Chiral phase transition of three flavor QCD with nonzero magnetic field

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Collaborate with:
- Heng-Tong Ding
- Swagato Mukherjee
- Christian Schmidt
- Xiao-Dan Wang
Background & Motivation

QCD phase transition at physical mass is crossover
Background & Motivation

QCD phase transition at physical mass is crossover

Crossover for the physical mass
Background & Motivation

QCD phase transition depends on Magnetic field?

Colombia plot: the order of QCD phase transition
Background & Motivation

QCD phase transition depends on Magnetic field?

Colombia plot: the order of QCD phase transition

* A. Tomiya et al. 1612.01908
S. Sharma et al. 1510.03930

Confinement/Deconfinement phase transition (Polyakov loop)

** Xiao-Yong Jin et al. 1706.01178
*** H. Saito et al. 1106.0974

Chiral phase transition
(Chiral cond.)

Tri. Crit.

Physical pt

$N_f=3$

$\leftarrow U(1) ~\text{axial}$

Cross over

$m_s$

$m_{ud}$
Background & Motivation

QCD phase transition depends on Magnetic field?

Colombia plot: the order of QCD phase transition

Confinement/Deconfinement phase transition (Polyakov loop)

Chiral phase transition (Chiral cond.)

Cross over

Physical pt

$U(1)$ axial*

$N_f=3$

$m_s$

$m_{ud}$

Tri. Crit.

1st

1st

Other than the physical point info. is useful for model building

*A. Tomiya et al. 1612.01908
S. Sharma et al. 1510.03930

** Xiao-Yong Jin et al. 1706.01178
*** H. Saito et al. 1106.0974
Background & Motivation
QCD phase transition depends on Magnetic field?

Colombia plot: the order of QCD phase transition

Off-central AA collisions generate huge magnetic field (15 mπ*)

Does it affect phase structure?

*V. V. SKOKOV et al (arXiv:0907.1396)
Background & Motivation

QCD phase transition depends on Magnetic field?

Colombia plot: the order of QCD phase transition

- Physical pt
- N_f=3
- Cross over
- 1st
- Tri. Crit.
- eB
- m_s
- m_{ud}
Background & Motivation
Magnetic field breaks symmetries
Background & Motivation
Magnetic field breaks symmetries

\[ \mathbf{B} \]

U(1) mag. \[ a_{\mu} = (0, xB, 0, 0) \]
Background & Motivation
Magnetic field breaks symmetries

U(1) mag. \( a_\mu = (0, xB, 0, 0) \)

Magnetic field breaks:
- Lorentz symmetry
- Flavor symmetry
- Time reversal

Phase diagram will change
Outline

1. Introduction (Background)
2. Previous studies (5 studies as example)
3. Setup
4. Preliminary results
5. Summary
Background & Motivation

Previous studies: Model & Lattice

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Lattice</th>
</tr>
</thead>
</table>
- Tc goes up along with mag. field.
- Order does not depend on eB? (not discussed)

**Background & Motivation**

**Previous studies: Model & Lattice 1/5**

**Model(1/3) : PNJL (Nf=2)**

Kenji Fukushima, Marco Ruggieri, Raoul Gatto 2010 (arXiv: 1003.0047)

![Graph](image_url)

**Physical mass** $T/T_c$

- $eB=0$
- $eB=4m^2$
- $eB=10m^2$
- $eB=20m^2$

**Chiral Condensate**

**Polyakov Loop**

**[300]**

**[250]**

**[200]**

**[150]**

**[100]**

**[50]**

- $T/T_c$
**Background & Motivation**

**Previous studies: Model & Lattice 2/5**

**Model(2/3) : ChPT (Nf=2) + gluon**


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**Figure 1:** Quark-hadron phase transition temperature vs external magnetic field. Critical point, $T^* = 104 \text{ MeV}$, is marked with the dot. Solid line corresponds to the first order phase transition, and dashed line corresponds to the crossover.

The dependence of $\Delta \varepsilon (H)$ is plotted in Fig. 2. The value of the magnetic field $\sqrt{eH} = 600 \text{ MeV}^2$ where at turnstozero, corresponds to the critical point, at which first order phase transition changes to the crossover.

**Conclusion**

We have studied the quark-hadron phase transition in QCD in the presence of external magnetic field, and have shown that the temperature of the phase transition decreases in comparison to the case of zero magnetic field. Equation (28) was solved numerically, the phase diagram in the plane temperature–magnetic field and critical point were found.

As was shown above, there are two phases in the presence of magnetic field: diamagnetic phase below $T_c$ and paramagnetic above $T_c$. Correspondingly magnetic susceptibility, $\chi = -\frac{\partial^2 P}{\partial H^2}$, changes its sign at the critical temperature. Thus, magnetic susceptibility may be considered as the order parameter of the left thermal QCD in the presence of magnetic field.

It is known from lattice calculations that there is a crossover for finite temperature QCD with physical quark masses. In the presence of magnetic field there are additional magnetic terms in the pressure, which give different contribution to the energy density in two phases. Thus we expect that a crossover is replaced by a first order phase transition.

Analogous phenomenon was found in [24], where it was shown that chiral transition changes from crossover to the weak first order transition in the linear sigma model in a magnetic field.
Massless 2f

- $T_c$ goes down and up along with mag. field.
- Order is not discussed
Background & Motivation

Previous studies: Model & Lattice 4/5

Lattice(1/2): Nf=2, standard KS

Massimo D’Elia, Swagato Mukherjee and Francesco Sanfilippo (arXiv: 1005.5365)

$m_\pi \approx 275$ MeV

- Tc goes up along with mag. field.
- Order depends on eB (Crossover to strong 1st)
Background & Motivation

Previous studies: Model & Lattice 5/5

Lattice(2/2) + model: Nf = 3, stout KS, physical mass

Gergely Endrődi 2015 (arXiv: 1504.08280)

- \( T_c \) goes **down** along with mag. field.
- Order depends on \( eB \) (crossover to 1st by a model)
Background & Motivation

Tc and order

QCD phase transition with external U(1) magnetic field has been investigated for long years, from J. Schwinger (1951)

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Ref.</th>
<th>Method</th>
<th>Tc w/ eB</th>
<th>Order w/ eB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenji Fukushima et al</td>
<td>arXiv: 1003.0047</td>
<td>PNJL(2f) at m_{phys}</td>
<td>Increase</td>
<td>No change?</td>
</tr>
<tr>
<td>N. O. Agasian et al</td>
<td>arXiv: 0803.3156</td>
<td>ChPT(2f) at m_{phys}</td>
<td>Decrease</td>
<td>1st to crossover</td>
</tr>
<tr>
<td>Jens Braun et al</td>
<td>arXiv: 1412.6025</td>
<td>RG, massless 2f</td>
<td>Decrease and increase</td>
<td>?</td>
</tr>
<tr>
<td>Massimo D'Elia et al</td>
<td>arXiv: 1005.5365</td>
<td>KS(2f)</td>
<td>Increase</td>
<td>Crossover to strong 1st</td>
</tr>
<tr>
<td>Gergely Endrődi</td>
<td>arXiv: 1504.08280</td>
<td>Stout KS (3f) at m_{phys} + model</td>
<td>Decrease</td>
<td>Crossover to 1st (model)</td>
</tr>
</tbody>
</table>

Ref. 1208.0917, 1209.0374, 1411.7176 and there in

We examine dependence of the order of phase transition on quark mass and external magnetic field
What we have done

Dependence of order of transition on ma and eB

Mass degenerated 3 flavor QCD with magnetic field

\[ ma = 0.024(1\text{st order}), 0.028(\text{critical pt}), 0.2, 0.4, 0.8 \]

Check the order of phase transition on eB
Setup

3Flavor Standard staggered + Wilson plaq. action

Setup: 3 Flavor degenerated mass staggered fermion (same as *)
4th rooted RHMC, Observables: chiral condensates, Polyakov loop
Resource: Fermi-lab GPU cluster

<table>
<thead>
<tr>
<th>m_q a</th>
<th>Size</th>
<th>β range</th>
<th>Nb (Magnetic flux)</th>
<th>#Conf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.024 (1st order for Nb=0*)</td>
<td>24³x4 (16³x4)</td>
<td>5.128-5.160</td>
<td>0-56</td>
<td>O(2000)</td>
</tr>
<tr>
<td>0.028 (Critical for Nb = 0*)</td>
<td>16³x4</td>
<td>5.130-5.170</td>
<td>0-56</td>
<td>O(1500)</td>
</tr>
<tr>
<td>0.2</td>
<td>16³x4</td>
<td>5.10-5.65</td>
<td>0-56</td>
<td>O(500)</td>
</tr>
<tr>
<td>0.4</td>
<td>16³x4</td>
<td>5.35-5.65</td>
<td>0-56</td>
<td>O(500)</td>
</tr>
<tr>
<td>0.8</td>
<td>16³x4</td>
<td>5.35-5.85</td>
<td>0-56</td>
<td>O(700)</td>
</tr>
</tbody>
</table>

$q B a^2 = \frac{2\pi N_b}{N_x N_y}$

Nb: Number of magnetic flux

* Dominik Smith et al 2010 (arXiv: 1109.6729)
In the Minkowski space time. If we move to Euclidean spacetime, then we can estimate a “comparable mass” as,

\[ a\sqrt{eB} = \sqrt{\frac{2\pi N_b}{N_x N_y |q|}} = am_{N_b} \]

q=2/3 Nx=Ny=16 case,

<table>
<thead>
<tr>
<th>( N_b )</th>
<th>1</th>
<th>4</th>
<th>17</th>
<th>24</th>
<th>32</th>
<th>56</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>( am_{N_b} )</td>
<td>0.19</td>
<td>0.38</td>
<td>0.79</td>
<td>0.9</td>
<td>1.1</td>
<td>1.4</td>
<td>1.5</td>
</tr>
</tbody>
</table>

For each \( N_b \) is consider to be compatible with the quark mass

We choose \( ma=0.2, 0.4, 0.8 \) to see effect of quark mass and the magnetic field
Preliminary Results

For 5 quark mass points:
ma = 0.024 (1st order),
ma = 0.028 (just above the critical pt, crossover regime),
ma = 0.2  (comparable with magnetic field Nb=1)
ma = 0.4  (comparable with magnetic field Nb=5)
ma = 0.8  (comparable with magnetic field Nb=24)
Preliminary Result (1/5)

\[ a_{m} = 0.024 (1\text{st for Nb}=0): \text{1st becomes stronger?} \]

L=24^{3\times4} 

Up quark cond.

Tc increases

Nb -> Large
Preliminary Result (1/5)
\(am = 0.024\) (1st for \(Nb=0\)): 1st becomes stronger?

L=24^3x4

Up quark cond.

Tc increases
\(Nb \rightarrow\) Large

Hysteresis (=1st order) and the gap of them becomes larger
Preliminary Result (1/5)

\[
\text{am} = 0.024 \text{ (1st for Nb=0): 1st becomes stronger?}
\]

L=24^3x4

Up quark cond.

\[
\text{ma} = 0.024, \text{ Nf=3, L=24}
\]

\[
\text{Tc increases}
\]

\[
\text{Nb \rightarrow Large}
\]
The situation is different for cumulants constructed from linear combinations of $\bar{\psi}\psi$ and $SG$, \[ B_4(x) = \frac{\langle (\delta M(x))^4 \rangle}{\langle (\delta M(x))^2 \rangle^2} \] M: order parameter

This quantity can distinguish the order of phase transition

$$ B_4 \begin{cases} = 1 & \text{First order} \\ \sim 1.6 & \text{Second with } Z_2 \\ \sim 3 & \text{Crossover} \end{cases} $$

But higher statistics are demanded
Preliminary Result (1/5)

\[ am = 0.024 \text{(1st for Nb=0): 1st becomes stronger?} \]

L=24^3x4

Up quark cond.

(For Nb-0, 1st order is already confirmed in Dominik Smith & Christian Schmidt 2010)
Statistics is not enough
Preliminary Result (2/5)

am = 0.028 (crossover for Nb=0): crossover to 1st

L=16^3x4

Up quark cond.

Tc increases
Nb -> Large
Preliminary Result (2/5)
\[\text{am} = 0.028, \text{crossover for Nb}=0: \text{crossover to 1st}\]

L=16^3x4

Up quark cond.

\[\text{Tc increases}\]
\[\text{Nb} \rightarrow \text{Large}\]
Preliminary Result (2/5)

am = 0.028 (crossover for Nb=0): crossover to 1st

L=16^3x4

Up quark cond. susceptibility

ma = 0.028, Nf=3, L=16

Tc increases
Nb -> Large
Preliminary Result (2/5)

$am = 0.028$ (crossover for Nb=0): crossover to 1st

$L = 16^3 \times 4$

**Up quark cond. susceptibility**

$ma = 0.028$, $N_f=3$, $L=16$

$Nb \to \text{Large},$

$B \to 1$? Statistics is not enough
Preliminary Result (3/5)
am = 0.2: Tc goes up

\( L=16^3 \times 4 \)

Up quark condensate

Polyakov loop

- Chiral condensate does not show phase transition for Nb = 0-56
- Behavior of the Polyakov loop is similar to that in PNJL results
- Tc for deconf/conf trans. goes up for increasing Nb (not clear)
Preliminary Result (4/5)
am = 0.4: No clear signal yet but Tc goes up

L=16^3x4

Up quark condensate

Polyakov loop

- Chiral condensate does not show phase transition (trivially scaled)
- Behavior of the Polyakov loop is similar to that in PNJL results
- Tc for deconf/conf trans. goes up for increasing Nb (not clear)
am = 0.8: No clear signal (1st order like for conf.)

- Chiral condensate does not show phase transition (trivially scaled)
- Conf/deconf. transition is not changed for Nb=0 to 56 (expected)
## Summary of preliminary results

### Tc goes up

<table>
<thead>
<tr>
<th>$a m_q$</th>
<th>Tc(chiral) dep. on Nb</th>
<th>Order(Chiral) dep. on Nb</th>
<th>Tc(Confinement) dep. on Nb</th>
<th>Order(Confinement) dep. on Nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.024 (1st order for Nb=0)</td>
<td><strong>Increase</strong></td>
<td>1st to strong 1st?</td>
<td><strong>Increase</strong></td>
<td>1st to strong 1st?</td>
</tr>
<tr>
<td>0.028 (Crossover for Nb = 0)</td>
<td><strong>Increase</strong></td>
<td>crossover to 1st</td>
<td><strong>Increase</strong></td>
<td>crossover to 1st</td>
</tr>
<tr>
<td>0.2</td>
<td>no critical behavior</td>
<td>-</td>
<td><strong>Increase</strong></td>
<td>crossover like</td>
</tr>
<tr>
<td>0.4</td>
<td>no critical behavior</td>
<td>-</td>
<td><strong>Increase</strong></td>
<td>crossover like</td>
</tr>
<tr>
<td>0.8</td>
<td>no critical behavior</td>
<td>-</td>
<td><strong>Increase?</strong></td>
<td>1st ?</td>
</tr>
</tbody>
</table>
Summary

QCD phase transition depends on Magnetic field

Summary:

1. We investigate 3 flavor QCD with U(1) external magnetic field for various mass using standard staggered fermion with $N\sigma = 16(24)$, $N_t = 4$
2. We observed strengthening of order of phase transition in light mass regime
3. Except for $m_a = 0.8$, $T_c$ goes up. $m_a = 0.8$, no clear response to $N_b$.

Tasks:

1. Increasing statistics
2. Improve resolution of beta
3. Scaling analysis to determine the order
5. Determination of the order of phase transition from the Binder ratio
6. Scale setting
7. Other cutoff scheme to check the cutoff effect on $T_c$ vs $N_b$
Backup
Tc for

$ma = 0.024$ (1st for Nb=0)

$ma = 0.028$ (crossover for Nb=0)
ma = 0.024 (1st for Nb=0) Up quark condensates

Consistent with 1st order
ma = 0.024 (1st for Nb=0) Polyakov loop

Consistent with 1st order
ma = 0.028 (crossover for Nb=0) Up quark condensates

Nb=14 is “critical” eB*
ma = 0.028 (crossover for Nb=0) Polyakov loop

Nb=14 is critical eB*
ma = 0.2 (crossover for Nb=0) Up quark condensate

No criticality
ma = 0.2 (crossover for Nb=0) Polyakov loop
ma = 0.4 (crossover for Nb=0) Up quark condensate

No criticality
ma = 0.4 (crossover for Nb=0) Polyakov loop
ma = 0.8 Up quark condensate

No criticality
ma = 0.8 Polyakov loop

ma = 0.8, Nf=3, L=16

Polyakov loop

Beta

Susceptibility for Polyakov loop

Border ratio for Polyakov loop