambda_{QCD}/m_b)$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Hadron moments $&lt;M^n_X&gt;$</th>
<th>Lepton moments $&lt;E^n&gt;$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>n=2 $c=0.9,1.1,1.3,1.5$</td>
<td>n=0 $c=0.6,1.2,1.5$</td>
<td>[1] Phys.Rev._D81 (2010) 032003</td>
</tr>
<tr>
<td></td>
<td>n=4 $c=0.8,1.0,1.2,1.4$</td>
<td>n=1 $c=0.6,0.8,1.0,1.2,1.5$</td>
<td>[2] Phys.Rev._D69 (2004) 111104</td>
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<tr>
<td></td>
<td>n=6 $c=0.9,1.3$</td>
<td>n=2 $c=0.6,1.0,1.5$</td>
<td>[3] Phys.Rev._D75 (2007) 032005</td>
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<tr>
<td>Belle</td>
<td>n=2 $c=0.7,1.1,1.3,1.5$</td>
<td>n=0 $c=0.6,1.4$</td>
<td>[5] Phys.Rev._D71 (2005) 051103</td>
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<tr>
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<td>n=4 $c=0.7,0.9,1.3$</td>
<td>n=1 $c=1.0,1.4$</td>
<td>[6] Phys.Rev._D70 (2004) 032002</td>
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<td>CDF</td>
<td>n=2 $c=0.7$</td>
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<tr>
<td></td>
<td>n=4 $c=0.7$</td>
<td>n=2 $c=0.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>n=6 $c=0.0$</td>
<td></td>
</tr>
<tr>
<td>CLEO</td>
<td>n=2 $c=1.0,1.5$</td>
<td>n=1 $c=0.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n=4 $c=1.0,1.5$</td>
<td>n=2 $c=0.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>n=6 $c=0.0$</td>
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<td>DELPHI</td>
<td>n=2 $c=0.0$</td>
<td>n=1 $c=0.0$</td>
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<tr>
<td></td>
<td>n=4 $c=0.0$</td>
<td>n=2 $c=0.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n=6 $c=0.0$</td>
<td>n=3 $c=0.0$</td>
<td></td>
</tr>
</tbody>
</table>

HFLAV
Exclusive $|V_{cb}|$ and Form Factors

- $B \to D \ell \nu$ and $B \to D^* \ell \nu$ provide a clean way to extract $|V_{cb}|$

\[ B \to D^* \ell \nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3} (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \chi(w) F^2(w) |V_{cb}|^2 \]

\[ B \to D \ell \nu \quad \frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} G^2(w) |V_{cb}|^2 \]

Assuming $m_\ell = 0$

\[ w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D} \]

$w = 0 \quad w_{\text{max}}$

Form Factor Parameterizations


\[ f_i(z) = \frac{1}{P_i(z) \phi_i(z)} \sum_{n=0}^{N} a_{i,n} z^n, \quad z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \]

Coefficient $a_{i,n}$ free parameters

The analyticity of the OPE assures bounds on the sum of the $a_{i,n}^2$


$B \to D^* \ell \nu$

\[ h_{A_1}(w) = h_{A_1}(1) \left[ 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3 \right], \]

\[ R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2 \]

Higher order coefficient connected with the slope $\rho^2$
Exclusive $|V_{cb}|$ and Form Factors

$B \rightarrow D^* \ell \nu$

Unquenched lattice FF calculation available only at zero-recoil
$F(1) = 0.906 \pm 0.013$

Quenched calculation extends to $w=1.1$

LCSR at $w_{\text{max}}$

$B \rightarrow D \ell \nu$

Unquenched lattice FF calculation also at moderately large recoil

Assuming $m_\ell = 0$

$$w = \frac{m_B^2 + m_D^2 - q^2}{2m_Bm_D}$$

$w=0$$$

$w_{\text{max}}$$$

$B \rightarrow D^* \ell \nu$

Coefficient $a_{i,n}$ free parameters

The analyticity of the OPE assures bounds on the sum of the $a_{i,n}^2$

$B \rightarrow D^* \ell \nu$

$h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$

$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2$

$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2$
B → Dℓν

- State of the art performed by BaBar and Belle with hadronic B tagging: improve kinematic resolution and reduce combinatorial backgrounds
- Use both B → D⁰ℓν ↔ K^+ B → D^±ℓν
- Signal extract in 10 bins of w from M_{miss}²
- Largest background
  - B → D^*ℓν

BaBar used 460M B̄B
Fit ~3200 signal events

Belle used 771M B̄B
Improved Hadronic B Tag based on NeuroBayes
Fit ~17000 signal events
## $G(1)|V_{cb}|$: results at B-Factories

<table>
<thead>
<tr>
<th></th>
<th>$B^- \to D^0 \ell^- \bar{\nu}_\ell$</th>
<th>$\bar{B}^0 \to D^+ \ell^- \bar{\nu}_\ell$</th>
<th>$\bar{B} \to D \ell^- \bar{\nu}_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G(1)</td>
<td>V_{cb}</td>
<td>\cdot 10^3$</td>
<td>$41.7\pm2.1 \pm 1.3$</td>
</tr>
<tr>
<td>$\rho^2$</td>
<td>$1.14\pm0.11 \pm 0.04$</td>
<td>$1.29\pm0.14 \pm 0.05$</td>
<td>$1.20\pm0.09 \pm 0.04$</td>
</tr>
<tr>
<td>$\rho_{corr}$</td>
<td>0.943</td>
<td>0.950</td>
<td>0.952</td>
</tr>
<tr>
<td>$\chi^2/ndf$</td>
<td>3.4/8</td>
<td>5.6/8</td>
<td>9.9/18</td>
</tr>
<tr>
<td>Signal Yield</td>
<td>$2147 \pm 69$</td>
<td>$1108 \pm 45$</td>
<td>-</td>
</tr>
<tr>
<td>Recon. efficiency</td>
<td>$(1.99 \pm 0.02) \times 10^{-4}$</td>
<td>$(1.09 \pm 0.02) \times 10^{-4}$</td>
<td>-</td>
</tr>
<tr>
<td>$B$</td>
<td>$(2.31\pm0.08 \pm 0.09)%$</td>
<td>$(2.23\pm0.11 \pm 0.11)%$</td>
<td>$(2.17\pm0.06 \pm 0.09)%$</td>
</tr>
</tbody>
</table>

### $G(1)|V_{cb}|$ Values

<table>
<thead>
<tr>
<th></th>
<th>$B^+ \to D^0 e^+ \nu_e$</th>
<th>$\bar{B}^+ \to \bar{D}^0 \mu^+ \nu_\mu$</th>
<th>$B^0 \to D^- e^+ \nu_e$</th>
<th>$B^0 \to D^- \mu^+ \nu_\mu$</th>
<th>$B \to D \ell^- \bar{\nu}_\ell$</th>
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</thead>
<tbody>
<tr>
<td>$\eta_{EW}G(1)</td>
<td>V_{cb}</td>
<td>[10^{-3}]$</td>
<td>$42.31 \pm 1.94$</td>
<td>$45.48 \pm 1.96$</td>
<td>$41.84 \pm 2.14$</td>
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<tr>
<td>$\rho^2$</td>
<td>$1.05\pm0.08$</td>
<td>$1.22\pm0.07$</td>
<td>$1.01\pm0.10$</td>
<td>$1.08\pm0.10$</td>
<td>$1.09\pm0.05$</td>
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<tr>
<td>Correlation</td>
<td>0.81</td>
<td>0.77</td>
<td>0.85</td>
<td>0.84</td>
<td>0.69</td>
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<tr>
<td>$\eta_{EW}</td>
<td>V_{cb}</td>
<td>[10^{-3}]$</td>
<td>$40.14 \pm 1.86$</td>
<td>$43.15 \pm 1.89$</td>
<td>$39.69 \pm 2.05$</td>
</tr>
<tr>
<td>$\chi^2/ndf$</td>
<td>2.19/8</td>
<td>2.71/8</td>
<td>9.65/8</td>
<td>4.36/8</td>
<td>4.57/8</td>
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<tr>
<td>Prob.</td>
<td>0.97</td>
<td>0.95</td>
<td>0.29</td>
<td>0.82</td>
<td>0.80</td>
</tr>
</tbody>
</table>
$G(1)|V_{cb}|$: effect of the parameterization

- Combined fit with Lattice data beyond zero-recoil using BGL parameterization
  
  Series truncated at $n=3$

| Lattice data          | $\eta_{EW}|V_{cb}| [10^{-3}]$ | $\chi^2/n_{df}$ | Prob. |
|-----------------------|---------------------------------|-----------------|-------|
| FNAL/MILC [15]        | $40.96 \pm 1.23$                | 6.01/10         | 0.81  |
| HPQCD [32]            | $41.14 \pm 1.88$                | 4.83/10         | 0.90  |
| FNAL/MILC & HPQCD [15, 32] | $41.10 \pm 1.14$                | 11.35/16        | 0.79  |

With the most recent FF normalization FNAL/MILC'15
$G(1)=1.0541 \pm 0.0083$

CLN fit: $|V_{cb}| = (39.86 +/- 1.33) \times 10^{-3}$
BGL fit: $|V_{cb}| = (40.83 +/- 1.13) \times 10^{-3}$

Critical discussion on the FF parameterizations, using both Belle and BaBar data reported in Bigi, Gambino Phys.Rev.D 94,094008(2016)

<table>
<thead>
<tr>
<th></th>
<th>$B^+ \rightarrow D^0 e^+ \nu_e$</th>
<th>$B^+ \rightarrow D^0 \mu^+ \nu_\mu$</th>
<th>$B^0 \rightarrow D^- e^+ \nu_e$</th>
<th>$B^0 \rightarrow D^- \mu^+ \nu_\mu$</th>
<th>$B \rightarrow D \ell \nu_\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{EW}G(1)</td>
<td>V_{cb}</td>
<td>[10^{-3}]$</td>
<td>42.31 $\pm$ 1.94</td>
<td>45.48 $\pm$ 1.96</td>
<td>41.84 $\pm$ 2.14</td>
</tr>
<tr>
<td>$\rho^2$</td>
<td>1.05 $\pm$ 0.08</td>
<td>1.22 $\pm$ 0.07</td>
<td>1.01 $\pm$ 0.10</td>
<td>1.08 $\pm$ 0.10</td>
<td>1.09 $\pm$ 0.05</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.81</td>
<td>0.77</td>
<td>0.85</td>
<td>0.84</td>
<td>0.69</td>
</tr>
<tr>
<td>$\eta_{EW}</td>
<td>V_{cb}</td>
<td>[10^{-3}]$</td>
<td>40.14 $\pm$ 1.86</td>
<td>43.15 $\pm$ 1.89</td>
<td>39.69 $\pm$ 2.05</td>
</tr>
<tr>
<td>$\chi^2/n_{df}$</td>
<td>2.19/8</td>
<td>2.71/8</td>
<td>9.65/8</td>
<td>4.36/8</td>
<td>4.57/8</td>
</tr>
<tr>
<td>Prob.</td>
<td>0.97</td>
<td>0.95</td>
<td>0.29</td>
<td>0.82</td>
<td>0.80</td>
</tr>
</tbody>
</table>
\[ \eta_{EW} \mathcal{F}(1) |V_{cb}| = (35.61 \pm 0.43) \times 10^{-3} \]
\[ \rho^2 = 1.205 \pm 0.026 \]
\[ R_1(1) = 1.404 \pm 0.032 \]
\[ R_2(1) = 0.854 \pm 0.020 \]

Only published unquenched calculation available is at zero-recoil from FANL/MILC


Unfortunately these old data cannot be re-analysed with a different parameterization
B → D*ℓν: news from Belle

- With the hadronic tag, similar to B → D
- Signal extracted from the missing mass distribution by a unbinned maximum likelihood fit
- Yields extracted in 4x10 bins of w and 3 angular variables: statistical correlations determined with bootstrapping technique
- For the first time published the Unfolded distributions
$B \rightarrow D^* \ell \nu$: news from Belle

- With the hadronic tag, similar to $B \rightarrow D$
- Signal extracted from the missing mass distribution by a unbinned maximum likelihood fit
- Yields extracted in $4 \times 10$ bins of $w$ and 3 angular variables: statistical correlations determined with bootstrapping technique
- For the first time published the Unfolded distributions

Belle fit with CLN parameterization consistent with world average
- Bigi, Gambino, Schacht Phys.Lett B 769 (2017) 441: Critical analysis of parameterization with the Belle data

<table>
<thead>
<tr>
<th></th>
<th>CLN Fit: Data + lattice</th>
<th>CLN Fit: Data + lattice + LCSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/dof</td>
<td>34.3/36</td>
<td>34.8/39</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>BGL Fit: Data + lattice</th>
<th>BGL Fit: Data + lattice + LCSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/dof</td>
<td>27.9/32</td>
<td>31.4/35</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$</td>
</tr>
</tbody>
</table>

- This result points to a systematic difference between CLN and a model-independent parameterization
- Similar analysis in Grinstein, Kobach arXiv.1703.0817, who claimed
  - “strong possibility that the tension between inclusive and exclusive $|V_{cb}|$ .... is due to the use of the CLN parameterization...”
New global picture?

$B \rightarrow D \ell \nu$

$B \rightarrow D^* \ell \nu$

Belle BGL

Inclusive

$|V_{cb}|$, global fit in KS

$|V_{ub}|$, HFLAV GGOU

${\text{UTFit}}$

$B \rightarrow \pi^+ \ell \nu$

$\Lambda_b \rightarrow p \mu \nu$

$|V_{ub}| [10^{-3}]$

$|V_{cb}| [10^{-3}]$
Remarks

- CLN can be affected by underestimated uncertainties
  - An uncertainty of “better than 2%” on the FF quoted in CLN paper, with the increasing precision, cannot be neglected anymore
- It is crucial to move to a model-independent parameterization, CLN is too constrained
  - Unfortunately existing HFLAV average uses measurements based on CLN
- But the difference BGL-CLN of about ~8% on $|V_{cb}|$ from the recent Belle data, cannot be considered the only systematic missed in the existing average that fill the gap with the inclusive: CLN fit well old precise data!

Remarks

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- We should not neglect that there is only one lattice calculation for F(1)
- Recently HPQCD F(1)=0.862(35) C.Davie at CKM2016
  - Lower than FNAL/MILC!
  - HQSum-Rule, F(1) = 0.86(2)
- Calculations at non-zero recoil could be desirable
Conclusione

- **Exclusive Vub**
  - huge progressed on lattice
  - LHCb is a new player: opened the route to $B_s \rightarrow K \ell \nu$ (cleanest on Lattice!)

- **Inclusive Vub**
  - It is still a puzzle: internally consistent but above CKM fit and exclusive
  - Partial rates that include the $b \rightarrow c$ region depends on the signal model: crucial to consider this and use the same model for both signal extraction and $|V_{ub}|$
  - Theory/parameters uncertainties dominate: need to constrain the SF (global fit $V_{cb}$-like from spectra measurements: SIMBA, NNVUB)

- **Inclusive Vcb**
  - Everything consistent and it gives inputs to $V_{ub}/SF$: it would be desirable an update of the 1S scheme framework

- **Exclusive Vcb**
  - General agreement to move to model independent FF parameterizations
  - New Lattice-FF calculation for $B \rightarrow D^*$ (even a non-zero recoil) are on the way from MILC/FNAL and HPQCD
**Semileptonic Decays**

- **Inclusive decays** $B \rightarrow X_u \ell \nu$:
  - QCD corrections to parton level decay rate
  - Operator Production Expansion in $\alpha_s$ and $\Lambda/m_b$

- **Exclusive decays** $B \rightarrow \pi/\rho \ell \nu$:
  - QCD correction parameterized in the Form Factors
  - Lattice-QCD, LCSR
Experiments: B-Factories

B-Factories: hermetic detectors, low background, access (mainly) at B^{0/+}

@ KEK Japan: 1999-2009

@ SLAC: 1999-2008

About \((771 + 467) \times 10^6\) e^+e^- → Y(4S) → BB events in the Belle+Babar data

Belle-II aims to collect 50ab^{-1} by 2024

Belle and KEK is being upgraded
LHCb: forward spectrometer for flavor physics
Excellent tracking and vertexing capabilities.
Excellent PID performances
Access to all hadrons with b- and c-quarks

Collected 3.0 fb\(^{-1}\) in 2011-2012
The gap problem

<table>
<thead>
<tr>
<th>charm state $X_c$</th>
<th>$B(B \to X_c \ell \bar{\nu})$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>2.29 ± 0.09</td>
</tr>
<tr>
<td>$D^*$</td>
<td>5.43 ± 0.17</td>
</tr>
<tr>
<td>$\sum D^{(*)}$</td>
<td>7.71 ± 0.19</td>
</tr>
<tr>
<td>$D_0^{*} \to D \pi$</td>
<td>0.41 ± 0.08</td>
</tr>
<tr>
<td>$D_1^{<em>} \to D^{</em>} \pi$</td>
<td>0.45 ± 0.09</td>
</tr>
<tr>
<td>$D_1 \to D^{*} \pi$</td>
<td>0.43 ± 0.03</td>
</tr>
<tr>
<td>$D_2^{<em>} \to D^{(</em>)} \pi$</td>
<td>0.41 ± 0.03</td>
</tr>
<tr>
<td>$\sum D^{**} \to D^{(*)} \pi$</td>
<td>1.70 ± 0.12</td>
</tr>
<tr>
<td>$D_s^{(*)^-} K^+$</td>
<td>0.06 ± 0.01</td>
</tr>
<tr>
<td>$D^{\pi}$</td>
<td>0.66 ± 0.08</td>
</tr>
<tr>
<td>$D^{*} \pi$</td>
<td>0.87 ± 0.10</td>
</tr>
<tr>
<td>$\sum D^{(*)} \pi$</td>
<td>1.53 ± 0.13</td>
</tr>
<tr>
<td>$\sum D^{(<em>)} + \sum D^{**} \to D^{(</em>)} \pi + D_s^{(*)^-} K^+$</td>
<td>9.47 ± 0.22</td>
</tr>
<tr>
<td>$\sum D^{(<em>)} + \sum D^{(</em>)} \pi + D_s^{(*)^-} K^+$</td>
<td>9.30 ± 0.23</td>
</tr>
</tbody>
</table>

Inclusive – $\Sigma$exclusive = (1.51 ± 0.26) %

From T.Lueck @EPS2015
Status of the “gap”

\[
\Delta BF = BF_{\text{incl}} - \sum \left[ BF(\overline{B} \to D^{(*)}(\pi)K \ell \nu) + BF(\overline{B} \to D_s^{(*)}(K)\ell \nu) \right]
\]

Extrapolation to $\pi\pi$ with implicit isospin assumptions

\[
\Delta BF \rightarrow
\]

gap reduced from $\approx 7\sigma$ to $\approx 3\sigma$

extrapolation to full $B$ assumed $\Gamma(D^{(*)}\pi^+\pi^-\ell\nu)/\Gamma(D^{(*)}\pi\pi\ell\nu) = 0.50 \pm 0.17$

From T. Lueck
@EPS2015