2017/6/28 @ XQCD 2017, Pisa

Thermodynamics of QCD at physical point with (2+1)-flavors of improved Wilson quarks using gradient flow

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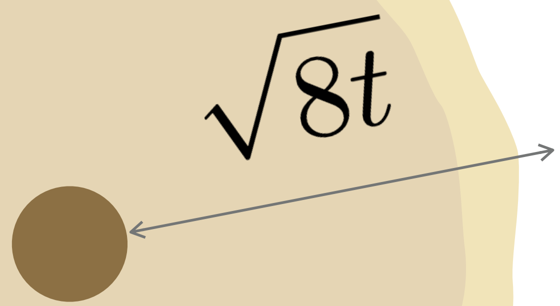
QCD thermodynamics with gradient flow

Gradient flow

Narayanan-Neuberger (2006), Lüscher (2009–)

Imaginary evolution of the system in terms of a fictitious "time" t preserving gauge sym. etc.:

(ex) pure gauge theory $\dot{B}_\mu = D_\nu G_{\nu\mu}, \qquad B_\mu\big|_{t=0} = A_\mu$ ⟵ original gauge field



We may view the flowed field B_μ as a smeared A_μ over a physical range of $\sqrt{(8t)}$.

It was shown that operators of flowed fields have no UV divergences nor short-distance singularities at $t > 0$.

Lüscher-Weisz (2011)

GF provides us with a new physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the $a \to 0$ limit.

This opened many possibilities to drastically simplify lattice evaluation of physical observables.

=> Using the finiteness of flowed observables, H. Suzuki proposed **a new method to calculate the EMT on the lattice**. EMT is the generator of continuous Poincare transf., and thus has not been simple to define/evaluate on the lattice.

QCD thermodynamics with gradient flow

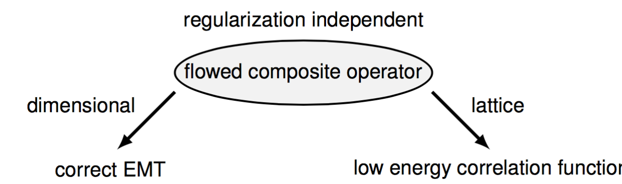
Energy-momentum tensor using GF

H.Suzuki (2013)

To avoid violation of Poincare sym. on the lattice,

1) **Define EMT in a continuum scheme, using a W-T identity of Poincare inv., as usual.**

2) **When we flow this EMT to $t > 0$, because it is finite,**
it becomes directly calculable on the lattice in the $a \to 0$ limit.



Through the GF evolution, however, higher-d operators can contaminate at $t > 0$.

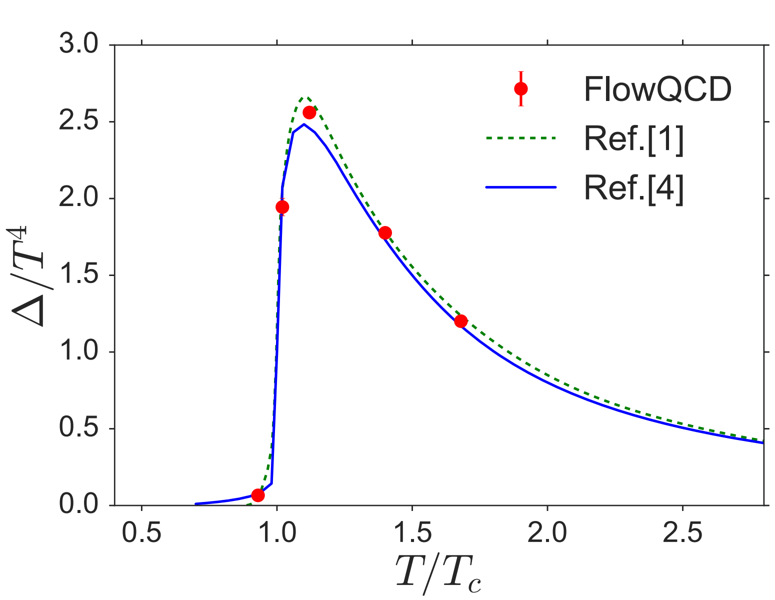
3) **Remove the unwanted contributions by another extrapolation of $t \to 0$.**

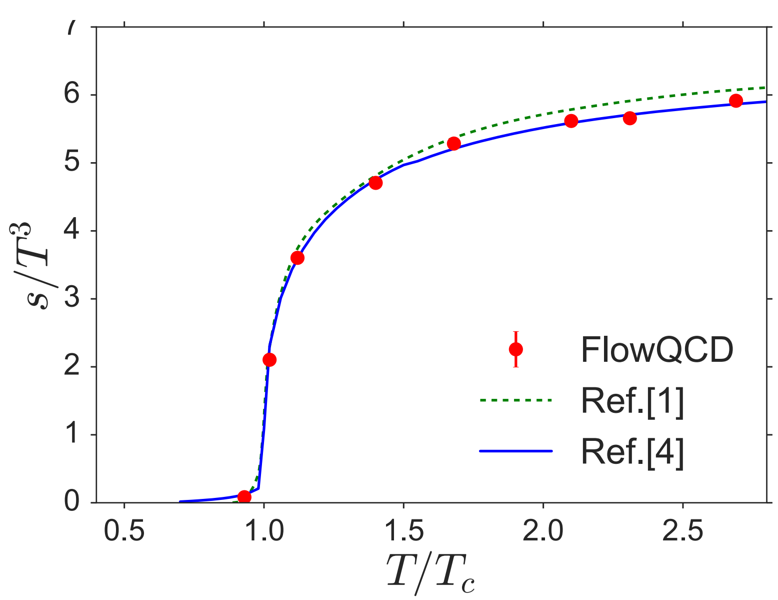
Can make this extrapol. smoother using a small-t oper. expansion by Lüscher:
The mixing coeff's. c_i near the $t \to 0$ limit can be calculated by PT in AF theories.

$$\tilde{O}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t) O_i^{\mathrm{R}}(x)$$

Test in quenched QCD

FlowQCD, PRD94,114512(2016); D90,011501(2014) [E:D92,059902]





$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$\epsilon = -\langle T_{00}\rangle,\ p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$$

The EOS' from the conventional methods reproduced in $t \to 0$ and $a \to 0$.

(2+1)-flavor QCD thermodynamics with GF

Our project: Application to (2+1)-flavor QCD

GF with quarks : Lüscher, JHEP 1304, 123 (2013)

- * We can adopt pure gauge actions for GF,
- * at the price of a non-trivial field renormalization of quarks.

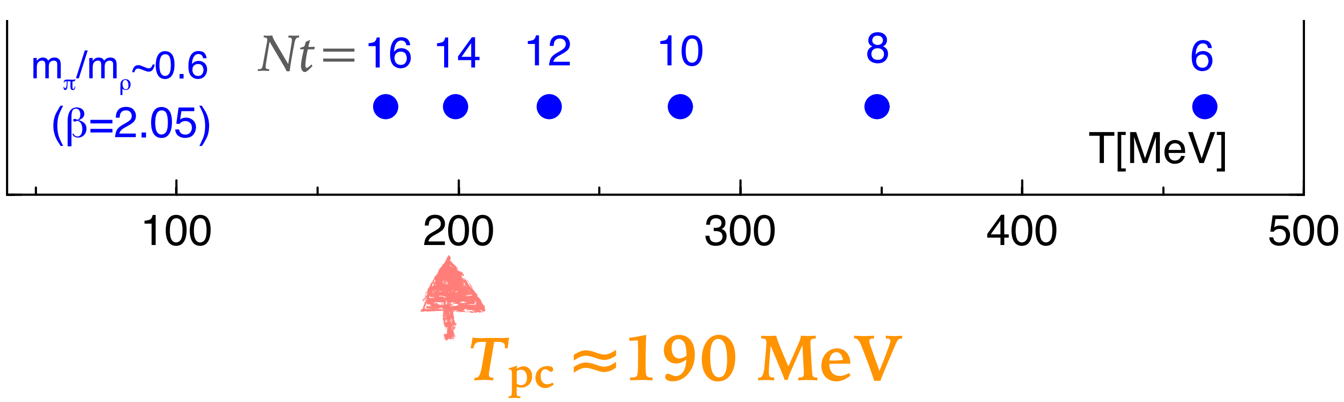
Full QCD EMT by GF : Makino-Suzuki, PTEP 2014, 063B02 (2014)

Chiral condensate by GF : Hieda-Suzuki, Mod.Phys.Lett. A31, 1650214 (2016)

Topological charge / susceptibility, etc. etc.

1st step:

- ➤ Heavy ud quarks with ≈physical s quark (m_{PS}/m_V ≈0.63).
- ➤ Fine lattice (a≈0.07fm) with the fixed-scale approach.
- ➤ Compare with the results of the conventional methods.



- ✱ Nf=2+1 QCD, Iwasaki gauge + NP-clover
- ✱ CP-PACS+JLQCD's $T = 0$ config. (β=2.05, 28^3x56, a≈0.07fm)
- ✱ $T > 0$ by fixed-scale approach, (32^3xNt, Nt = 4, 6, ... , 14, 16): T≈174--697MeV
- ✱ EoS by T-integration method available (WHOT-QCD, PRD85)
- ✱ gauge meas. at every config., quark meas. every 10 config's.

Gauge and Quark Flows

Lüscher, JHEP 1008, 071 ('10); 1304, 123 ('13)

We adopt the simplest one suggested by Lüscher.

Gauge flow: standard Wilson flow

original gauge field at t = 0

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)],$$

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)],$$

Quark flow: as suggested by Lüscher

original quark field at t = 0

$$\partial_t \chi_f(t,x) = \Delta \chi_f(t,x), \qquad \chi_f(t=0,x) = \psi_f(x),$$

$$\partial_t \bar\chi_f(t,x) = \bar\chi_f(t,x)\overleftarrow{\Delta}, \qquad \bar\chi_f(t=0,x) = \bar\psi_f(x),$$

$$\Delta\chi_f(t,x) \equiv D_\mu D_\mu \chi_f(t,x), \qquad D_\mu \chi_f(t,x) \equiv [\partial_\mu + B_\mu(t,x)]\,\chi_f(t,x),$$

$$\bar\chi_f(t,x)\overleftarrow{\Delta} \equiv \bar\chi_f(t,x)\overleftarrow{D}_\mu\overleftarrow{D}_\mu, \qquad \bar\chi_f(t,x)\overleftarrow{D}_\mu \equiv \bar\chi_f(t,x)\left[\overleftarrow{\partial}_\mu - B_\mu(t,x)\right]$$

only gauge fields involved

Quark field renormalizattion

$$\chi_R(t,x) = Z_\chi \chi_0(t,x) \qquad Z_\chi = \sqrt{\varphi(t)} \qquad \varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar\chi_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

Makino-Suzuki ('14)

VEV (T=0)

No more renormalization needed for any composite op's.

Nf=2+1 QCD EMT by GF

Makino-Suzuki, PTEP 2014, 063B02 (2014)

EMT in full QCD

Operators on the lattice

$$\tilde{\mathcal{O}}_{1\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x),$$
$$\tilde{\mathcal{O}}_{2\mu\nu}(t,x) \equiv \delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x),$$
$$\tilde{\mathcal{O}}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{\mathcal{O}}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{\mathcal{O}}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

Quark field renormalization

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

$$\begin{aligned} T_{\mu\nu}(x) = \lim_{t\to 0}\Big\{ & c_1(t)\left[\tilde{\mathcal{O}}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right] \\ & + c_2(t)\left[\tilde{\mathcal{O}}_{2\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}_{2\mu\nu}(t,x)\right\rangle_0\right] \\ & + c_3(t)\sum_{f=u,d,s}\left[\tilde{\mathcal{O}}^f_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^f_{4\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}^f_{3\mu\nu}(t,x) - 2\tilde{\mathcal{O}}^f_{4\mu\nu}(t,x)\right\rangle_0\right] \\ & + c_4(t)\sum_{f=u,d,s}\left[\tilde{\mathcal{O}}^f_{4\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}^f_{4\mu\nu}(t,x)\right\rangle_0\right] \\ & + \sum_{f=u,d,s} c^f_5(t)\left[\tilde{\mathcal{O}}^f_{5\mu\nu}(t,x) - \left\langle\tilde{\mathcal{O}}^f_{5\mu\nu}(t,x)\right\rangle_0\right]\Big\}, \end{aligned}$$

VEV-subtraction
(T = 0 subtraction)

using coefficients by Makino-Suzuki evaluated in one-loop PT.

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right],$$
$$c_2(t) = \frac{1}{(4\pi)^2}\frac{33}{16},$$
$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\},$$
$$c_4(t) = \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2,$$
$$c^f_5(t) = -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}$$

Physics extracted by $t\to 0$ extrapolation.

At $a\neq 0$, additional mixing with unwanted operators

$$T_{\mu\nu}(t,x,a) = T_{\mu\nu}(t,x) + A_{\mu\nu}\frac{a^2}{t} + \sum_f B_{f\mu\nu}(am_f)^2 + C_{\mu\nu}(aT)^2 + D_{\mu\nu}(a\Lambda_{\rm QCD})^2 + a^2 S'_{\mu\nu}(x) + \mathcal{O}(a^4),$$

Note: lattice artifacts of NP-clover is O(a^2).

Singular terms at t = 0 due to mixing with D=4 ops.
=> should be handled properly in the $t\to 0$ extrapolation.

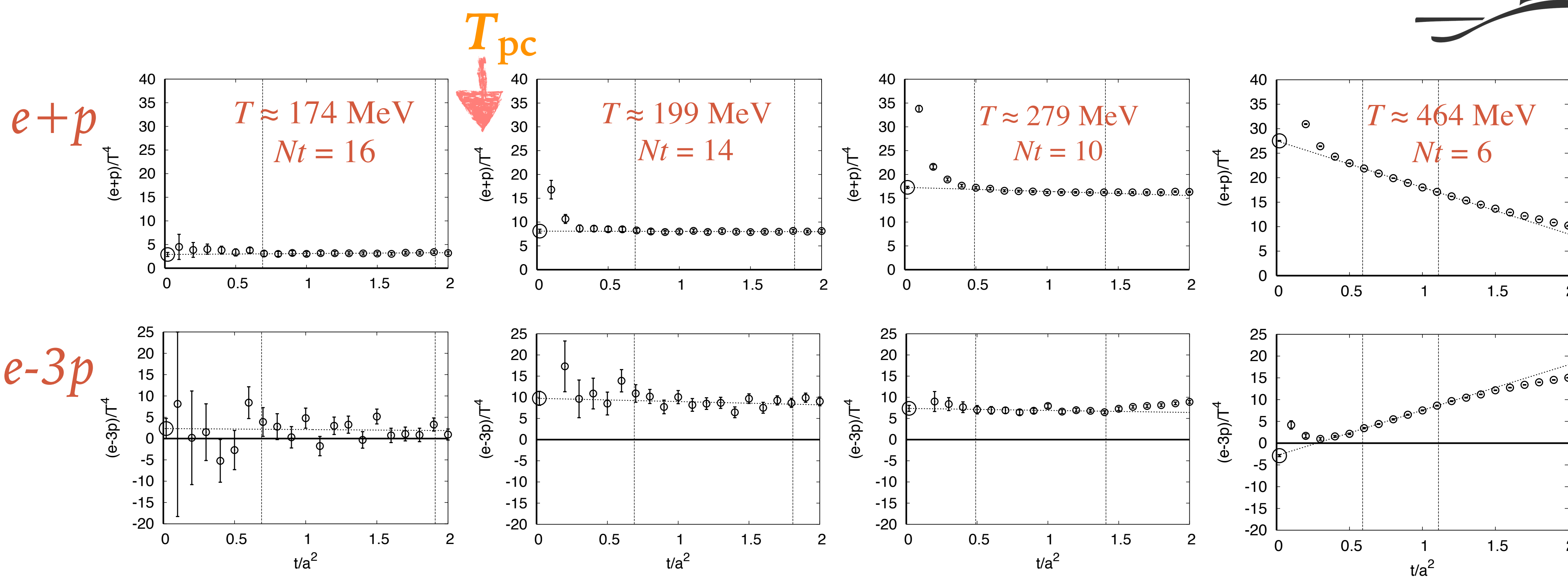
Nf=2+1 heavy QCD EoS by GF

$$\epsilon = -\langle T_{00} \rangle, \ p = \frac{1}{3} \sum_i \langle T_{ii} \rangle$$

1st step:

- Heavy ud quarks with ≈physical s quark (m_{PS}/m_V ≈0.63), but on a fine lattice (a≈0.07fm) with the fixed-scale approach.
- Preliminary results presented at xQCD 2016, Plymouth:

XQCD 2016



- a^2/t-like behavior close to t = 0.
- Linear behavior within meaningful range of t. <= $\sqrt{(8t/a^2)} \leq \min(Ns/2, Nt/2)$ to avoid oversmearing.
- a^2/t term looks negligible in the "linear windows" => Linear fit using the windows.
- At T ≈ 697 MeV (Nt=4), no linear windows found.

Nf=2+1 heavy QCD EoS by GF

1st step:

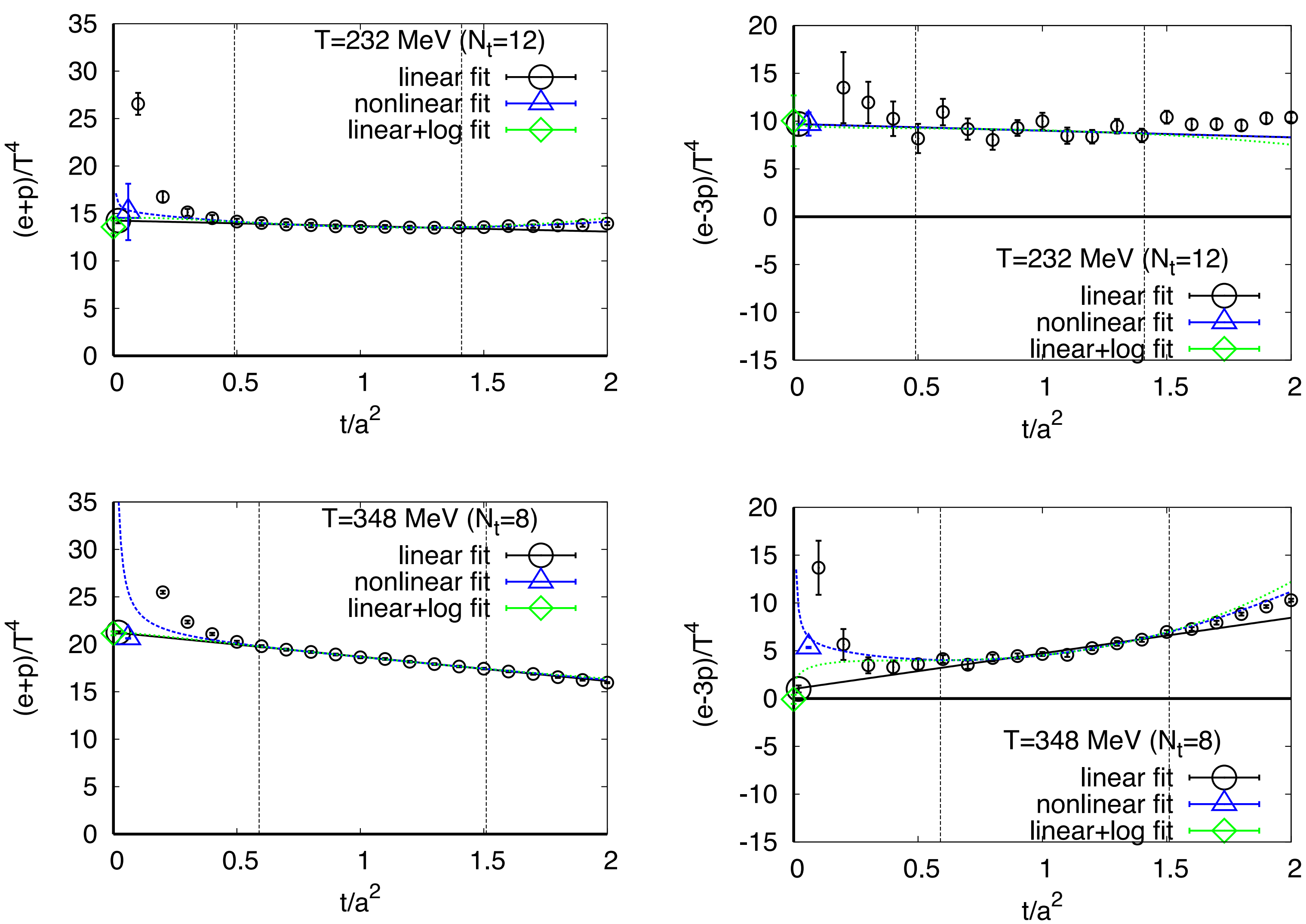
- After XQCD 2016, we made a series of additional anlyses
  - to confirm the linear extrapolation procedure at a>0
  - to estimate systematic error due to the fit ansatz

$$\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + A_{\mu\nu}\frac{a^2}{t} + t\,S_{\mu\nu} + t^2 R_{\mu\nu}$$

- nonlinear fit, inspired from a^2/t as well as next-leading t corrections.

$$\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$$

- linear+log fit, inspired from higher order PT corrections in the one-loop Suzuki coeff's. c_i.



- In most cases, all the fits are consistent with each other using the same window.
- Take the deviations as an estimate of systematic error due to the fit ansatz.

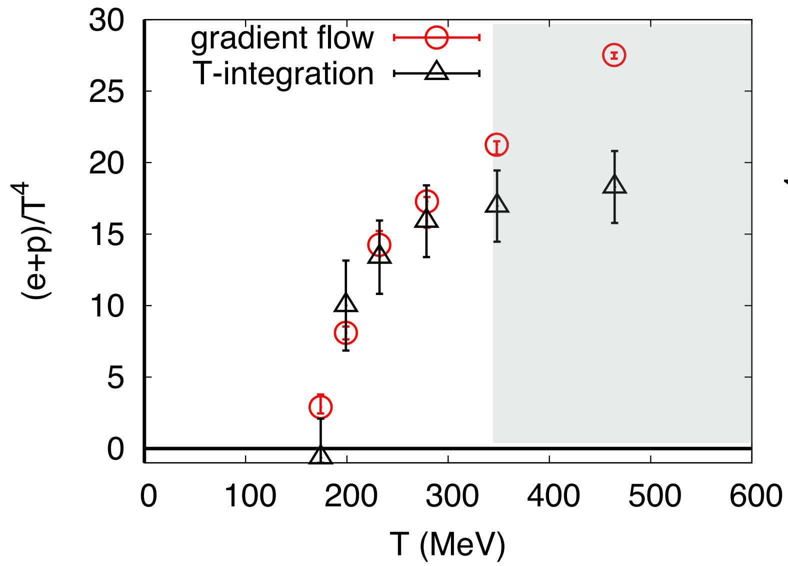
Nf=2+1 heavy QCD EoS / chiral transition by GF

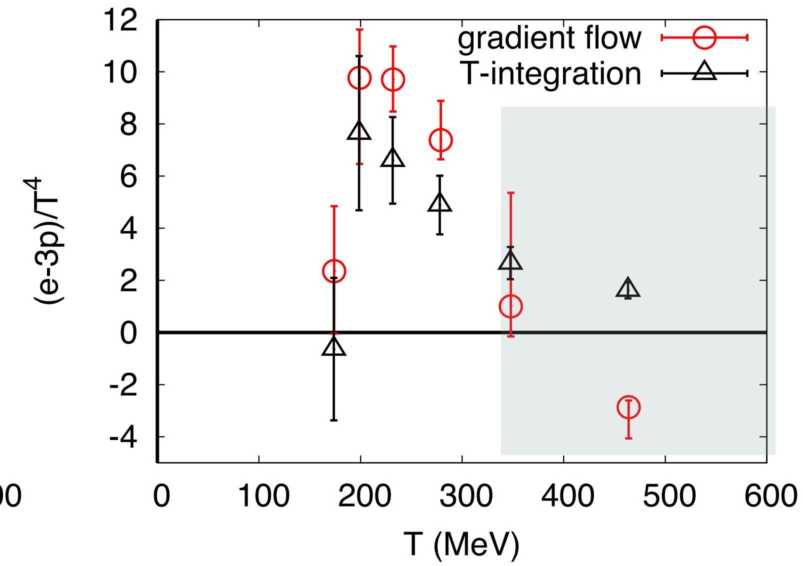
1st step: Results

to be published in Phys.Rev.D

➤ EoS

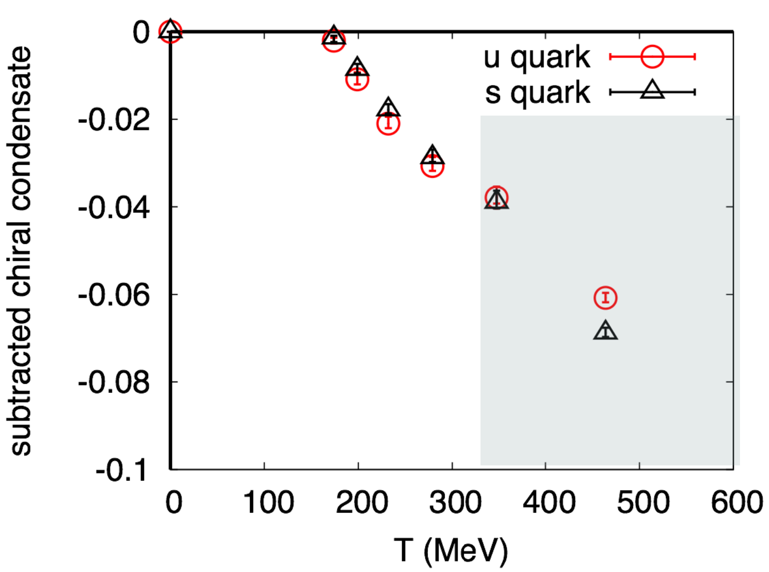
Errors include statical as well as systematical ones due to fit ansatz etc.

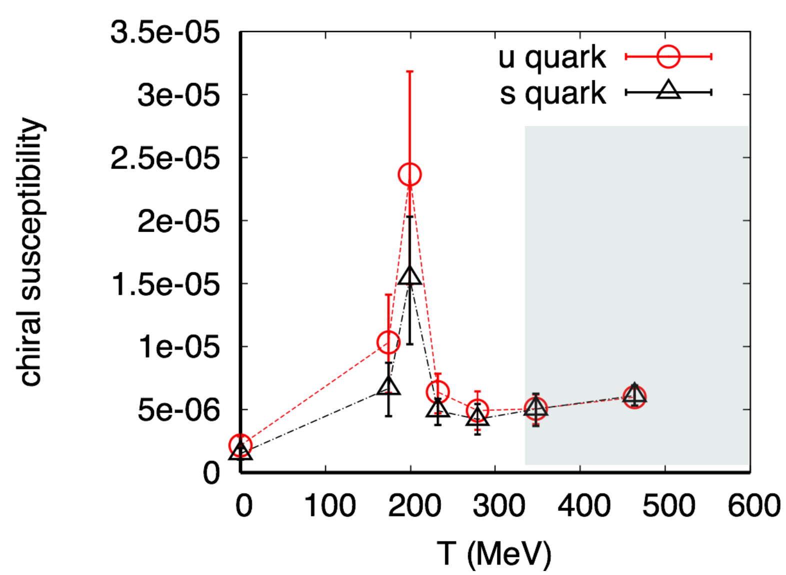




- ☑ EoS by GF agrees with conventional method at $T \lesssim 300$ MeV ($Nt \geq 10$). Suggest $a \approx 0.07$fm close to the cont. limit.
- ☑ Disagreement at $T \gtrsim 350$ MeV due to $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt \lesssim 8$.

➤ Chiral cond. / disconnected susceptibility





$\overline{\text{MS}}$ scheme at μ=2 GeV

- ☑ Crossover suggested around $T_{pc} \approx 190$ MeV, consistent with previous study.
- ☑ Peak higher with decreasing m_q, as expected.
- ☑ Physically expected results even with Wilson-type quarks. GF method powerful to extract physical properties.

Nf=2+1 heavy QCD thermodynamics by GF

1st step: Topological charge / susceptibility — Phys.Rev.D95, 054502 (2017)

- Gluonic definition *vs.* fermonic definition

(a) *gluonic definition* — Lüscher('10), Consonni-Engel-Giusti('15)

$$Q(t) = \int d^4x\, q(t,x) \qquad q(t,x) = \frac{1}{64\pi^2}\epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu}(t,x) G^a_{\rho\sigma}(t,x)$$

(b) *fermionic definition* — Guisti-Rossi-Testa('04)

$$N_f^2 \langle Q^2 \rangle = m^2 \left(\langle P^0 P^0 \rangle - N_f \langle P^a P^a \rangle \right)$$

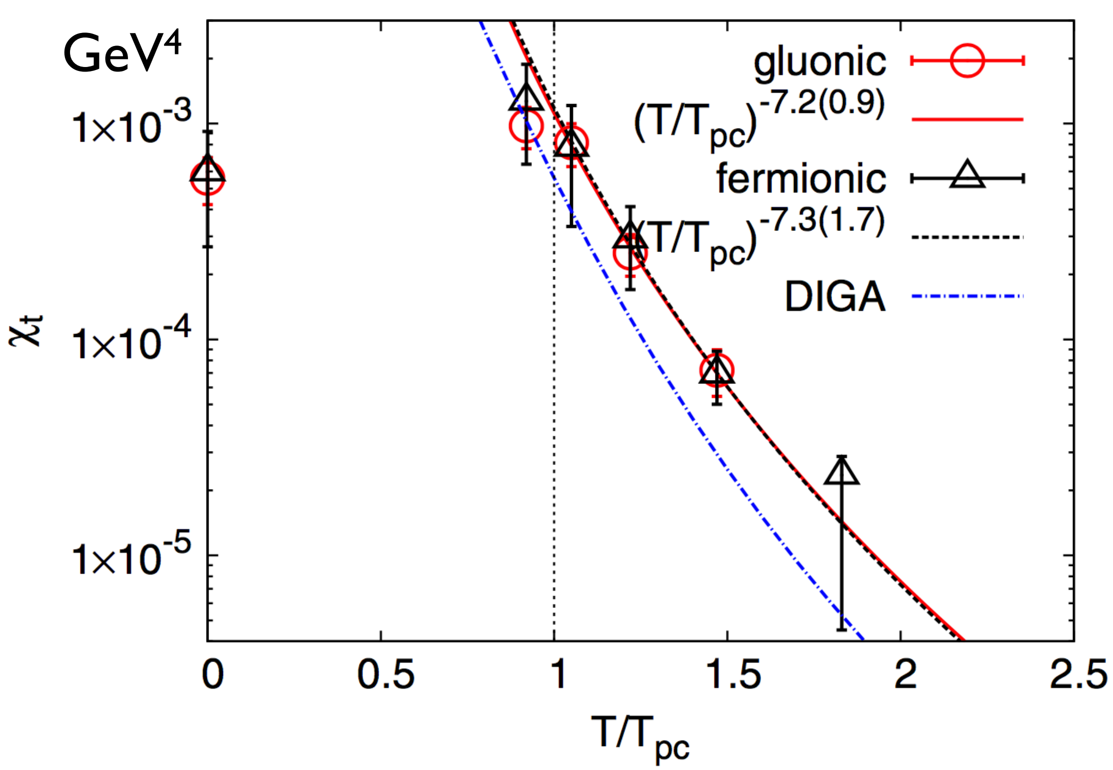
using chiral W-T identities

$$P^0 = \int d^4x \bar\psi(x)\gamma_5\psi(x)$$
$$P^a = \int d^4x \bar\psi(x) T^a \gamma_5 \psi(x)$$

Equivalence shown with GW quarks, but large discrepancy found with non-chiral quarks.

E.g. Petreczky et al.(1606.03145): factor ≈2^4 different χ at *Nt*=12 with HISQ.

- **Topological susceptibility by GF**



☑ Gluonic and fermionic definitions agree well with each other at *T*/*T*pc ≤ 1.5, even with Wilson-type quarks!

☑ Power-low behavior consistent with the dilute instant gas approximation (DIGA) which predicts the exponent to be –8.

Next Steps

- Look fine, in spite of the use of Wilson-type quarks with explicit chiral violation!
- But, $m_\pi \sim 400$ MeV and $a \neq 0$ yet. A definite conclusion possible only after continuum extrapolation and at the physical point.
- Our good results suggests that our lattice is already close to the cont. limit, while the lattice artifact of $O((aT)^2 = 1/Nt^2)$ visible at $Nt \leq 8$.

Application to other physical quantities

$T_{\mu\nu}$ correlation functions (towards specific heat, shear/bulk viscosities, ...)

=> Yusuke Taniguchi

Continuum extrapolation by adding *a* points

Available CP-PACS+JLQCD T=0 configurations have slightly different m_{PS}/m_V etc. To fine-tune on the same line of constant physics, we have decided to generate a new T=0 configuration near the CP-PACS+JLQCD simulation point. => on-going

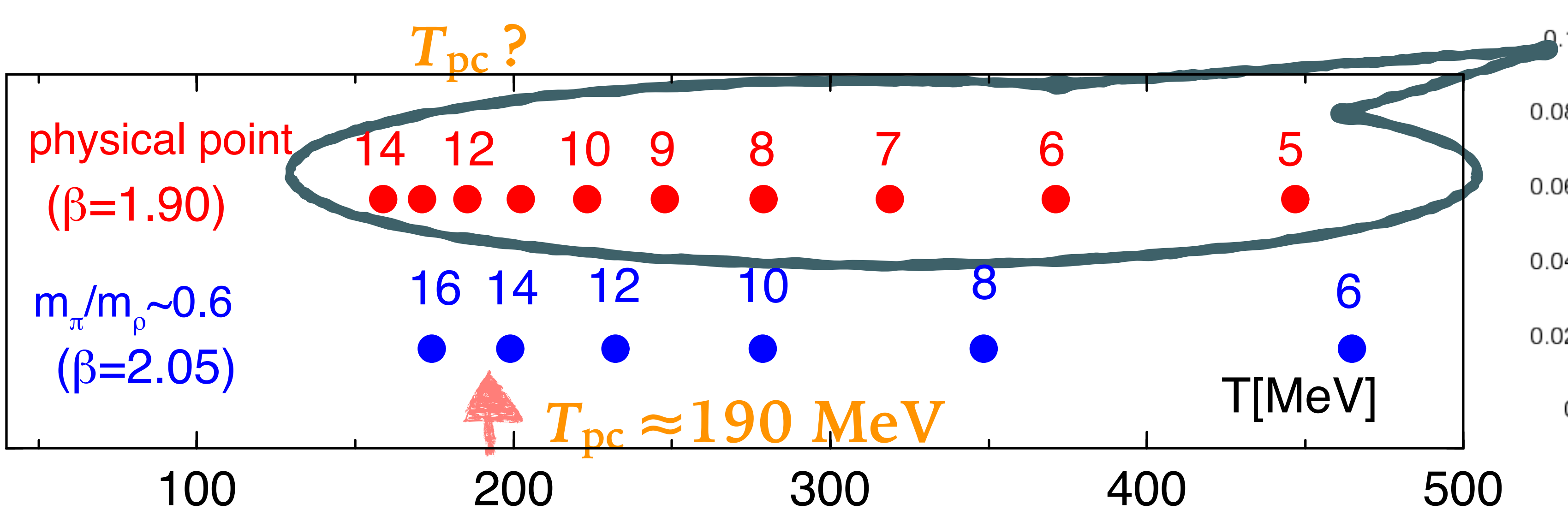
Physical point

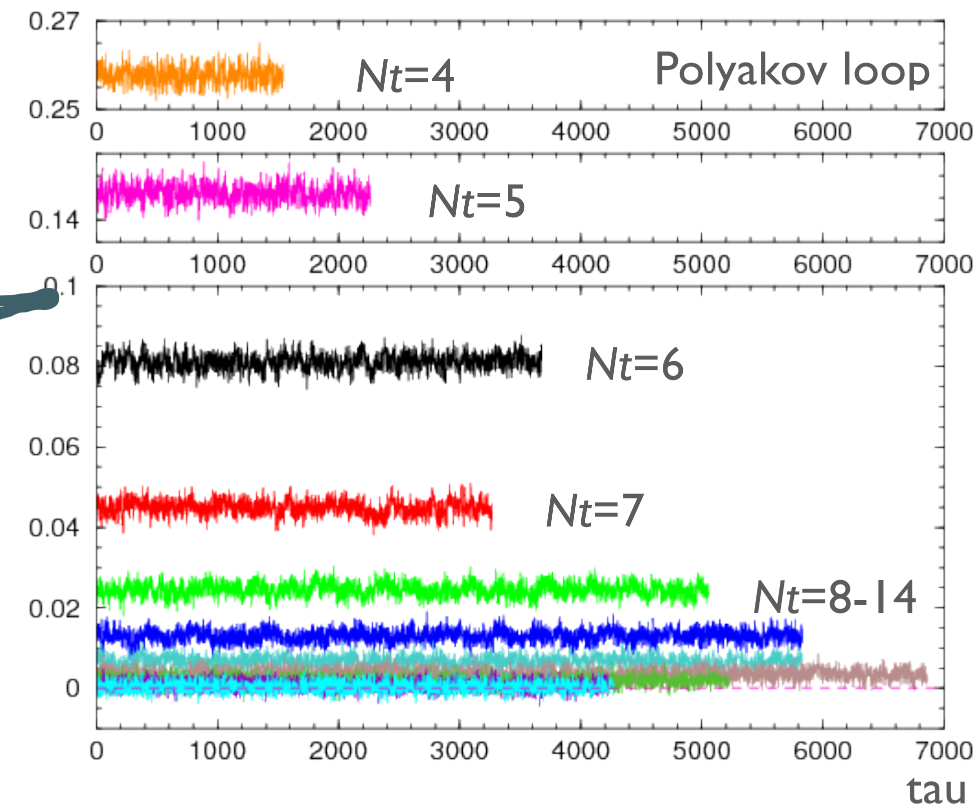
T=0 configuration available from PACS-CS; T>0 configurations also in part available from WHOT-QCD's on-going project.

this talk

(2+1)-flavor phys.pt. QCD thermodynamics with GF

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- T=0 configs. of PACS-CS (β=1.9, 32^3x64, a≈0.09fm) [Phys.Rev.D79, 034503 (2009)] 80 configs. @ILDG/JLDG
  Fine-tuned to the phys.pt. by reweighting. [Phys.Rev.D81, 074503 (2010)] using m_π, m_K, m_Ω inputs.
- T>0 by fixed-scale approach, (32^3xN_t, N_t = 4, 5, ... , 14): T≈157--549MeV, on-going
  Odd N_t too, to have a finer T-resolution.
  Generated directly at the phys.pt. w/o reweighting.
  β=1.9, Kud=0.13779625, Ks=0.13663377
- Gauge meas. every 5 tau, quark meas. every 50 tau.





- Where is T_{pc} for physical m_q? Expect T_{pc}^{phys} < 190 MeV.
- Lattice slightly coarser than the heavy case: a≈0.09fm > 0.07fm.
- May have the lattice artifact of $O((aT)^2=1/N_t^2)$ at N_t < 8.

But we have some configurations. Let us try!

GF with reweighting

- *T*=0 configs. of PACS-CS (β=1.9, 32^3x64, $a\approx$0.09fm)
  Fine-tuned to the phys.pt. by reweighting.

  Simulation: β=1.9, Kud=0.137785, Ks=0.136600
  => Phys.pt.: β=1.9, Kud=0.13779625, Ks=0.13663377

$$\langle \mathcal{O}[U](\kappa_{\rm ud}^*, \kappa_{\rm s}^*)\rangle_{(\kappa_{\rm ud}^*, \kappa_{\rm s}^*)} = \frac{\langle \mathcal{O}[U](\kappa_{\rm ud}^*, \kappa_{\rm s}^*) R_{\rm ud}[U] R_{\rm s}[U]\rangle_{(\kappa_{\rm ud}, \kappa_{\rm s})}}{\langle R_{\rm ud}[U] R_{\rm s}[U]\rangle_{(\kappa_{\rm ud}, \kappa_{\rm s})}},$$

$$R_{\rm ud}[U] = \left| \det\left[\frac{D_{\kappa_{\rm ud}^*}[U]}{D_{\kappa_{\rm ud}}[U]}\right]\right|^2$$

$$R_{\rm s}[U] = \det\left[\frac{D_{\kappa_{\rm s}^*}[U]}{D_{\kappa_{\rm s}[U]}}\right]$$

Reweighting factors provided for each of 80 configs.

Because GF is introduced to define renormalized observables, we compute flowed fields/observables on each config. as usual, and average them over configurations using the provided reweighting factors,

i.e., we do not treat the reweighting factors as a part of observables.

- In this talk, errors from the reweighing factors not estimated yet.

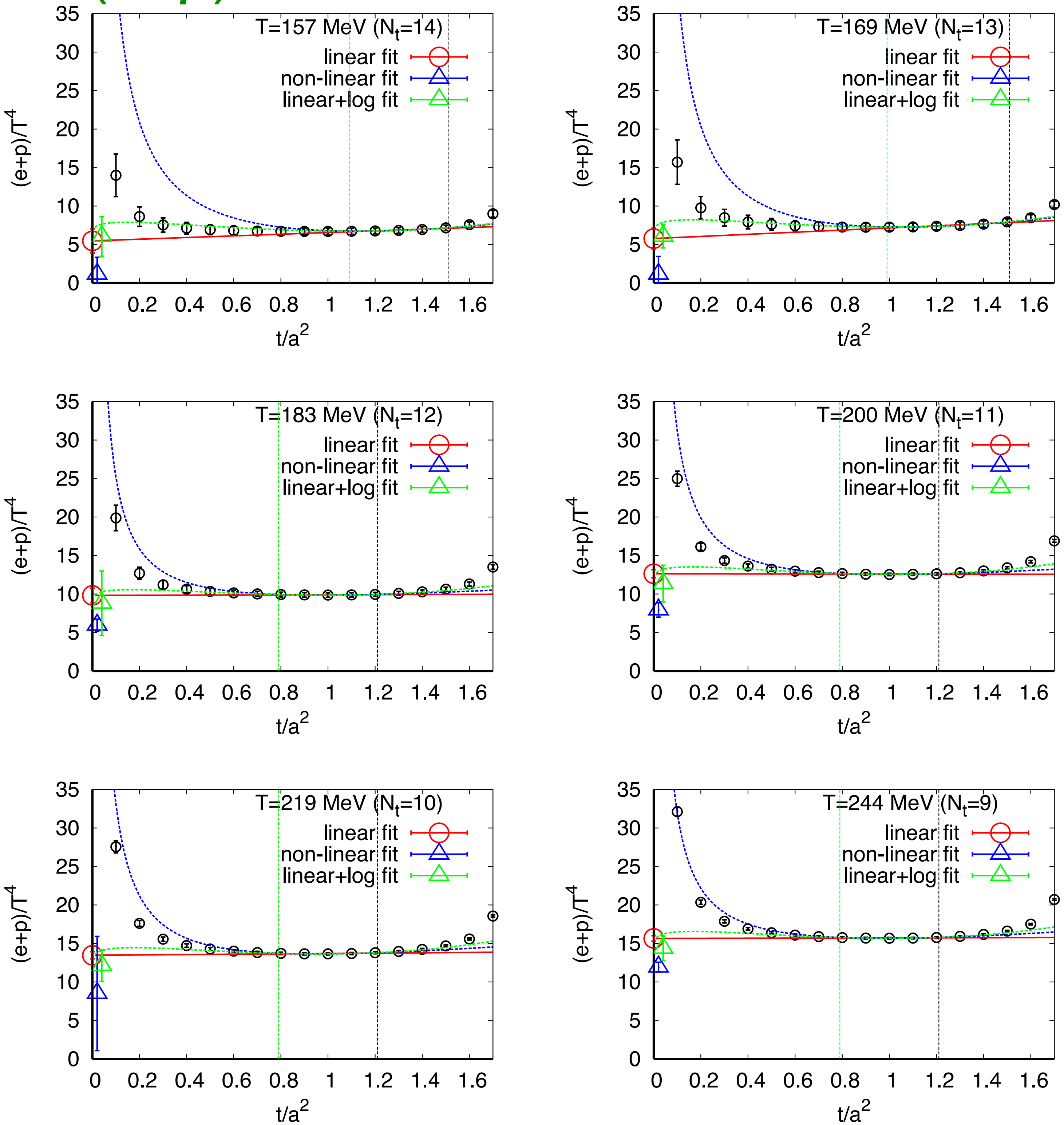
- *T*>0 by fixed-scale approach (32^3xNt, Nt = 4, 5, ..., 13, 14)
  Generated directly at the phys.pt. w/o reweighting.

(2+1)-flavor phys.pt. QCD EoS with GF

Results of 1st trial fits: *Preliminary*

- linear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + O(a^2, t^2)$
- nonlinear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + A_{\mu\nu}\frac{a^2}{t} + t\,S_{\mu\nu} + t^2 R_{\mu\nu}$
- linear+log $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$

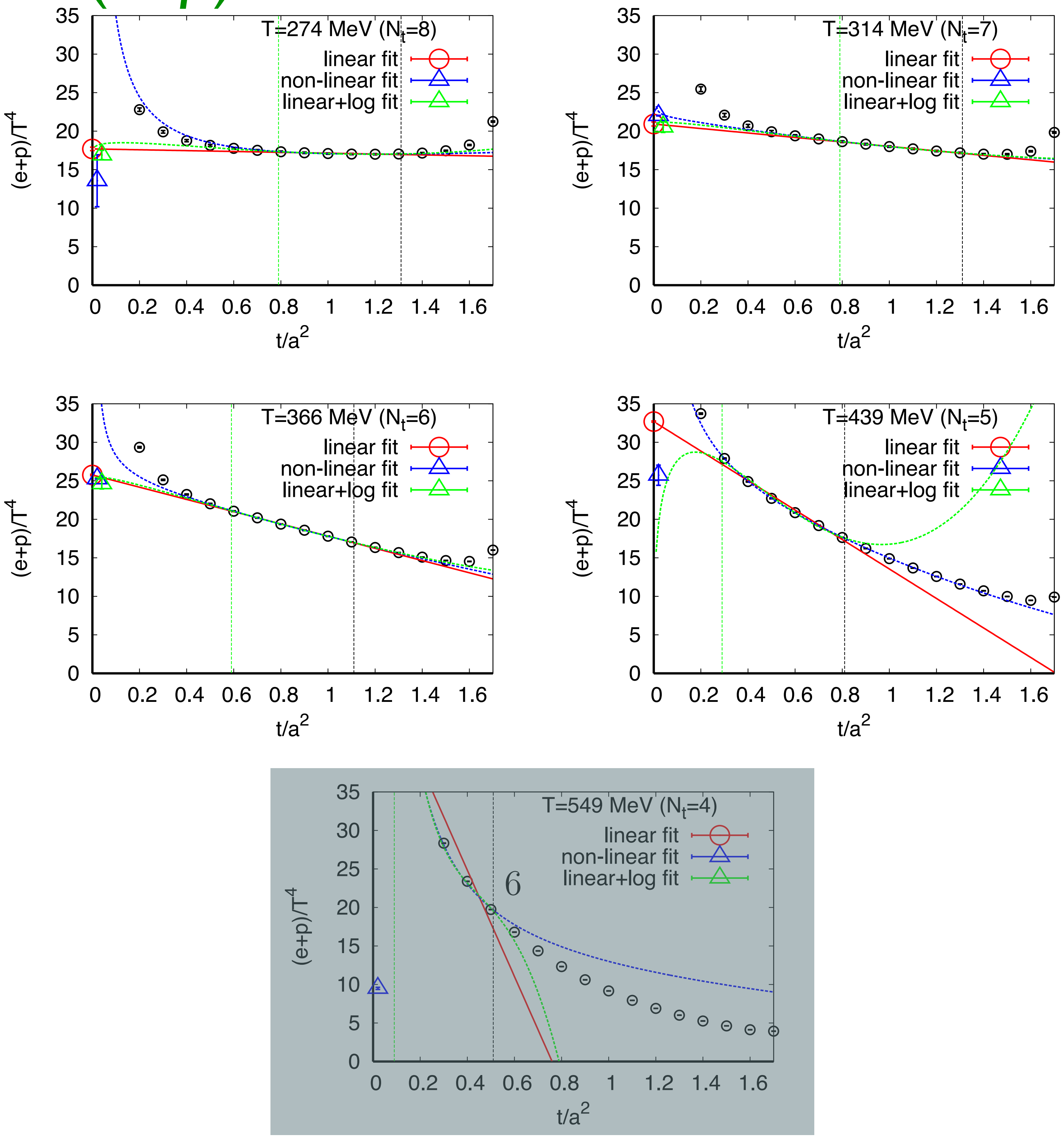
➤ $(e+p)/T^4$



(2+1)-flavor phys.pt. QCD EoS with GF

Preliminary

➤ $(e+p)/T^4$



- linear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + O(a^2,t^2)$
- nonlinear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + A_{\mu\nu}\frac{a^2}{t} + t\,S_{\mu\nu} + t^2 R_{\mu\nu}$
- linear+log $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$

☑ Linear windows narrower than the heavy QCD case.

<= coarser lattice and/or smaller m_q?

☑ Take the difference among fits as an estimate of the systematic error due to the fit ansatz.

☑ No linear windows at Nt=4 ($T\approx$549MeV). like the heavy case.

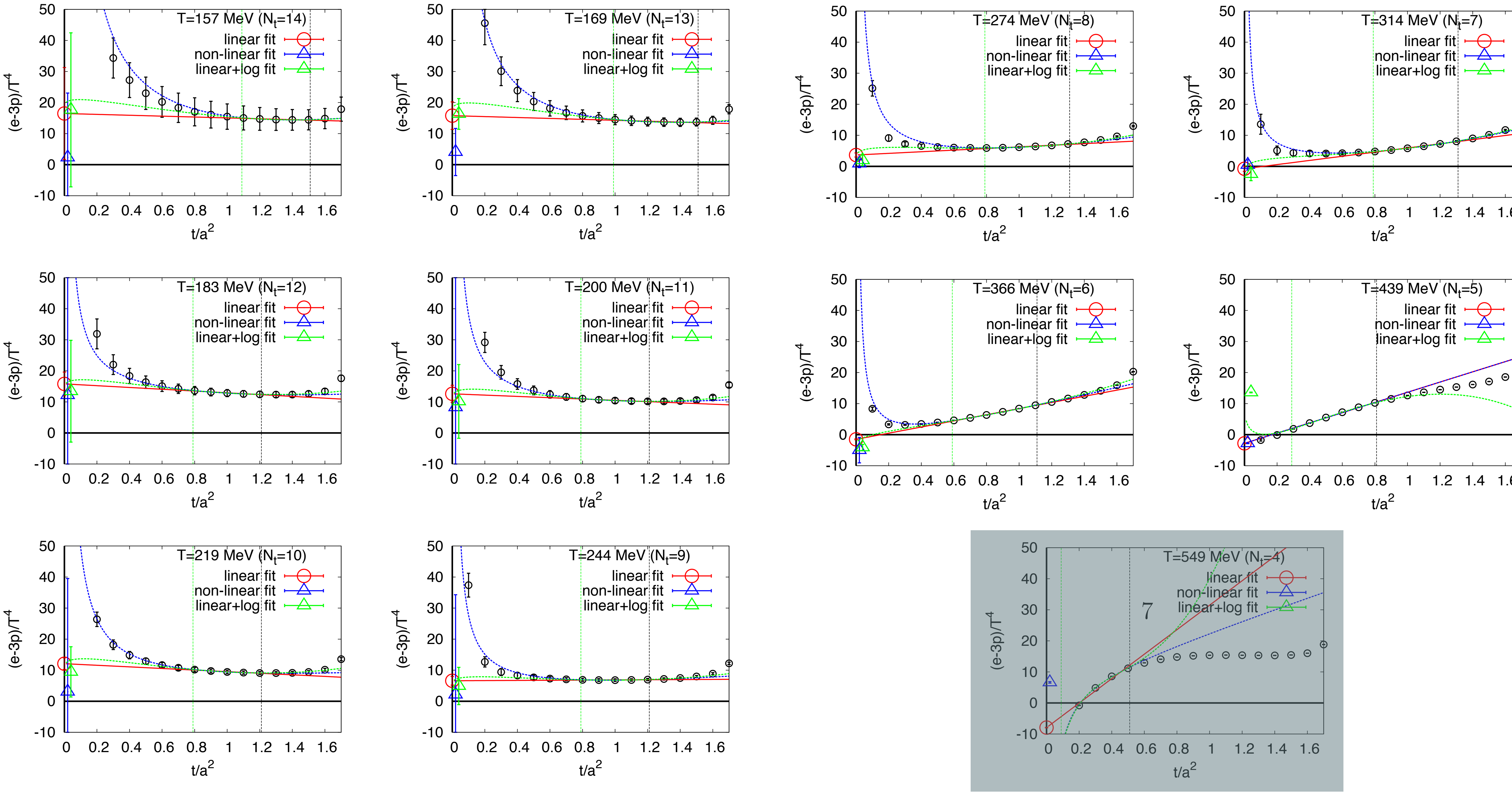
<= $t/a^2 \le t_{1/2}=[\min(Ns/2, Nt/2)]^2/\sqrt{8}$ to avoid overlapped smearing.

(2+1)-flavor phys.pt. QCD EoS with GF

Preliminary

➤ $(e-3p)/T^4$

- linear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + O(a^2, t^2)$
- nonlinear $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + A_{\mu\nu}\dfrac{a^2}{t} + t\,S_{\mu\nu} + t^2 R_{\mu\nu}$
- linear+log $\langle T_{\mu\nu}(t,a)\rangle = \langle T_{\mu\nu}\rangle + t\,S_{\mu\nu} + \dfrac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$



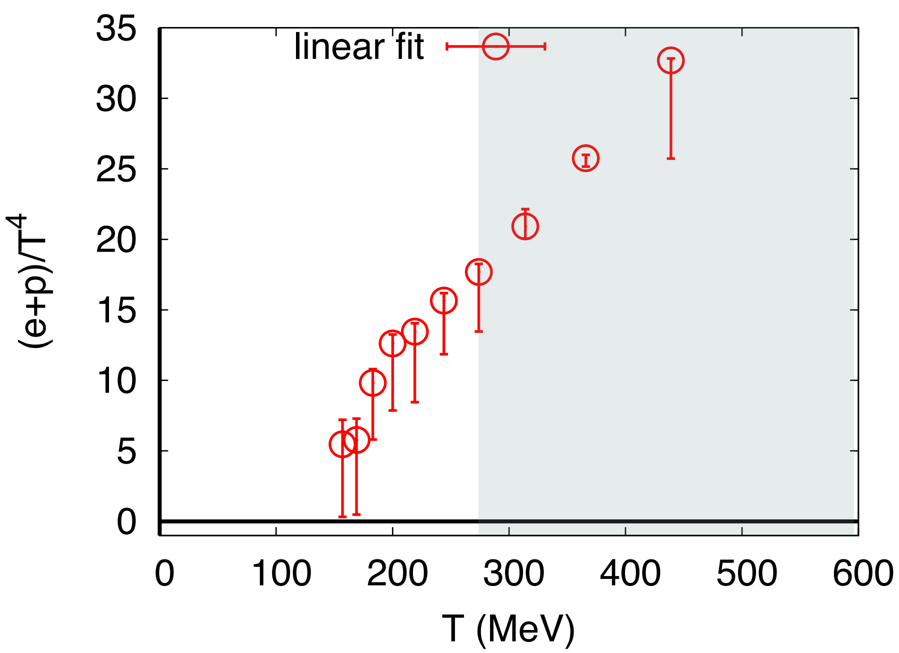
(2+1)-flavor phys.pt. QCD EoS with GF

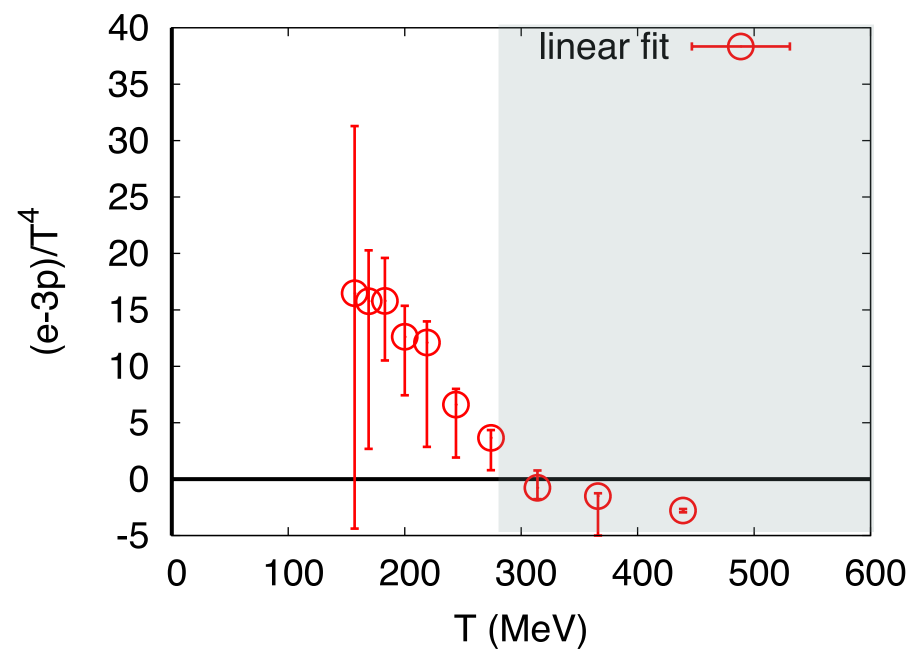
Preliminary

➤ EoS

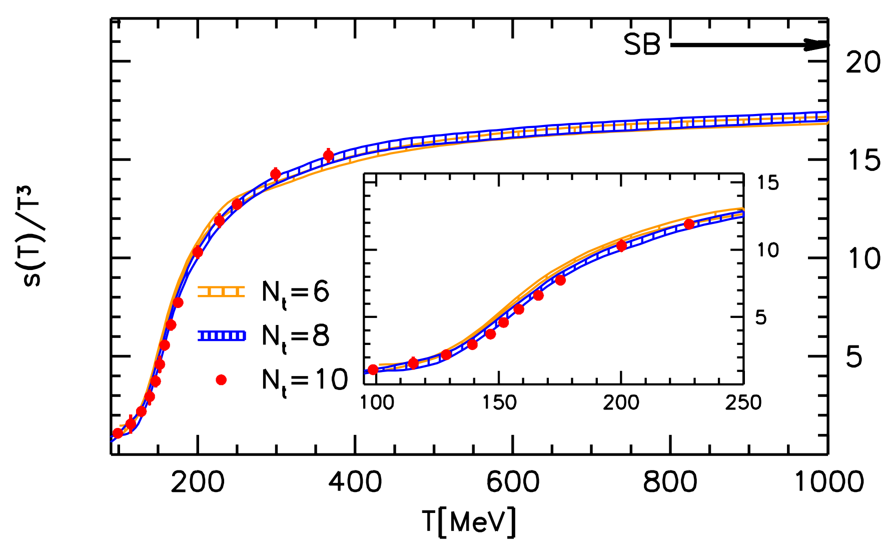
Central from the linear fit.

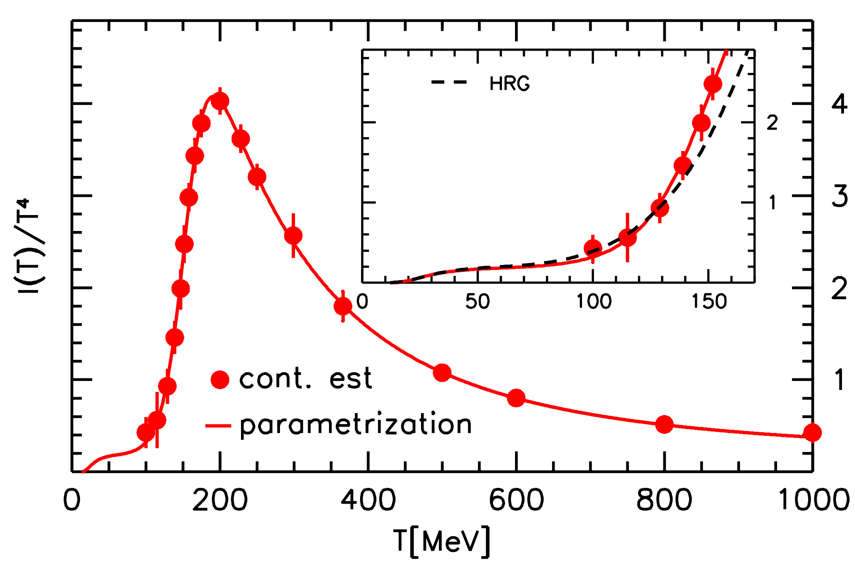
Errors include statical as well as systematical one due to fit ansatz.





- Experience with the heavy case suggests that T>274 MeV may be contaminated by the $O((aT)^2 = 1/Nt^2)$ lattice artifact at Nt < 8.
- Results of a conventional method on the same configurations not available yet.





- Borsany et al., JHEP 1011, 077 (2010), stout.
- HISQ/asqtad agree.

- ❑ Definite comparison possible only after continuum extrapolation.
- ❑ More statistics? VEV-subtraction with reweighting??

Chiral Condensate by GF

Hieda-Suzuki, Mod.Phys.Lett. A31, 1650214 (2016)

From axial W-T identity $$\left\{\bar{\psi}_f\psi_f\right\}^{(0)}(t,x) = \left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + 8 + \frac{4}{3}\ln(432)\right]\right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f}\left[\varphi_f(t)\bar{\chi}_f(t,x)\chi_f(t,x)\right]$$

At $m_f > 0$, chiral cond. in usual lattice simulation can have m_f/a^2 singularity.

With GF, such divergence is prohibited by the finiteness of flowed operators, but m_f/t can appear, instead.

In fact, to the lowest order of PT, we do encounter such m_f/t term.

$$\sum_{f,f'=u,d,s}\sqrt{\varphi_f(t)}\sqrt{\varphi_{f'}(t)}\,\bar{\chi}_f(t,x)\left\{\{t^A,M\},t^B\right\}_{ff'}\chi_{f'}(t,x)$$
$$\overset{t\to 0}{\sim}\left[-\frac{12}{(4\pi)^2}\sum_{f=u,d,s}\left(\{\{t^A,M\},t^B\}M\left\{\frac{1}{2t} + M^2\left[\gamma + \ln(2M^2t)\right] + \mathcal{O}(t)\right\}\right)_{ff} + \mathcal{O}(g^2)\right]\mathbb{1}$$
$$+\left[1 + \mathcal{O}(g^2)\right]\bar{\psi}(x)\{\{t^A,M\},t^B\}\psi(x) + \mathcal{O}(t).$$

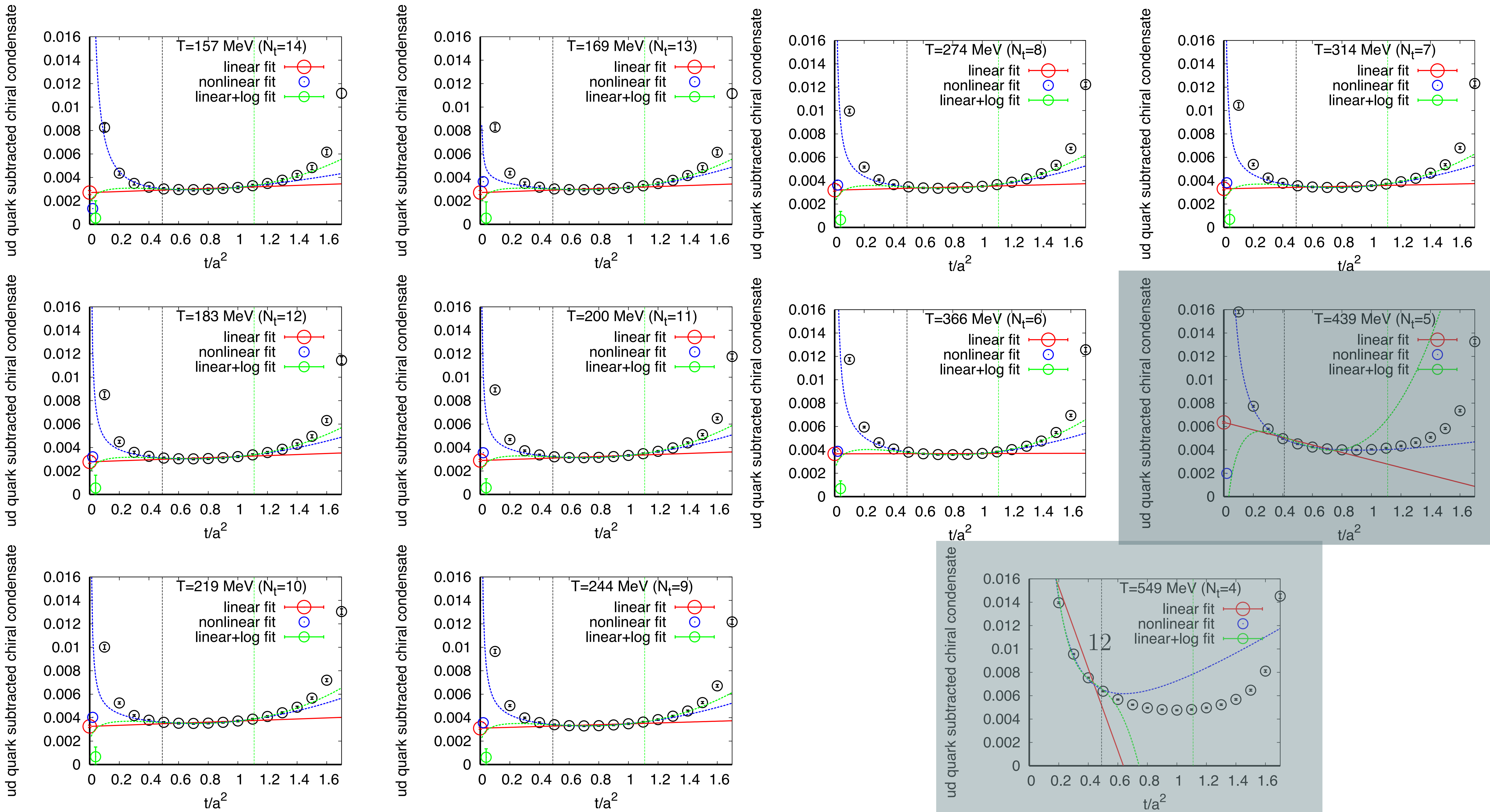
To remove this obstacle in the $t \to 0$ extrapolation, Hieda-Suzuki suggests a VEV-subtraction.

$$\left\{\bar{\psi}_f\psi_f\right\}(x) = \lim_{t\to 0}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + 8 + \frac{4}{3}\ln(432)\right]\right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f}\left[\varphi_f(t)\,\bar{\chi}_f(t,x)\,\chi_f(t,x) - \mathrm{VEV}\right].$$

(2+1)-flavor phys.pt. QCD chiral cond. with GF

Preliminary

➤ ud-quark chiral cond. w/ VEV-subtraction

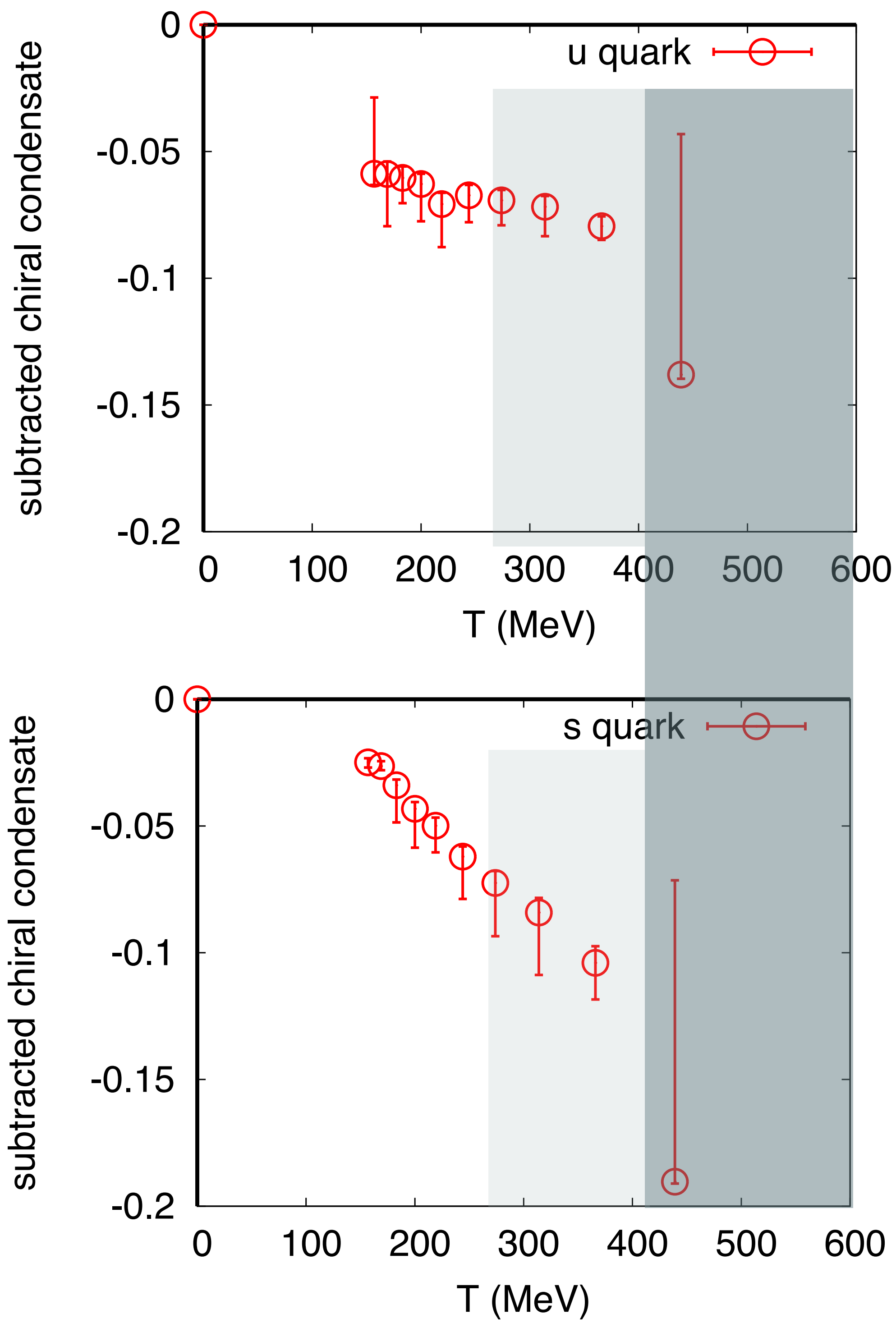


Some higher-order fits are not converged yet.

(2+1)-flavor **phys.pt.** QCD chiral cond. with GF

Preliminary

➤ **chiral conds. w/ VEV-subtraction**



- ∗ Larger difference between ud and s quarks than the heavy case.
- ∗ Sharper crossover/transition with lighter ud quarks.
- ∗ Experience with the heavy case suggests that T>274 MeV may be contaminated by the $O((aT)^2=1/Nt^2)$ lattice artifact at Nt < 8.

Some higher-order fits are not converged yet.

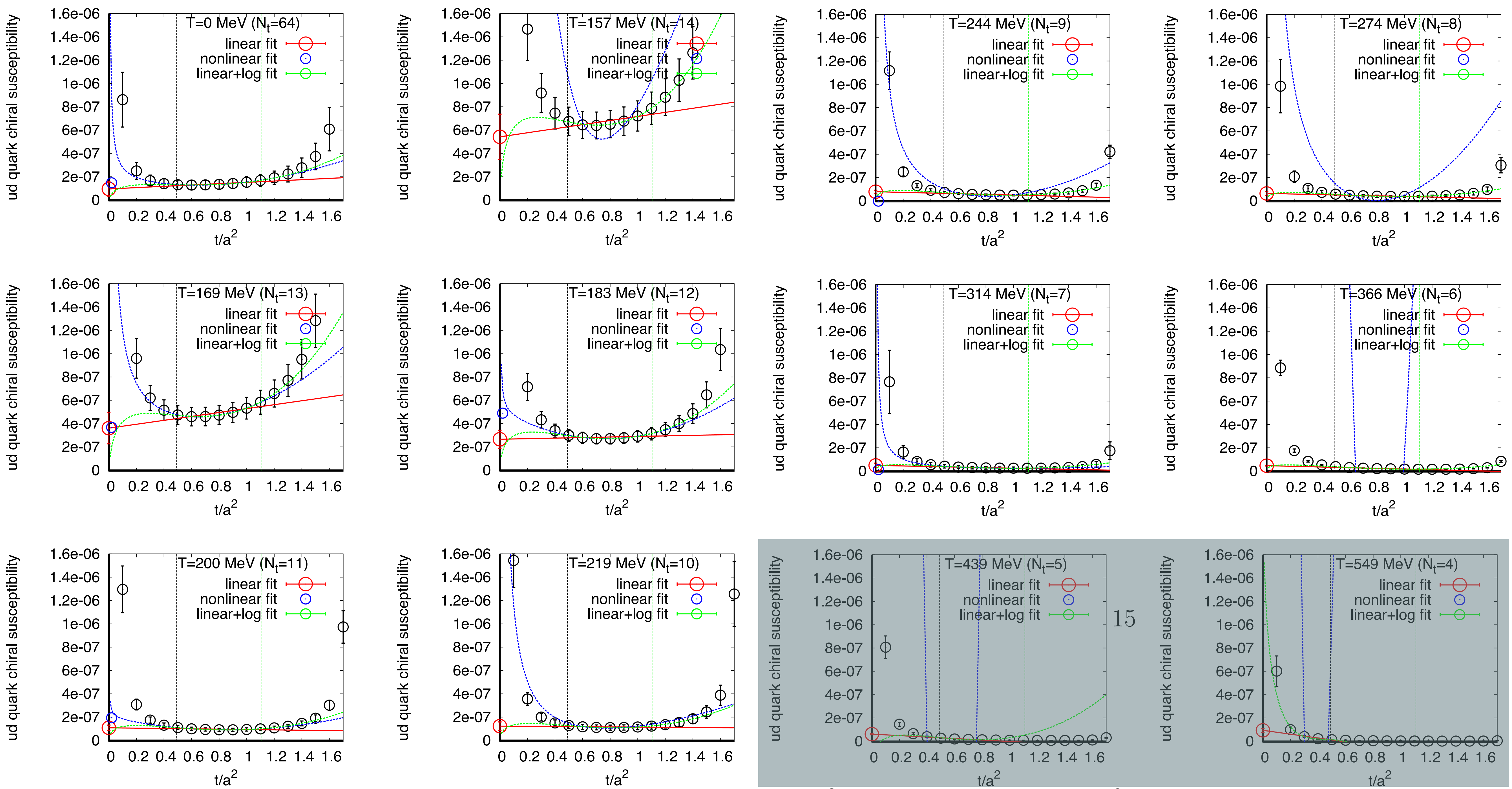
(2+1)-flavor phys.pt. QCD chiral suscept. with GF

Preliminary

- **disconnected ud chiral susceptibility**

$$\chi^{\rm disc.}_{\bar f f} = \left\langle \left[\frac{1}{N_\Gamma} \sum_x \{\bar\psi_f \psi_f\}(x) \right]^2 \right\rangle_{\rm disconnected} - \left[\left\langle \frac{1}{N_\Gamma} \sum_x \{\bar\psi_f \psi_f\}(x) \right\rangle \right]^2$$

Note: no VEV-subtraction needed in the susceptibility.



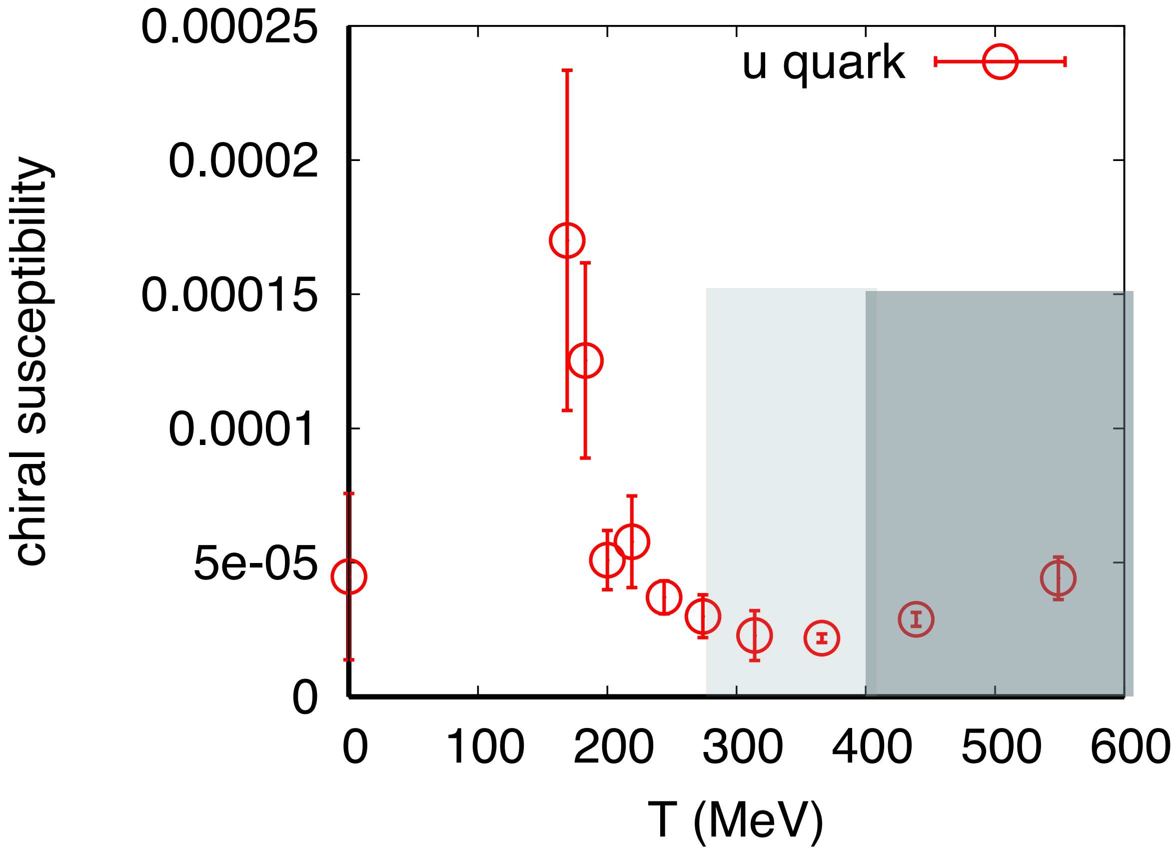
Some higher-order fits are not converged yet.

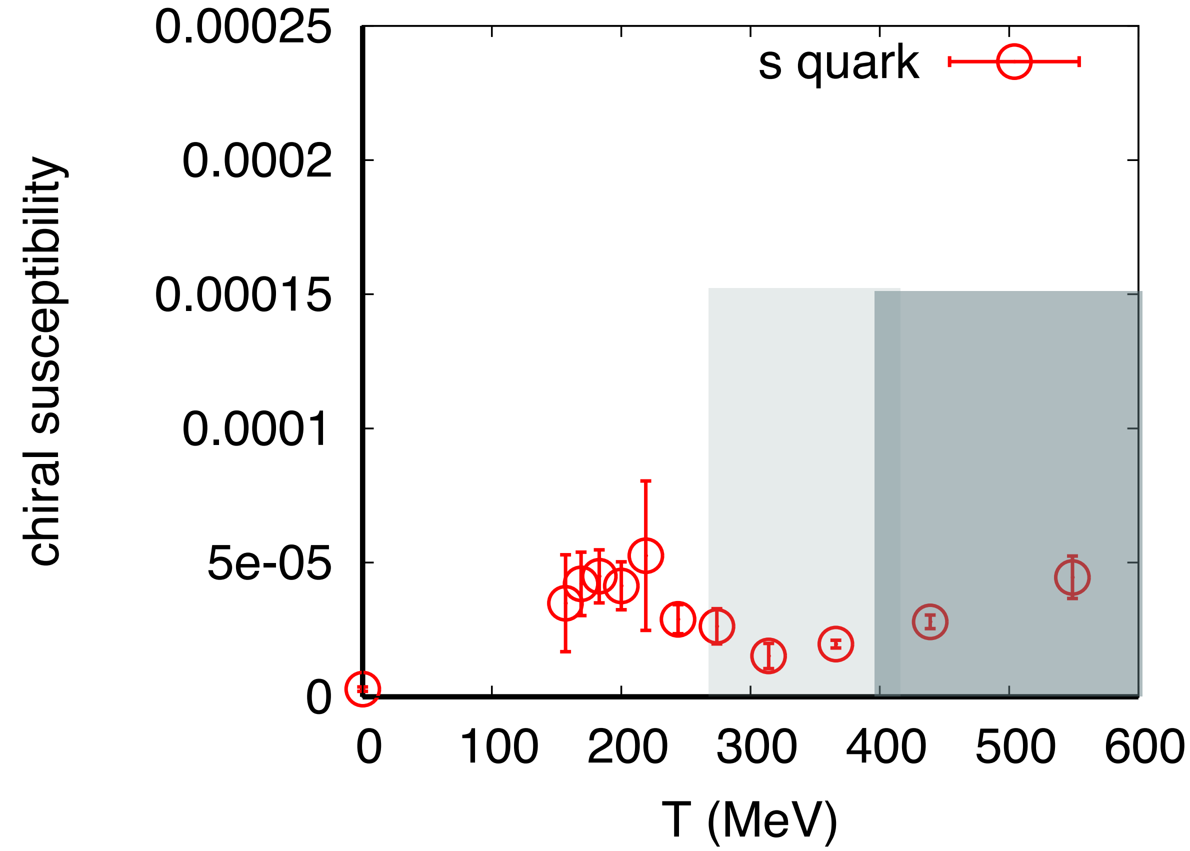
(2+1)-flavor phys.pt. QCD chiral transition with GF

Preliminary

➤ **disconnected chiral susceptibilities** (very preliminary)

Errors are statistical only,
because nonlinear/linear+log fits sometimes failed to converge yet.





▶ $T_{pc} \lesssim 169$ MeV (around 157 ? or lower?)

need lower T + more statics + more trials of the fits

* Experience with the heavy case suggests that T>274 MeV may be contaminated by the $O((aT)^2 = 1/Nt^2)$ lattice artifact at Nt < 8.

Summary

(2+1)-flavor **heavy** QCD thermodynamics with GF

Phys.Rev.D95, 054502 (2017), and to be published.

➤ Heavy ud ($m_{PS}/m_V \approx 0.63$), fine lattice ($a \approx 0.07$fm), $32^3 \times Nt$ (Nt=4,6,...,16): $T \approx 174$-697MeV

- ☑ EoS consistent with conventional method.
- ☑ Chiral suscept. shows peak at expected $T_{pc} \sim 190$MeV even with Wilson-type quark.
- ☑ $a \approx 0.07$fm seems to be close to the continuum limit, but $O((aT)^2)$ lattice artifacts at $Nt \lesssim 8$.
- ☑ The GF method works well.

(2+1)-flavor **phys.pt.** QCD thermodynamics with GF

on-going

➤ Physical point, slightly coarser lattice ($a \approx 0.09$fm), $32^3 \times Nt$ (Nt=4,5,...,14): $T \approx 157$-549MeV

Preliminary results suggest

- ☑ Similar to the heavy case. The method seems to work.
  However, . . .
- ☑ Windows for linear fit narrower. <= Coarser lattice and/or lighter quarks?
- ☑ $T_{pc}^{phys} \lesssim 169$ MeV (around 157 or lower?)

- ☐ Definite conclusions possible only after continuum extrapolation.
- ☐ Need more work on the fits and the VEV subtraction procedure with reweighting.
- ☐ Need more statistics at this light m_q. (T=0 also).
- ☐ Need lower T (larger Nt) too.
- ☐ Errors due to the reweighting factor not estimated yet.