Scale Invariant Resummed Thermal Perturbative Expansion

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Context: QCD phase diagram/ Quark Gluon Plasma

Complete QCD phase diagram far from being confirmed:

$T \neq 0, \mu = 0$ well-established from lattice: no sharp phase transition, continuous crossover at $T_c \simeq 154 \pm 9$ MeV

Goal: more analytical approximations, ultimately in regions not much accessible on the lattice: large density (chemical potential) due to the (in)famous “sign problem”

Introduction/Motivations

Context: (unconventional) resummation of perturbative expansions

Very general: relevant both at $T = 0$ or $T \neq 0$ (and finite density)
$\rightarrow$ addresses well-known problems of unstable +badly scale-dependent thermal perturbative expansions:
Illustrate here $T \neq 0$ σ model, + (preliminary) QCD (pure glue)

NB Previous results ($T = 0$):
estimate with our RGOPT approach the order parameter
$F_\pi(m_q = 0)/\Lambda^{\rm QCD}_{\overline{MS}}$:
$F_\pi \simeq 92.2\text{MeV} \rightarrow F_\pi(m_q = 0) \rightarrow \Lambda^{n_f=3}_{\overline{MS}} \rightarrow \alpha_S^{\overline{MS}}(\mu = m_Z)$.

N^3LO: $F_\pi^{m_q=0}/\Lambda^{n_f=3}_{\overline{MS}} \simeq 0.25 \pm .01 \rightarrow \alpha_S(m_Z) \simeq 0.1174 \pm .001 \pm .001$
(JLK, A.Neveu, PRD88 (2013))
(compares well with latest (2016) α_S lattice and world average values [PDG2016])

Also applied to $\langle \bar{q}q \rangle$ at N^3LO (using spectral density of Dirac operator):
$\langle \bar{q}q \rangle^{1/3}_{m_q=0}(2\,\text{GeV}) \simeq -(0.84 \pm 0.01)\Lambda_{\overline{MS}}$ (JLK, A.Neveu, PRD 92 (2015))
(compares well with latest most precise lattice value.)

(Variationally) Optimized Perturbation (OPT)

Trick ($T = 0$): add and subtract a mass, consider $m\,\delta$ as interaction:

$$\mathcal{L}_{QCD}(g, m) \to \mathcal{L}_{QCD}(\delta\, g, m(1-\delta)) \;\; \text{(e.g. in QCD } g \equiv 4\pi\alpha_S)$$

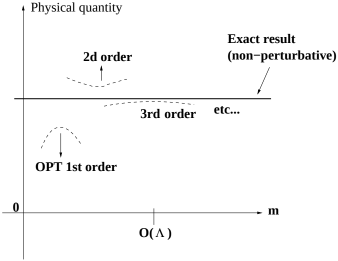
where $0 < \delta < 1$ interpolates between $\mathcal{L}_{free}$ and *massless* $\mathcal{L}_{int}$;
e.g. (quark) mass $m_q \to m$: arbitrary trial parameter

• Take any standard (renormalized) QCD pert. series, expand in δ *after:*
$m_q \to m\,(1-\delta)$; $\quad g \to \delta\, g$
then take $\delta \to 1$ (to recover original massless theory):

BUT a m-dependence remains at any finite δ^k-order:
fixed typically by stationarity prescription: optimization (OPT):
$\frac{\partial}{\partial m}$(physical quantity) $= 0$ for $m = \bar{m}_{opt}(\alpha_S) \neq 0$:
• $T = 0$: exhibits *dimensional transmutation*: $\bar{m}_{opt}(g) \sim \mu\, e^{-const./g}$

•At $T \neq 0$, same idea dubbed "screened perturbation" (SPT), or "hard thermal loop (HTLpt) resummation", etc.
But does this 'cheap trick' always work? and why?

Expected behaviour (Ideally...)



But not quite what happens... except in simple models:
•Convergence proof of this procedure for $D = 1$ $g\phi^4$ oscillator (cancels large pert. order factorial divergences!) Guida et al '95

particular case of 'order-dependent mapping' Seznec, Zinn-Justin '79

•But in QFT: multi-loop calculations (specially $T \neq 0$) (very) difficult beyond first order:
→ what about convergence? not much apparent in fact

•Main pb at higher order: OPT: $\partial_m(...) = 0$ has multi-solutions (some complex!), how to choose right one, if no nonperturbative "insight"??

RG compatible OPT ($\equiv$ RGOPT)

Our main additional ingredient to OPT (JLK, A. Neveu 2010):

Consider a physical quantity (i.e. perturbatively RG invariant)
(in present context, will be the pressure $P(m, g, T)$):

in addition to OPT Eq: $\frac{\partial}{\partial m} P^{(k)}(m, g, \delta = 1)|_{m \equiv \tilde{m}} \equiv 0$,
Require (δ-modified!) series at order δ^k to satisfy a standard (perturbative) Renormalization Group (RG) equation:

$$\mathrm{RG}\left(P^{(k)}(m, g, \delta = 1)\right) = 0$$

with standard RG operator ($g = 4\pi\alpha_S$ for QCD):

$$\mathrm{RG} \equiv \mu \frac{d}{d\,\mu} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g)\, m \frac{\partial}{\partial m}$$

$\beta(g) \equiv -b_0 g^2 - b_1 g^3 + \cdots$, $\gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots$

$\rightarrow$ Additional nontrivial constraint (even if started from RG invariant standard perturbation): contains a priori more consistent RG 'information' than simple $\partial_m P(m)$ optimization.

RG compatible OPT (RGOPT)

$\rightarrow$ Combined with OPT, RG Eq. reduces to massless form:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]P^{(k)}(m,g,\delta=1)=0$$

Note: using OPT AND RG completely fix $m \equiv \bar{m}$ and $g \equiv \bar{g}$.

But $\Lambda_{\overline{\rm MS}}(g)$ satisfies by def.:
$[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}]\,\Lambda_{\overline{\rm MS}} \equiv 0$ consistently at a given pert. order for $\beta(g)$.

Thus equivalent to:

$$\frac{\partial}{\partial\, m}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{\rm MS}}(g)}\right)=0\ ;\quad \frac{\partial}{\partial\, g}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{\rm MS}}(g)}\right)=0 \text{ for } \bar{m},\bar{g}$$

Optimal $\bar{m},\bar{g}=4\pi\bar{\alpha}_S$ unphysical: final (physical) result from $P(\bar{m},\bar{g},T)$

At $T=0$ reproduces at first order exact nonperturbative results in simpler models [e.g. Gross-Neveu model]

OPT + RG = RGOPT main new features

•Standard OPT: embarrassing freedom (a priori) in interpolating form:
e.g. why not $m \to m\,(1-\delta)^a$?
Most previous works: linear case $a = 1$ for simplicity
but generally (we have shown) $a = 1$ spoils RG invariance!

•OPT,RG Eqs: many solutions at increasing δ^k-orders

→ Our approach restores RG, +requires OPT, RG sol. to match standard perturbation (e.g. Asymptotic Freedom for QCD ($T = 0$)):
$\alpha_S \to 0$, $\mu \to \infty$: $\bar{g} = 4\pi\bar{\alpha}_S \sim \frac{1}{2b_0 \ln \frac{\mu}{\bar{m}}} + \cdots$

→ At arbitrary order, AF-compatible RG + OPT branch, often unique, *only* appear for a critical universal a:
$m \to m\,(1-\delta)^{\frac{\gamma_0}{b_0}}$ (e.g. $\frac{\gamma_0}{b_0}(\mathrm{QCD}, n_f = 3) = \frac{4}{9}$)

→ Goes beyond simple "add and subtract" trick
+ It removes spurious solutions incompatible with AF
− But does not always avoid complex solutions
(if those (perturbative artifacts) occur, are possibly cured by renormalization scheme change [JLK, Neveu '13])

Problems of thermal perturbation (QCD and generic)

Main culprit: mix up of *hard* $p \sim T$ and *soft* $p \sim \alpha_S T$ modes.

Thermal 'Debye' screening mass $m_D^2 \sim \alpha_S T^2$ gives IR cutoff,
BUT $\Rightarrow$ perturbative expansion in $\sqrt{\alpha_S}$ in QCD
$\rightarrow$ advocated reason for slower convergence

Yet many interesting QGP physics features happen at not that large $\alpha_S(\gtrsim 2\pi T_c) \simeq .5$ or lower values.

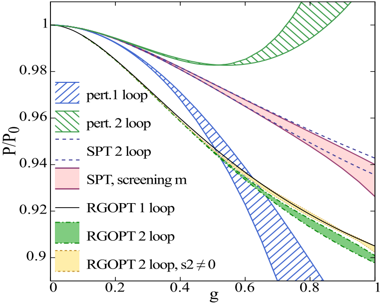
Many efforts to improve this (review e.g. Blaizot, Iancu, Rebhan '03):

Screened PT (SPT) (Karsh et al '97), $\sim$ Hard Thermal Loop (HTL) resummation (Andersen, Braaten, Strickland '99); Functional RG, 2-particle irreducible (2PI) formalism (Blaizot, Iancu, Rebhan '01; Berges, Borsanyi, Reinosa, J. Serreau '05)

RGOPT $T \neq 0$: essentially treats thermal mass 'RG consistently':
$\rightarrow$ UV divergences also induce its anomalous dimension.

(NB some qualitative connections with 2PI results, also with recent "massive scheme" approach (Blaizot, Wschebor '14)

Previous $T \neq 0$: two-loop RGOPT($g\phi^4$) vs standard PT and SPT



[JLK, M.B Pinto, PRL 116 (2016) [1507.03508]; PRD92 (2015)]

•Definite scale-dependence improvement (a factor ~ 3) w.r.t. SPT [J.O. Andersen et al '01]

•Improvement should be more drastic at 3-loops, where SPT scale dependence strongly increases.

How this is obtained: details next for the nonlinear σ model

One step closer to QGP: $O(N)$ nonlinear σ model (NLSM)

[G. Ferreri, JLK, M.B. Pinto, R.0 Ramos, to appear on arXiv very soon]

(1+1)D NLSM shares many properties with QCD: asymptotic freedom, mass gap, $T \neq 0$ pressure, trace anomaly have QCD-similar shape
Other nonperturbative $T \neq 0$ results available for comparison
(lattice [Giacosa et al '12], $1/N$ expansion [Andersen et al '04], others)

$$\mathcal{L}_0 = \frac{1}{2}(\partial\pi_i)^2 + \frac{g(\pi_i\partial\pi_i)^2}{2(1-g\pi_i^2)} - \frac{m^2}{g}\left[\left(1-g\pi_i^2\right)^{1/2} - 1\right]$$

two-loop pressure from:

•Advantage w.r.t. QCD: exact T-dependence at 2-loops:

$$P_{\rm pert.2loop} = -\frac{(N-1)}{2}\left[I_0^{\rm r}(m,T) + \frac{(N-3)}{4}m^2 g I_1^{\rm r}(m,T)^2\right] + \mathcal{E}_0,$$

$$I_0(m,T) = T\sum\!\!\!\!\!\!\!\int_{n,\mathbf{p}} \ln\left[(2\pi n T)^2 + \mathbf{p}^2 + m^2\right] = \frac{1}{2\pi}\left(m^2(1-\ln\frac{m}{\mu}) + 4T^2 J_0(\frac{m}{T})\right)$$

$$J_0(x) = \int_0^\infty dz \ln\left(1 - e^{-\sqrt{z^2+x^2}}\right), \quad I_1(m,T) = \partial I_0(m,T)/\partial m^2$$

First crucial step: standard perturbative RG invariance

$\mathcal{E}_0$ in P_{2-loop}: *finite* (T-independent) vacuum energy contribution:

$\mathcal{E}_0(g,m) = -m^2\left(\frac{s_0}{g} + s_1 + s_2 g + \cdots\right)$ such that $\mu\frac{d}{d\,\mu}\mathcal{E}_0$ cancels the remnant M dependence:

$s_0 = \frac{(N-1)}{4\pi(b_0-2\gamma_0)} = 1\,,\;\; s_1 = (b_1 - 2\gamma_1)\frac{s_0}{2\gamma_0} = 0$ (NB: accident of NLSM)

•Next step: $m^2 \to m^2(1-\delta)^a\;;\;\; g \to \delta g$;
expand in δ; then $\delta \to 1$:

•RG only consistent for $a = 2\gamma_0/b_0 = (N-3)/(N-2)/2$ for NLSM ($\neq 1$ as in SPT/HTLpt)

•Extra bonus from RG: non-trivial OPT mass gap $\tilde{m}(g,T)$ already at one-loop

•Aim: illustrate in NLSM the scale dependence (and other) improvements wrt former SPT$\sim$ HTLpt

One-loop RGOPT ($\mathcal{O}(\delta^0)$) for NLSM pressure

Exact (arbitrary T) OPT "thermal mass gap" $\bar{m}$ from $\partial_m P(m) = 0$:

$$\ln \frac{\bar{m}}{\mu} = -\frac{1}{b_0\, g(\mu)} - 2J_1(\frac{\bar{m}}{T}), \qquad (b_0^{\rm nlsm} = \frac{N-2}{2\pi})$$

or more explicitly, for $T=0$: $\bar{m} = \mu e^{-\frac{1}{b_0\, g(\mu)}} = \Lambda_{\overline{\rm MS}}^{1-{\rm loop}}$
and for $T \gg m$:

$$\frac{\bar{m}}{T} = \frac{\pi\, b_0\, g}{1 - b_0\, g\, L_T} \simeq \pi\, b_0\, g(\mu) + \mathcal{O}(g^2), \quad (L_T \equiv \ln \frac{\mu\, e^{\gamma_E}}{4\pi T})$$

$$P_{1L}^{\rm RGOPT} = -\frac{(N-1)}{\pi} T^2 \left[J_0(\bar{x}) + \frac{\bar{x}^2}{8}\, (1 + 4J_1(\bar{x})\,) \right], \quad (\bar{x} \equiv \bar{m}/T)$$

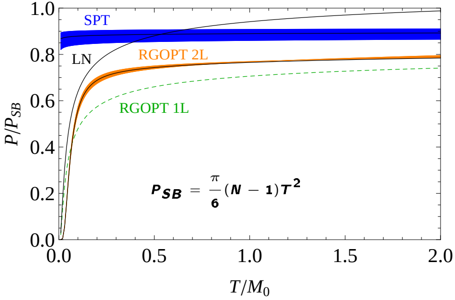
- Standard one-loop running: $g^{-1}(\mu) = g^{-1}(M_0) + b_0 \ln \frac{\mu}{M_0}$

$\Rightarrow \bar{m}, P(\bar{m})$ **are explicitly 'exactly' (one-loop) scale-invariant**

•+ It reproduces exact (all orders) known large N (LN) results
(Andersen et al '04)

RGOPT NLSM mass and pressure: two-loop order

$P/P_{SB}(N=4, g(M_0)=1)$ vs standard perturbation (PT), large N (LN), and SPT $\equiv$ ignoring RG-induced subtraction; $m^2 \to m^2(1-\delta)$:



(shaded range: scale-dependence $\pi T < \mu \equiv M < 4\pi T$)

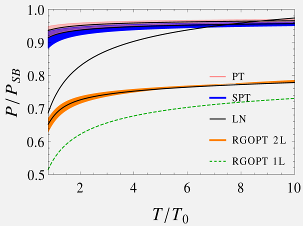
$\to$ A moderate scale-dependence reappears, from unperfectly matched 2-loop $T=0$ standard running coupling.

NB for 2-loop NLSM, alternative $\bar{g}(\mu)$ from combining RG+OPT accidentally gives $g=0$... (traced to 2-loop subtraction $s_1=0$)

(not expected in other models, and nontrivial NLSM $\bar{g}(\mu)$ appears at 3-loop)

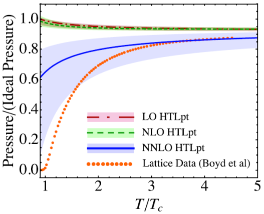
High T: pressure shape more comparable to QCD HTLpt

$$\frac{P_{PT}^{2-\text{loop}}}{P_{SB}} \simeq \frac{P_{SPT}^{2-\text{loop}}}{P_{SB}} = 1 - \frac{3}{2}\frac{(N-3)}{8\pi} g(\mu) + \mathcal{O}(g^2)$$



(NB: RGOPT 1,2L reach SB limit for $T \to \infty$ but more slowly than PT)

HTLpt (beyond 1-loop only $T \gg m$ approximation): QCD (pure glue)
[Andersen, Strickland, Su '10]:

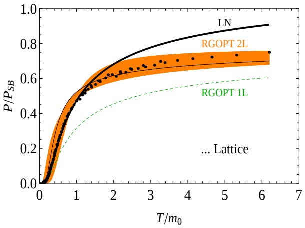


RGOPT(NLSM) lattice comparison

•NLSM $T \neq$ lattice simulations: (apparently) only available for $N = 3$ [E. Seel, D. Smith, S. Lottini, F. Giacosa '12]

•Remind: at 2-loop NLSM combined RG +OPT Eqs. gives no nontrivial $\bar{g}(\bar{m})$ by accident (traced to $s_1 = 0$), yet one remarkable value: $g(M_0) = 2\pi \Rightarrow \bar{m}(g) = M_0$ (NB similar feature in Gross-Neveu model)

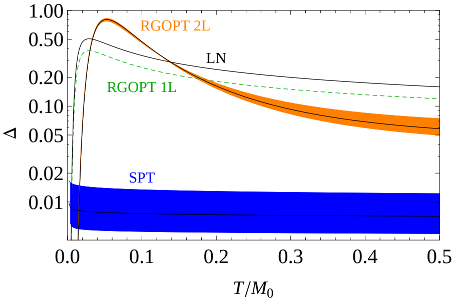
•Drawback: for such large coupling , 2-loop RGOPT remnant scale dependence becomes much more sizable.
(at 3-loop order $\bar{g}(m)$ would likely be more reasonable)



shaded regions: scale-dependence $\pi T < \mu = M < 4\pi T$
(Notice: $N = 3$ lattice pressure appears very close to large-N for low $T \lesssim M_0$)

NLSM interaction measure (trace anomaly)

NB $\Delta_{\text{2D NLSM}} \equiv \mathcal{E} - P = S\,T - 2P \equiv T^3 \partial_T(\frac{P}{T^2})$



$N = 4, g(M_0) = 1$ (shaded regions: scale-dependence $\pi T < \mu = M < 4\pi T$)

•2-loop SPT Δ small, monotonic behaviour + sizeable scale dependence.

•RGOPT shape 'qualitatively' comparable to QCD, showing a peak
(but no spontaneous sym breaking/phase transition in 2D NLSM
(Mermin-Wagner-Coleman theorem): reflects broken conformal
invariance (mass gap).

Thermal (pure glue) QCD: hard thermal loop perturbation (HTLpt) (see J.O Andersen talk)

QCD generalization of OPT = HTLpt [Andersen, Braaten, Strickland '99]: same "OPT" trick operates on a gluon "mass" term [Braaten-Pisarski '90]:

$$\mathcal{L}_{QCD}(\text{gauge}) - \frac{m_D^2}{2} Tr \left[G_{\mu\alpha} \langle \frac{y^\alpha y^\beta}{(y.D)^2} \rangle_y G^\mu_\beta \right], \quad D^\mu = \partial^\mu - ig\, A^\mu, \quad y^\mu = (1, \hat{\mathbf{y}})$$

(effective, gauge-invariant):

describes screening mass $m_D^2 \sim \alpha_S T^2$, but also many more 'hard thermal loop' contributions [modifies vertices and gluon propagators]

Other gluon "mass prescriptions" exist [e.g. Reinosa et al '15] but HTLpt nice advantage: calculations up to 3-loop α_S^2 (NNLO) [Andersen et al '99-'15]: highly nontrivial, available analytically as m_D/T expansions, neglecting consistently higher orders [e.g. $m_D^4 \alpha_S = \mathcal{O}(\alpha_S^3)$].

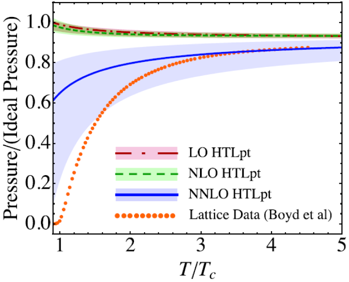
e.g. $P^{HTLpt}_{\text{1-loop},\overline{MS}} =$

$$P_{\text{ideal}} \left[1 - \frac{15}{2} \hat{m}_D^2 + 30 \hat{m}_D^3 + \frac{45}{4} \hat{m}_D^4 (\ln \frac{\mu}{4\pi T} + \gamma_E - \frac{7}{2} + \frac{\pi^2}{3}) \right]$$

$\hat{m}_D \equiv \frac{m}{2\pi T}$, $P_{\text{ideal}} = (N_c^2 - 1)\pi^2 \frac{T^4}{45}$

standard HTLpt results:

(pure glue) [Andersen, Strickland, Su '10]



Reasonable agreement with lattice simulations (Boyd et al '96) at NNLO (3-loop), down to $T \sim 2 - 3T_c$, for low scale $\mu \sim \pi T - 2\pi T$.

RGOPT adaptation of HTLpt =RGOHTL

Main issue of HTLpt however: odd increasing scale dependence at higher (NNLO) order

Our main RGOPT changes:

• Crucial RG-invariance restoring subtractions in Free energy: reflects its anomalous dimension.
• interpolate with $m_D^2(1-\delta)^{\frac{\gamma_0}{b_0}}$, where gluon 'mass' anomalous dimension defined (as it should) from its (available) counterterm.

RGOPT scale dependence should improve at higher orders from basically consistent RG invariance:
both from subtraction terms (prior to interpolation), and from above interpolation maintaining RG invariance.

• HTLpt does not include the subtractions: yet scale dependence moderate up to 2-loops,
because the (leading order) RG-unmatched term, of $\mathcal{O}(m^4)$, is formally like a (3-loop order) α_S^2 term:

$\rightarrow$ Explains why HTLpt scale dependence plainly resurfaces at 3-loops.

Preliminary RGO(HTL) results (1- and 2-loop, pure glue)

One-loop: obtain exactly scale-invariant pressure (like for ϕ^4 and NLSM):

$\frac{P}{P_{ideal}}(G) = 1 - \frac{15}{4}\hat{m}^2 + \frac{15}{2}\hat{m}^3 + \mathcal{O}(m^6)$

where $\hat{m} = G\,(1+\sqrt{1-\frac{1}{3G}})$, and $G \sim$ a coupling: $G^{-1} = \ln\frac{4\pi T}{\Lambda_{\overline{MS}}} + const.$

But this OPT (i.e. $\partial_m P(m) \equiv 0$) solution is complex for $T/T_c \gtrsim 2$ (with small imaginary parts):
a complex pressure is unphysical, but here largely an artefact of $\overline{\text{MS}}$-scheme +high-T approx:

• by renormalization scheme change, can push complex solution to much higher T/T_c, where to match standard PT.

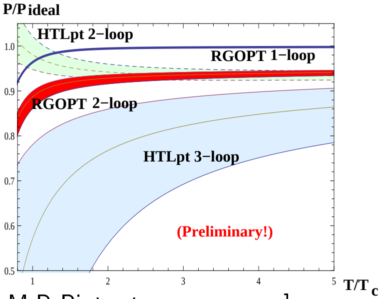
• Yet physically consistent with standard $P(g)$:
for $P(m \to m^{PT}_{Debye})$ for $g \to 0$ (SB limit)

• Our attitude: crude one-loop approximation not final stage, so better scale invariant and complex than conversely.
Pragmatic: at one-loop we take $Re[P(g)]$ in larger T/T_c range.

• 2-loops: RG Equation gives a real unique solution.

Preliminary RGO(HTL) results (1- and 2-loop, pure glue)

2-loops: a moderate scale-dependence reappears, similar to ϕ^4, NLSM case: a factor ~ 2 improvement w.r.t. HTLpt 2-loops:



[JLK, M.B Pinto, to appear soon]

NB scale dependence improvement should be more drastic at 3-loops:

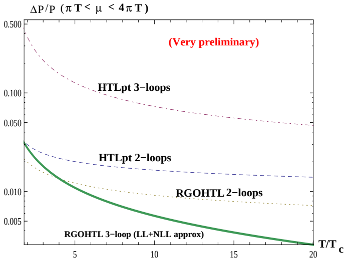
Generically: RGOPT at $\mathcal{O}(g^k) \to \bar{m}(\mu)$ appears at $\mathcal{O}(g^{k+1})$ for *any* $\bar{m}$, but $\bar{m}_G^2 \sim gT \to P \simeq \bar{m}_G^4/g + \cdots$ has leading μ-dependence at $\mathcal{O}(g^{k+2})$.

• however low $T \sim T_c$ genuine pressure shape needs determining higher order subtraction terms of $\mathcal{O}(m_D^4 \alpha_S^2 \ln \mu)$:

new calculations of 3-loop HTL integrals (neglected in standard HTLpt since formally $\mathcal{O}(\alpha_S^4)$)

(Very!) preliminary RGO(HTL) approximate 3-loop results

3-loops: exact missing $m^4 \alpha_S^2$ terms need extra nontrivial calculations, but
$P^{3l}_{RGOHTL} \sim$
$RGOPT(P^{3l}_{HTLpt}) + m^4 \alpha_S^2 (C_{30} \ln^3 \frac{\mu}{2\pi T} + C_{31} \ln^2 \frac{\mu}{2\pi T} + C_{32} \ln \frac{\mu}{2\pi T} + C_{33})$:
leading logarithms (LL) and next-to-leading (NLL) C_{30}, C_{31} determined for free from lower orders from RG invariance!
Within this LL, NLL approximation and in the $T/T_c \gtrsim 2$ range where more trustable:



We assume/expect unknown terms will not spoil this improved scale dependence
(But $P(T/T_c)$ shape not shown since not precisely known for low $T \sim T_c$ (for the time being): will be sensitive to those missing terms)

Summary and Outlook

•OPT gives a simple procedure to resum perturbative expansions, using only perturbative information.

•Our RGOPT version includes 2 major differences w.r.t. previous OPT/SPT/HTLpt... approaches:

1) OPT +/or RG minimizations fix optimized $\tilde{m}$ *and possibly* $\tilde{g} = 4\pi\tilde{\alpha}_S$

2) Requiring AF-compatible solutions uniquely fixes the basic interpolation $m \to m(1-\delta)^{\gamma_0/b_0}$: discards spurious solutions and accelerates convergence.

Applied to $T \neq 0$: exhibits improved stability + much improved scale dependence (with respect to standard PT, but also wrt SPT $\sim$ HTLpt)

•Paves the way to extend such RG-compatible methods to full QCD thermodynamics, (work in progress, starting with $T \neq 0$ pure gluodynamics) specially for exploring also finite density