Department of Physics, KU Leuven Campus Kortrijk - KULAK

Thermal entropy of the quark-antiquark pair from dynamical holographic QCD

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In collaboration with D. Dudal

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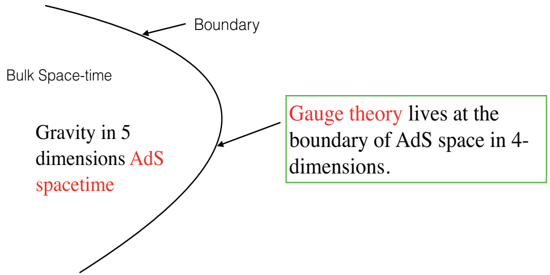
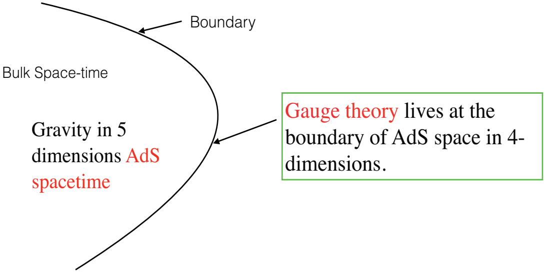
Outline

General Introduction

- AdS/CFT correspondence or the gauge/gravity duality is a conjectured duality between a gravity theory on anti de-Sitter (AdS) spacetime and a gauge theory living on its boundary[1].
- The gauge/gravity duality has become a valuable method for investigating strongly coupled gauge theory.
- It offers an intrinsically non-perturturbative framework that allows one to study strongly coupled gauge theories both at vanishing and at finite temperature, with and without chemical potential.

[1] **J. Maldacena, Adv. Theor. Math. Phys 2 (1998) 231.**


Boundary
Bulk Space-time
Gravity in 5 dimensions AdS spacetime
Gauge theory lives at the boundary of AdS space in 4-dimensions.



- It is therefore a duality between gravitational and non - gravitational theories.

Dictionary

One can relate the observable in one theory to the observable of its dual theory[2]

Gravity side	Gauge theory side
Field ϕ	Operator $\hat{O}$
Metric g_{MN}	EM tensor $T_{\mu\nu}$
Hawking temperature T	Gauge theory temperature T

Boundary value ϕ_0 of the field ϕ corresponds to the source for the corresponding operator $\hat{O}$.

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The duality is, then, stated as

$$Z[\phi_0]_{AdS} = Z_{\hat{O}}[\phi_0]_{boundary}$$

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Applications

- Strongly coupled limit of one side corresponds to weakly coupled limit of its dual side.
- This strong-weak nature of the gauge/gravity duality can be exploited to compute useful quantities in a strongly coupled field theory from relatively simpler calculations in its dual gravity theory.
- The duality has been successfully applied to gain useful insights into a number of fields like hydrodynamics, QGP, superconductivity, entanglement entropy etc.
- Our aim in this work is to study some of lattice QCD results qualitatively from the phenomenological bottom-up models of gauge/gravity duality.

Lattice Results

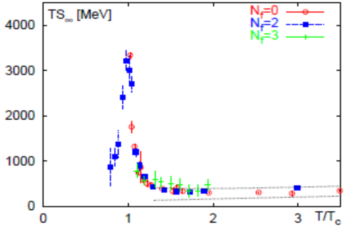
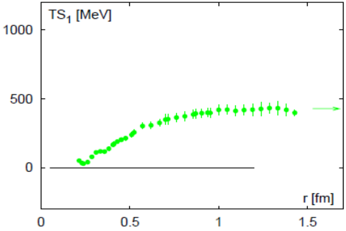


Figure: Lattice QCD result for the entropy of the quark-antiquark pair as function of temperature T/T_c for large quark-antiquark separation. The result is taken from Kaczmarek et al [PoS LAT2005 (2005) 192].



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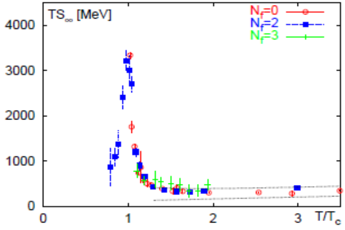

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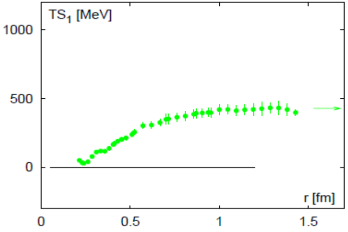

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- Lattice data predicts sharp peak in the entropy near the transition temperature.

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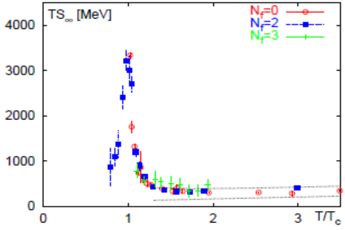

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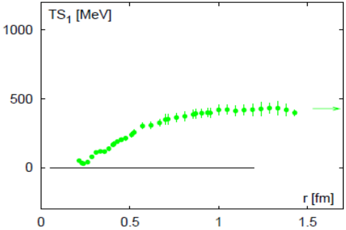

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- It predicts growth of the entropy with the inter-quark distance. [Starting point in the "Deconfinement as an entropic self destruction" scenario of Kharzeev.]

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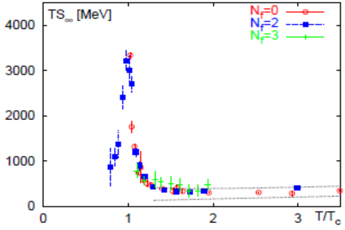

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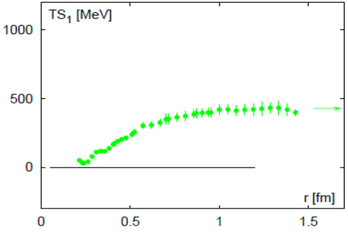

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- Lattice data predicts sharp peak in the entropy near the transition temperature.
- It predicts growth of the entropy with the inter-quark distance. [Starting point in the "Deconfinement as an entropic self destruction" scenario of Kharzeev.]
- The entropy saturates to a constant temperature dependent value at large distances.

- Our Aim is to construct a holographic QCD model with dual parameters fixed to some QCD observables (string tension and light meson spectrum), and then investigate how this model can predict similar results as the lattice data for e.g. entropy of the $Q - \bar{Q}$ pair, P-loop etc.

- For this purpose, we consider a phenomenological bottom-up approach, where the gravity theory is constrained by hand as to reproduce the desirable features of the boundary gauge theory, without actually deriving them from a consistent truncation of an underlying string theory.

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- While a priori this assumption is not guaranteed to be true, however there are also strong numerical evidences that suggest this might be the case.[3]

[3] **M. Teper**, **PoS LATTICE2008, 010 (2008); M. Panero**, **Phys. Rev. Lett. 103 (2009) 232001**.

Holographic model

We start with the Einstein-Maxwell-Dilaton action in five dimensions

$$S_{EM} = \frac{1}{16\pi G_5} \int \mathrm{d}^5 x \sqrt{-g} \ \left[R - \frac{f(\phi)}{4} F_{MN} F^{MN} - \frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right],$$

where G_5 is the Newton constant in five dimension, $V(\phi)$ is the potential of the dilaton field and $f(\phi)$ is a gauge kinetic function which represents the coupling between dilaton (ϕ) and gauge field (A_M).

In order to simultaneously solve Einstein-Maxwell-Dilaton equations, we consider the following ansatz,

$$
\begin{aligned}
ds^2 &= \frac{L^2 e^{2A(z)}}{z^2}\left(-g(z)dt^2 + \frac{dz^2}{g(z)} + dy_1^2 + dy_3^2 + dy_3^2\right),\\
A_M &= A_t(z), \quad \phi = \phi(z)
\end{aligned}
$$

where we have assumed that various fields depend only on the redial coordinate z. Here L is AdS length scale and in our notation $z = 0$ corresponds to the asymptotic boundary of the spacetime.

EOMs

$$\phi'' + \phi'(-\frac{3}{z} + \frac{g'}{g} + 3A') - \frac{L^2 e^{2A}}{z^2 g}\frac{\partial V}{\partial \phi} + \frac{z^2 e^{-2A} A_t'^2}{2L^2 g}\frac{\partial f}{\partial \phi} = 0$$

$$A_t'' + A_t'(-\frac{1}{z} + \frac{f'}{f} + A') = 0$$

$$g'' + g'\left(-\frac{3}{z} + 3A'\right) - \frac{e^{-2A} A_t'^2 z^2 f}{L^2} = 0$$

$$A'' + \frac{g''}{6g} + A'(-\frac{6}{z} + \frac{3g'}{2g}) - \frac{1}{z}(-\frac{4}{z} + \frac{3g'}{2g}) + 3A'^2 + \frac{L^2 e^{2A} V}{3z^2 g} = 0$$

$$A'' - A'(-\frac{2}{z} + A') + \frac{\phi'^2}{6} = 0$$

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These equations can be solved analytically.

A particular gravity solution

$$g(z) = 1 - \frac{1}{\int_0^{z_h} dx\ x^3 e^{-3A(x)}} \left[\int_0^z dx\ x^3 e^{-3A(x)} + \frac{2c\mu^2}{(1 - e^{-cz_h^2})^2} det\mathcal{G} \right]$$

$$\phi'(z) = \sqrt{6(A'^2 - A'' - 2A'/z)}$$

$$A_t(z) = \mu \frac{e^{-cz^2} - e^{-cz_h^2}}{1 - e^{-cz_h^2}}$$

$$V(z) = -3L^2 z^2 g e^{-2A} \left[A'' + A' \left(3A' - \frac{6}{z} + \frac{3g'}{2g} \right) - \frac{1}{z} \left(-\frac{4}{z} + \frac{3g'}{2g} \right) + \frac{g''}{6g} \right]$$

where,

$$det\mathcal{G} = \begin{vmatrix} \int_0^{z_h} dx\ x^3 e^{-3A(x)} & \int_0^{z_h} dx\ x^3 e^{-3A(x) - cx^2} \\ \int_{z_h}^{z} dx\ x^3 e^{-3A(x)} & \int_{z_h}^{z} dx\ x^3 e^{-3A(x) - cx^2} \end{vmatrix}$$

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$$c = 1.16$$

Thermodynamics of the gravity solution

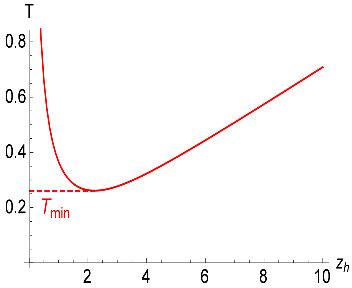


Figure: T as a function of z_h for $\mu = 0$. In units GeV.

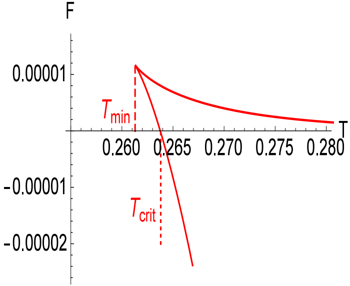


Figure: F as a function of T for $\mu = 0$. In units GeV.

- AdS phase is dual to confinement.
- Black hole phase is dual to deconfinement.

Thermodynamics of the gravity solution

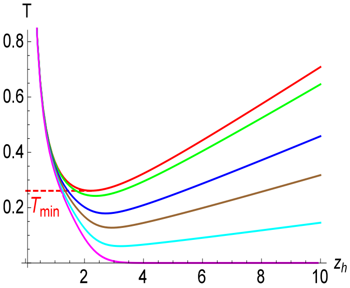
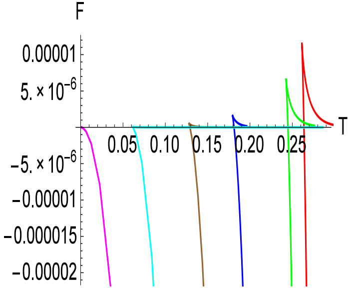


Figure: T as a function of z_h for various values of the chemical potential μ. Here red, green, blue, brown, cyan and magenta curves correspond to $\mu = 0$, 0.2, 0.4, 0.5, 0.6 and 0.673 respectively. In units GeV.



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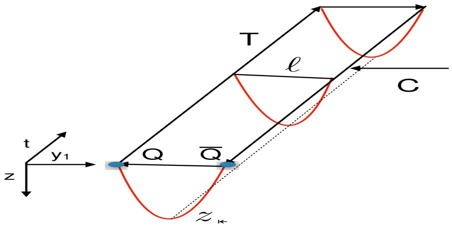
Wilson loop

- Consider a rectangular Wilson loop C living on the boundary ($z = 0$) of five-dimensional space. The quark and antiquark are set at $y_1 = \ell/2$ and $y_1 = -\ell/2$ respectively. Taking the limit $T \to \infty$ allows one to read off the energy of such a pair from the expectation value of the Wilson loop, namely,

$$\langle W(C) \rangle = e^{-TF(\ell)}$$



Holographic Wilson loop

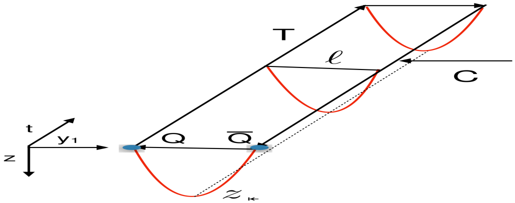
- In gauge/gravity duality, the expectation value of the Wilson loop is given by[4]

$$\langle W(C)\rangle = e^{-S_{NG}}$$

where

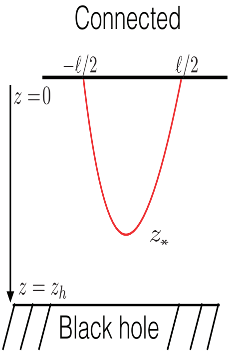
$$S_{NG} = -\frac{1}{2\pi l_s^2}\int d\tau d\sigma \sqrt{-\det g_s}, \quad (g_s)_{\alpha\beta} = (g_s)_{MN}\partial_\alpha X^M \partial_\beta X^N$$

is an area of a string world-sheet bounded by a curve C at the boundary of AdS space.



[4] J. Maldacena, Phys. Rev. Lett. 80 (1998) 4859.

EOM for string world sheet - connected solution

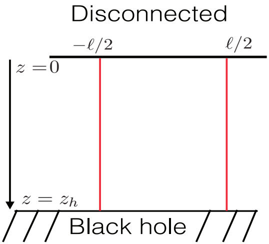


$$\mathcal{F}_{con} = \frac{L^2}{\pi l_s^2} \int_0^{z_*} dz \frac{z_*^2}{z^2} \frac{\sqrt{g(z)} e^{2A_s(z) - 2A_s(z_*)}}{\sqrt{g(z) z_*^4 e^{-4A_s(z_*)} - g(z_*) z^4 e^{-4A_s(z)}}}$$

where z_* is the turning point of the connected world sheet. This turning point is related to separation length between the quark-antiquark pair as

$$\ell = 2 \int_0^{z_*} dz\, z^2 \sqrt{\frac{g(z_*)}{g(z)}} \frac{e^{-2A_s(z)}}{\sqrt{g(z) z_*^4 e^{-4A_s(z_*)} - g(z_*) z^4 e^{-4A_s(z)}}}$$

EOM for string world sheet - disconnected solution



On the other hand, the disconnected configuration consists of two lines which are separated by distance ℓ and are extended from the boundary to the horizon (or to $z = \infty$).

$$\mathcal{F}_{discon} = \frac{L^2}{\pi l_s^2} \int_0^{z_h} dz \frac{e^{2A_s(z)}}{z^2}$$

- $\mathcal{F}_{discon}$ is independent of z_* and therefore of quark-antiquark separation length ℓ as well.

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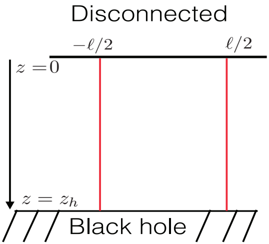

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- $\mathcal{F}_{discon}$ is independent of z_* and therefore of quark-antiquark separation length ℓ as well.
- Both $\mathcal{F}_{con}$ and $\mathcal{F}_{discon}$ are divergent quantities. The divergence arises from $z = 0$ part of the integration. We use Kaczmarek et al [arXiv:1605.07181] prescription to regularize the free energy.

Free energy $\mathcal{F}$ of $Q\bar{Q}$ pair with AdS background

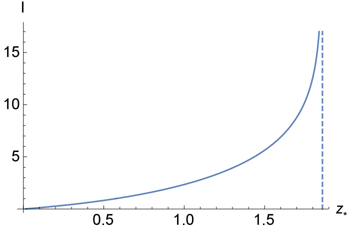

Figure: ℓ as a function of z_* in AdS background. In units GeV.

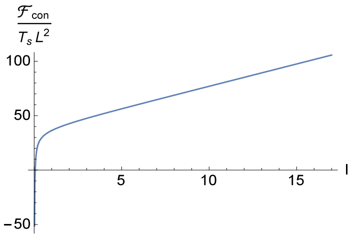

Figure: $\mathcal{F}_{con}$ as a function of ℓ in AdS background. In units GeV.

- The string world sheet does not penetrate deep into the bulk and saturates near $z = z_s$, suggesting some kind of an “imaginary” wall in the bulk AdS which can not penetrated by string world sheet.
- In the AdS phase, quark-antiquark pair is always connected by an open string and forms a confined state.

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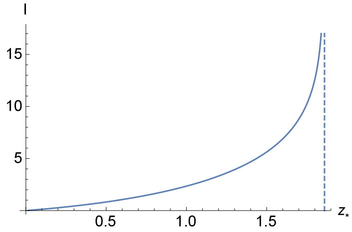


Figure: ℓ as a function of z_* in thermal-AdS background. In units GeV.

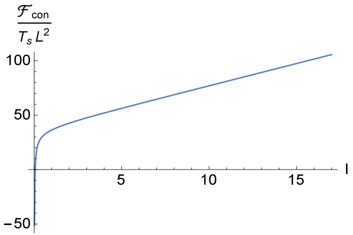


Figure: $\mathcal{F}_{con}$ as a function of ℓ in thermal-AdS background. In units GeV.

- $\mathcal{F} \propto -1/\ell$ for small ℓ exhibiting Coulomb potential, and $\mathcal{F} = \sigma_s \ell$ for large ℓ exhibiting confinement. These properties can be shown analytically.
- For AdS phase we have the famous Cornell expression $\mathcal{F} = -\frac{\kappa}{\ell} + \sigma_s \ell$ for the quark-antiquark pair.

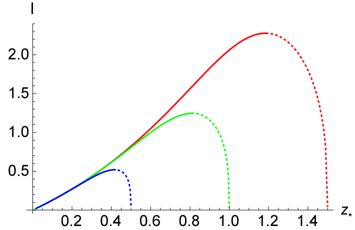


Figure: ℓ as a function of z_*. Here $\mu = 0$ and red, green and blue curves correspond to $z_h = 1.5$, 1.0 and 0.5 respectively.

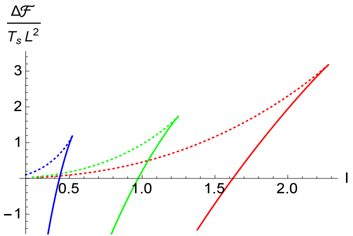
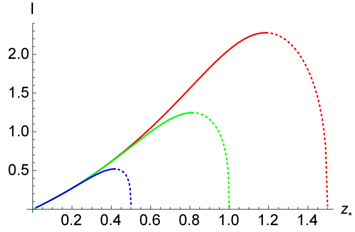


Figure: $\Delta\mathcal{F} = \mathcal{F}_{con} - \mathcal{F}_{discon}$ as a function of ℓ. Here $\mu = 0$ and red, green and blue curves correspond to $z_h = 1.5$, 1.0 and 0.5 respectively.

- No “imaginary” wall appears in the AdS-BH background. There exist an ℓ_{max} above which connected string configuration does not exist.
- Phase transition from connected string solution to disconnected string solution as we increase the string length ℓ

Free energy $\mathcal{F}$ of $Q\bar{Q}$ pair with AdS-BH background



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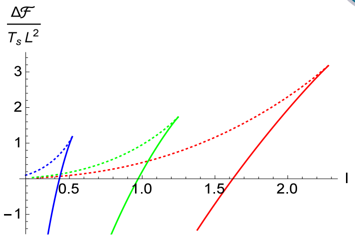

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- The behaviour that ℓ_{crit} decreases with temperature is consistent with the physical expectation that at higher and higher temperatures the boundary meson state would eventually melt to a free quark and antiquark (deconfined phase) which on the dual gravity side is described by the disconnected string configuration.
- Since for large separations, this disconnected string configuration which is independent of separation length ℓ is more favorable, therefore the corresponding free energy of the quark-antiquark pair is also independent of ℓ. It implies that the string tension is zero and there is no linear law confinement in the boundary theory dual to black hole phase.

- For AdS phase: linear law confinement
  : Polyakov loop expectation value vanishes

AdS phase is dual to confinement.

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- For AdS-BH phase: no linear law confinement
  : non-zero Polyakov loop expectation value.

  AdS-BH phase is dual to deconfinement.

Entropy of the $Q\bar{Q}$ pair

The entropy can be calculated from the quark-antiquark free energy $\mathcal{F}$ via the relation,

$$S = -\frac{\partial \mathcal{F}}{\partial T}$$

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However, for AdS black hole background we have two choices for S corresponding to two different behaviours of $\mathcal{F}$ with respect to quark-antiquark separation length. For large separation, we have

$$S_{decon}(\ell > \ell_{crit}) = -\frac{\partial \mathcal{F}_{discon}}{\partial T}$$

On the other hand for small separation, we have

$$S_{decon}(\ell < \ell_{crit}) = -\frac{\partial \mathcal{F}_{con}}{\partial T}$$

We find that these two distinct behaviours of $\mathcal{F}$ as a function of quark-antiquark separation qulitatively capture the QCD result in their respective regime.

Entropy in the deconfined phase

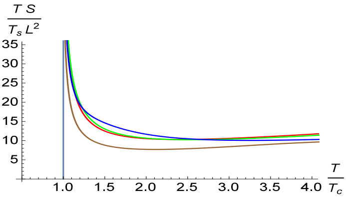

Figure: Entropy of the quark-antiquark pair as a function of temperature in the deconfined phase for various values of μ. Here red, green and blue curves correspond to $\mu = 0$, 0.2, 0.4 and 0.6 respectively.

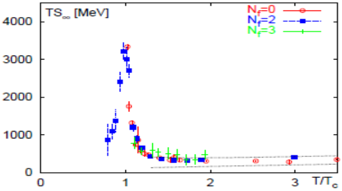

Figure: Lattice QCD result for the entropy of the quark-antiquark pair as function of temperature T/T_c for large quark-antiquark separation. The result is taken from Kaczmarek et al [PoS LAT2005 (2005) 192]

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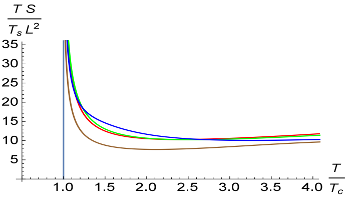

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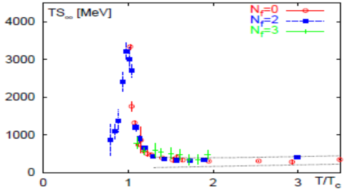

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- Drawback: Entropy in the confined phase is zero.

Entropy in the deconfined phase

- Another important lattice QCD result which our holographic model qualitatively reproduces is to predict increase in the entropy of quark-antiquark pair as a function of distance between them.

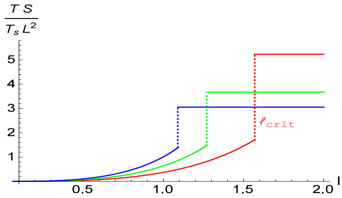

Figure: Entropy of the quark-antiquark pair as a function of distance in the deconfined phase for various temperatures. Here $\mu = 0$ and red, green and blue curves correspond to $T/T_{crit} = 1.1, 1.2$ and 1.3 respectively.

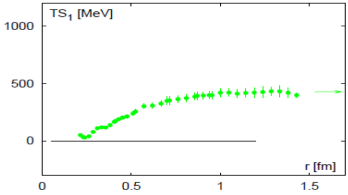

Figure: Lattice QCD result for the entropy of the quark-antiquark pair as function of quark-antiquark separation at temperature $T \simeq 1.3T_c$. The result is taken from Kaczmarek et al [PoS LAT2005 (2005) 192]

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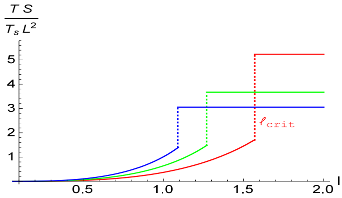

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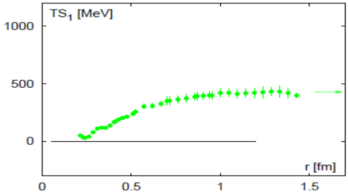

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- However, as opposed to lattice QCD, the entropy in our model does not smoothly go to saturation. There is a discontinuity in the entropy at ℓ_{crit}. This discontinuity in the entropy arises again due to first order transition between different string surfaces at ℓ_{crit}.
- Drawback: Entropy in the confined phase is zero.

With another form of scale factor

$$A(z) = -\frac{3}{4}\ln\left(az^2 + 1\right) + \frac{1}{2}\ln\left(bz^3 + 1\right) - \frac{3}{4}\ln\left(az^4 + 1\right)$$

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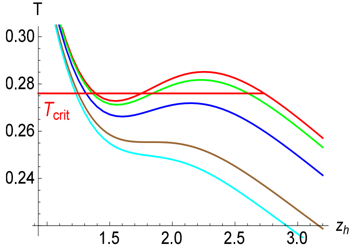

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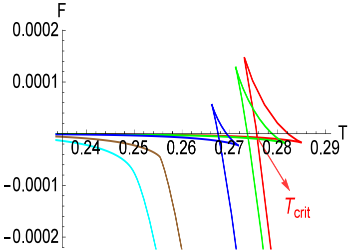

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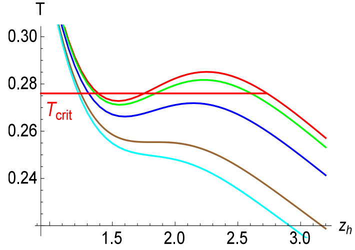

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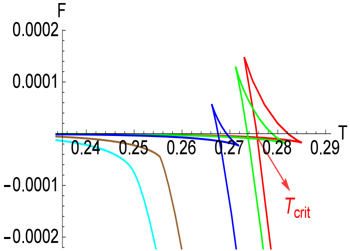

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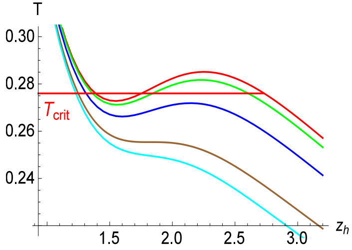

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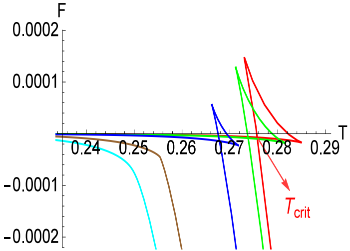

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- Large black hole phase is dual to deconfinement.
- The phase dual to small black hole is quite similar to confined phase !.

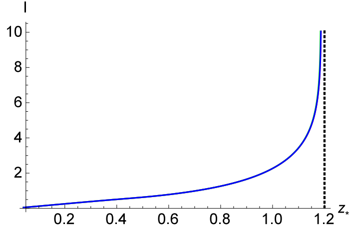


Figure: ℓ as a function of z_* in thermal-AdS background.

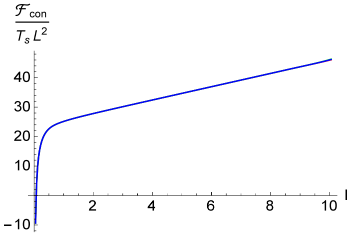


Figure: $\mathcal{F}_{con}$ as a function of ℓ in thermal-AdS background.

- The boundary phase dual to small black hole phase seems to show linear confinement at low temperatures. For too large ℓ, the linear area law is gone again, because of the dominance of the disconnected string configuration.

- Polyakov loop expectation value is extremely small, however it is strictly non-zero.

Entropy in the phase dual to small black hole

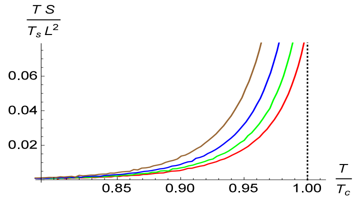
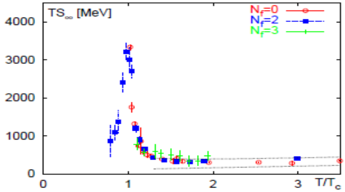


Figure: Entropy of the quark-antiquark pair as a function of temperature in the *specious-confined* phase for various values of μ. Here red, green, blue and brown curves correspond to $\mu = 0.10$, 0.15, 0.20 and 0.25 respectively.



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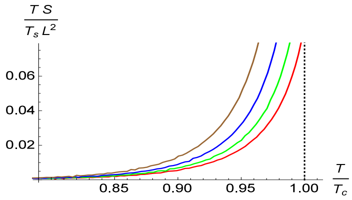

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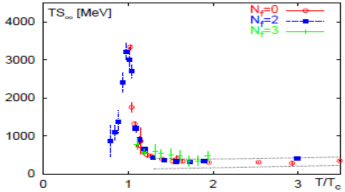

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- A large amount of entropy associated with the quark-antiquark pair near the critical temperature, as also observed in lattice QCD.
- The non-zero entropy in the *specious-confined* phase arises precisely due to the fact that the dual gravity background of *specious-confined* phase is a (small) black hole, which depends on temperature. In the usual AdS/CFT correspondence, the confined phase is generally dual to pure AdS (without horizon and temperature) and therefore the entropy of the quark pair is inherently zero in those confined phases. However, in our model, temperature dependence of the small black hole phase naturally leads to temperature dependence in the quark-antiquark entropy in the dual confined phase.

Conclusions

- We use the gauge/gravity to study the entropy of the quark-antiquark pair.
- We constructed a holographic model for confined and deconfined phases. In some cases, these phases can be described by black holes in the gravity side.
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- We are also planning to study the entanglement entropy in terms of growing quark-separation.

Thank You