

# Quasiparticle anisotropic hydrodynamics for ultrarelativistic heavy-ion collisions

Based on arXiv:1703.05808, and arXiv:1705.10191

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# Motivation

- Viscous hydrodynamics is derived by linearization around an isotropic equilibrium distribution function

$$f(x, p) = f_{\text{eq}} \left( \frac{p^\mu u_\mu}{T} \right) [1 + \delta f(x, p)]$$

- However, QGP in the local rest frame is a highly anisotropic plasma.
  - At early times,  $P_L/PT \ll 1$  which relaxes slowly towards isotropy at late times.
  - At all times, large momentum-space anisotropies at the system edges.
- To take this into account in anisotropic hydrodynamics (aHydro), momentum-space anisotropies are included from the beginning

$$f(x, p) = f_{\text{eq}} \left( \frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)} \right) + \delta f(x, p)$$

# Motivation

- Anisotropic hydrodynamics is expected to do better at
    - Early time dynamics and near the system edges
    - Small systems (p+A, p+p)
    - Temperature dependent  $\frac{\eta}{s}(T)$
- See , M. Strickland, arXiv:1410.5786
- Some situations where aHydro does well compared with other methods:
    - Exact Boltzmann solutions in 0+1d systems
    - Cold atoms.
- See: M. Bluhm and T. Schaefer, arXiv:1512.00862
- The ultimate goal: To extract transport coefficients from data: the shear viscosity to entropy density ratio ( $\eta/s$ ) and the bulk viscosity ( $\zeta/s$ ) in a more reliable way, more accurately than other methods.
  - As we will see, we were able to fit the data with a much smaller bulk viscosity than other methods suggest.

# Anisotropic hydro distribution function

- The distribution function in aHydro is

$$f(x, p) = f_{\text{eq}} \left( \frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)} \right) + \delta f(x, p)$$

- Here  $\Xi_{\mu\nu}$  is the anisotropy tensor which in aHydro replaces the viscous stress tensor in viscous hydro.

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu} \Phi$$

$u^\mu$  LRF four velocity

$\xi^{\mu\nu}$  the traceless anisotropy tensor

$\Delta^{\mu\nu}$  the transverse projector

$\Phi$  the degree of freedom associated with bulk

- In leading-order aHydro,  $\delta f$  is assumed to be zero. In this case, the distribution function is positive definite in both the hydro stage and at freeze-out.

# The dynamical equations for aHydroQP

- The dynamical equations can be found by taking moments of the Boltzmann equation

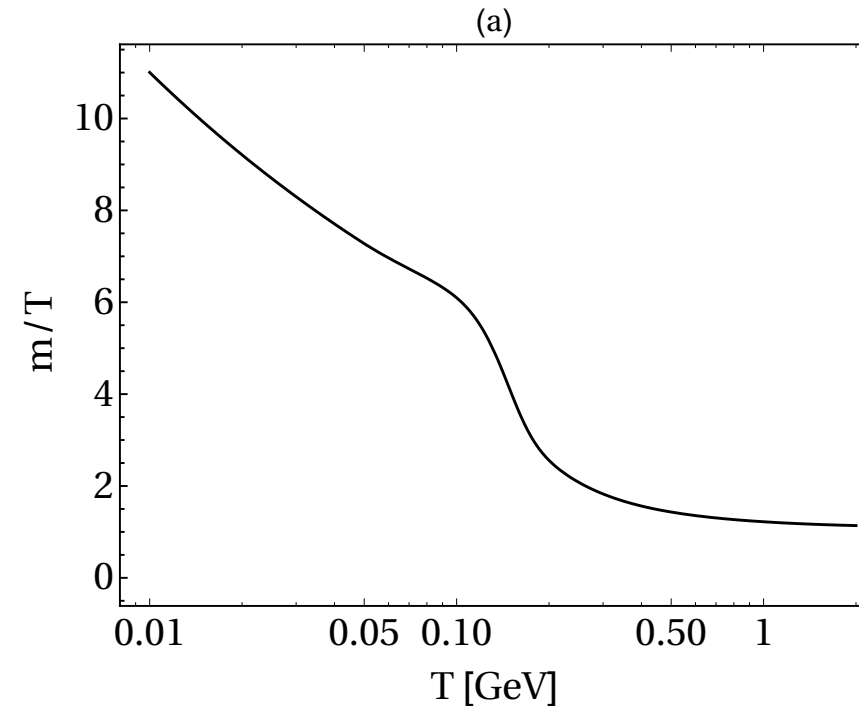
$$p^\mu \partial_\mu f + \frac{1}{2} \partial_i m^2 \partial_{(p)}^i f = -\mathcal{C}[f]$$

- Thermodynamic consistency is not guaranteed when including thermal mass  $m(T)$ , i.e.,  $S \neq \frac{\partial P}{\partial T}$ . So, a background field is introduced

$$T^{\mu\nu} = T_{\text{kinetic}}^{\mu\nu} + g^{\mu\nu} B$$
$$\partial_\mu B = -\frac{1}{2} \partial_\mu m^2 \int dP f(x, p)$$

- From lattice results  $m(T)$  can be obtained

$$\mathcal{S}_{\text{eq}} = 4\pi \tilde{N} m^3 K_3(m/T)$$



# 3+1d aHydroQP equations of motion

- In leading-order aHydro, we have eight degrees of freedom  $\alpha_x, \alpha_y, \alpha_z, u_x, u_y, u_z, \lambda$ , and  $T$ .
- We use 7 equations from the first and second moment and the matching condition to determine the effective temperature.

- First moment equations:

$$D_u \mathcal{E} + \mathcal{E} \theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu = 0$$

$$D_x \mathcal{P}_x + \mathcal{P}_x \theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu = 0$$

$$D_y \mathcal{P}_y + \mathcal{P}_y \theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu = 0$$

$$D_z \mathcal{P}_z + \mathcal{P}_z \theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu = 0$$

- Second moment equations:

$$D_u \mathcal{I}_x + \mathcal{I}_x (\theta_u + 2u_\mu D_x X^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_x]$$

$$D_u \mathcal{I}_y + \mathcal{I}_y (\theta_u + 2u_\mu D_y Y^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_y]$$

$$D_u \mathcal{I}_z + \mathcal{I}_z (\theta_u + 2u_\mu D_z Z^\mu) = \frac{1}{\tau_{\text{eq}}} [\mathcal{I}_{\text{eq}}(T, m) - \mathcal{I}_z]$$

- The matching condition:

$$\mathcal{E}(\alpha, \lambda) = \mathcal{E}_{\text{eq}}(T)$$

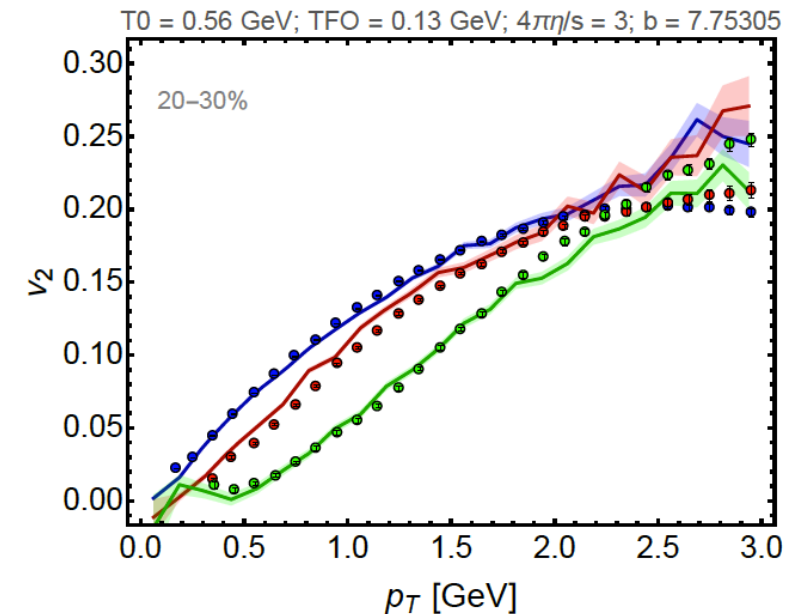
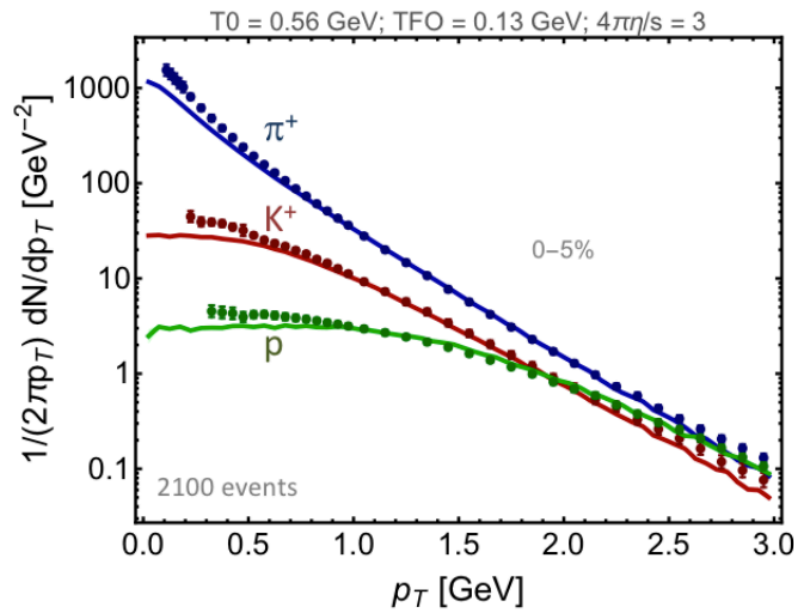
# Comparisons with ALICE data using aHydro

- Here, the authors used the standard way for imposing the equation of state which is based on the factorization, the system is taken to be massless but the equation of state enters through the bulk variables.

$$\mathcal{E} = \mathcal{H}_3(\alpha)\mathcal{E}_{\text{iso}}(\lambda)$$

- We can replace the energy density then by the equilibrium one coming from realistic EoS.

- M. Nopoush, R. Ryblewski, and M. Strickland, 1610.10055
- M. Strickland, 1611.05056



- This model fits the data reasonably well when compared the spectra and differential  $v_2$ .
- One troubling problem, the underestimation of the spectra at low  $p_T$ .

# Comparisons with ALICE data using aHydroQP

- M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1703.05808
- M. Alqahtani, M. Nopoush, R. Ryblewski, and M. Strickland, arXiv:1705.10191

- We solve 3+1d quasiparticle anisotropic hydrodynamics.
- The system is assumed initially to be isotropic in momentum space.
- We used smooth Glauber initial conditions.
- We consider ALICE 2.76 TeV Pb-Pb collisions.
- We use anisotropic Cooper-Frye freeze-out to extract the freeze-out hypersurface.
- Then, we use THERMINATOR 2 to perform the hadronic production decays.
- The parameters we obtained from our fit are

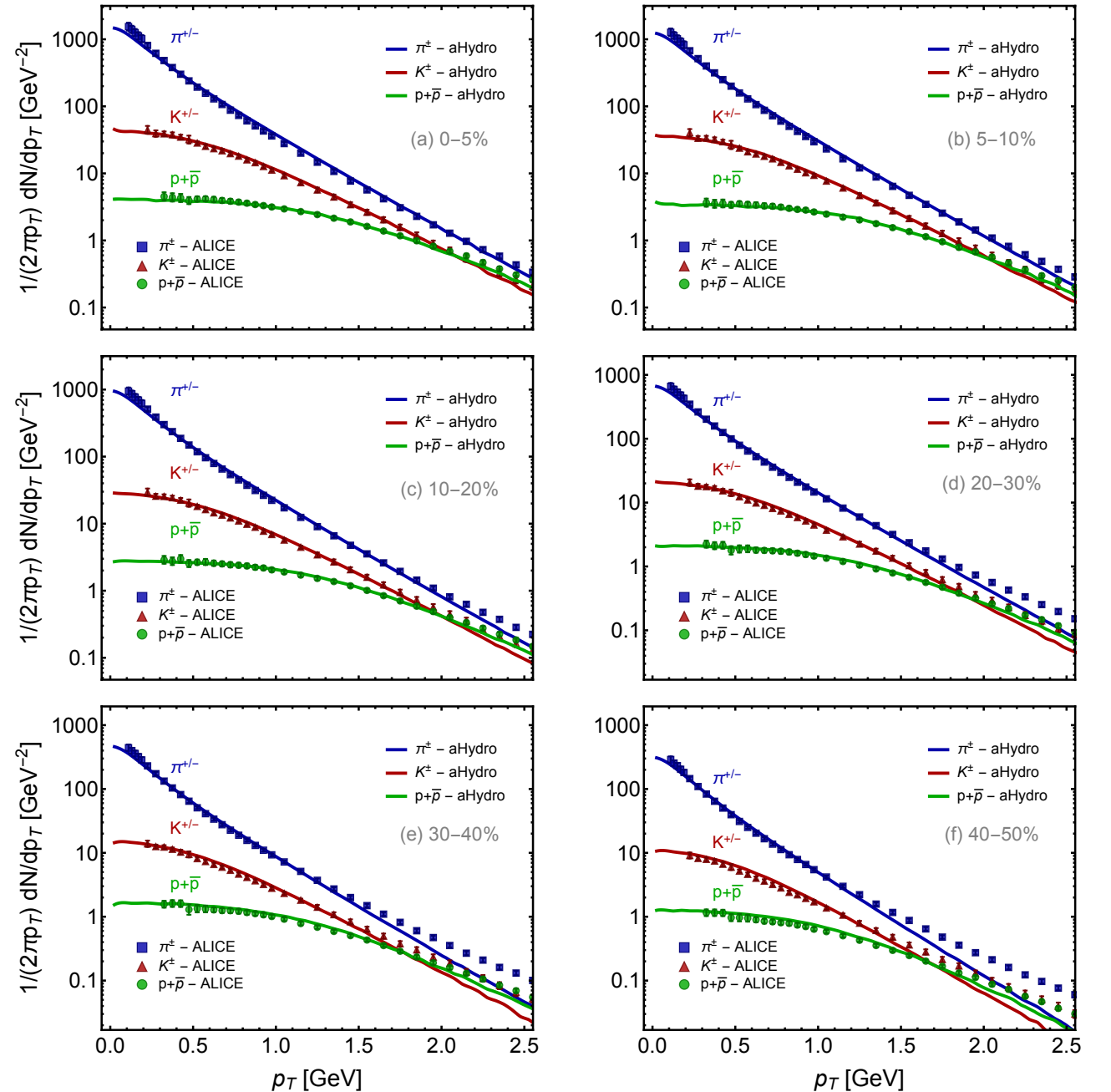
$$\tau_0 = 0.25 \text{ fm}/c$$

$$\begin{aligned} T_0 &= 600 \text{ MeV} \\ \frac{\eta}{s} &= 0.159 \\ T_{\text{FO}} &= 130 \text{ MeV} \end{aligned}$$

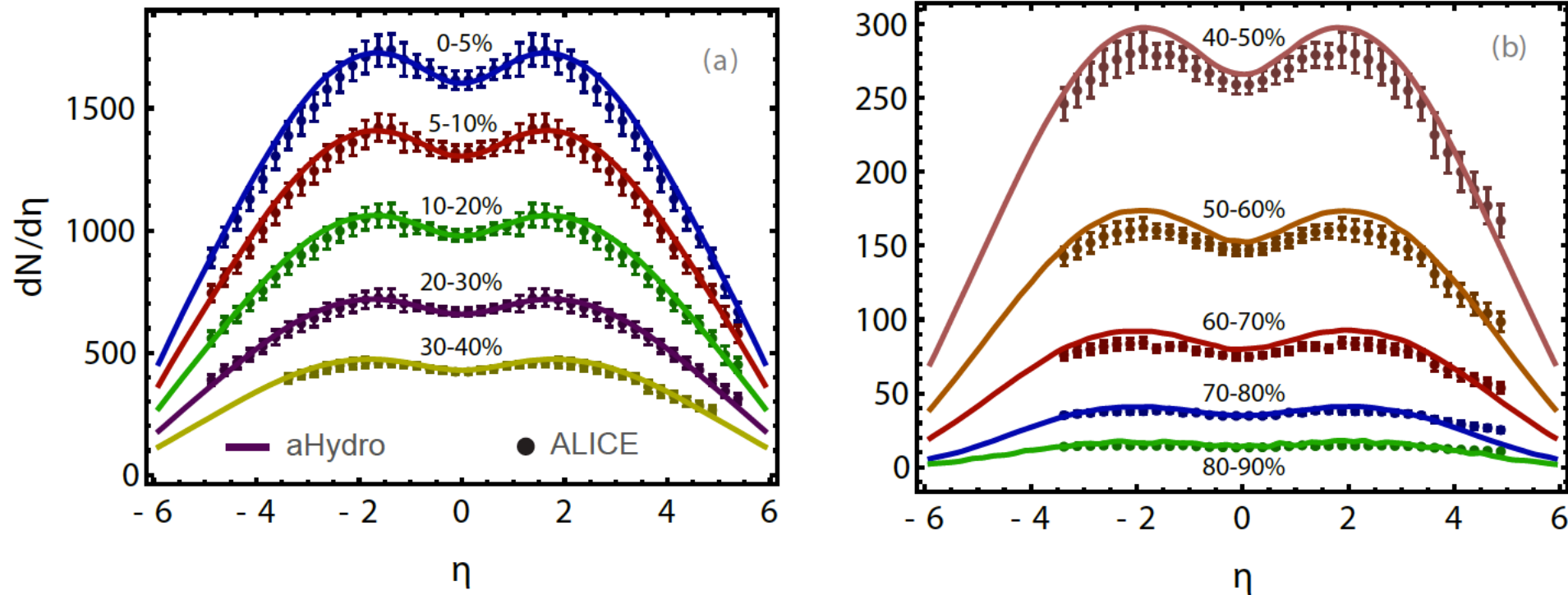


# Pions, kaons, and protons spectra

- We fix the underestimation of the spectra predicted by the standard way for imposing EoS.
- Our fits to the data show very good agreement including the mass splitting between different hadrons (pion, kaon, protons).
- The largest differences appear at relatively high centrality classes, e.g, our model shows good agreement only up to  $p_T \approx 1.5$  GeV in 40-50%, where in 0-5% up to  $p_T \approx 2.5$  GeV.



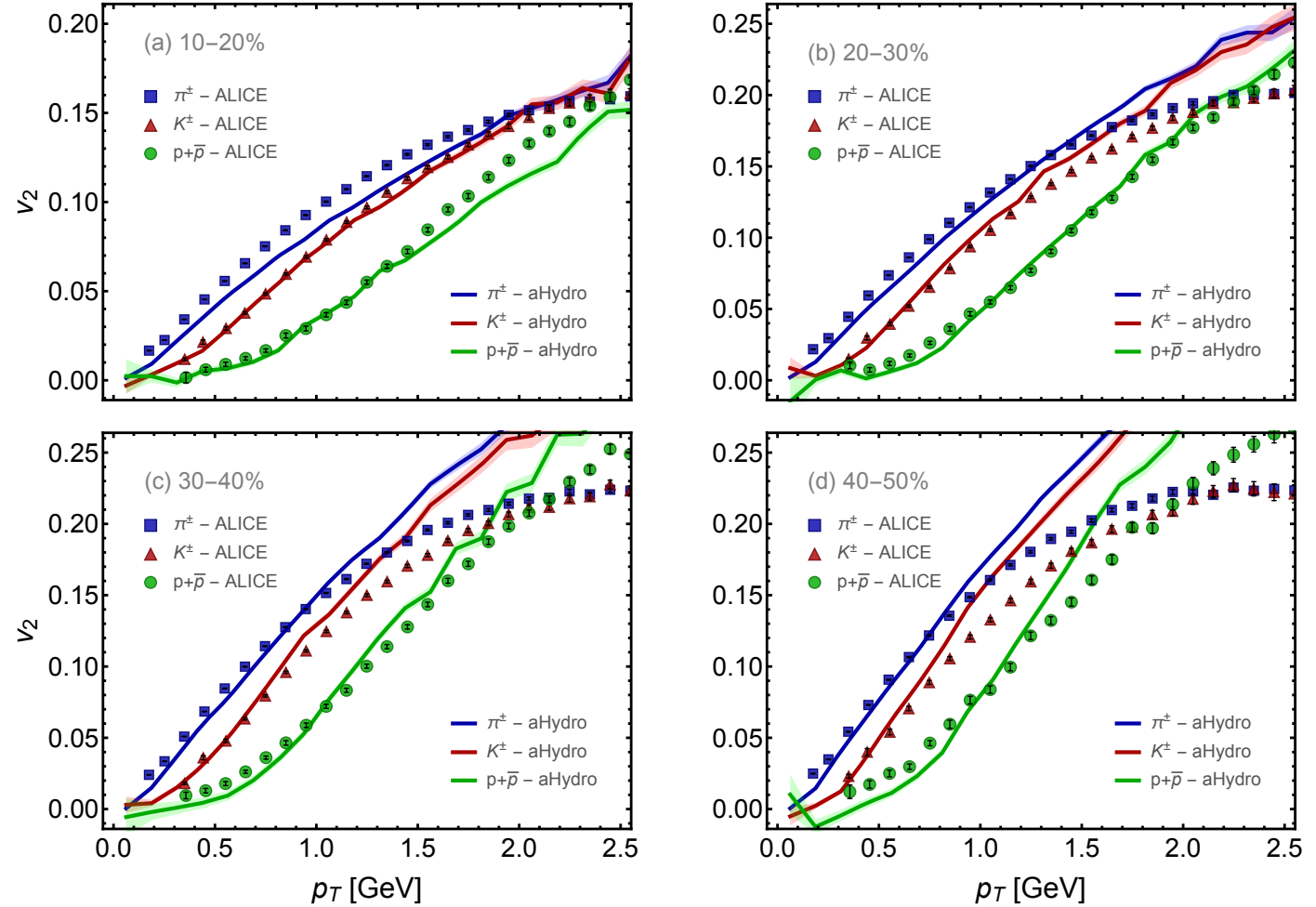
# Charged-hadron multiplicity



- Our model shows good agreement with data for the charged hadron multiplicity as a function of pseudorapidity in many different centrality classes.

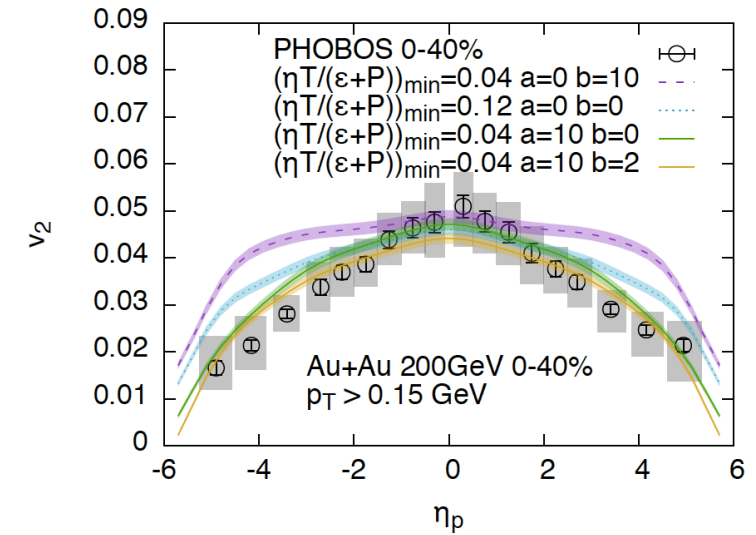
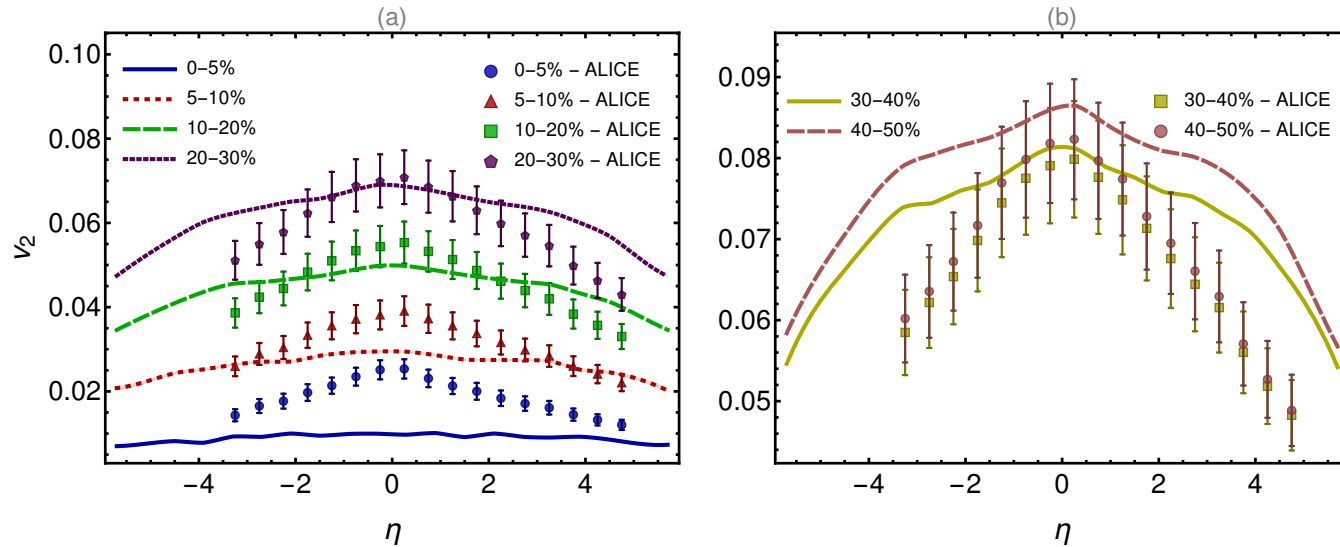
## $V_2$ for identified hadrons as a function of $p_T$

- Overall, our model shows a quite reasonable description of the data.
- In panels (b) and (c), the agreement is quite good, our model reproduces the data for pion, kaon, and proton out to  $p_T \sim 1.5$ , 1.5, and 2.5 GeV.
- In panel (a) and (d), the agreement is less.
- This difference can be related to using smooth Glauber initial conditions and/or constant  $\eta/s$ .



# The pseudorapidity dependence of $v_2$

G. Denicol, A. Monnai, and B. Schenke arXiv:1512.01538



- We show the pseudorapidity dependence of  $v_2$  for different centrality classes.
- At large pseudorapidity, we note that our model results do not fall fast enough compared with experimental data.
- This can be related to using a constant  $\eta/s$ , using a temperature-dependent  $\eta/s(T)$  can improve the agreement (work in progress).

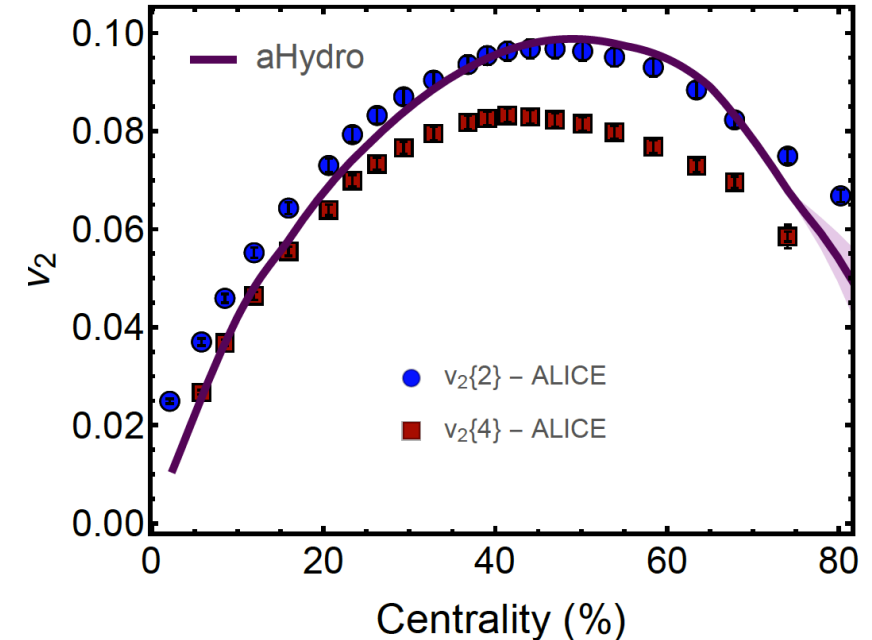
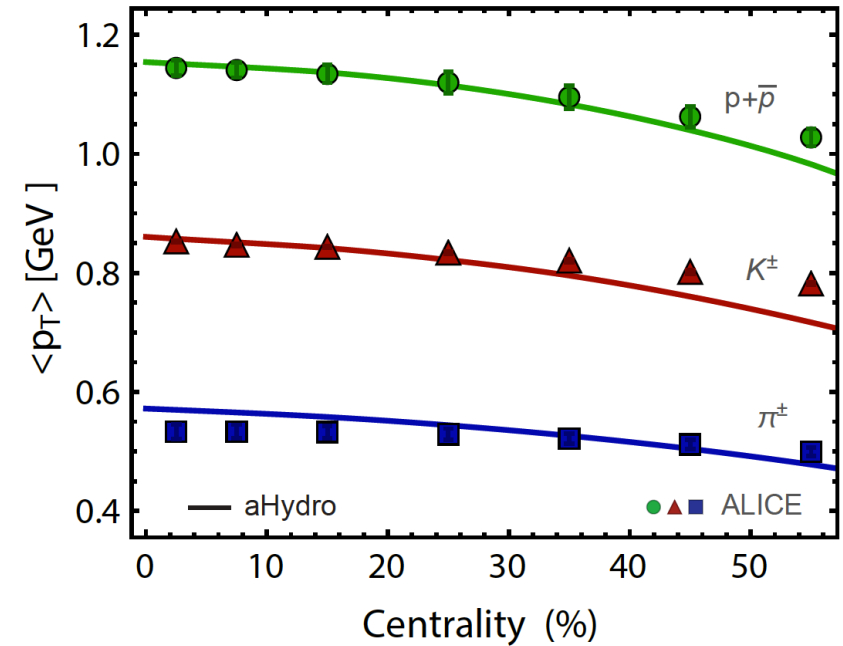
## The average transverse momentum

- Our model reproduces the data quite well for the mean transverse momentum as a function of centrality.
- The agreement that our model provides is similar to what other models find.

See: S. Ryu, J. F. Paquet, etc , arXiv: 1502.01675

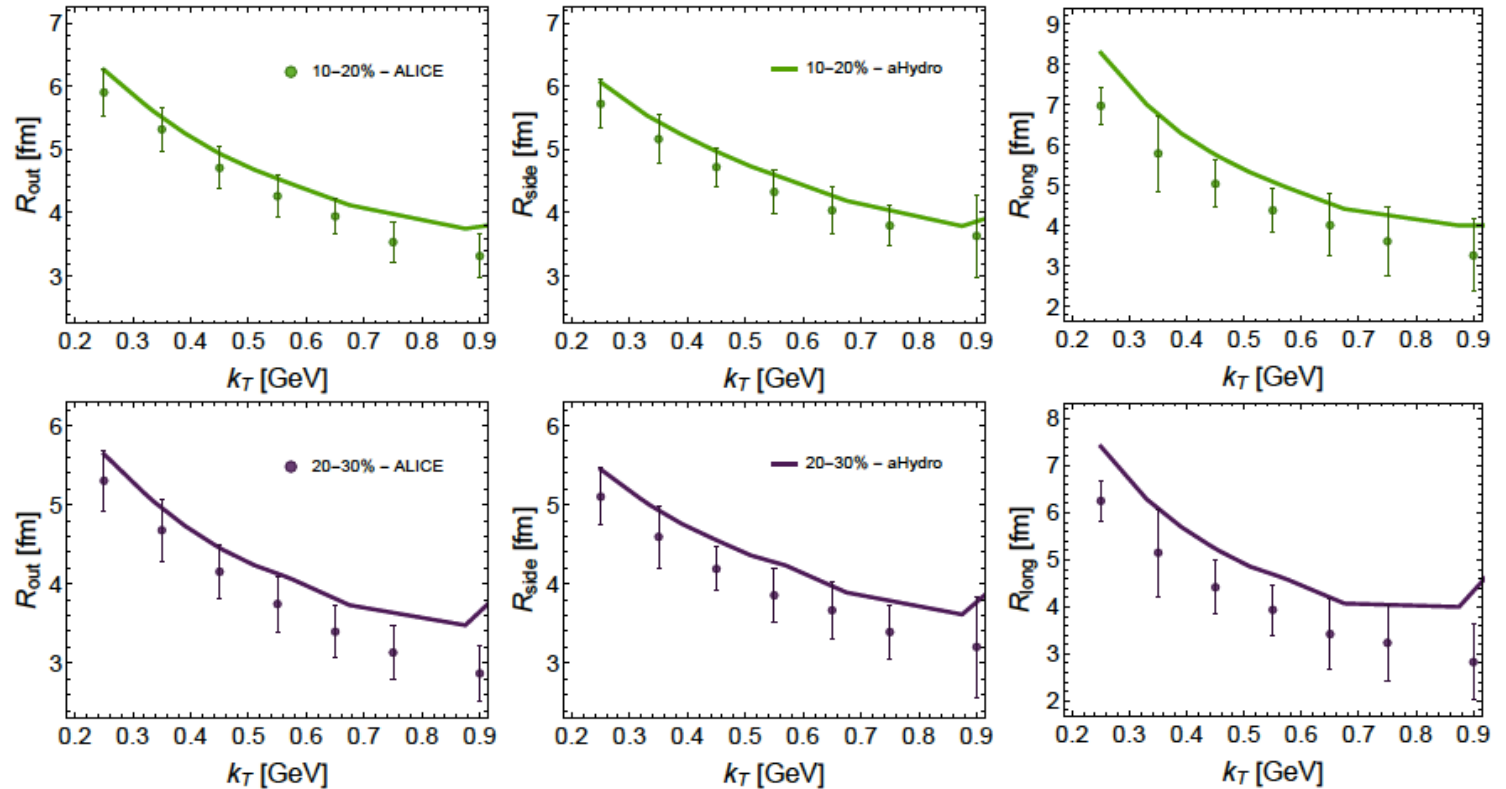
## $V_2$ as a function of centrality

- We compared also the integrated elliptic flow for charged hadrons as a function of centrality where our model shows a quite good agreement.
- Our model underestimates the  $v_2$  at very central collisions, this is due to using smooth initial conditions.



# HBT radii

- We show comparisons for HBT radii as a function of mean transverse momentum of the pair  $\pi^+\pi^+$ .
- Our model is able to describe trends seen in the data with the same parameters used to describe other observables.



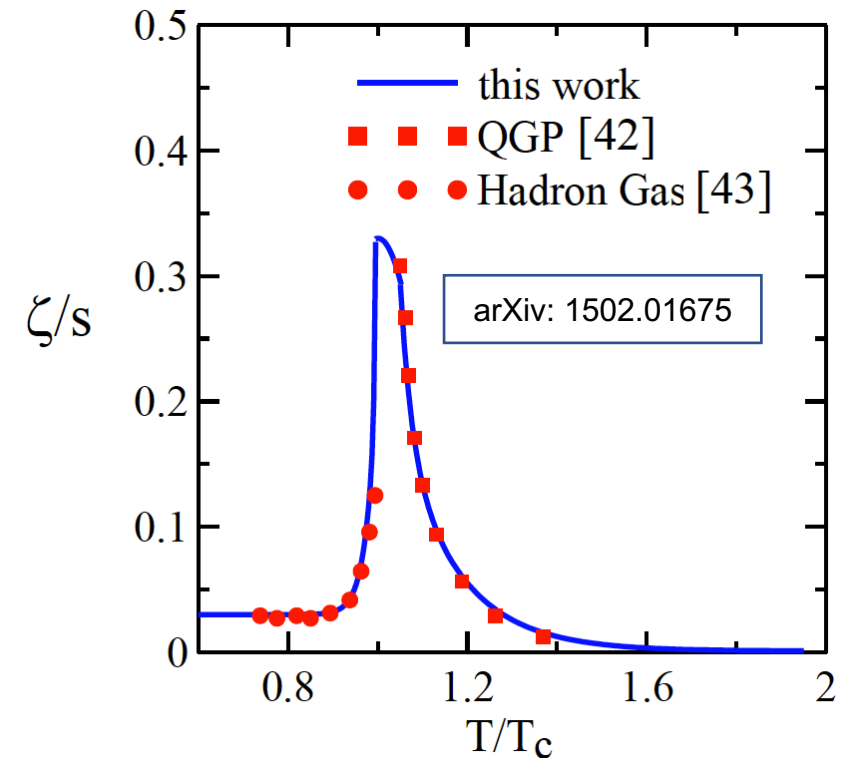
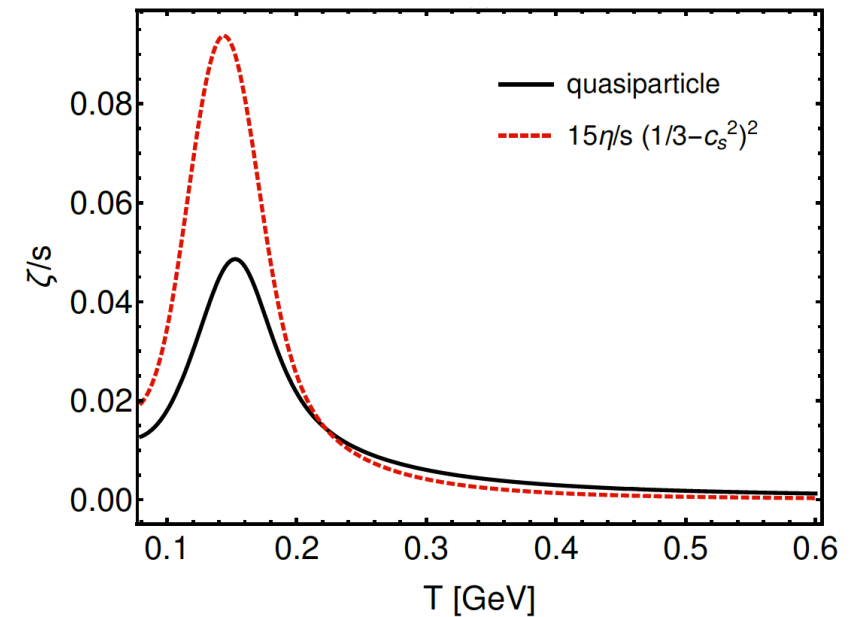
## The bulk viscosity

- The peak value of the bulk viscosity predicted by our model (quasiparticle) is much lower than other models predictions. For example, the peak value predicted was  $\zeta/s \sim 0.3$  in

See: S. Ryu, J. F. Paquet, etc , arXiv: 1502.01675

- Note that we use  $\eta/s = 0.159$ , however they use  $\eta/s = 0.09$ .
- However, using Bayesian analyses it was found that the peak value is similar to our results.

See: S. A. Bass, J. E. Bernhard, and J. S. Moreland, arXiv: 1704.07671



# Conclusions and outlook

- Anisotropic hydrodynamics takes into account momentum-space anisotropies of the QGP from the beginning.
- Quasiparticle anisotropic hydrodynamics more self-consistently treats the non-conformality of the QGP than prior approaches.
- Using aHydroQP, we were able to fit the ALICE 2.76 TeV Pb-Pb collisions quite well.
- Our model predicts  $\frac{\eta}{s} = 0.159$ ,  $T_0 = 600$  MeV at  $\tau_0 = 0.25$  fm/c,  $T_{FO} = 130$  MeV, and the peak value of  $\zeta/s \sim 0.05$  which is smaller than what other viscous studies found  $\sim 0.3$ .
- Looking to future, we are working on improving our code in many ways
  - Including fluctuating initial conditions.
  - Including temperature dependence of  $\frac{\eta}{s}$
  - Realistic collisional kernels instead of relaxation time approximation (RTA).
  - Including elastic hadronic collisions using some available codes on the market like URQMD or SMASH.



Backup slides

# HBT radii in more centrality classes

