Extreme QCD 2017

Taylor series expansions for higher order cumulants

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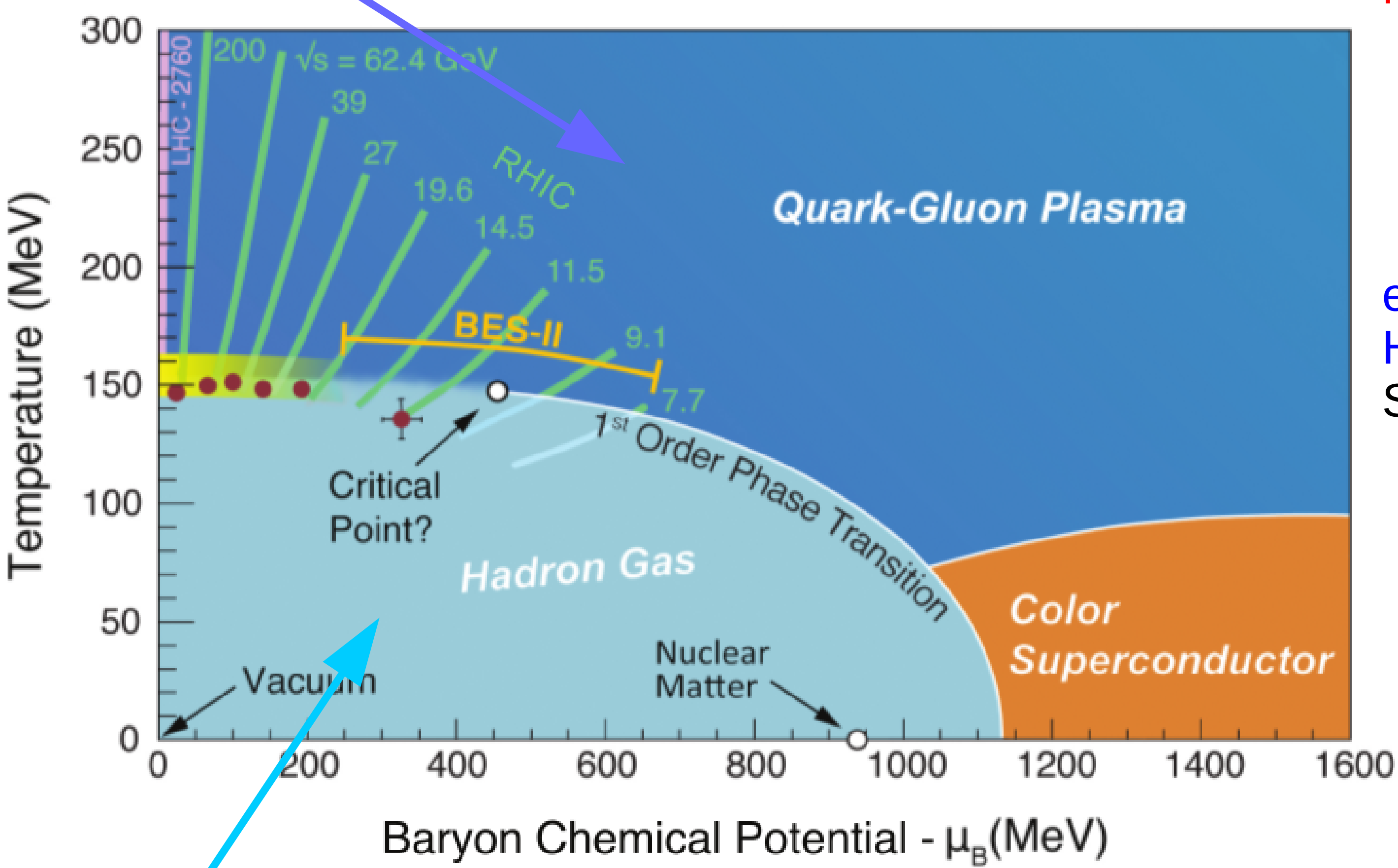
- Limiting the validity range of HRG model calculations
  characterization of bulk thermodynamics and
  fluctuations of conserved charges in the crossover region

- Taylor expansion of cumulant ratios

Phases of strong-interaction matter

thermodynamics of quarks and gluons:
high-T – QCD perturbation theory



thermodynamics of a hadron gas

physics of the early universe
hot $T \sim 10^{12}$K

experimentally accessible in Heavy Ion Collisions at SPC, RHIC, LHC, FAIR,NICA

may exist in the interior of compact stars,
dense $n_B \sim 10 n_{NM}$

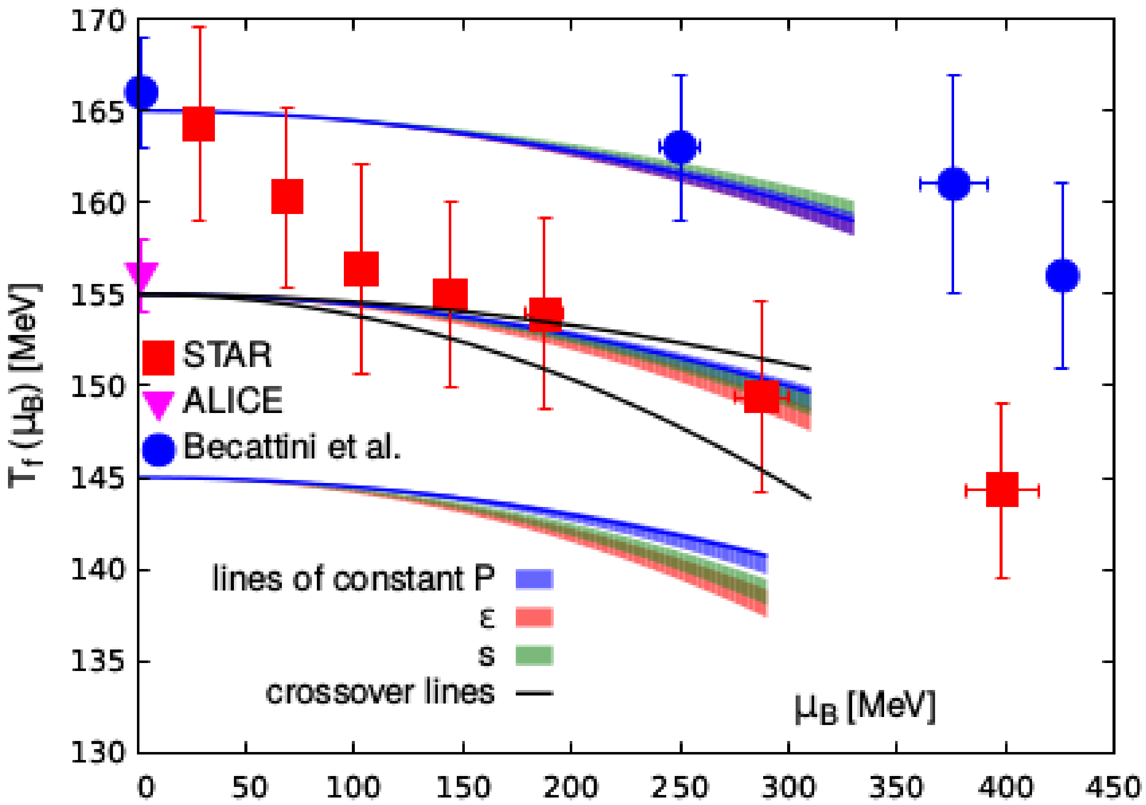
Chiral transition, hadronization and freeze-out

$\mu_B = 0$: – pseudo-critical temperature $T_c = 154(9)\mathrm{MeV}$

– hadronization temperatures $T_h = 164(3)$ MeV

– freeze-out temperatures: $T_{fo} = 156(3)$ MeV

$T_{fo} = [164(5) - 168(4)]$ MeV



HOWEVER

physics is quite different at lower and upper end of the current error bar on Tc

probed with net-charge correlations&fluctuations

lines of constant physics from a 6th order Taylor expansion of the QCD equation of state

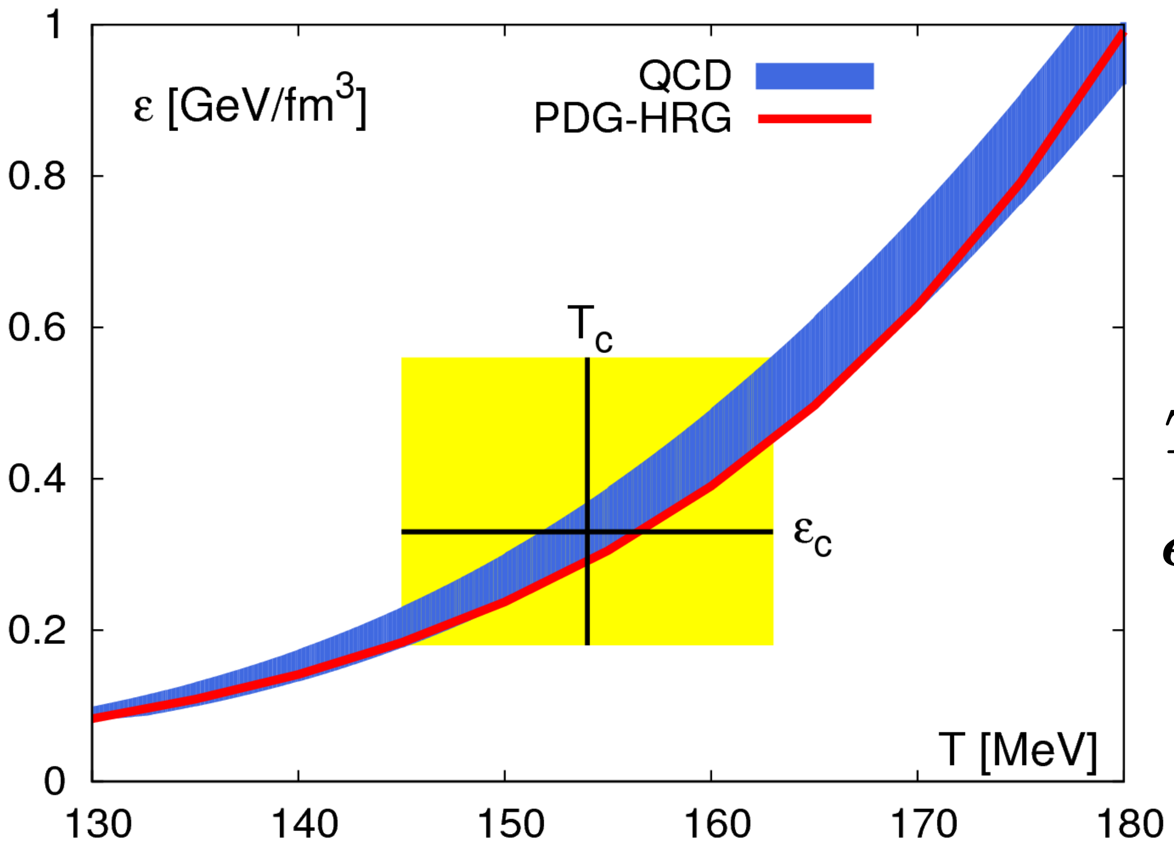
A. Bazavov et al. (Bielefeld-BNL-CCNU) arXiv:1701.04325

crossover transition lines:

G. Endrodi et al., arXiv:1102.1356, O. Kaczmarek et al., arXiv:1011.31.30
C. Bonati et al., arXiv:1507.03571, P. Cea et al., arXiv:1403.0821

Crossover transition parameters

PDG: Particle Data Group hadron spectrum



$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm^3}$$

compare with:

$$\epsilon^{\text{nucl. mat.}} \simeq 150\ \mathrm{MeV/fm^3}$$

$$\epsilon^{\text{nucleon}} \simeq 450\ \mathrm{MeV/fm^3}$$

A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

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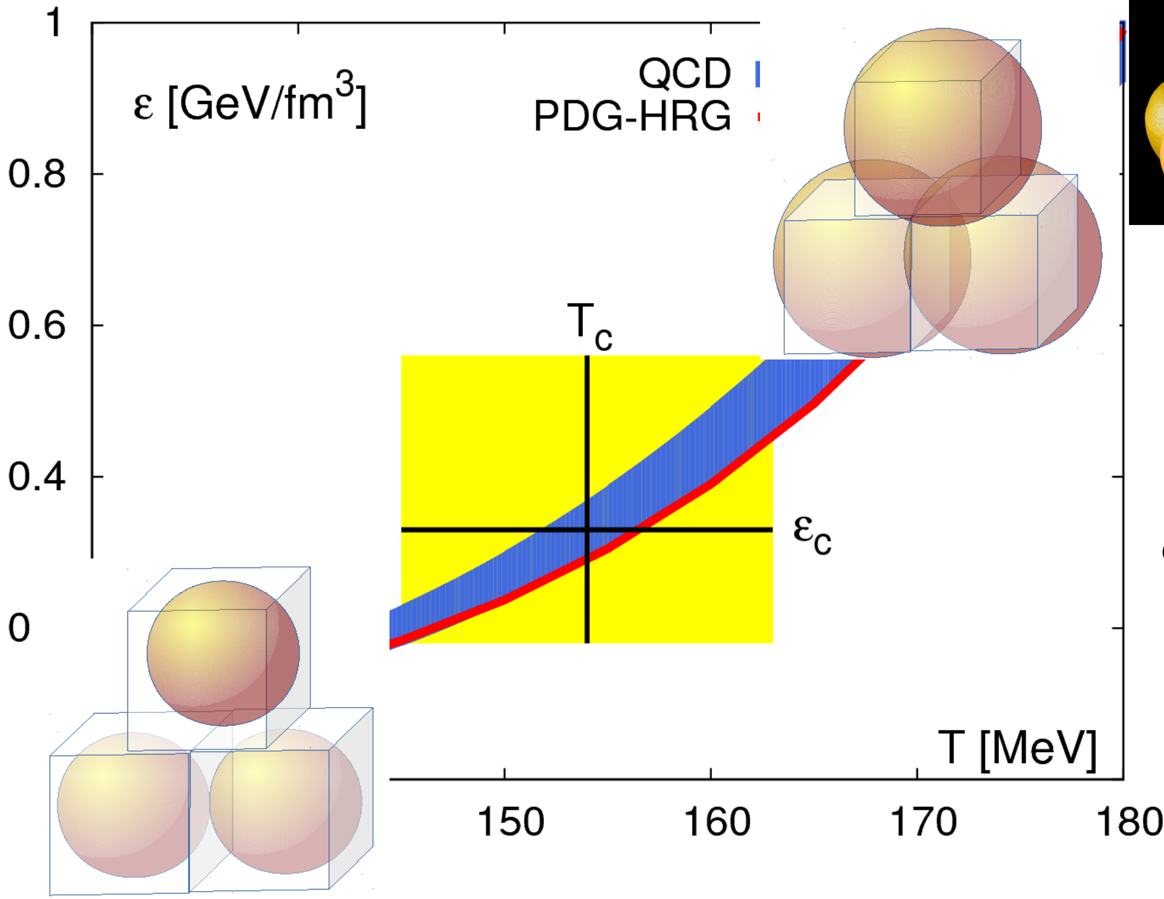

A. Bazavov et al. (hotQCD) ,
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dense packing of spheres (DPS)

$$\epsilon^{\mathrm{DPS}} = 0.74\epsilon^{\mathrm{nucleon}}$$
$$\simeq 0.33\ \mathrm{GeV/fm}^3$$
$$(R_p \simeq 0.8\mathrm{fm})$$

overlapping hadrons = QGP ??

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$
$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm}^3$$

compare with:

$$\epsilon^{\mathrm{nucl.\ mat.}} \simeq 150\ \mathrm{MeV/fm}^3$$
$$\epsilon^{\mathrm{nucleon}} \simeq 450\ \mathrm{MeV/fm}^3$$

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\mu=0}, \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$

conserved charge fluctuations: $\chi_n^X(T, \mu_B, ...) = \frac{\partial^n P/T^4}{\partial\hat{\mu}_X^n}$ $\quad X = B,\ Q,\ S$

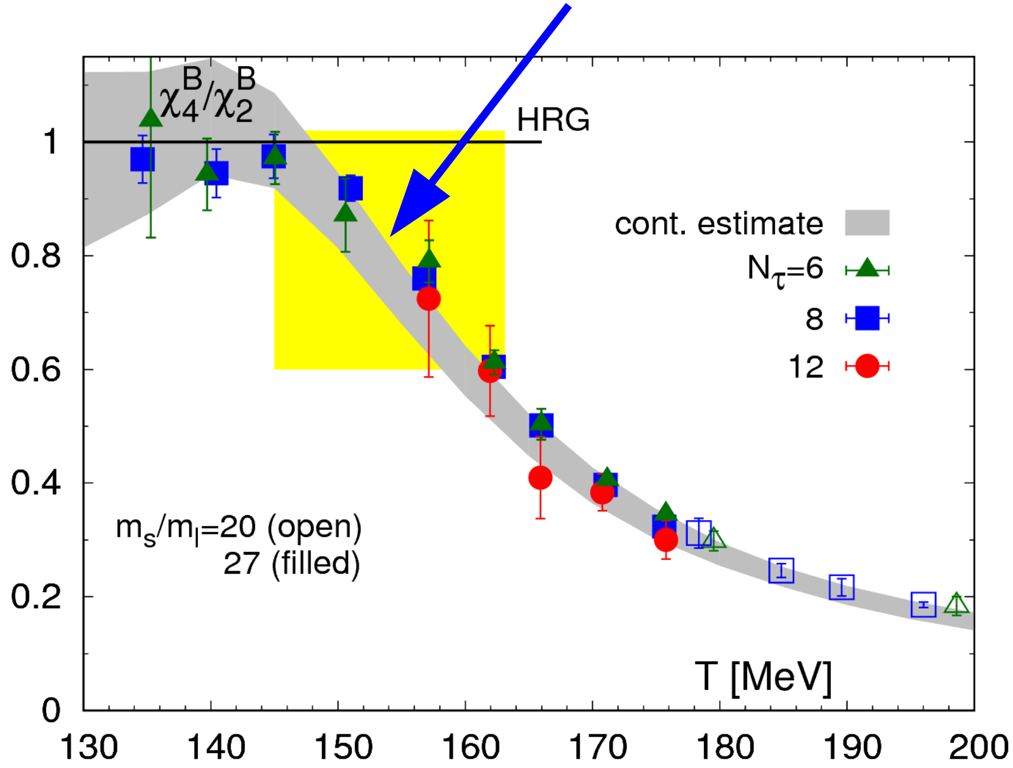
cumulant ratios:

$$\boxed{\frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \; , \quad S_X\sigma_X = \frac{\chi_3^X(T,\mu)}{\chi_2^X(T,\mu)} \; , \quad \kappa_X\sigma_X^2 = \frac{\chi_4^X(T,\mu)}{\chi_2^X(T,\mu)}}$$

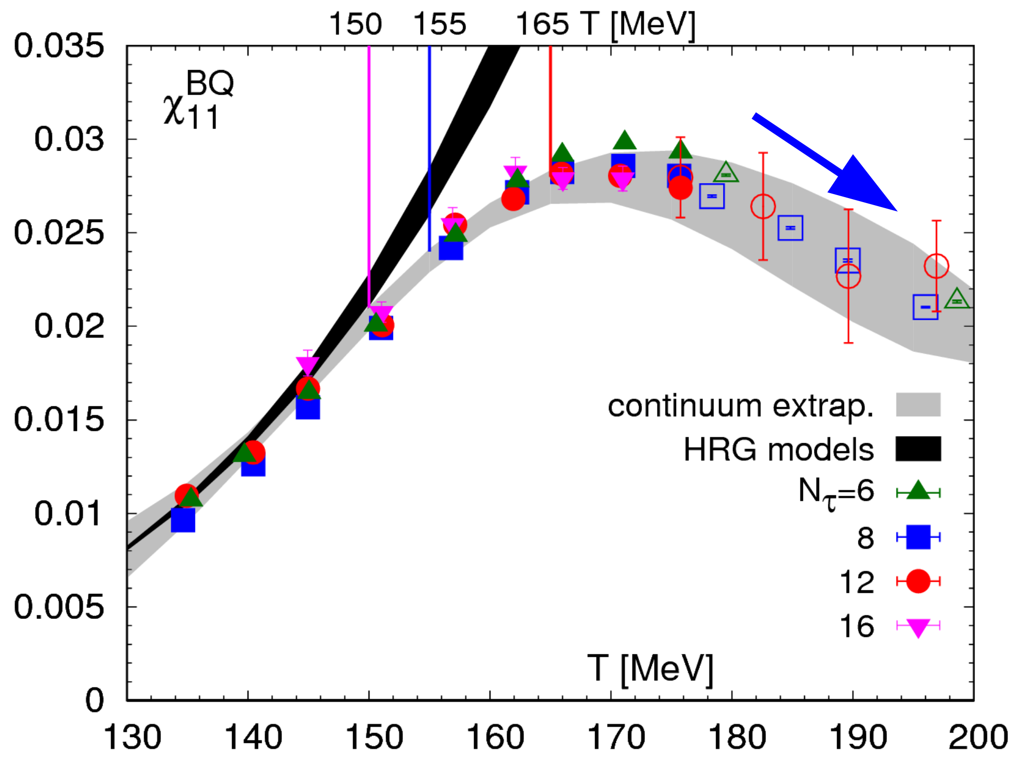
$\mu \equiv (\mu_B, \mu_Q, \mu_S)$

HRG vs. QCD
baryon number – electric charge correlations

$$\kappa_B \sigma_B^2 \equiv \chi_4^B / \chi_2^B < 1$$



$$\chi_{11}^{BQ} = \frac{\partial^2 P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q} = \langle B \cdot Q \rangle - \langle B \rangle \langle Q \rangle$$

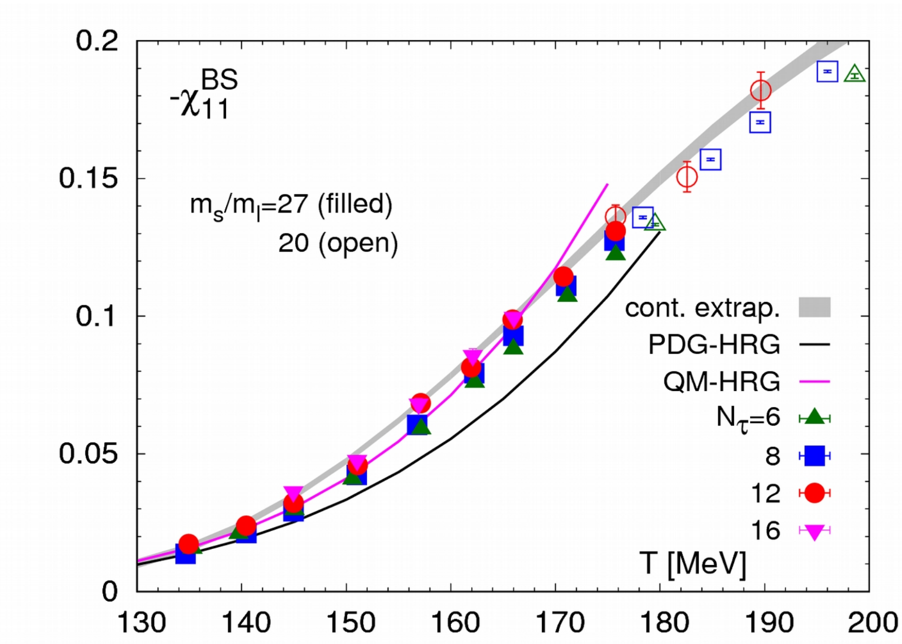


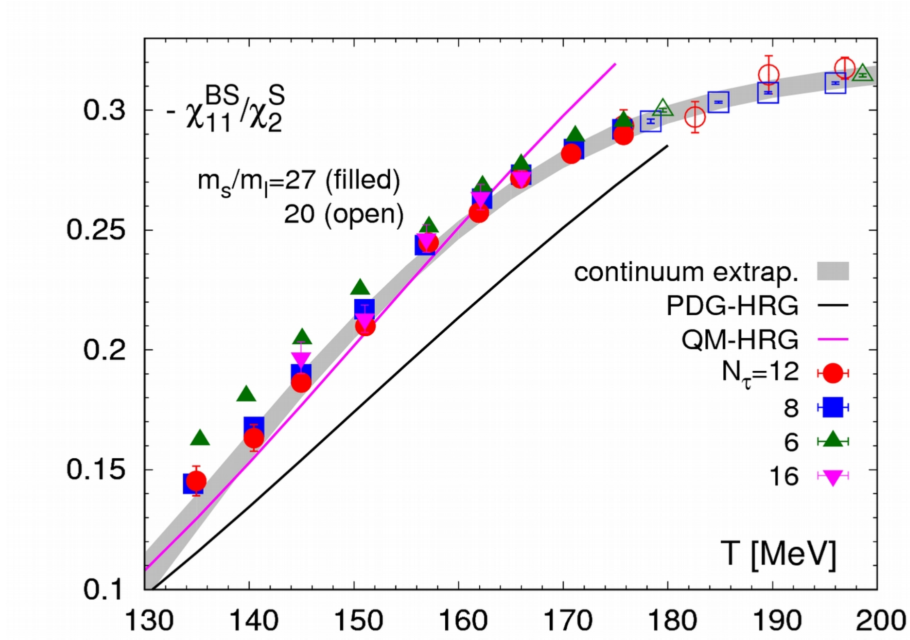
free quark gas: $\frac{1}{3}\left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) = 0$

– change in composition of the thermal medium
is detected through conserved charge correlations
=> this gets reflected in cumulant ratios of, e.g.
net-baryon number fluctuations => no longer Skellam for $T \gtrsim 155\mathrm{MeV}$

HRG vs. QCD charge correlations at $\mu_B = 0$

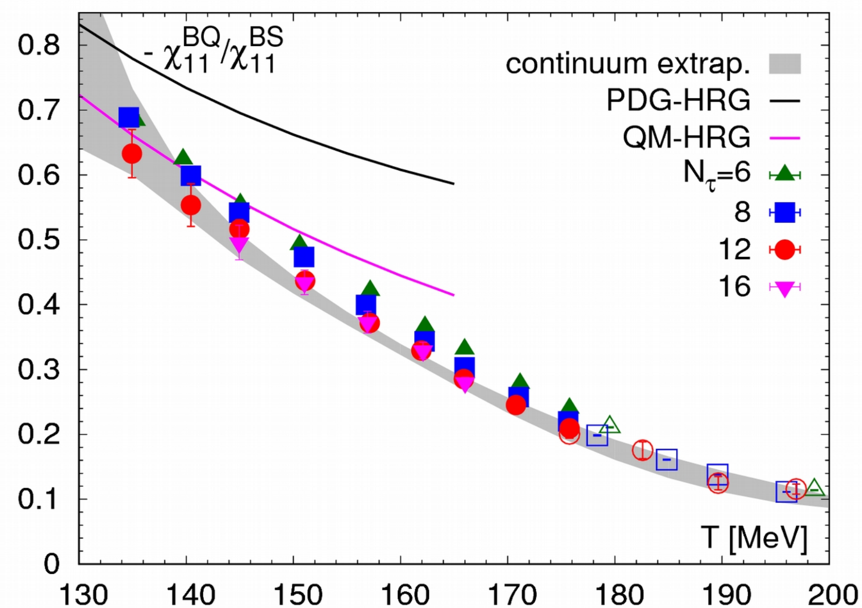
--evidence for additional strange hadrons (baryons) observable at the LHC??





all fits and results are taken from:
lattice QCD calculation of 6th order Taylor expansion of the QCD equation of state, using the Highly Improved Staggered Quark (HISQ) action,

A. Bazavov et al. (Bielefeld-BNL-CCNU)
arXiv:1701.04325



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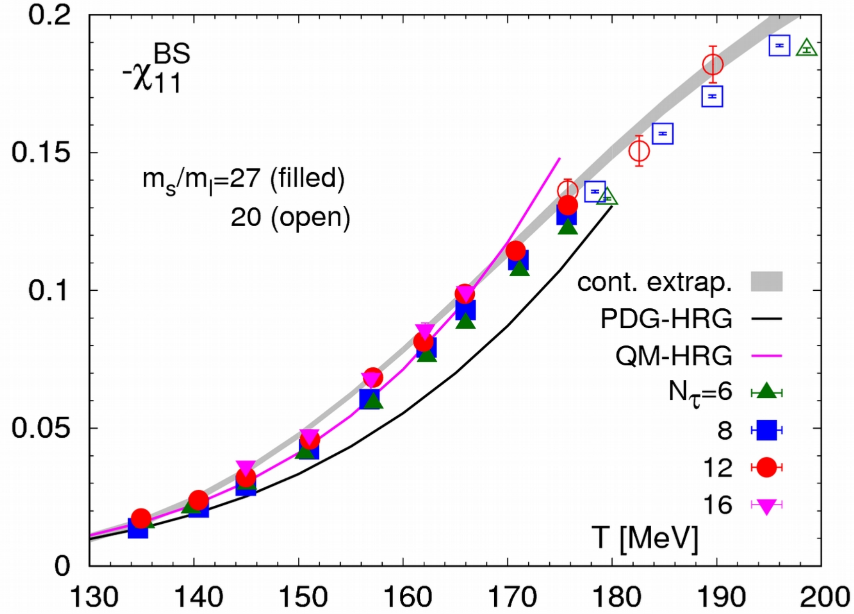
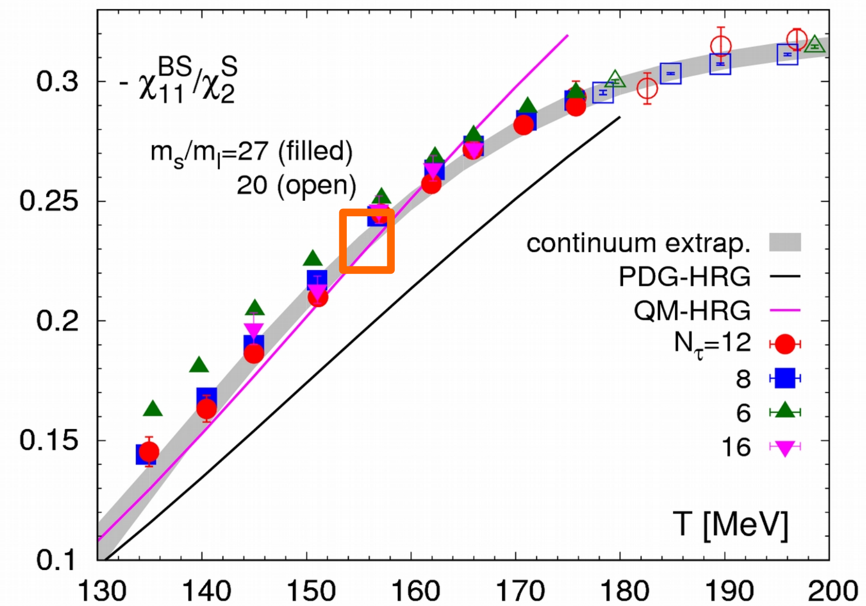




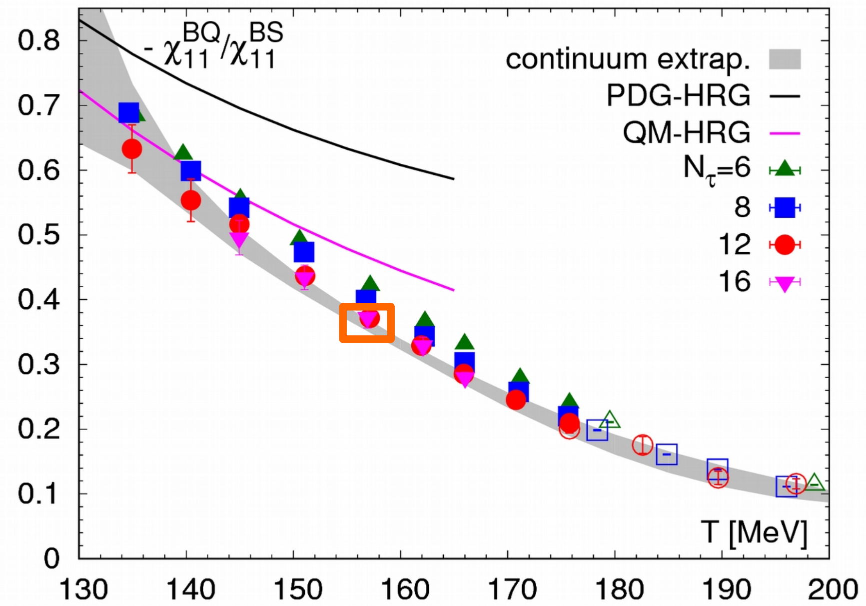
at ALICE freeze-out temperature

$$T_{fo} = 156(3)\mathrm{MeV}$$

$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.37(3)$$

HRG: -0.65 !!



HRG vs. QCD charge correlations at $\mu_B = 0$

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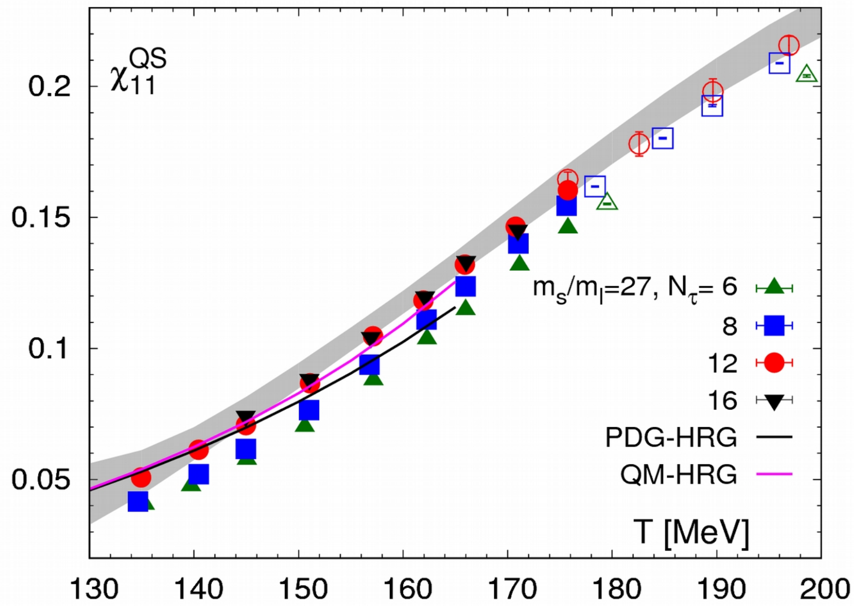
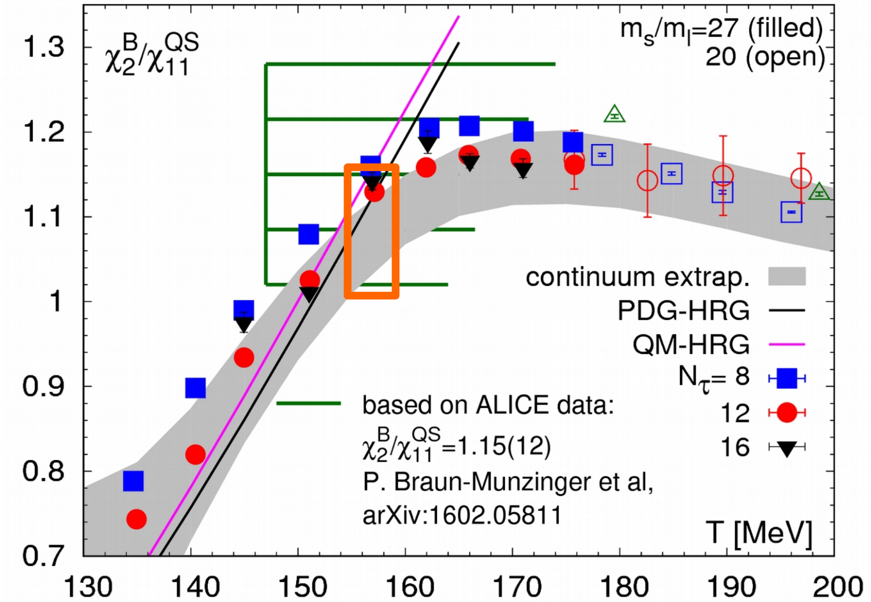


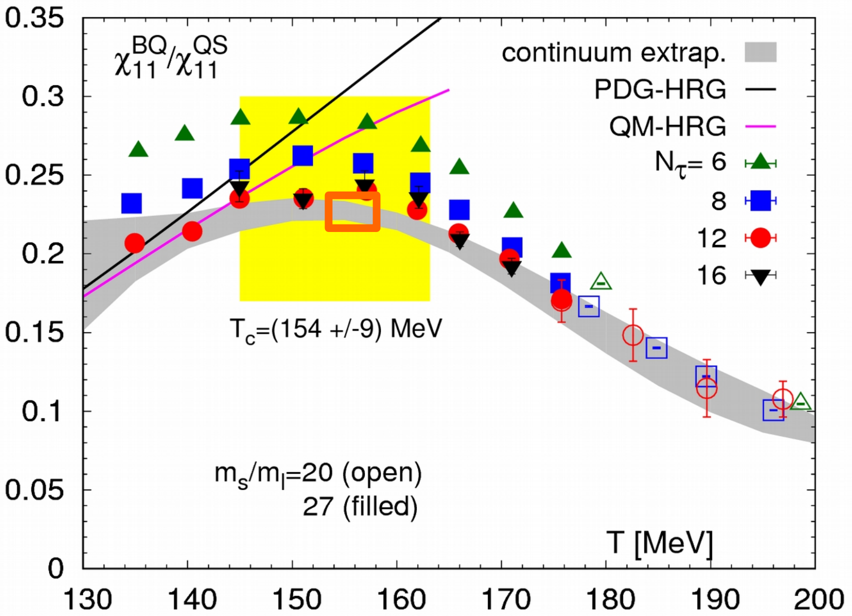


at ALICE freeze-out temperature

$T_{fo} = 156(3)\mathrm{MeV}$

$$\chi_2^B/\chi_{11}^{QS} = 1.10(8)$$

$$\chi_{11}^{BQ}/\chi_{11}^{QS} = 0.225(15)$$



a QCD bound on ratios of charge correlations:

$$\chi_{11}^{BQ}/\chi_{11}^{QS} \leq 0.24$$

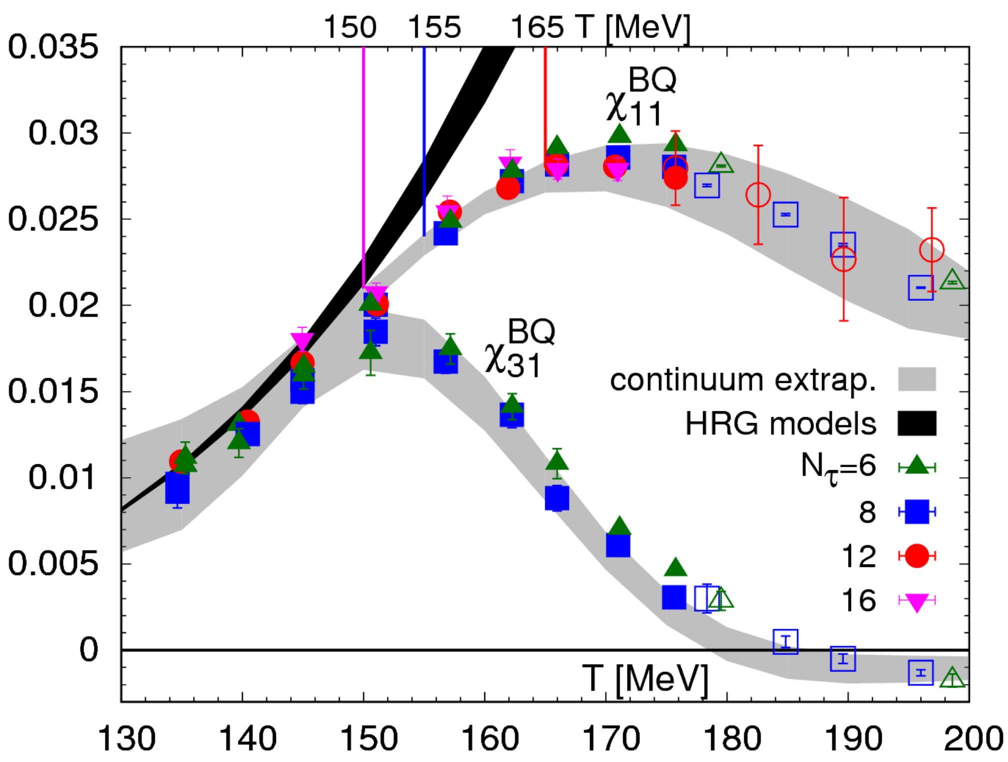
HRG vs. QCD
electric charge-baryon number correlations

$$\boxed{\mu_B/T > 0}$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_{11}^{BQ}(T,\mu_B) = \chi_{11}^{BQ} + \frac{1}{2}\chi_{31}^{BQ}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

– agreement between HRG and QCD will start to deteriorate for T>150 MeV



HRG: $\chi_{11}^{BQ} \equiv \chi_{31}^{BQ}$

– increase of BQ-correlation with baryon chemical potential smaller in QCD than HRG

$\Rightarrow$ deviations from HRG become larger with increasing baryon chemical potential

HRG vs. QCD
electric charge-baryon number correlations

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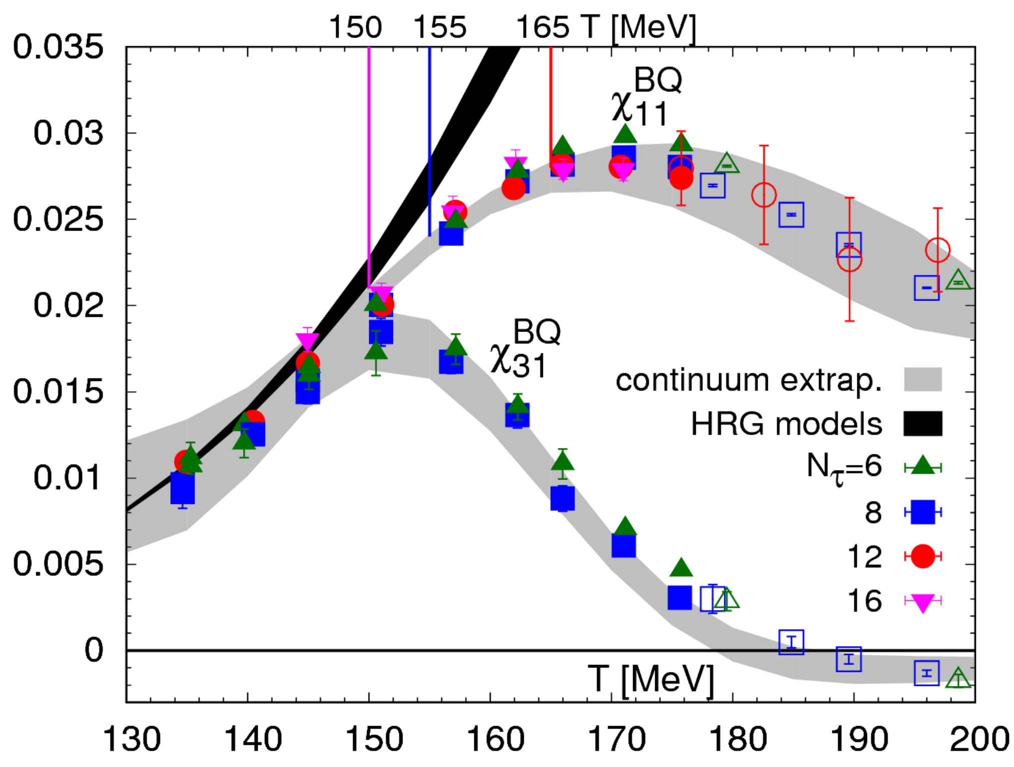
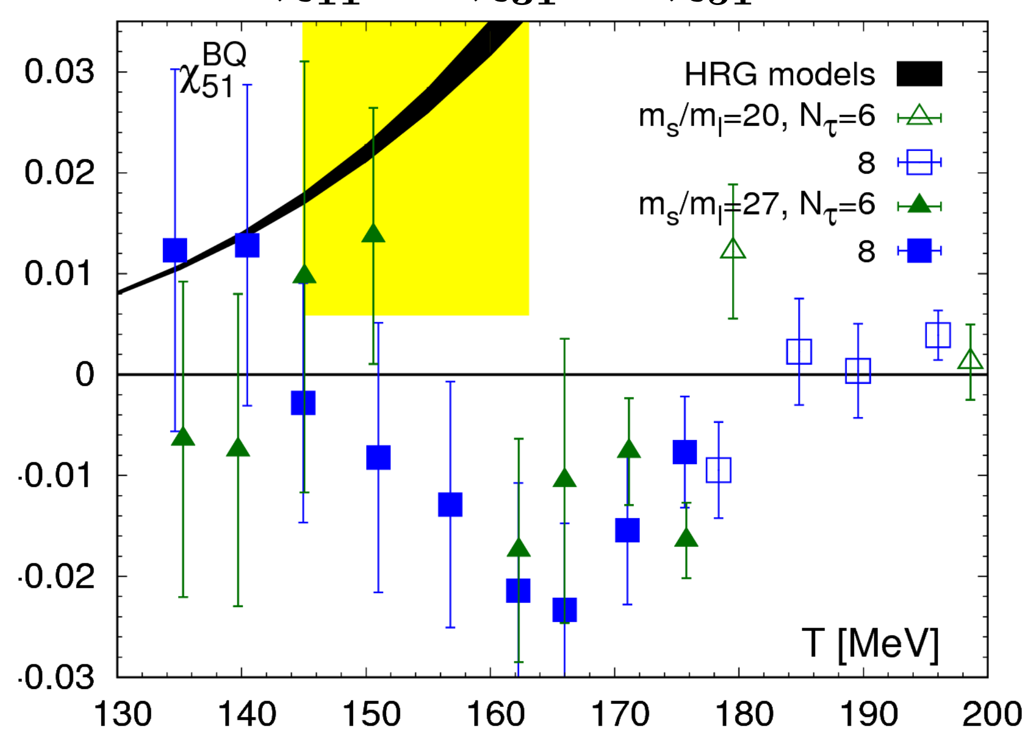


HRG: $\chi_{11}^{BQ} \equiv \chi_{31}^{BQ} \equiv \chi_{51}^{BQ}$



HRG vs. QCD
net baryon-number fluctuations

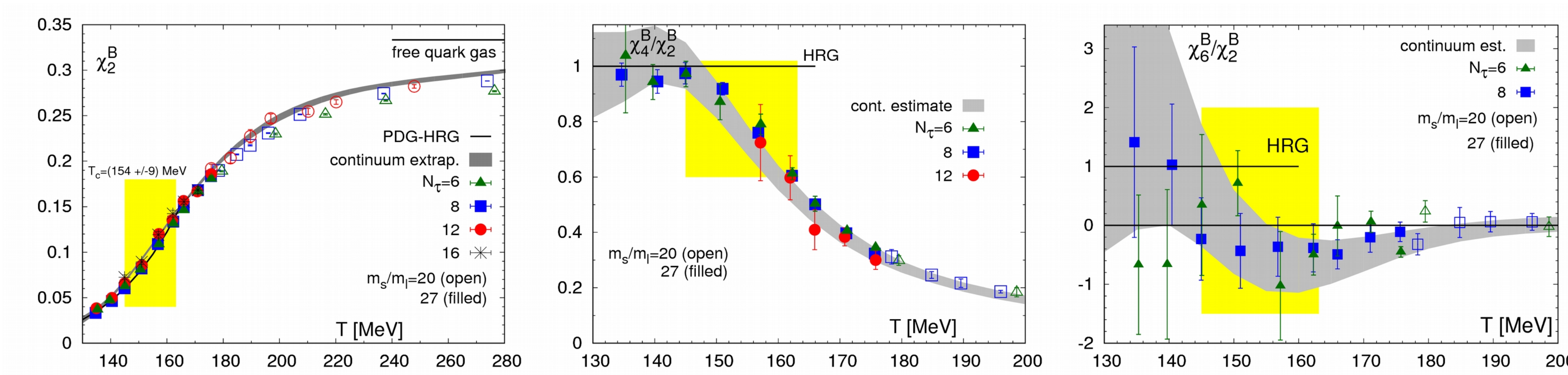
$$\boxed{\mu_B/T > 0}$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**



fits: A. Bazavov et al. (Bielefeld-BNL-CCNU)
arXiv:1701.04325

data are updated:
Bielefeld-BNL-CCNU preliminary

HRG vs. QCD
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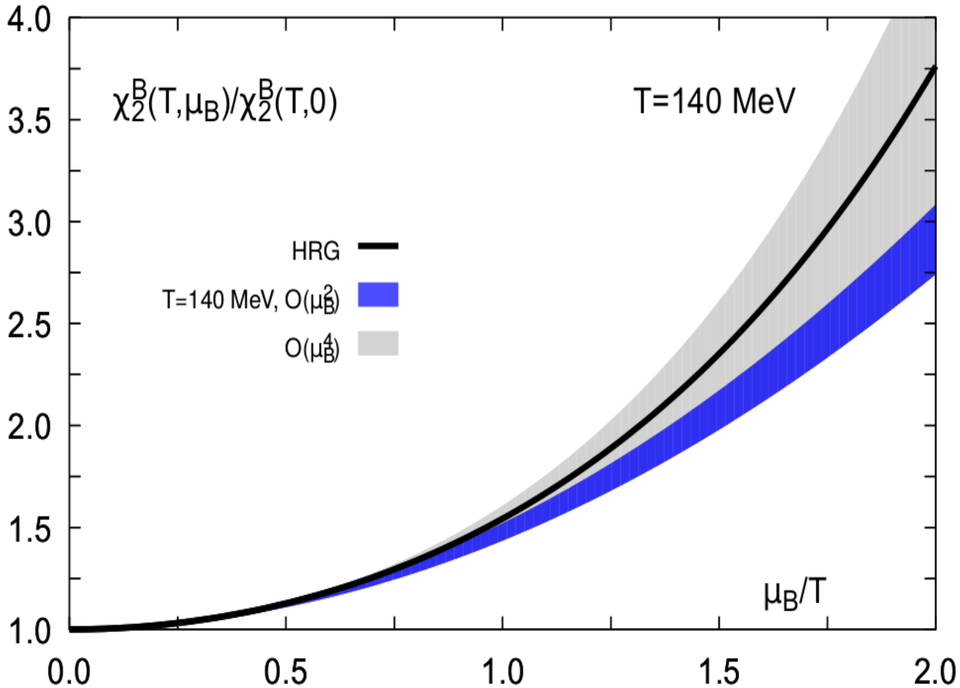
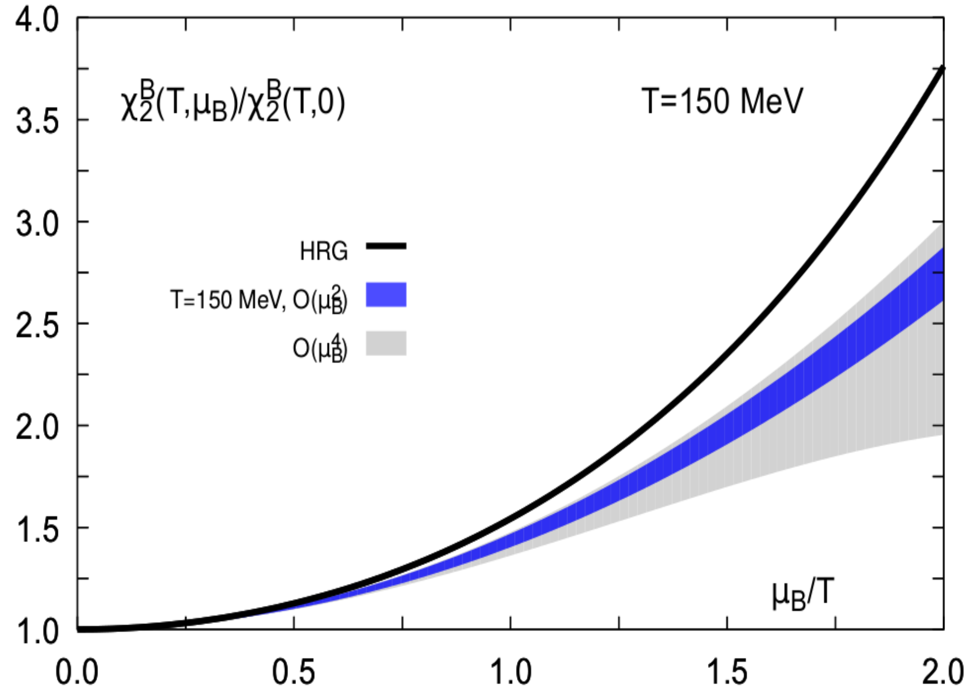




HRG vs. QCD
net baryon-number fluctuations

$$\boxed{\mu_B/T > 0}$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B \left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\chi_6^B \left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^6)$$

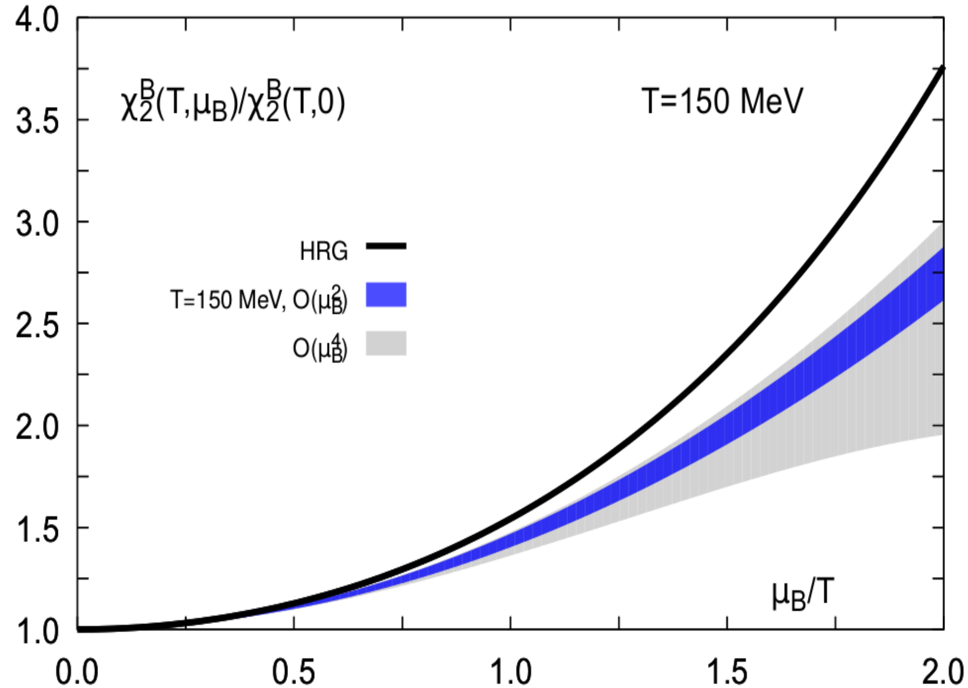
– **agreement between HRG and QCD will start to deteriorate for T>150 MeV**

– **net baryon-number fluctuations in QCD always smaller than in HRG for T>150 MeV**

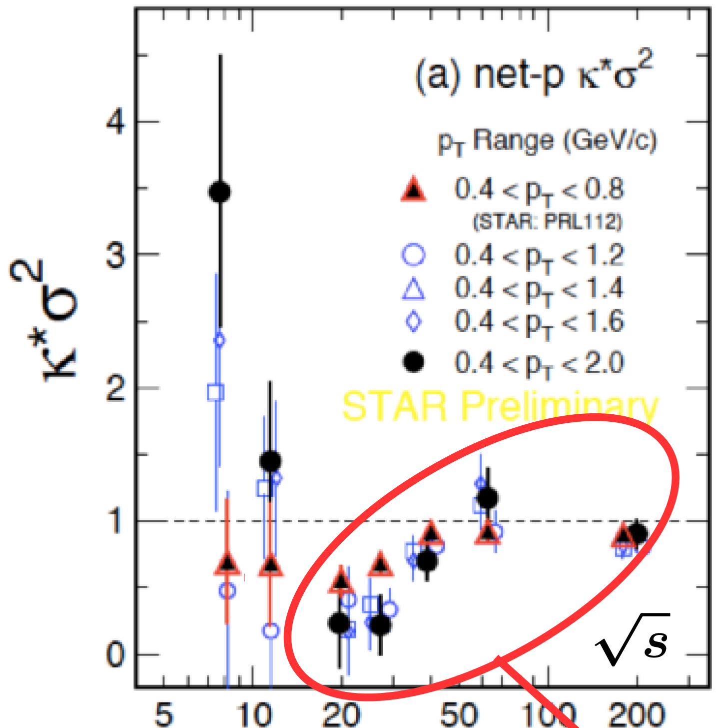
no evidence for enhanced
net baryon-number fluctuations
for

$$T \geq \ 135\text{MeV} \ , \ \mu_B \leq 2T$$

no evidence for getting closer
to a "critical region"



Exploring the QCD phase diagram



3 2 1.2 0.7 μ_B/T

$$\kappa_X \sigma_X^2 = \frac{\chi_4^X}{\chi_2^X}$$

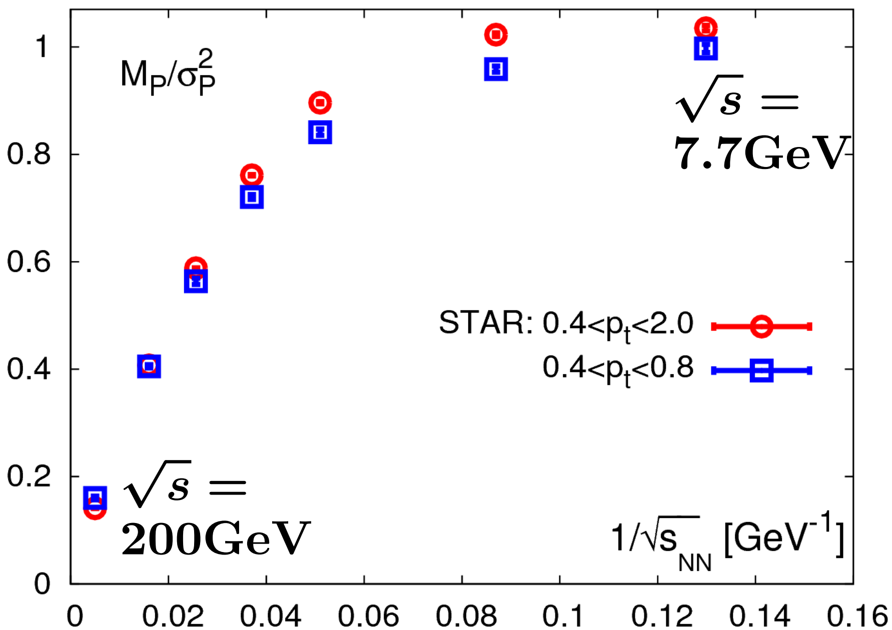
More moderate questions:

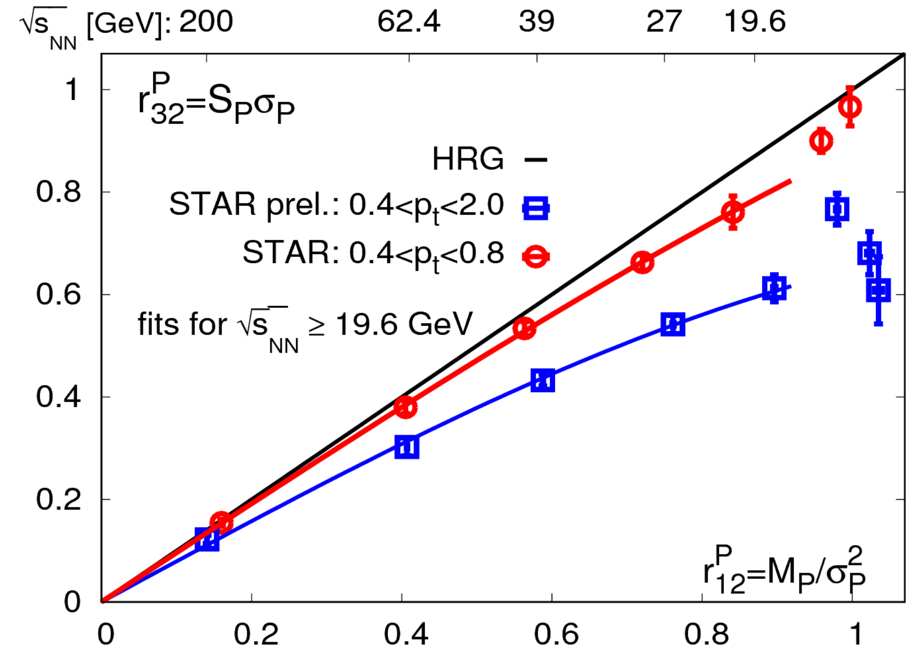
- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

?

- For $\sqrt{s} \geq 27$ GeV : Structure of net-electric charge and net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion? – rather small** μ_B/T

STAR and PHENIX data on cumulant ratios of net-proton number and net-electric charge fluctuations





$r_{12}^P \equiv \frac{M_P}{\sigma_P^2}$ is monotonic functions of $\sqrt{s_{NN}}$

$r_{32}^P \equiv S_P\sigma_P < r_{12}^P$

and hence also of μ_B

(this is not trivial;
It will not hold close to a critical point !)

replace μ_B/T in favor of r_{12}^B , e.g.

$$\boxed{\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3 + \mathcal{O}\left(\left(r_{12}^B\right)^5\right)}$$, e.g. $m_1^B = \frac{1}{r_{12}^{B,1}}$

Conserved charge fluctuations and freeze-out
mean and variance

for simplicity: $\mu_S = \mu_Q = 0$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B \sigma_B = \frac{\mu_B}{T} \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

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$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

$\mu_S = \mu_Q = 0$

$$\Rightarrow \quad S_B\sigma_B \equiv r_{32} = \frac{\chi_4^B}{\chi_2^B} r_{12}^B + \mathcal{O}((r_{12}^B)^3)$$

Conserved charge fluctuations and freeze-out
mean and variance

for simplicity: $\mu_S = \mu_Q = 0$

freeze-out line

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \left(\frac{\mu_B}{T}\right)^2\right)$$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T}\,\frac{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T}\,\frac{\chi_4^B}{\chi_2^B}\,\frac{1+\frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

$$\left(\frac{\chi_4^B}{\chi_2^B}\right)_{T_{f,0}} - \kappa_2^f T_{f,0}\left(\frac{\chi_4^B}{\chi_2^B}\right)' \left(\frac{\mu_B}{T}\right)^2$$

all $\chi_n \equiv \chi_n(T)$ eventually need to be expanded in T

$\mu_S = \mu_Q = 0$

$$\Rightarrow\quad S_B\sigma_B \equiv r_{32} = \frac{\chi_4^B}{\chi_2^B} r_{12}^B + \mathcal{O}((r_{12}^B)^3)$$

Conserved charge fluctuations and freeze-out
mean, variance and skewness

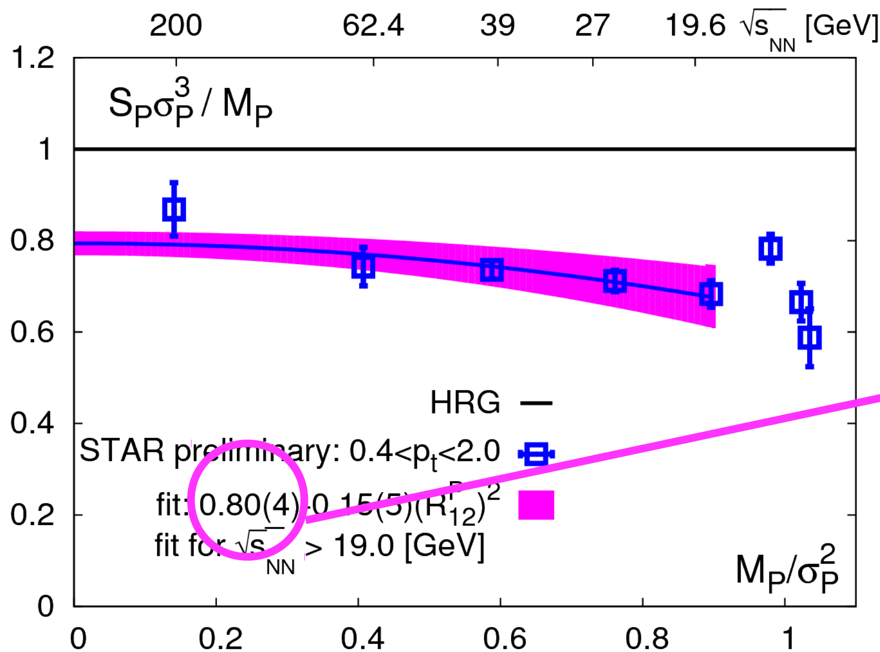
NLO Taylor expansion

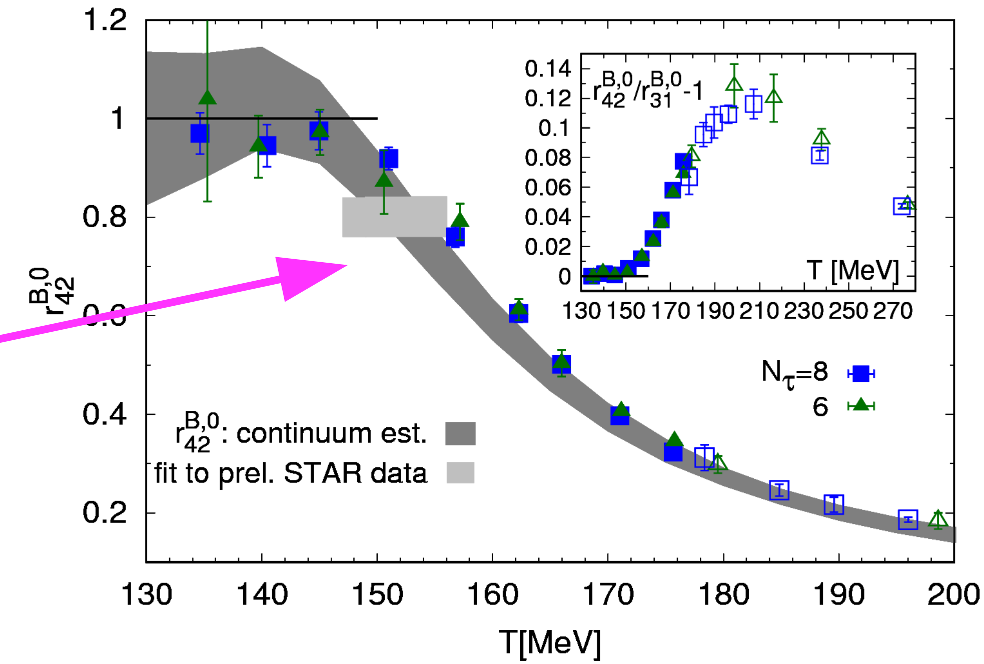
$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots \qquad \mu_Q = \mu_S = 0$$

$$(\text{and } \kappa_f = 0)$$

$$\Longleftrightarrow \quad r_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = r_{31}^{B,0} + \frac{r_{31}^{B,2}}{(r_{12}^{B,1})^2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

F. Karsch et al., arXiv:1512.06987





Bielefeld-BNL-CCNU, preliminary

– intercept consistent with QCD, if freeze-out at Tf<160 MeV.

– if $T_f \simeq 165\text{MeV}$ at $\mu_B \simeq 0$: skewness ratio should be small $\frac{S_B\sigma_B^3}{M_B} \simeq 0.6$

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $r_{31}^B \equiv S_B\sigma_B^3/M_B$, $r_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

F. Karsch et al., arXiv:1512.06987

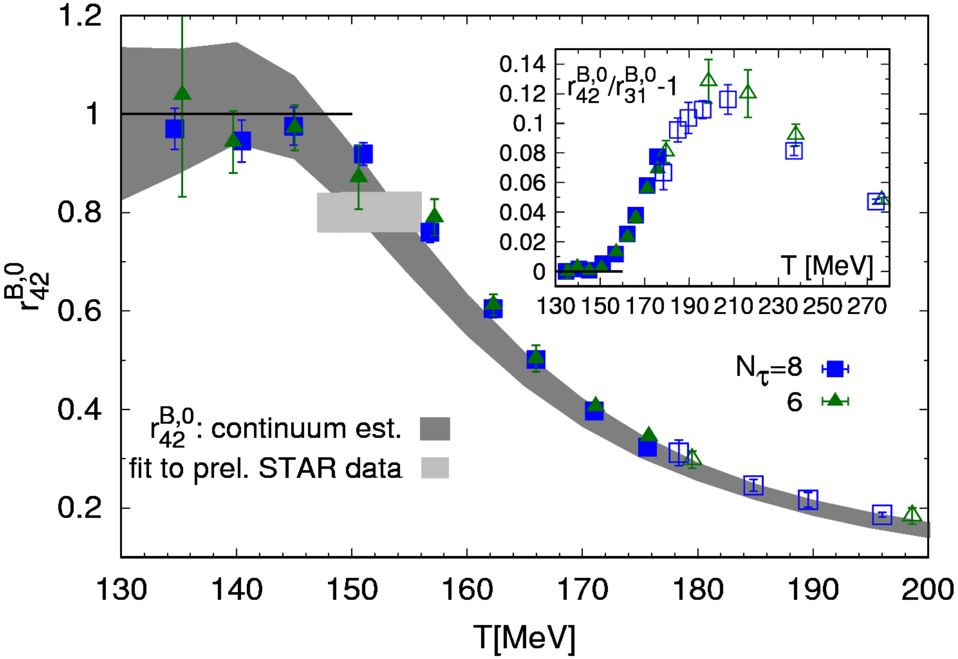
$$\left.\begin{aligned} r_{31}^B &= r_{31}^{B,0} + r_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2 \\ r_{42}^B &= r_{42}^{B,0} + r_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2 \end{aligned}\right\}$$

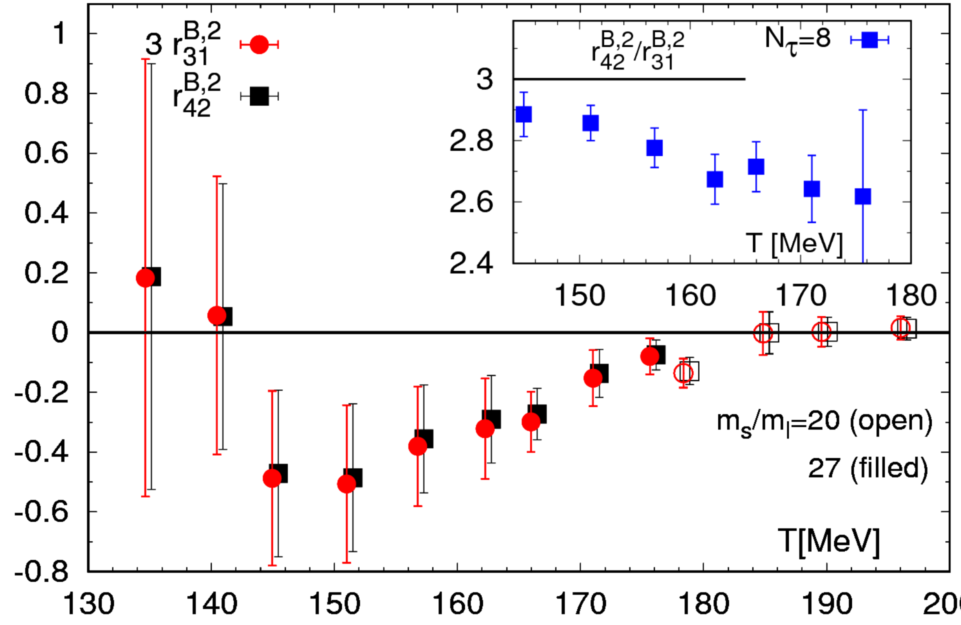
$\Rightarrow$ $r_{42}^{B,0} \simeq r_{31}^{B,0}$

$\underline{\mu_S = \mu_Q = 0:}$

$$r_{42}^{B,2} = 3r_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$

$\underline{\mu_S \neq \mu_Q \neq 0:}$





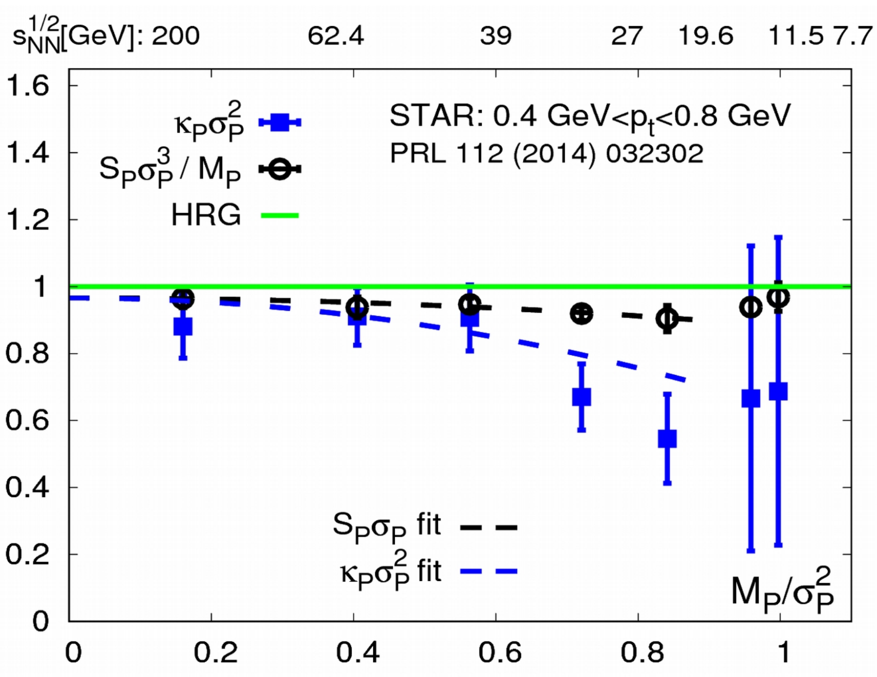
Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

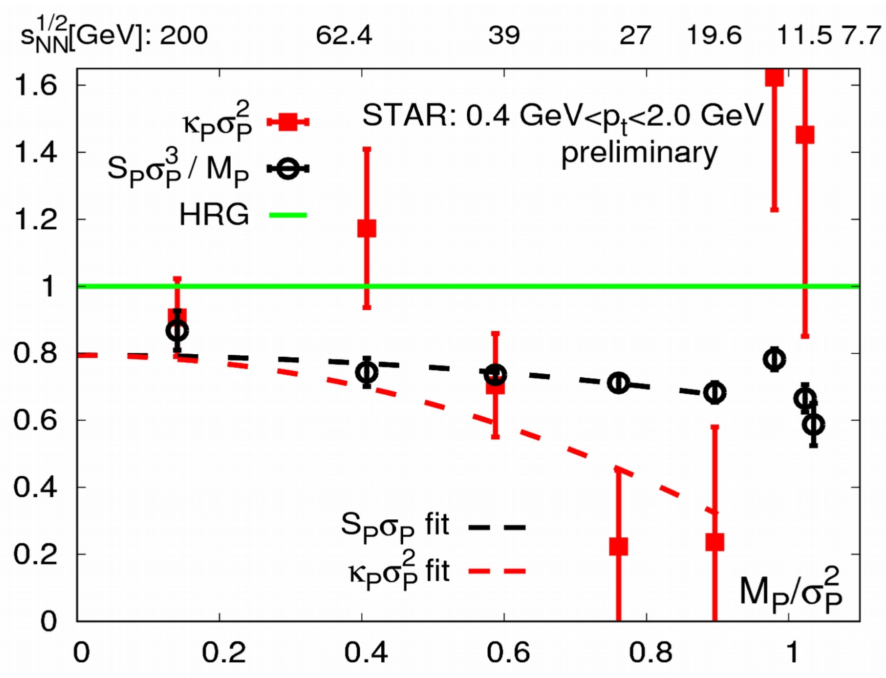
in a NLO Taylor expansion $r_{31}^B \equiv S_B\sigma_B^3/M_B$ and $r_{42}^B \equiv \kappa_B\sigma_B^2$ are closely related

$$\left.\begin{aligned} r_{31}^B &= r_{31}^{B,0} + r_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2 \\ r_{42}^B &= r_{42}^{B,0} + r_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2 \end{aligned}\right\}$$

$\underline{\mu_S = \mu_Q = 0:}$

$$r_{42}^{B,2} = 3r_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$





Conclusions

– attempts to understand freeze-out/hadronization in terms of HRG model based calculations at temperatures T > 160 MeV are difficult to conciliate with QCD;

at $\mu_B = 0$ QCD thermodynamics is quite different from HRG thermodynamics for T > 160 MeV;

for $\mu_B > 0$ differences between HRG and QCD become larger

– no evidence for large enhancement of net-baryon number or electric charge fluctuations in the parameter range $T \gtrsim 135\ \mathrm{MeV}\ ,\ 0 \le \mu_B/T \le 2$

– leading order Taylor expansions provide good approximations for cumulant ratios measured at RHIC for $\sqrt{s_{NN}} \ge 19\ \mathrm{GeV}$

Taylor expansion coefficients may be measured by ALICE

Universität Bielefeld

The Collaborative Research Center CRC-TR211 has an immediate opening for a

research position in Theoretical Physics

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– (4-5) post-doc positions in lattice QCD
– 2 PhD positions in lattice QCD

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