Temperature dependence of bulk viscosity of SU(3) gluodynamics

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Outline:

- Introduction
- Details of the calculation
- Bulk viscosity
  - Middle point method
  - Backus-Gilbert method
- Conclusion

Relativistic Hydrodynamics

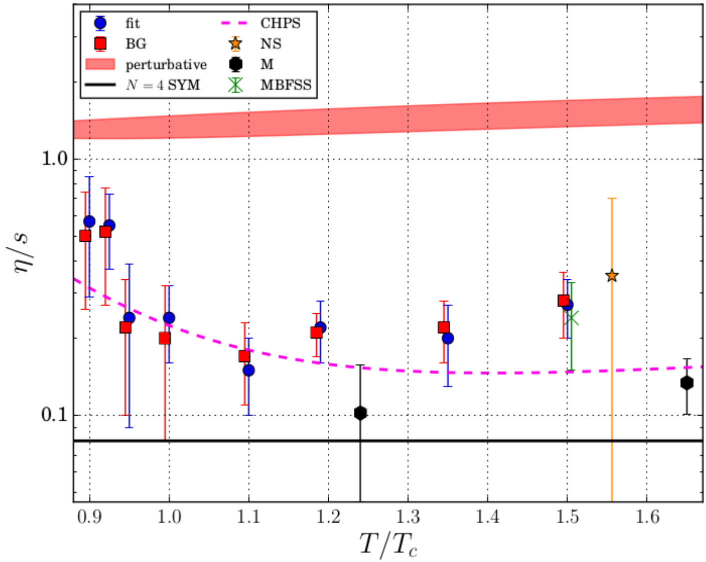
- $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + (\eta\nabla^{\langle\mu}u^{\nu\rangle} + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \ldots$
  $\nabla^\alpha = \Delta^{\alpha\nu}\partial_\nu, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
  $\nabla^{\langle\mu}u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$

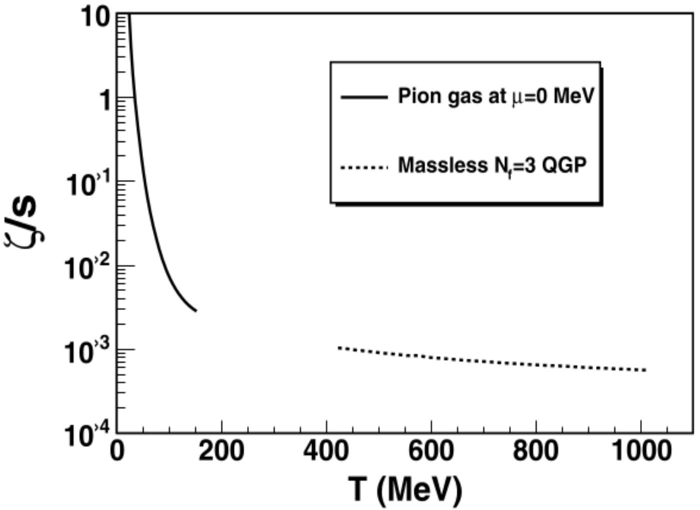
- EOM $\partial_\mu T^{\mu\nu} = 0$

- Non-relativistic limit ($u^\mu = (1, \vec{v})$)
  - *Continuity equation:* $\partial_t\rho + \rho(\vec{\partial}\vec{v}) + \vec{v}\vec{\partial}\rho = 0$
  - *Navier–Stokes equation:* $\frac{\partial v^i}{\partial t} + v^k\frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$
  - *Viscous stress tensor:* $\Pi^{ik} = -\eta\left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ik}\frac{\partial v^l}{\partial x^l}\right) - \zeta\delta^{ik}\frac{\partial v^l}{\partial x^l}$

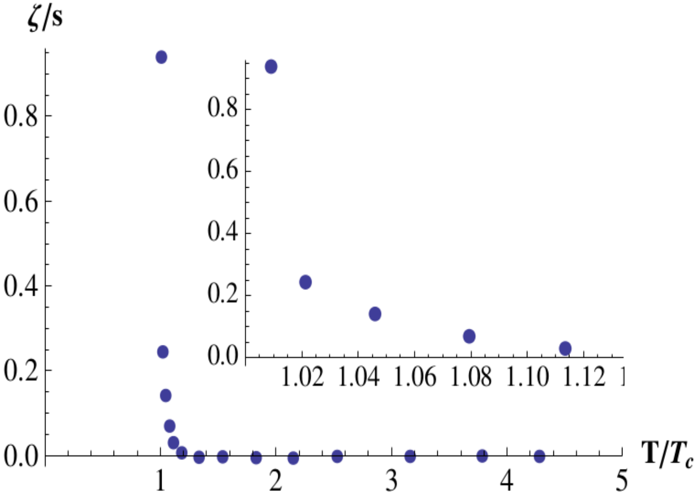
- η–shear viscosity, ζ–bulk viscosity

Shear viscosity (JHEP 1704 (2017) 101)





- **CHPT:** A. Dobado, F.J. Llanes-Estrada, J.M. Torres-Rincon, Physics Letters B 702 (2011) 43
- **Perturbative QCD:** P. Arnold, C. Dogan, G. Moore , Physical Review D 74, 085021 (2006)

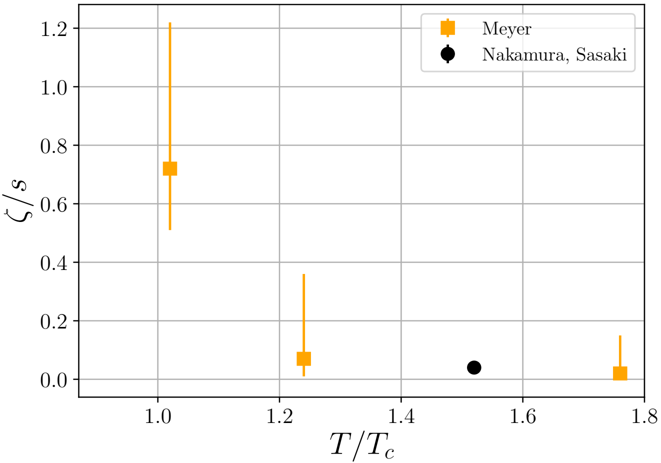


- Low energy theorems of QCD: $\zeta = \frac{1}{9\omega_0}\left(T^5 \frac{\partial}{\partial T}\frac{e-3p}{T^4} + 16\epsilon_v\right)$

  D. Kharzeev, K. Tuchin, JHEP 0809 (2008) 093,
  D. Kharzeev, F. Karsch, K. Tuchin, Phys.Lett. B663 (2008) 217

Previous works (SU(3) gluodynamics):

- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev.Lett. 100 (2008) 162001



Lattice calculation of bulk viscosity

The first step:

Measurement of the correlation function:

$$C_E(t) = \langle T^{\mu}_{\mu}(t) T^{\nu}_{\nu}(0) \rangle$$

The second step:

Calculation of the spectral function $\rho(\omega)$:

$$C_E(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

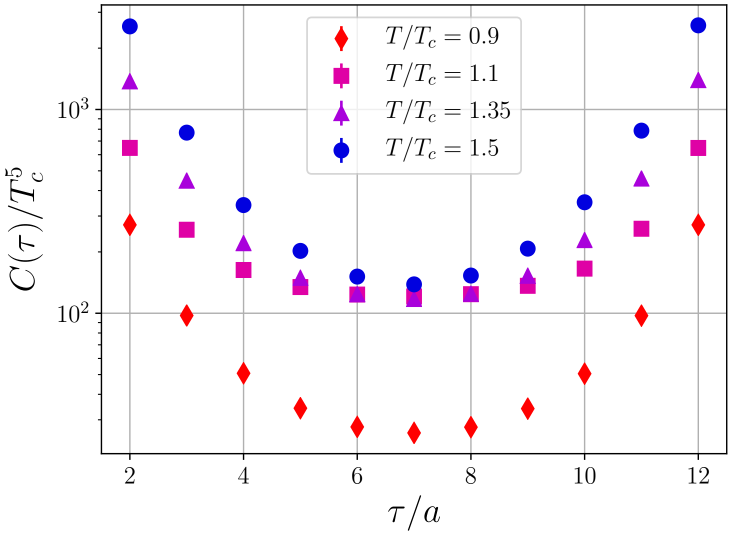
$$\zeta = \frac{\pi}{9} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.5$
- Accuracy $\sim$ 3% at $t = \frac{1}{2T}$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300
- ...

Our results are preliminary !

Correlation functions



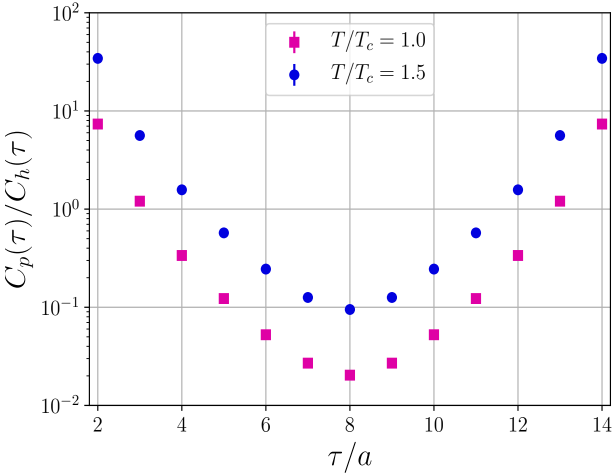
Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0,\ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|^{NLO}_{\omega\to\infty} = d_A \left(\frac{11\alpha_s}{(4\pi)^2}\right)^2 \omega^4$

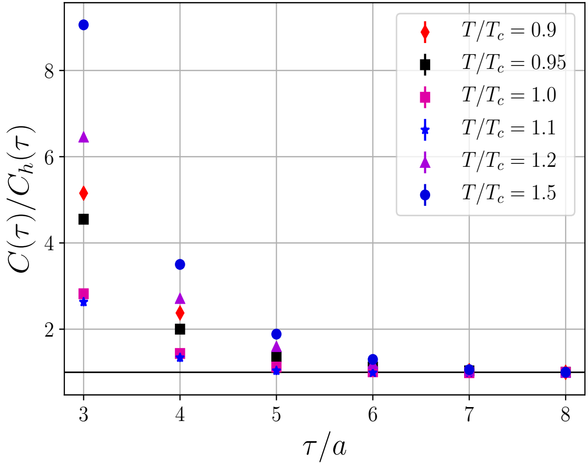
  compare with shear channel $\sim d_A \frac{1}{10(4\pi)^2}\omega^4$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{9}{\pi}\zeta\omega$

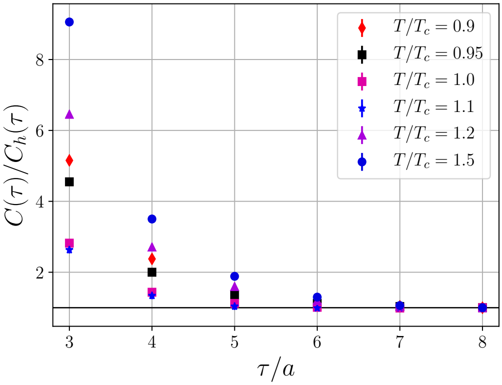


- In the region $\tau/a \sim \frac{\beta}{2}$ hydrodynamics is dominant
- In the region $\tau/a \sim$ few perturbative contribution is dominant

Hydrodynamical approximation

$$C_h(\tau) = \int_0^\infty d\omega \rho_h(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega\tau\right)}{sh\left(\frac{\omega}{2T}\right)}, \quad \rho_h(\omega) = \frac{9}{\pi}\zeta\omega\theta(\omega_0 - \omega)$$





Middle point estimation of bulk viscosity

- In the vicinity of the phase transition hydrodynamics very works well!
- $C_h\left(\frac{\beta}{2}\right) = \frac{9}{\pi}\zeta \int_0^{\omega_0} d\omega \frac{\omega}{sh\left(\frac{\omega}{2T}\right)}$
- ω_0 is varied within the interval $1.5 - 3$ GeV

Backus-Gilbert method for the spectral function

- Problem: find $f(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega f(\omega) K(x_i,\omega), \quad K(x_i,\omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{f}(\bar{\omega})$ ($\delta(\bar{\omega},\omega)$ - resolution function):

$$\tilde{f}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega},\omega) f(\omega)$$

- Let us expand $\delta(\bar{\omega},\omega)$ as

$$\delta(\bar{\omega},\omega) = \sum_i b_i(\bar{\omega}) K(x_i,\omega) \quad \tilde{f}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

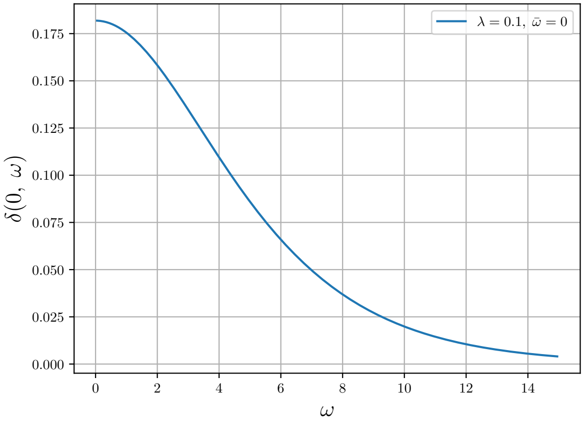
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i,\omega)(\omega - \bar{\omega})^2 K(x_j,\omega), R_i = \int d\omega K(x_i,\omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution function $\delta(0,\omega)$ ($T/T_c = 1.5,\ \lambda = 0.1$)



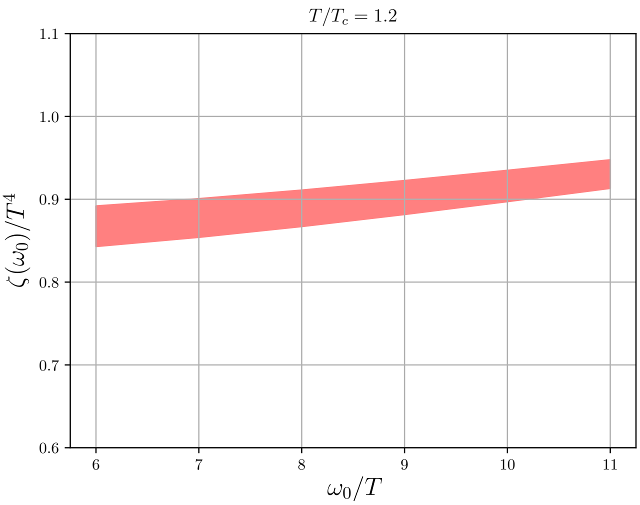
- Width of the resolution function $\omega/T \sim 5$

Removal of the ultraviolet contribution

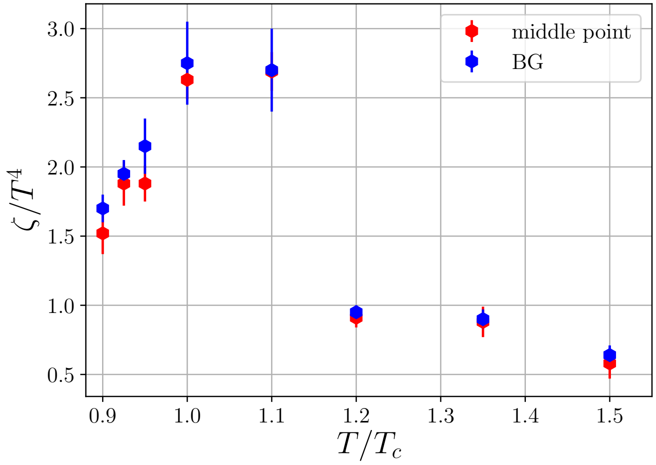
- Take ultraviolet contribution in the form:

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

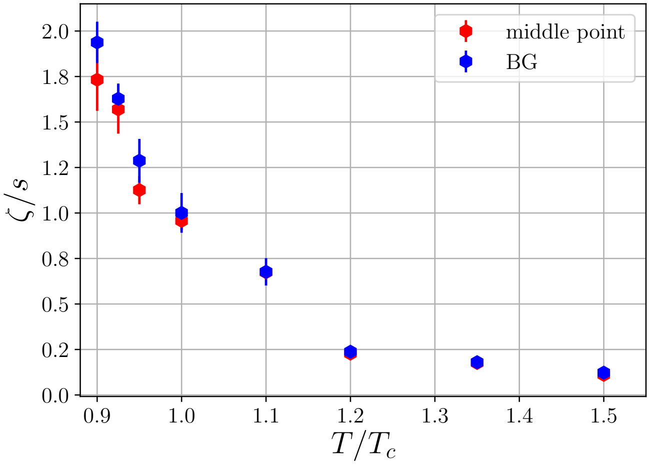
- Determine the value of the constant A from the $C(\tau/a = 2)$
- Subtract ultraviolet contribution and obtain ζ/T^4 as a function of ω_0


$T/T_c = 1.2$
1.1
1.0
0.9
0.8
0.7
0.6
$\zeta(\omega_0)/T^4$
6
7
8
9
10
11
ω_0/T

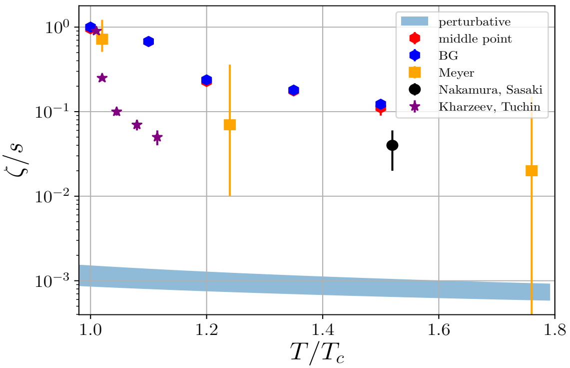
Our results (preliminary!)



Our results (preliminary!)

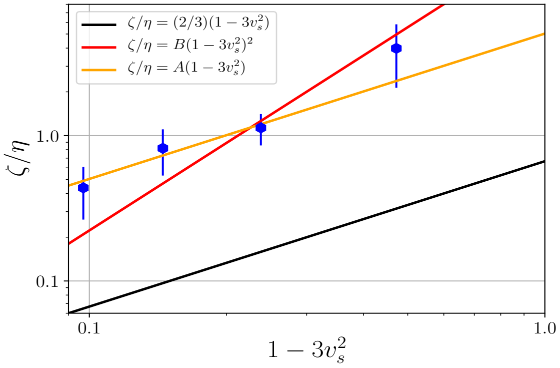


Comparison with other approaches

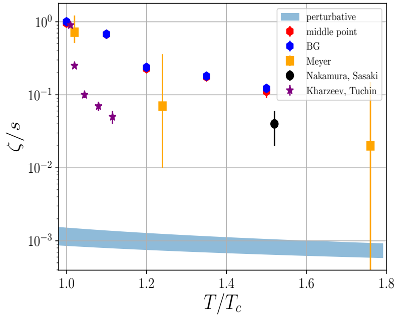


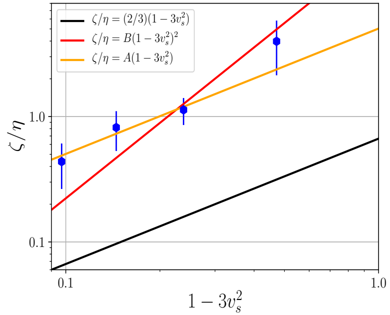
- Agreement with other lattice studies
- Large deviation from perturbative results

Weakly or strongly coupled QGP?



- Weakly coupled system $\zeta/\eta \sim (1-3v_s^2)^2$ $(\chi^2/dof \sim 4)$
- Strongly coupled system $\zeta/\eta \sim (1-3v_s^2)$ $(\chi^2/dof \sim 1)$
- $\zeta/\eta \geq \frac{2}{3}(1-3v_s^2)$ (A. Buchel, Physics Letters B663, 286 (2008))





Conclusion:

- We calculated ζ/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Agreement with previous lattice results
- Large deviation from perturbative calculation
- QGP reveals the properties of strongly coupled system