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# Preliminary QCD phase transition line with 695 MeV dynamical staggered fermions from effective Polyakov line actions

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- hard to compute $\exp[S_P(P_x)]$ directly, but action ratios are easily computed as expectation values $\rightarrow$ relative weights via derivatives of $S_P$ w.r.t. Fourier components $a_k$ of $P_x$

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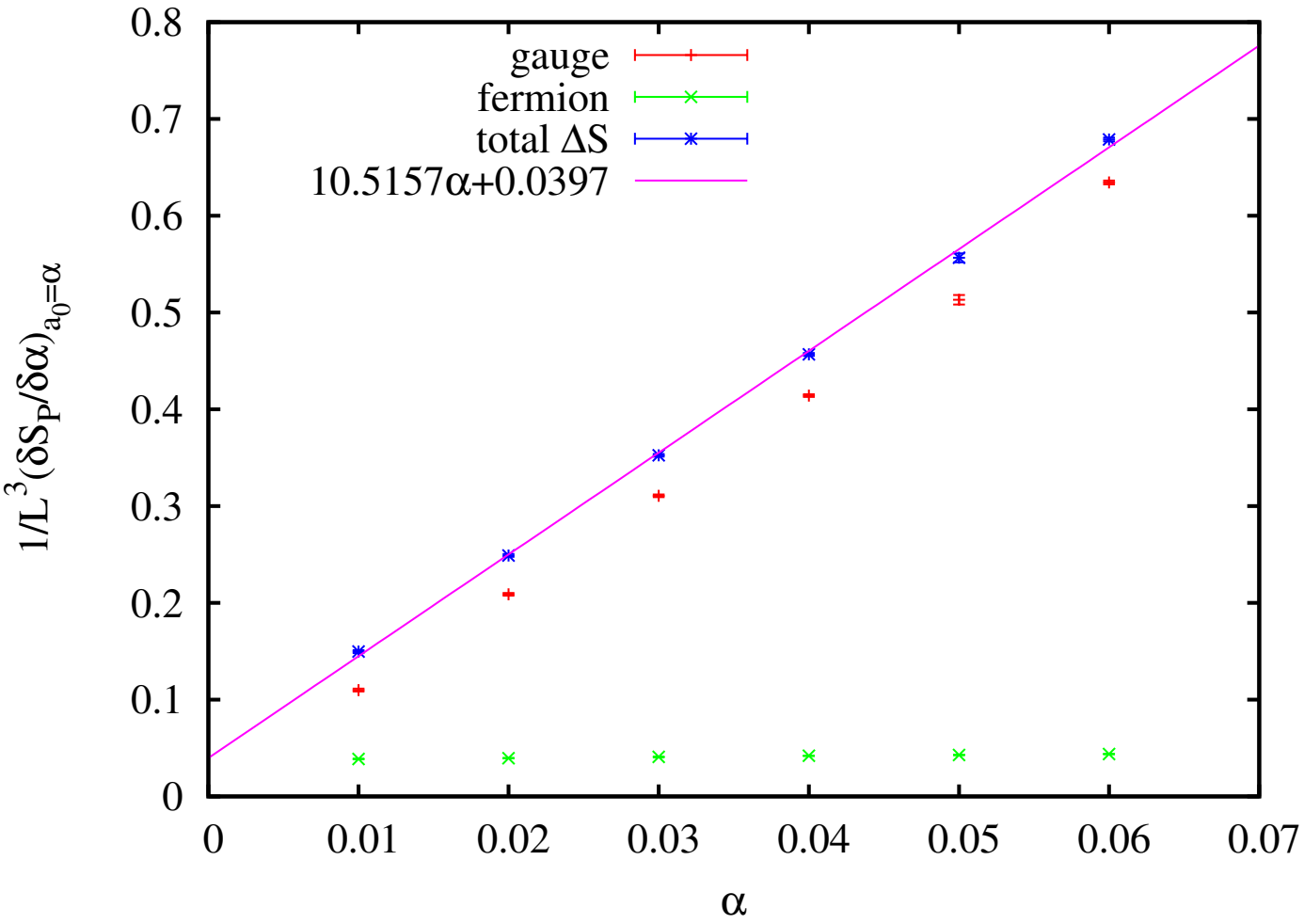
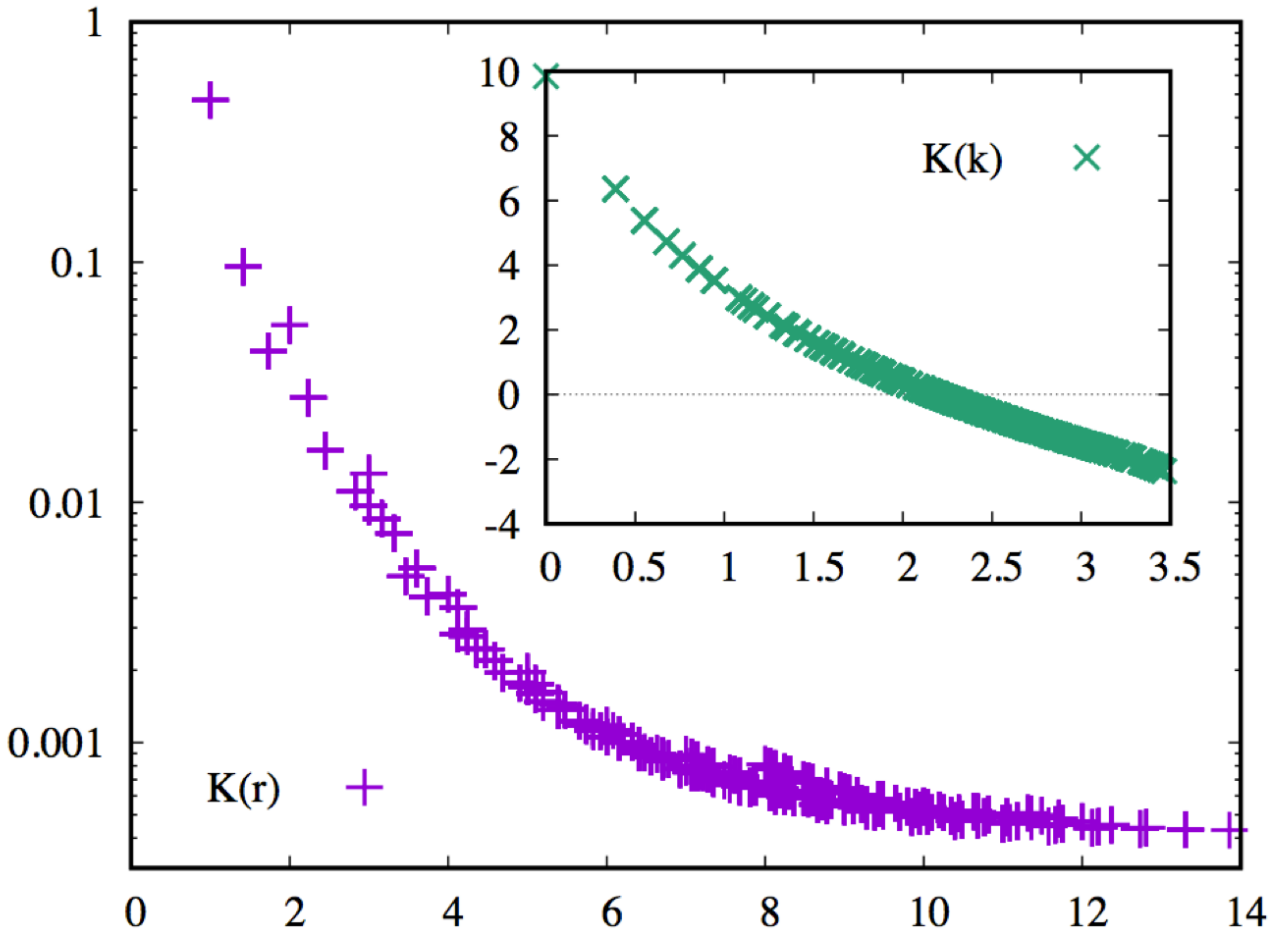
$$\frac{1}{L^3}\left(\frac{\partial S_P}{\partial a_k}\right)_{a_k=\alpha} = 2K(k)\alpha + \frac{p}{L^3}\sum_x (3he^{ikx} + 3h^2 e^{-ikx} + c.c.)$$

# Fitting to lattice data

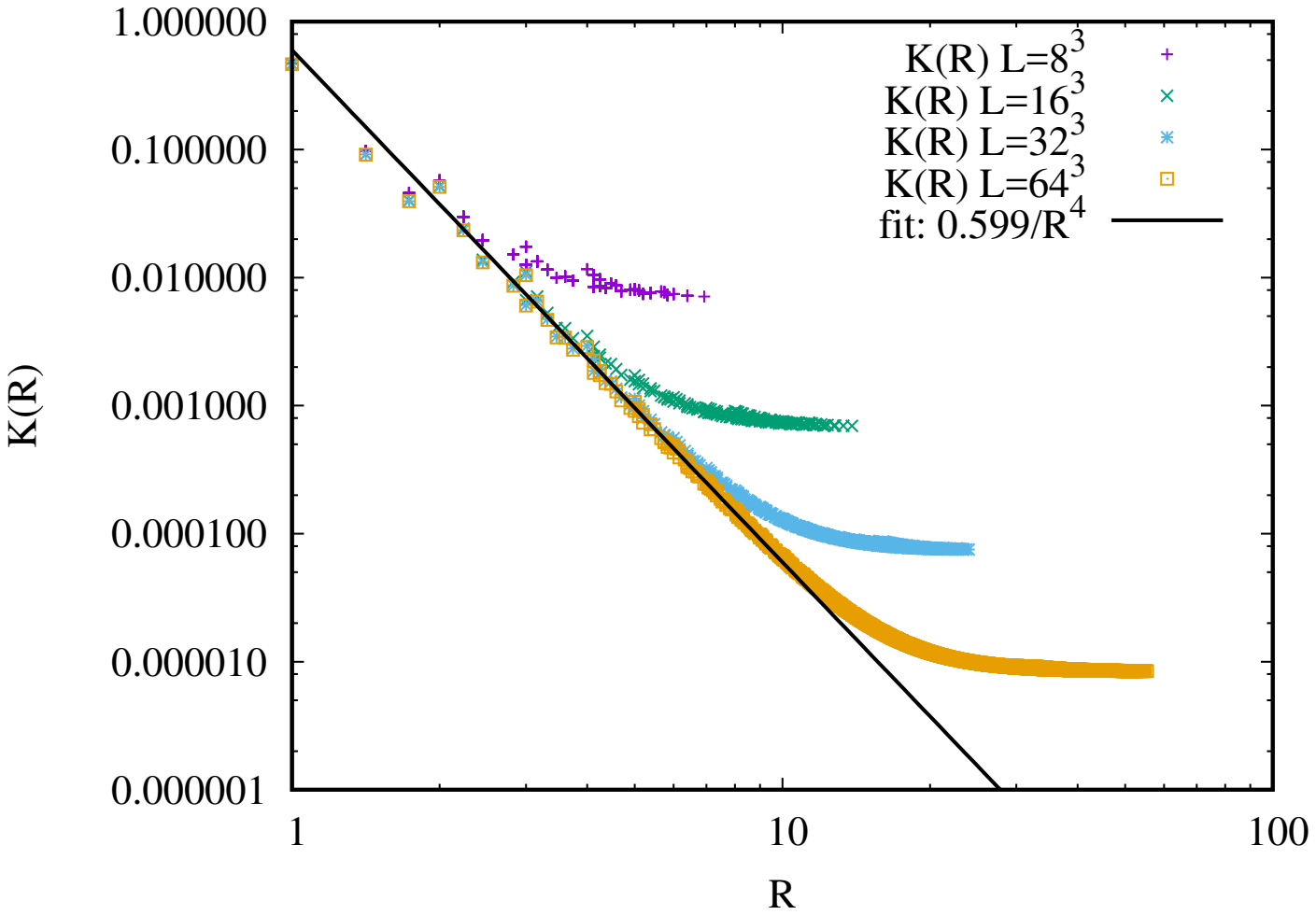


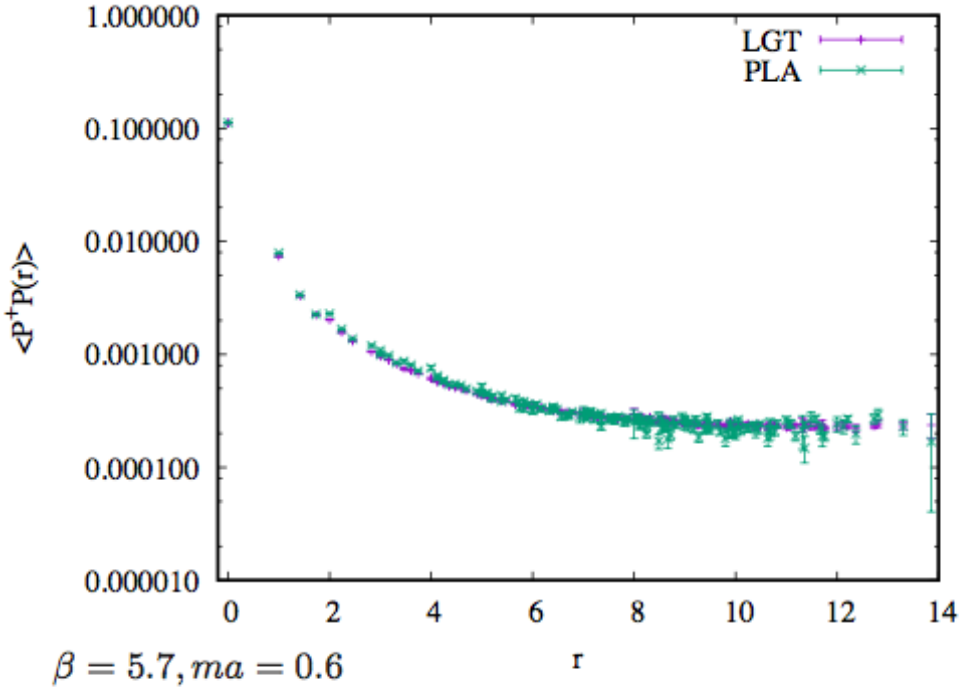

k=0

# Fourier transform $K(k)$ to $K(r)$

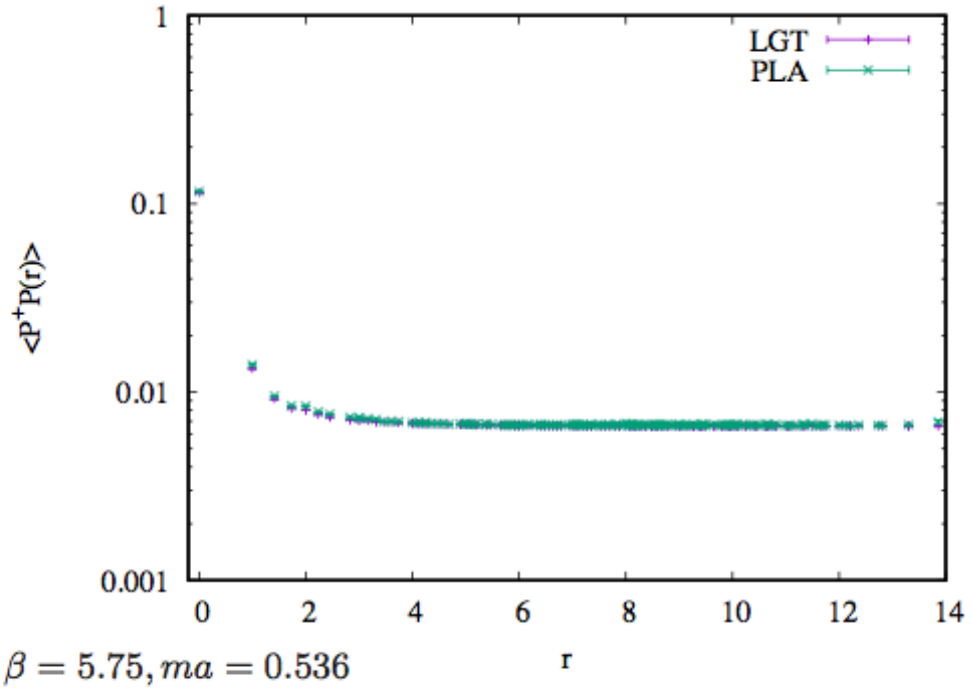


# Finite size cutoff $R_{cut}$ for $K(r)$






$\beta = 5.7, ma = 0.6$

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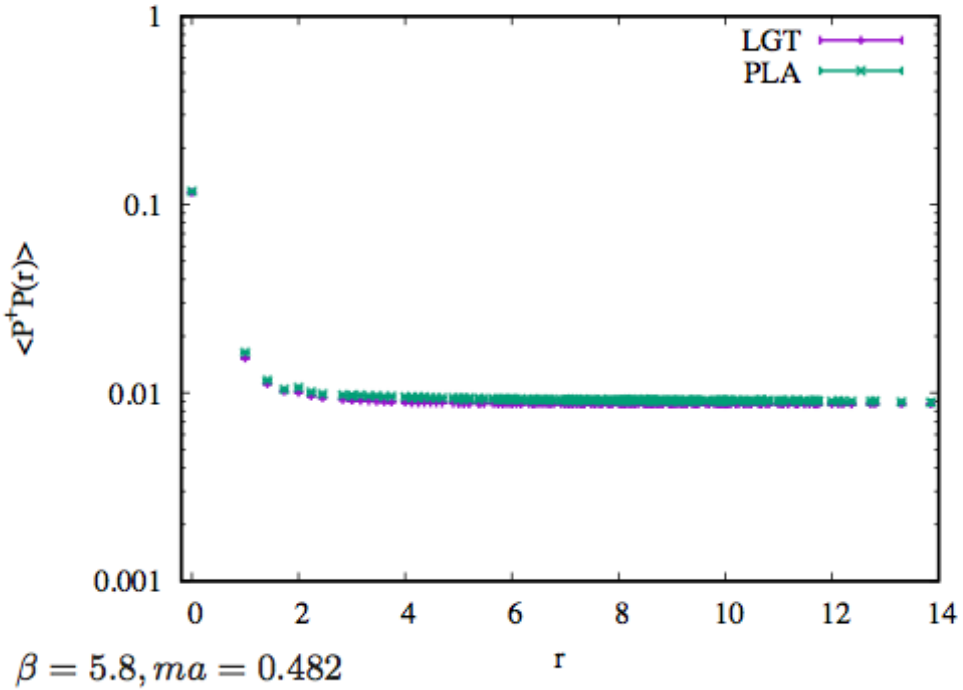
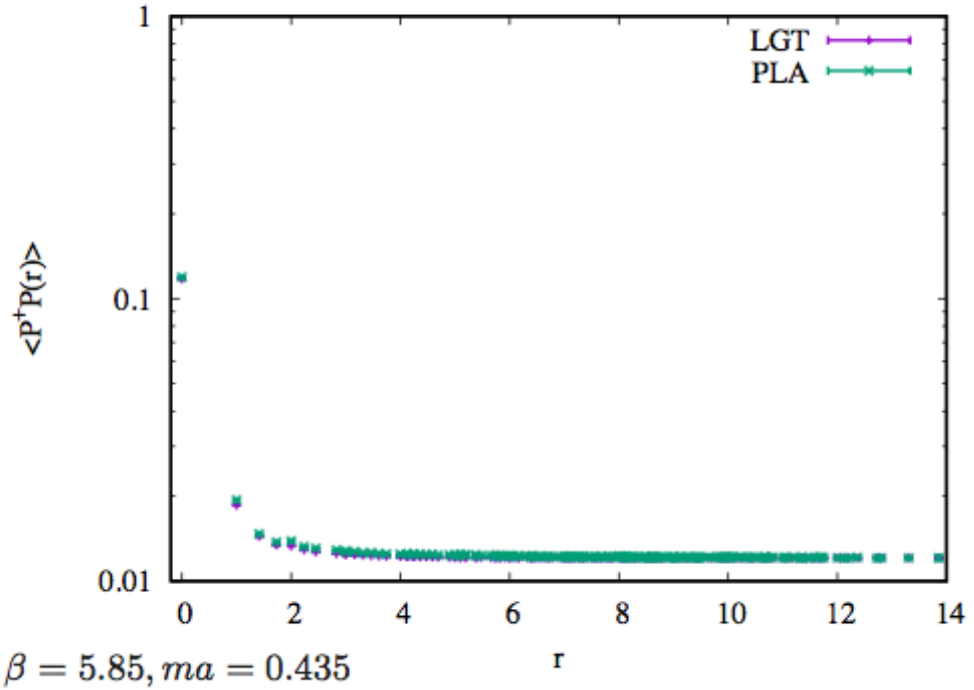



$\beta = 5.8, ma = 0.482$

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$$\mathrm{Tr}U_x = (\mathrm{Tr}U_x - u) + u \ , \ \mathrm{Tr}U_x^\dagger = (\mathrm{Tr}U_x^\dagger - v) + v$$

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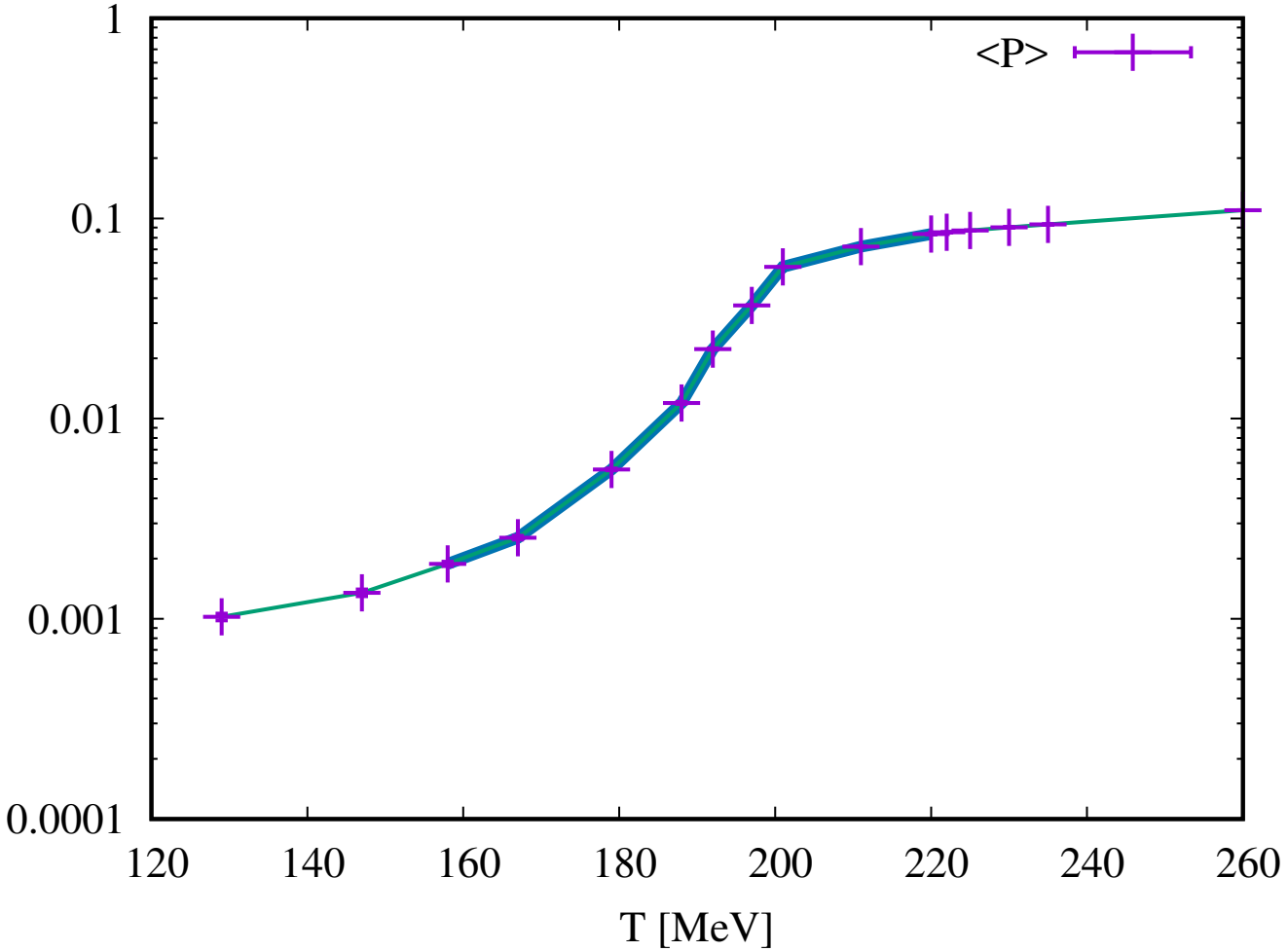
- equivalent to the stationarity of the mean field free energy with respect to variations in $u$ and $v$ $\rightarrow$ solve numerically

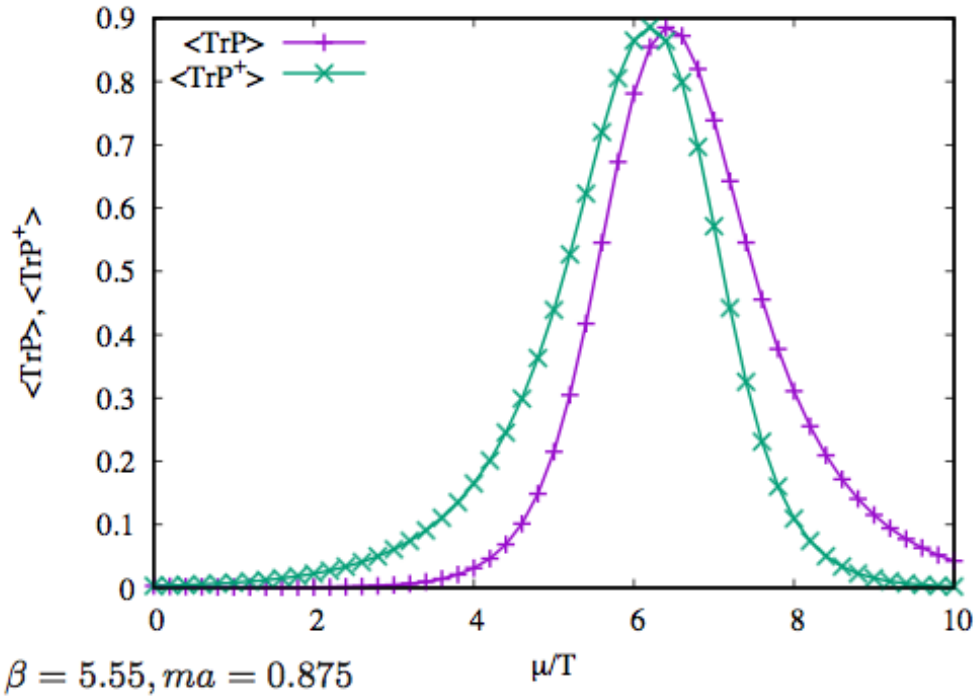
# Simulation parameters and mean field results

- for effective Polyakov line actions derived from LGT
- on $16^3$x6 lattices with Wilson gauge action and
- dynamical staggered fermions with $m_q = 695$MeV
- note good agreement of Polyakovs and small $h$!!!

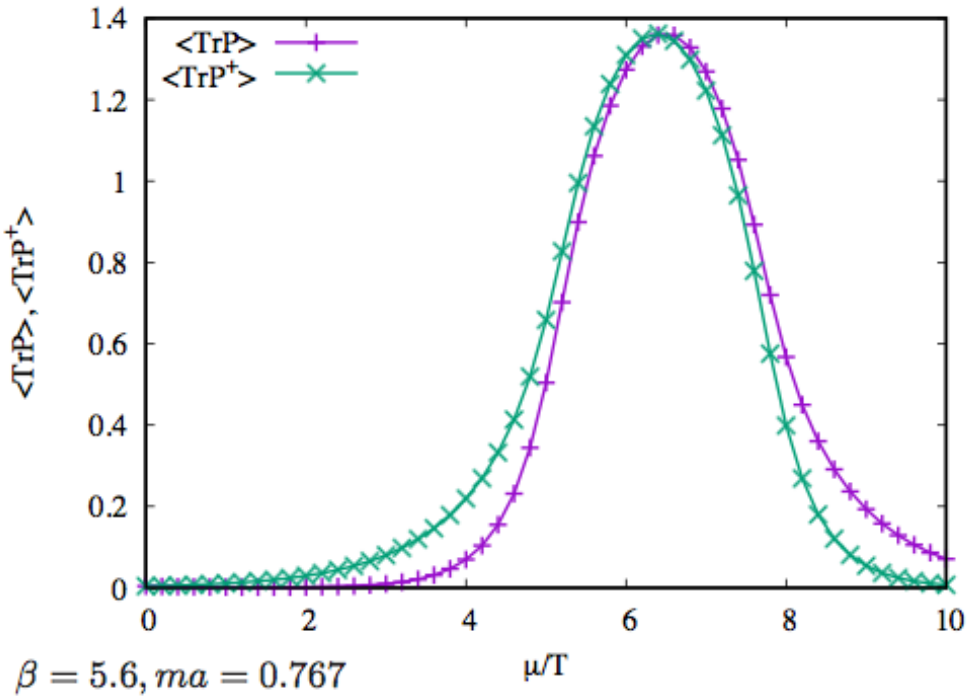
| $\beta$ | $T$[MeV] | $a$[fm] | $ma$ | P (LGT) | P (mfd) | $R_{cut}$ | $h$ | $\mu_1/T$ | $\mu_2/T$ |
|---|---|---|---|---|---|---|---|---|---|
| 5.55 | 129 | 0.248 | 0.875 | 0.00102 | 0.00101 | 5.5 | 0.0018 | - | - |
| 5.60 | 147 | 0.217 | 0.767 | 0.00135 | 0.00133 | 5.5 | 0.0014 | - | - |
| 5.63 | 158 | 0.201 | 0.711 | 0.00188 | 0.00189 | 5.5 | 0.0017 | 4.4 | 8.5 |
| 5.65 | 167 | 0.192 | 0.677 | 0.00254 | 0.00249 | 5.5 | 0.0019 | 4.2 | 8.3 |
| 5.70 | 188 | 0.170 | 0.601 | 0.01198 | 0.01195 | 5.0 | 0.0069 | 3.5 | 6.5 |
| 5.73 | 201 | 0.159 | 0.561 | 0.05734 | 0.05731 | 5.0 | 0.0220 | 2.9 | 5.5 |
| 5.75 | 211 | 0.152 | 0.536 | 0.07235 | 0.07110 | 5.0 | 0.0272 | 2.1 | 5.3 |
| 5.77 | 220 | 0.145 | 0.513 | 0.08354 | 0.08347 | 5.0 | 0.0387 | 1.1 | 5.1 |
| 5.775 | 222 | 0.144 | 0.508 | 0.08522 | 0.08523 | 4.0 | 0.0565 | - | - |
| 5.78 | 225 | 0.142 | 0.502 | 0.08703 | 0.08679 | 4.0 | 0.0610 | - | - |
| 5.80 | 235 | 0.136 | 0.482 | 0.09332 | 0.09437 | 4.0 | 0.0710 | - | - |
| 5.85 | 260 | 0.123 | 0.435 | 0.10992 | 0.01115 | 4.0 | 0.0900 | - | - |

# Finite temperature transition at $\mu = 0$

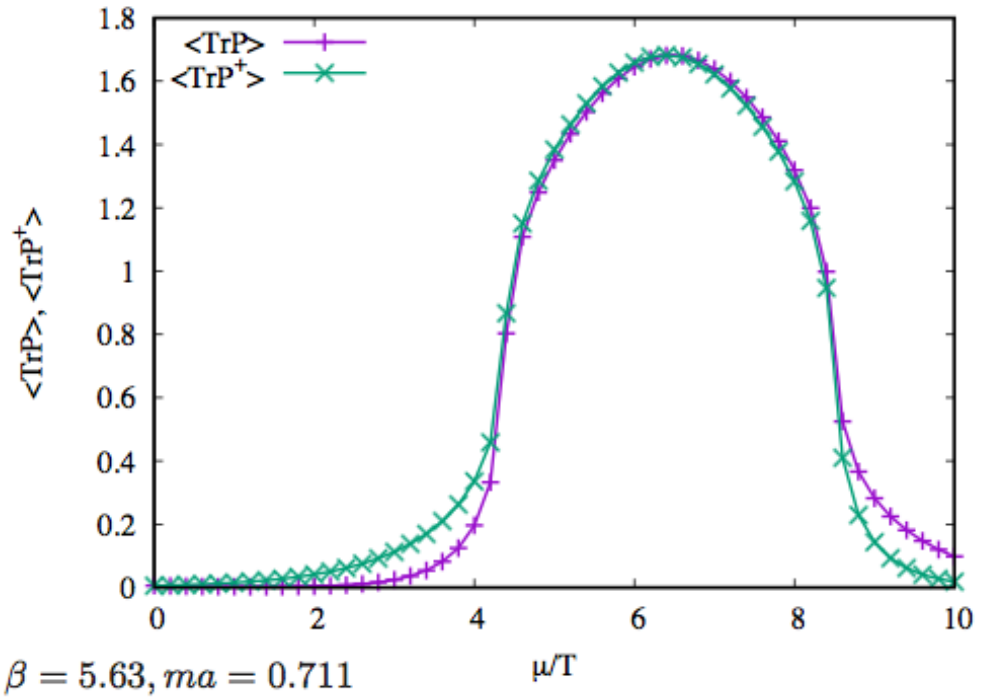




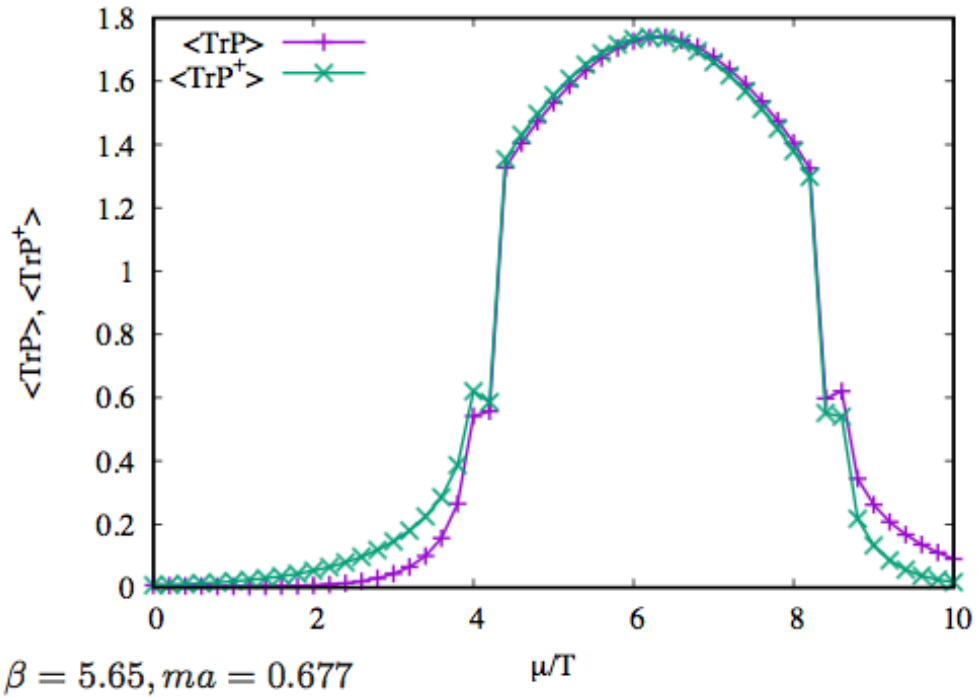

$\beta = 5.55, ma = 0.875$



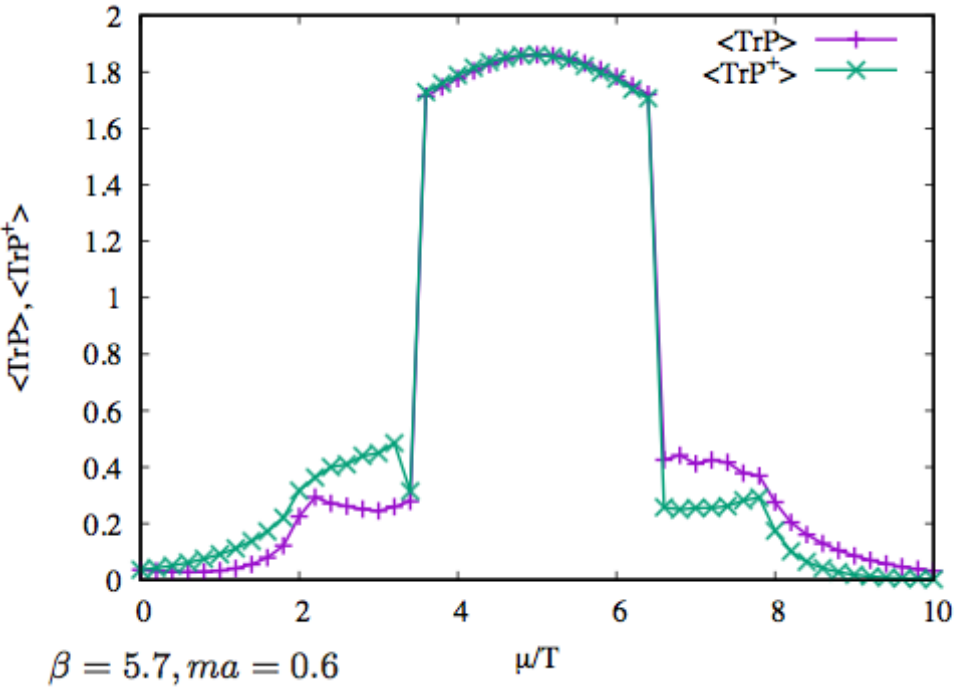

$\beta = 5.6, ma = 0.767$




$\beta = 5.63, ma = 0.711$




$\beta = 5.65, ma = 0.677$

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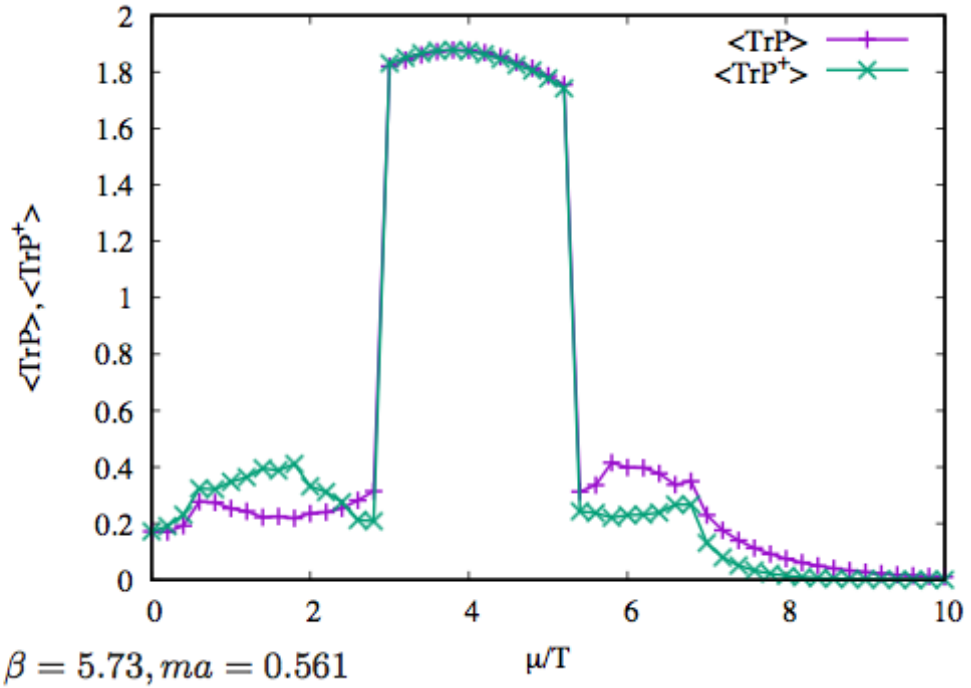
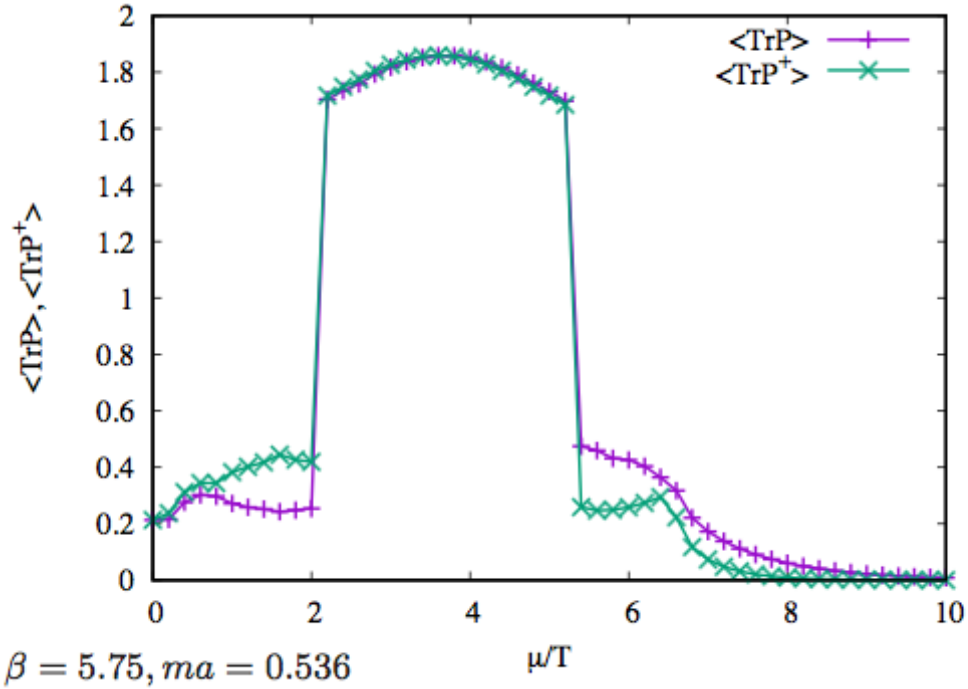



$\beta = 5.73, ma = 0.561$

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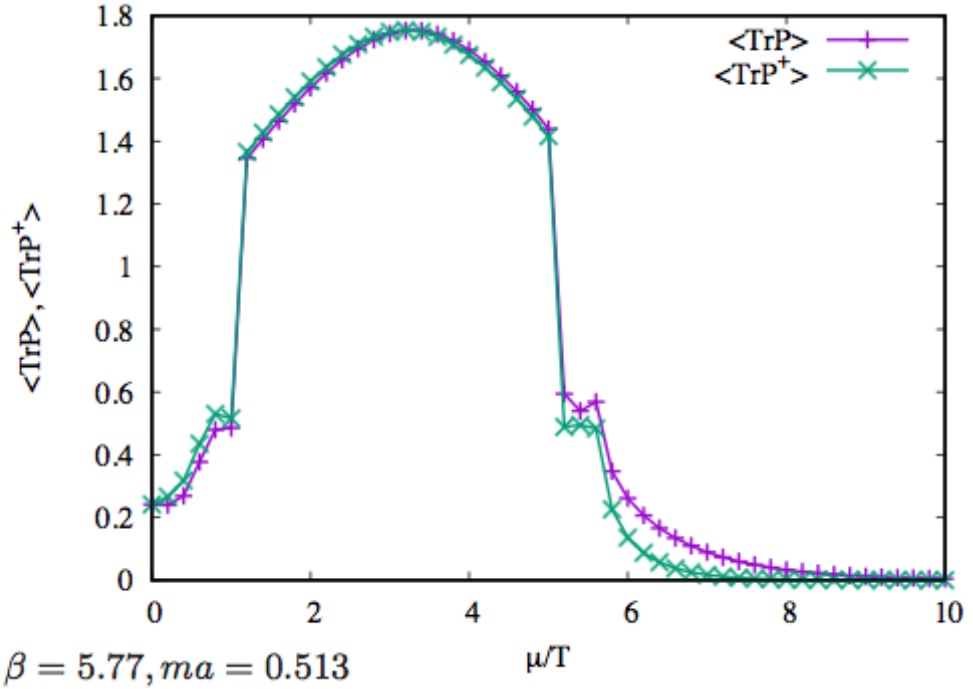
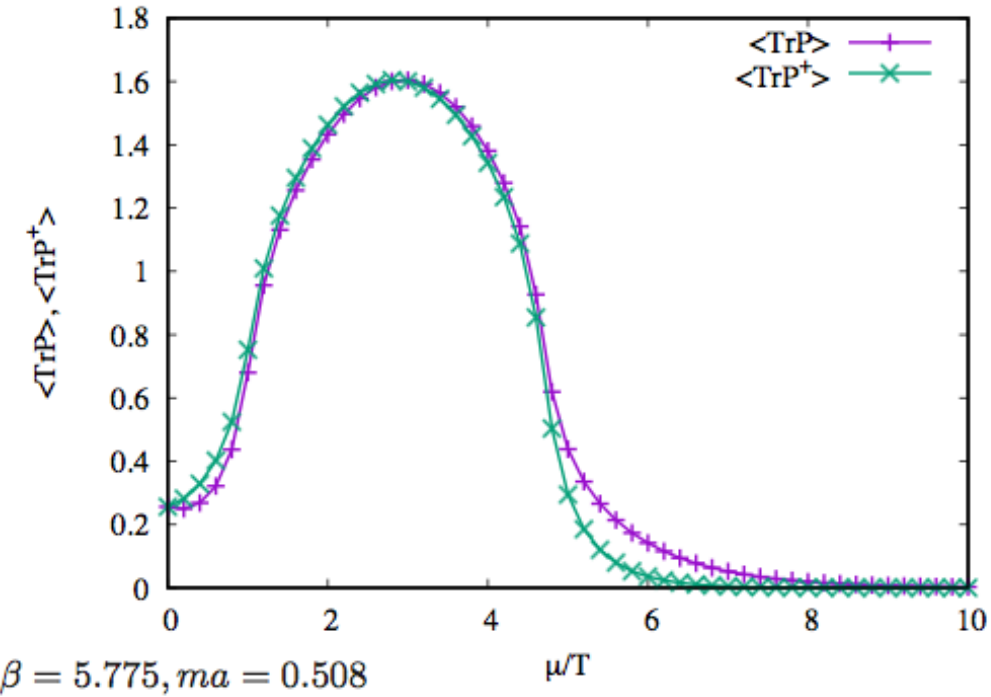
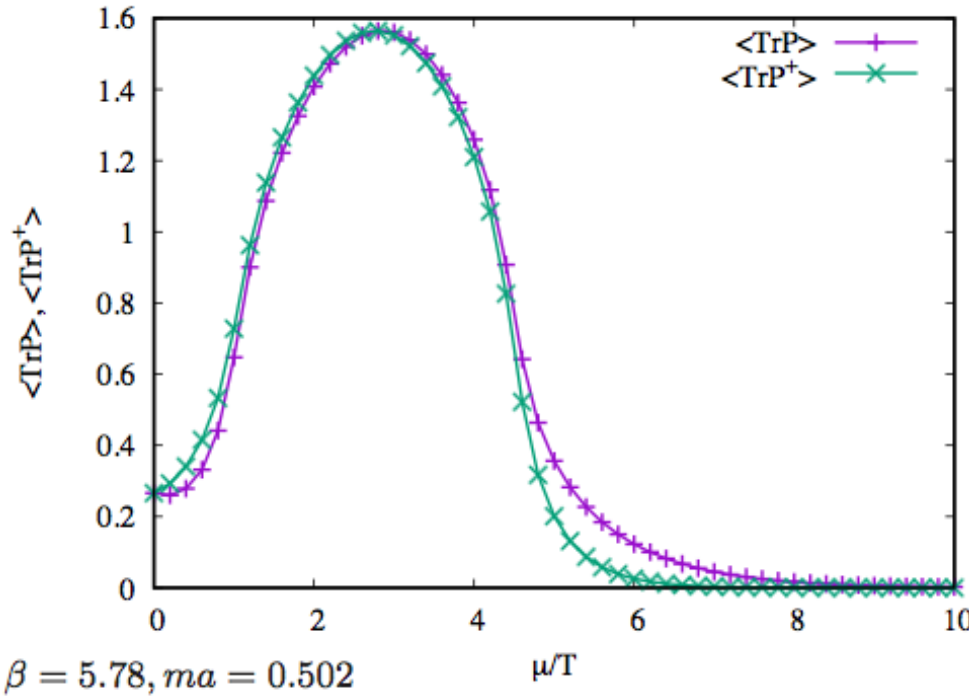


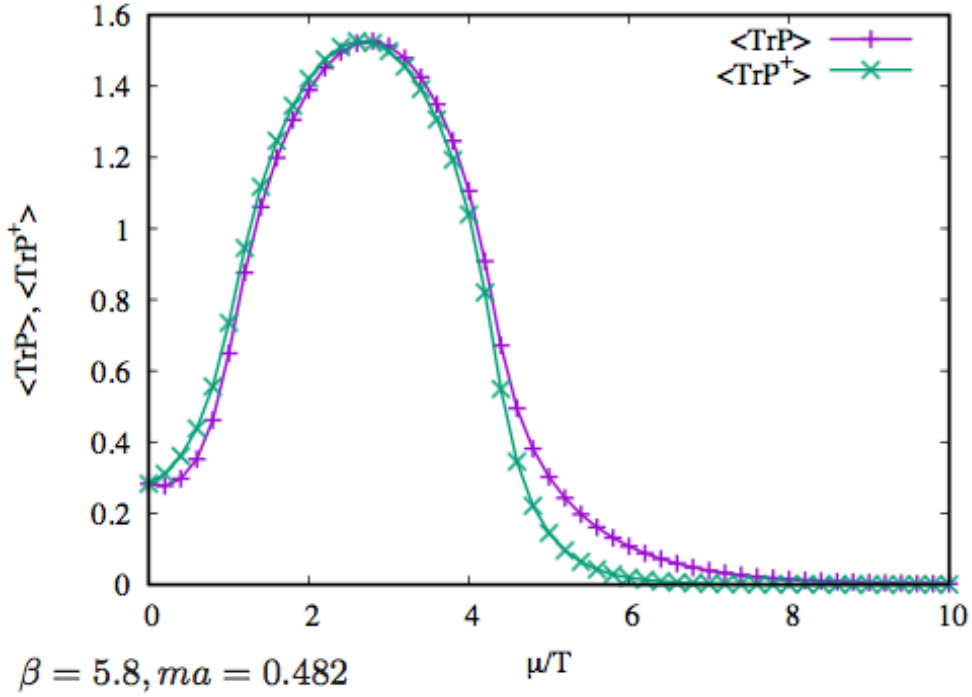

$\beta = 5.77, ma = 0.513$




$\beta = 5.775, ma = 0.508$




$\beta = 5.78, ma = 0.502$

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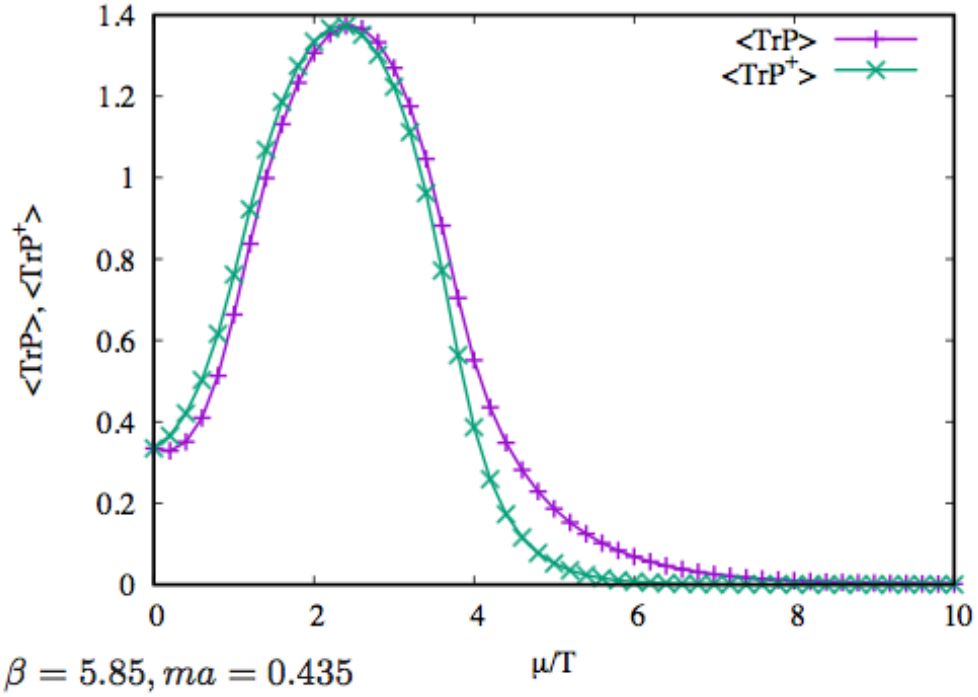


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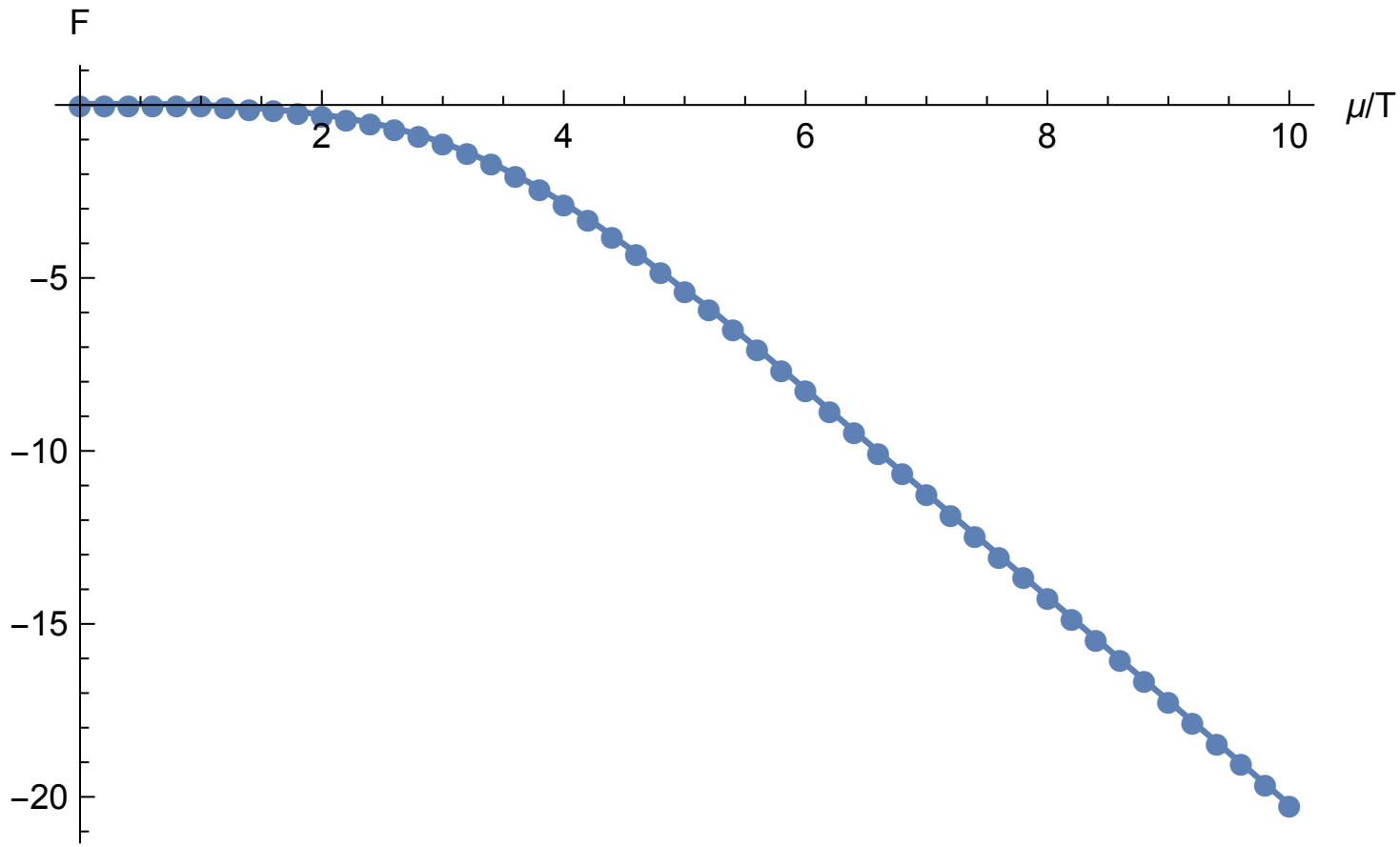
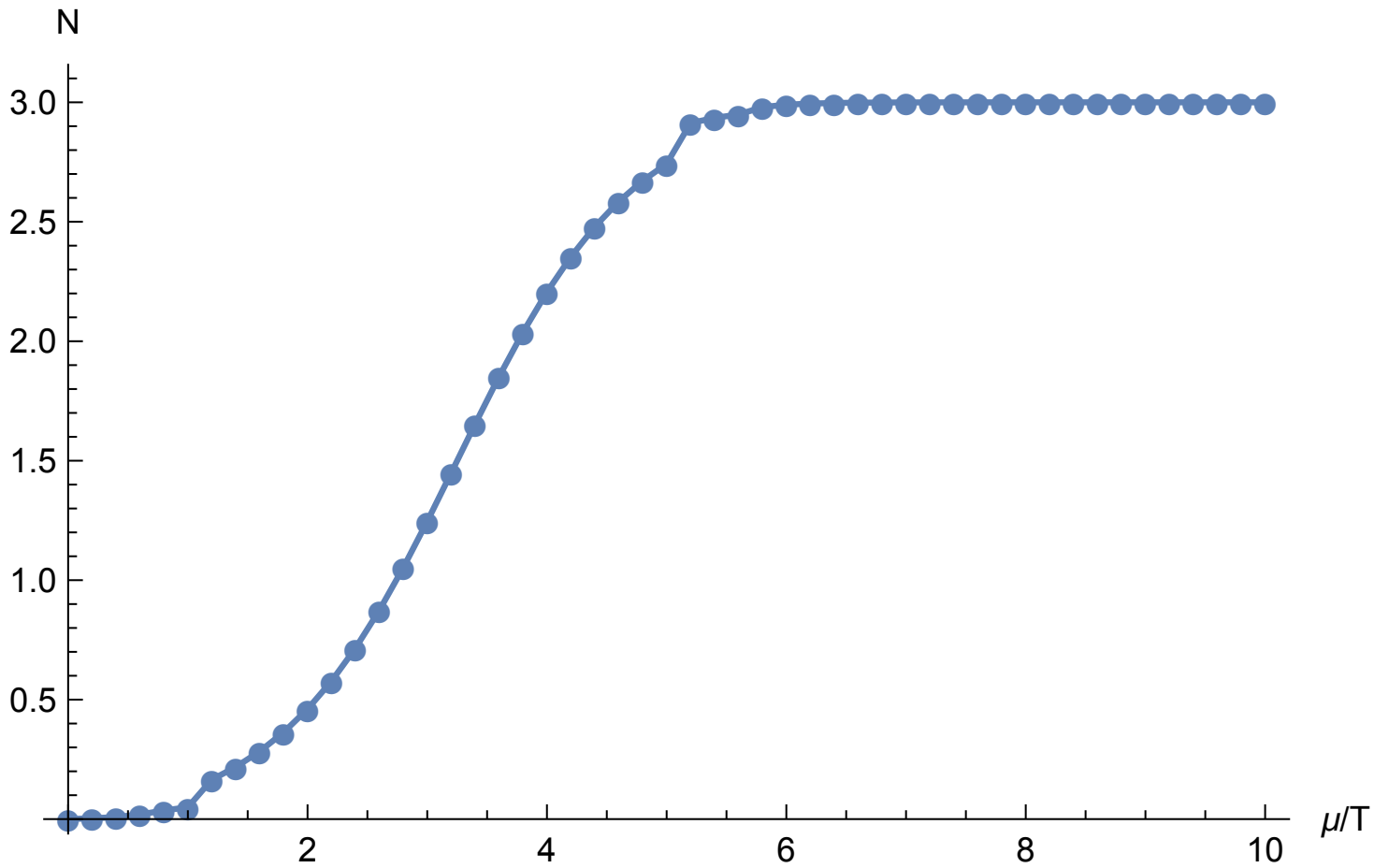
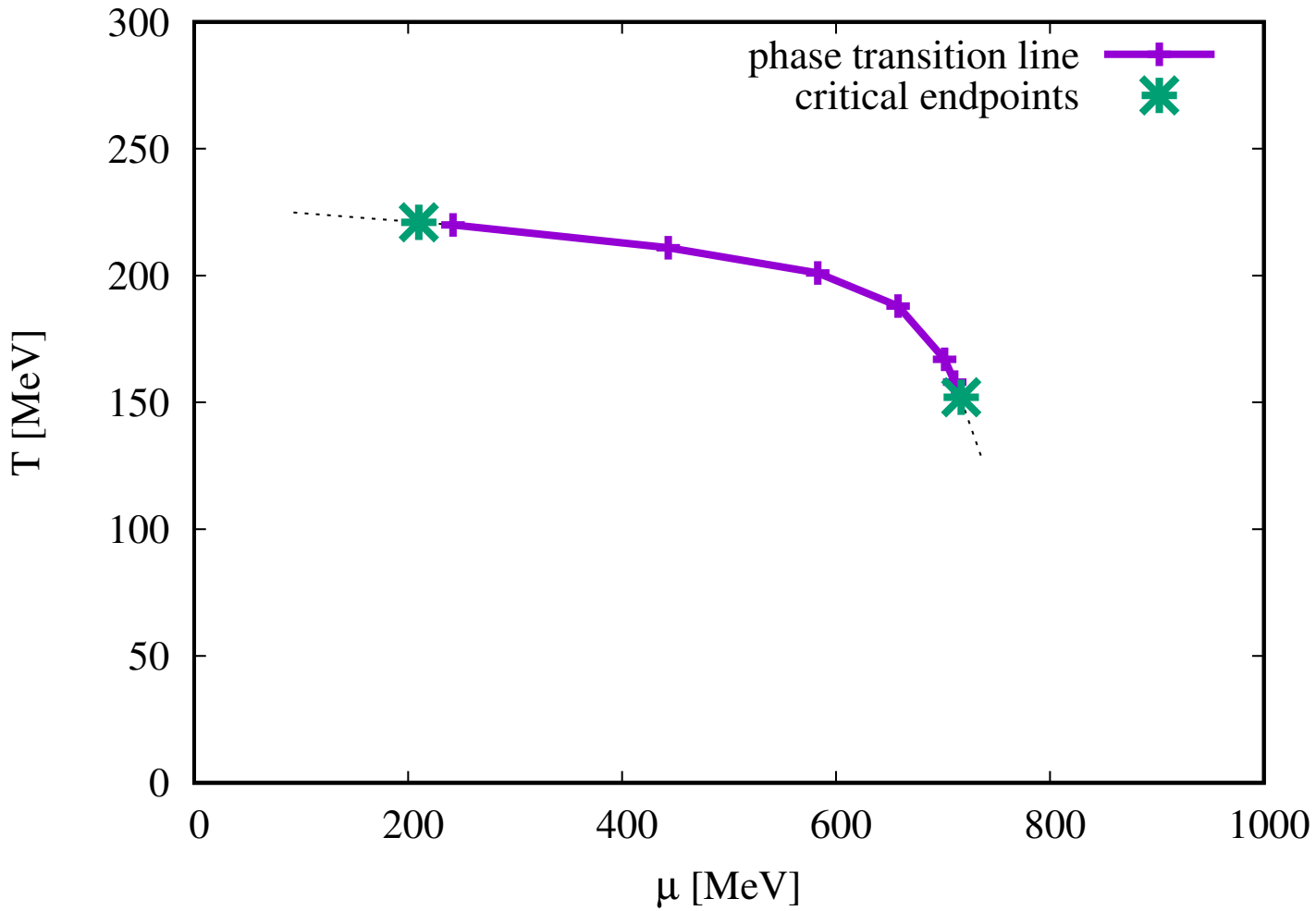
# Free energy



# Number density



# Preliminary Phase Diagram



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- can the mean-field PLA still locate transition lines and determine critical properties reliably?

# Questions?

## Thank You &

Tareq Alhalholy, Derar Altarawneh, Michael Engelhardt, Manfried Faber, Martin Gal, Jeff Greensite, Urs M. Heller, James Hettrick, Andrei Ivanov, Thomas Layer, Štefan Olejnik, Luis Oxman, Mario Pitschmann, Jesus Saenz, Thomas Schweigler, Wolfgang Söldner, David Vercauteren, Markus Wellenzohn

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# Preliminary QCD phase transition line with 695 MeV dynamical staggered fermions from effective Polyakov line actions

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