General equilibrium second-order hydrodynamic coefficients for quantum fields

Matteo Buzzegoli[a], Eduardo Grossi[b] and Francesco Becattini[a]

[a] Dipartimento di Fisica e Astronomia, Universitá di Firenze, Italy   INFN - Sezione di Firenze, Italy

[b] Institut für Theoretische Physik, Universität Heidelberg

Introduction

In high energy nuclear collisions, the Quark Gluon Plasma is the fluid with the largest acceleration and vorticity ever produced in laboratory. The hydrodynamic simulations and recent experimental measurements indicate [1]:

$$|a| \approx 0.05\ c^2/\mathrm{fm} \approx 5\,10^{30}\ m/s^2$$
$$|\omega| \approx 0.06\ c/\mathrm{fm} \approx 2\,10^{22}\ s^{-1}$$

We studied how acceleration and vorticity affects the thermodynamics of the system and showed that the stress-energy tensor gets non-dissipative quantum corrections which are quadratic in vorticity and acceleration [2, 3, 4], which may not be negligible for the hydrodynamic simulation of the Quark Gluon Plasma.

General global equilibrium

The most general equilibrium distribution in relativistic quantum statistical mechanics is described by the covariant statistical operator [5, 6]:

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\widehat{j}^\mu(x)\right)\right]$$

where β is the four-temperature vector and defines a hydrodynamical frame [7] $u = \beta/\sqrt{\beta^2}$, $\zeta = \mu/T$, μ the chemical potential and Σ is an arbitrary spacelike 3D hypersurface, provided that β is a Killing vector:

$$\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu = 0, \quad \partial_\mu\zeta = 0.$$

The Minkowski spacetime solution ζ =constant and

$$\beta^\mu = b^\mu + \varpi^{\mu\nu}x_\nu \rightarrow \varpi^{\mu\nu} = \frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$$

describes a system with constant **thermal vorticity** ϖ (hence with acceleration and rotation) and simplify $\widehat{\rho}$ into [7]

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-b_\mu\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right]$$

where $\widehat{P}^\mu$, $\widehat{J}^{\mu\nu}$ are the Poincaré group generators.

Expansion for small vorticity ϖ

$$\langle\widehat{T}_{\mu\nu}(x)\rangle = \frac{1}{Z}\mathrm{tr}\left[\exp\left(-b\cdot\widehat{P} + \frac{1}{2}\varpi : \widehat{J} + \zeta\widehat{Q}\right)\widehat{T}_{\mu\nu}(x)\right]$$

The mean value of an operator in general equilibrium can be calculated through an expansion in ϖ if the thermal correlation length is much smaller than the length over which the fields β and ζ significantly vary (hydrodynamic limit), that is $\partial\beta/\beta \ll 1/\beta$, $1/m$ and $\varpi \ll 1$

$$\langle\widehat{T}_{\mu\nu}(x)\rangle = (\rho+p)u_\mu u_\nu - p\,g_{\mu\nu} + \frac{\varpi_{\rho\sigma}}{2|\beta|}\langle\widehat{J}^{\rho\sigma}\,\widehat{T}_{\mu\nu}(0)\rangle_{\beta(x)} + \frac{\varpi_{\rho\sigma}\varpi_{\lambda\kappa}}{8|\beta|^2}\langle\widehat{J}^{\rho\sigma}\,\widehat{J}^{\lambda\kappa}\,\widehat{T}_{\mu\nu}(0)\rangle_{\beta(x)} + \mathcal{O}(\varpi^2)$$

where $\langle\ldots\rangle_{\beta(x)}$ is the mean value with familiar homogeneous thermodynamic equilibrium at constant four-temperature equal to $\beta(x)$ in the point x, that is with the density operator: $\widehat{\rho} = \frac{1}{Z}\exp\left[-\beta_\mu\widehat{P}^\mu + \zeta\widehat{Q}\right]$.

Acceleration and rotation components

We can decompose ϖ into two spacelike vectors proportional to acceleration and rotation by projecting onto the four-velocity u

$$\varpi^{\mu\nu} = \alpha^\mu u^\nu - \alpha^\nu u^\mu + \epsilon^{\mu\nu\rho\sigma}w_\rho u_\sigma$$

- $u^\mu = \beta^\mu/\sqrt{\beta^2}$ — u^μ four-velocity
- $\alpha^\mu = \varpi^{\mu\nu}u_\nu = a^\mu/T$ — a^μ acceleration
- $w^\mu = \epsilon^{\rho\sigma\nu\mu}u_\nu\,\varpi_{\rho\sigma} = w^\mu/T$ — w^μ angular velocity
- $\gamma^\mu = \epsilon^{\mu\nu\rho\sigma}w_\nu\,\alpha_\rho\,u_\sigma$ — γ^μ transverse vector

We can then adopt the non-normalized tetrad $\{u,\,\alpha,\,w,\,\gamma\}$. Restoring the natural units:

$$|\alpha| = \frac{\hbar\,|\vec{a}|}{c\,k_B\,T},\ |w| = \frac{\hbar\,|\vec{\omega}|}{k_B\,T} \xrightarrow[|\vec{a}|\simeq c|\vec{\omega}|\approx 0.05\ c^2/\mathrm{fm}]{T=300\ \mathrm{MeV}} |\varpi| \approx 10^{-2}$$

Non-dissipative second-order hydrodynamic coefficients

The final expression of the stress-energy tensor up to second order in ϖ [8]:

$$T_{\mu\nu}(x) \simeq (\rho - \alpha^2 U_\alpha - w^2 U_w)u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w)\Delta_{\mu\nu} + A\,\alpha_\mu\alpha_\nu + W\,w_\mu w_\nu + G\,(u_\mu\gamma_\nu + u_\nu\gamma_\mu)$$

The coefficients can be calculated systematically in the rest frame as **Euclidean connected correlators** between appropriate stress-energy tensor components and generators of the Poincaré group:

$$U_\alpha = \frac{1}{2}\langle\widehat{K}_3\,\widehat{K}_3\,\widehat{T}_{00}\rangle_T \qquad U_w = \frac{1}{2}\langle\widehat{J}_3\,\widehat{J}_3\,\widehat{T}_{00}\rangle_T \qquad A = \langle\widehat{K}_1\,\widehat{K}_2\,\widehat{T}_{12}\rangle_T$$

$$D_\alpha = \frac{1}{6}\sum_{i=1}^{3}\langle\widehat{K}_3\,\widehat{K}_3\,\widehat{T}_{ii}\rangle_T - \frac{1}{3}\langle\widehat{K}_1\,\widehat{K}_2\,\widehat{T}_{12}\rangle_T \qquad W = \langle\widehat{J}_1\,\widehat{J}_2\,\widehat{T}_{12}\rangle_T$$

$$D_w = \frac{1}{6}\sum_{i=1}^{3}\langle\widehat{J}_3\,\widehat{J}_3\,\widehat{T}_{ii}\rangle_T - \frac{1}{3}\langle\widehat{J}_1\,\widehat{J}_2\,\widehat{T}_{12}\rangle_T \qquad G = -\frac{1}{2}\langle\{\widehat{K}_1\,,\,\widehat{J}_2\}\,\widehat{T}_{03}\rangle_T.$$

All corrections to $T^{\mu\nu}$ are of **quantum origin**, as all the coefficients U, D, A, W, G turn out to have a finite classical limit for the free gas, while α ad ω have an $\hbar$ factor (see previous frame).

Results for free fields

The stress-energy tensor and current operators for free **charged scalar field** are

$$\widehat{T}_{\mu\nu} = \partial_\mu\widehat{\varphi}^*\partial_\nu\widehat{\varphi} + \partial_\nu\widehat{\varphi}^*\partial_\mu\widehat{\varphi} - g_{\mu\nu}(\partial\widehat{\varphi}^*\cdot\partial\widehat{\varphi} - m^2\widehat{\varphi}^*\widehat{\varphi}) - \xi(\partial_\mu\partial_\nu - g_{\mu\nu}\Box)\widehat{\varphi}^*\widehat{\varphi} \qquad \widehat{j}_\mu = \mathrm{i}(\widehat{\varphi}^*\partial_\mu\widehat{\varphi} - \widehat{\varphi}\partial_\mu\widehat{\varphi}^*),$$

instead for free **Dirac field** are

$$\widehat{T}_{\mu\nu} = \frac{\mathrm{i}}{4}\left[\widehat{\overline{\psi}}\,\gamma_\mu\,\partial_\nu\widehat{\psi} - \partial_\nu\widehat{\overline{\psi}}\,\gamma_\mu\,\widehat{\psi} + \widehat{\overline{\psi}}\,\gamma_\nu\,\partial_\mu\widehat{\psi} - \partial_\mu\widehat{\overline{\psi}}\,\gamma_\nu\,\widehat{\psi}\right] \qquad \widehat{j}_\mu = \widehat{\overline{\psi}}\,\gamma_\mu\widehat{\psi}.$$

From the previous operators we obtain the analytic expression for the coefficients, whose behavior in temperature is plotted in the figure in the case of massive fields with zero chemical potential [8, 9].

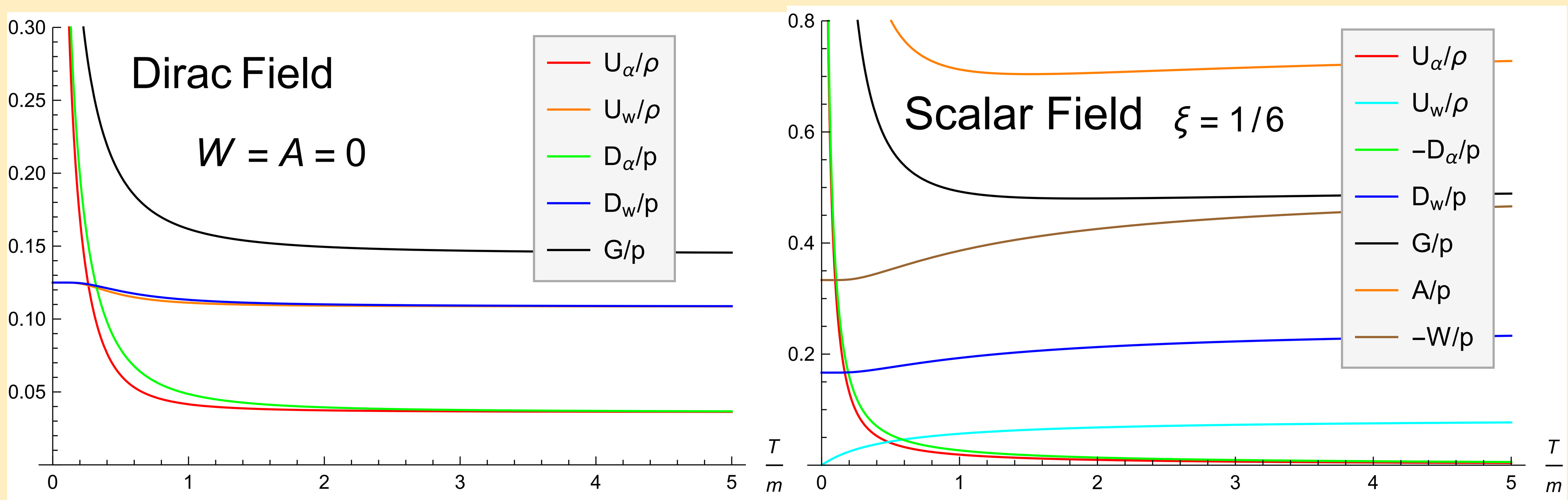

Figure: The coefficients divided by energy ρ or pressure p for $m > 0$ and $\mu = 0$

The coefficients in the massless case correspond to the asymptotic values at high temperature in figure.

In the same way, the mean values of the vector current $\widehat{j}_\mu$ and the axial current (for Dirac field: $\widehat{j}_{A\mu} = \widehat{\overline{\psi}}\,\gamma_\mu\gamma_5\widehat{\psi}$) have the corrections:

$$j_\mu(x) = n\,u_\mu + (\alpha^2 N_\alpha + w^2 N_\omega)\,u_\mu + G_V\,\gamma_\mu \qquad j_{A\mu}(x) = w_\mu\,W_A,$$

where the coefficients N_α, N_ω, G_V and W_A are also given by Euclidean connected correlators. In particular, W_A recover the Axial Vortical Effect [10], that for $m = 0$ is

$$\vec{j}_A = \frac{W_A}{T}\vec{\omega} = \frac{1}{T}\langle\widehat{J}_3\,\widehat{j}_{A3}\rangle\,\vec{\omega} = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\vec{\omega}$$

Consequences and conclusions

- The stress-energy tensor has **equilibrium non-dissipative corrections** if the fluid is rotating or accelerating. Such corrections may be phenomenologically relevant for system with very high acceleration, such as in the early stage of relativistic heavy ion collisions.
- These corrections are pure **quantum effects** (they vanish in classical limit).
- We have an easy prescription on how to evaluate them, as they are **Euclidean correlators** of conserved quantities ($\widehat{T}_{\mu\nu}$ and Poincaré groups generators).
- We recovered the **anomalous transport coefficients** describing the Axial Vortical Effect.

References

[1] STAR Collaboration, arXiv:1701.06657.
[2] R. Baier et al., JHEP **0804**, 100 (2008).
[3] S. Bhattacharyya, JHEP **1207**, 104 (2012).
[4] G. D. Moore and K. A. Sohrabi, JHEP **1211**, 148 (2012).
[5] C. G. Van Weert, Ann. Phys. **140**, 133 (1982).
[6] D. N. Zubarev et al., Theoret. and Math. Phys. **40**, 821 (1979).
[7] F. Becattini et al., Eur. Phys. J. C **75**, 5, 191 (2015).
[8] F. Becattini and E. Grossi., Phys. Rev. D **92**, 045037 (2015).
[9] M. Buzzegoli, E. Grossi and F. Becattini, arXiv:1704.02808.
[10] D. E. Kharzeev et al., Prog. Part. Nucl. Phys. **88** (2016).

E-mail: matteo.buzzegoli@fi.infn.it