# Phase structure of large-N gauge theory at finite temperature

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with R. Pisarski and V. Skokov

<in prepration>

# Outline

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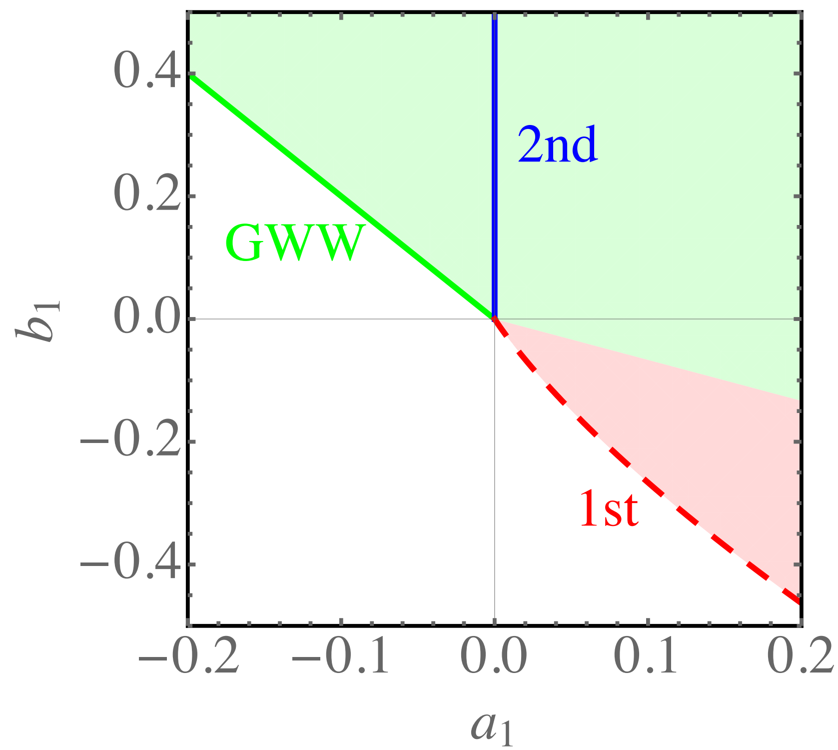


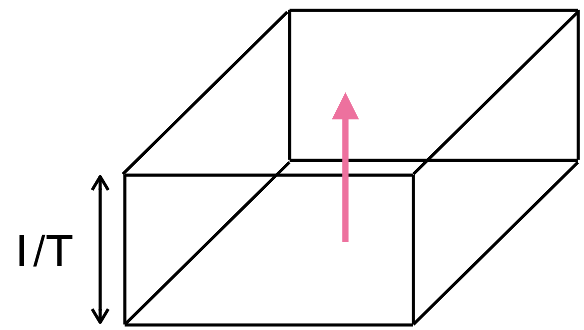
# Introduction

# Polyakov loop for SU(N)

- Order parameter

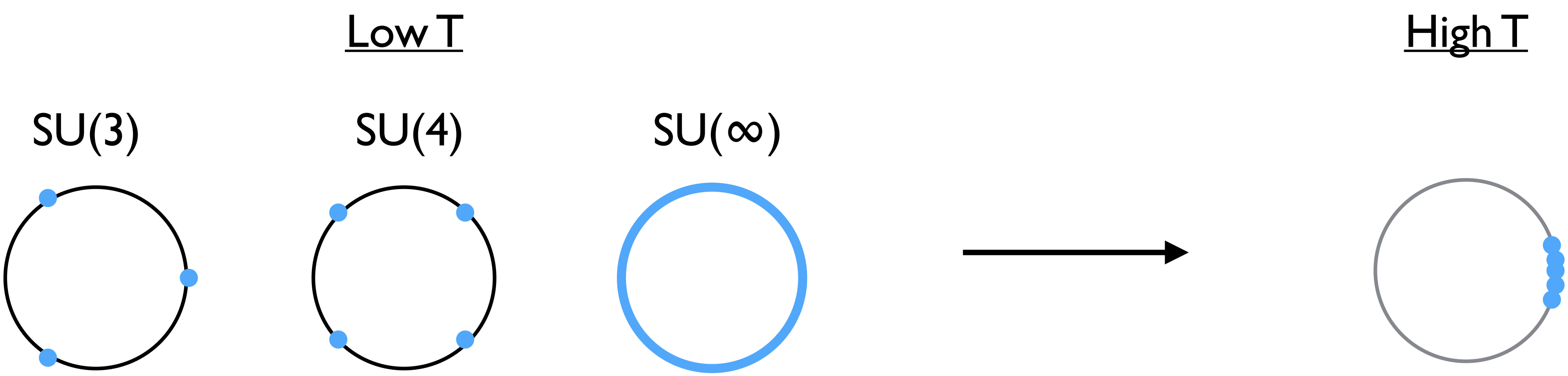
$$P(\vec{x}) = \mathcal{P} e^{i \int_0^{1/T} dx_4 A_4(x)}$$

Static quark, $P(\vec{x})$




Center symmetry Z(N): $P \rightarrow zP$

- Eigenvalue distribution $P = \mathrm{diag}\left\{ e^{i\theta_1}, e^{i\theta_2}, \cdots, e^{i\theta_N} \right\}$




$$\langle \mathrm{tr} P(\vec{x}) \rangle = 0$$

$$\langle \mathrm{tr} P(\vec{x}) \rangle \neq 0$$

- Another extreme: N=∞

$$\rho_n = \frac{1}{N}\mathrm{tr}_F P^n = \frac{1}{N}\sum_{i=1}^{N} e^{in\theta_i} \xrightarrow{N\to\infty} \int d\theta \rho(\theta) e^{in\theta}$$

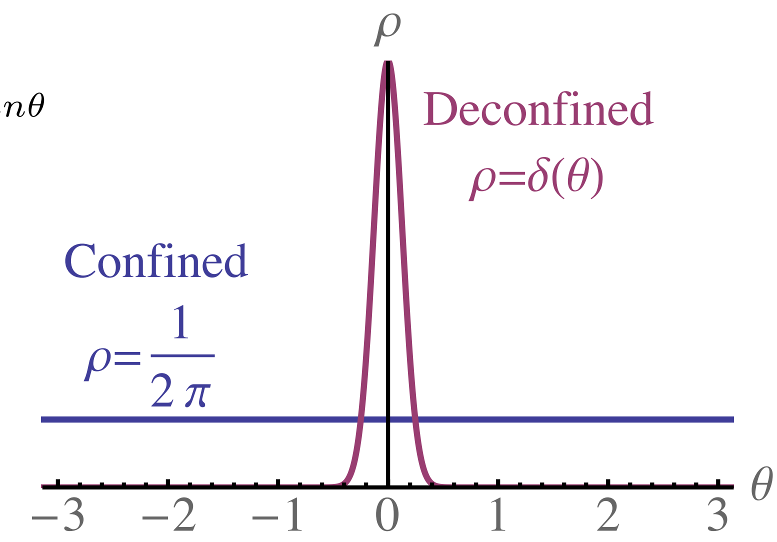
<E. Brezin, C. Itzykson, G. Parisi, and J. Zuber, 1978>

Two constraints:

1. Normalization: $\int_{-\pi}^{\pi} d\theta \rho(\theta) = 1$

2. Nonnegative: $\rho(\theta) \geq 0$

## Polyakov loop for SU(∞)




- Third-order phase transition

**- 2-D Wilson action**

<D. Gross and E. Witten, 1980>

<S. R. Wadia, 1980> etc..

**- Strong coupling**

<F. Green and F. Karsch, 1984>

<P.H. Damgaard and A. Paktos, 1986> etc..

**- $S^1$ X $S^3$**

<O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, 2004>

<S. Hands, T. Hollowood and J. Myers, 2010> etc..

**- Matrix Model**

<A. Dumitru, J. Lenaghan and R. Pisarski, 2005>

<R. Pisarski and V. Skokov, 2012> etc..

**- Lattice simulations**

<F. Bursa and M. Teper, 2006> etc..

See Review: <B. Lucini and M. Panero, 2013>

# Effective potential

# SU(N)

- Effective potential of the Polyakov loop

Constant background normalized Polyakov loop: $\rho_n = \frac{1}{N}\mathrm{tr}_F P^n$

$$\exp\left[-\beta\mathcal{V}N^2 V_{eff}(\rho_n)\right] = \int DA_\mu e^{-S_{YM}(A_\mu)} \prod_{m=1}^{N-1} \delta(\rho_m - \frac{1}{\mathcal{V}N}\int d\mathbf{x}\,\mathrm{tr}_F P^m)$$

<A. Dumitru, Y. Guo and C. P. Korthals Altes, 2014>

- Z(N) symmetric potential

Center symmetry Z(N): $\rho_n \to z^n \rho_n$

$$V_{eff} = \sum_n \tilde{a}_n \rho_n \rho_{-n} + \sum_{n,p} \tilde{a}_{n,p} \rho_n \rho_p \rho_{-n-p} + \ldots$$

## SU(∞)

- Perturbation theory up to 2-loops

$$V_{eff} = -T^4 (c_1 + c_2\lambda) \sum_{n=1}^{\infty} \frac{1}{n^4} |\rho_n|^2 + \mathcal{O}(3 \text{ loops})$$

<A. Dumitru, Y. Guo and C. P. Korthals Altes, 2014>

- Strong-coupling

  - Leading order: only the double trace terms.

    <L. Del Debbio and A. Patella, 2009>

  - Beyond leading order.

    <Work in progress>

<u>In this talk</u>

$$V_{eff} = \sum_{n=1}^{\infty} a_n |\rho_n|^2 + b_1 |\rho_1|^4 - h(\rho_1 + \rho_1^*)$$

# Phase Structure

# General structure

- Landau energy for each loop

$$V_{eff} = \sum_{n=1}^{\infty} V_n(\rho_n) \quad \text{where} \quad V_1 = a_1 |\rho_1|^2 + b_1 |\rho_1|^4 - h(\rho_1 + \rho_1^*)$$

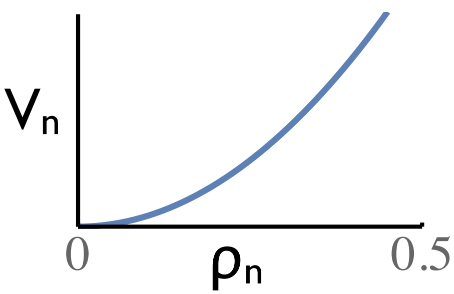
$$V_{n>1} = a_n |\rho_n|^2$$

- Assume $\rho_1$ drives the phase transition

<O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, 2004>

h=0, Positive $b_1$

| Confined (T<Tc) | Deconfined (T>Tc) | |
|---|---|---|
| $a_n>0$ | $a_1<0$ | GWW point |

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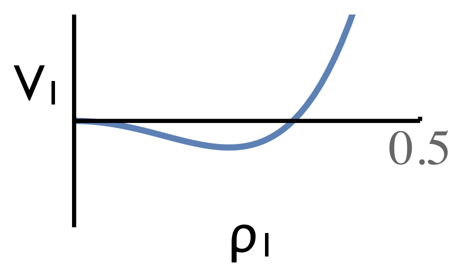
2nd-order P.T.

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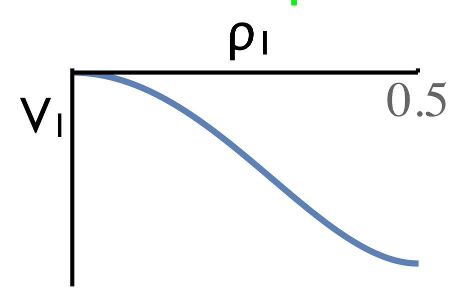



$$\rho_n^{min} = 0$$

$$0 < \rho_1^{min} < 1/2$$

$$\rho_1^{min} = 1/2$$

$$\longrightarrow \quad \rho = \frac{1}{2\pi}$$

$$\longrightarrow \quad \rho = \frac{1}{2\pi}\left(1 + 2\rho_1^{min}\cos\theta\right)$$

# General structure

- Landau energy for each loop

$$V_{eff} = \sum_{n=1}^{\infty} V_n(\rho_n) \qquad \text{where} \qquad V_1 = a_1 |\rho_1|^2 + b_1 |\rho_1|^4 - h(\rho_1 + \rho_1^*)$$

$$V_{n>1} = a_n |\rho_n|^2$$

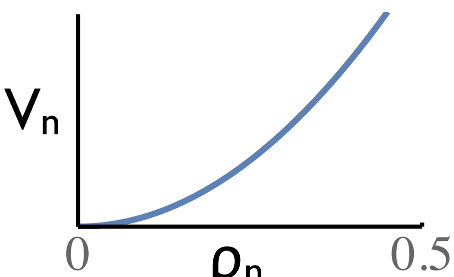
- Assume $\rho_1$ drives the phase transition

<O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, 2004>

h=0, Negative $b_1$

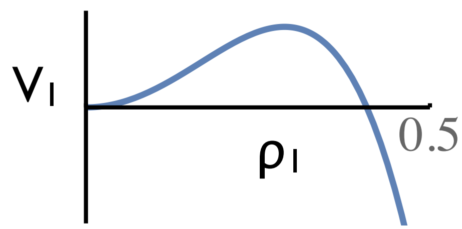
| Confined (T<Tc) | Deconfined (T>Tc) |
| --- | --- |

$a_n>0$




→ 1st-order P.T.

$a_1>0$




$$\rho_n^{min} = 0$$

$$\longrightarrow \quad \rho = \frac{1}{2\pi}$$

$$\rho_1^{min} > 1/2$$

For $\rho_1 > 1/2$,
need the Legendre transform.

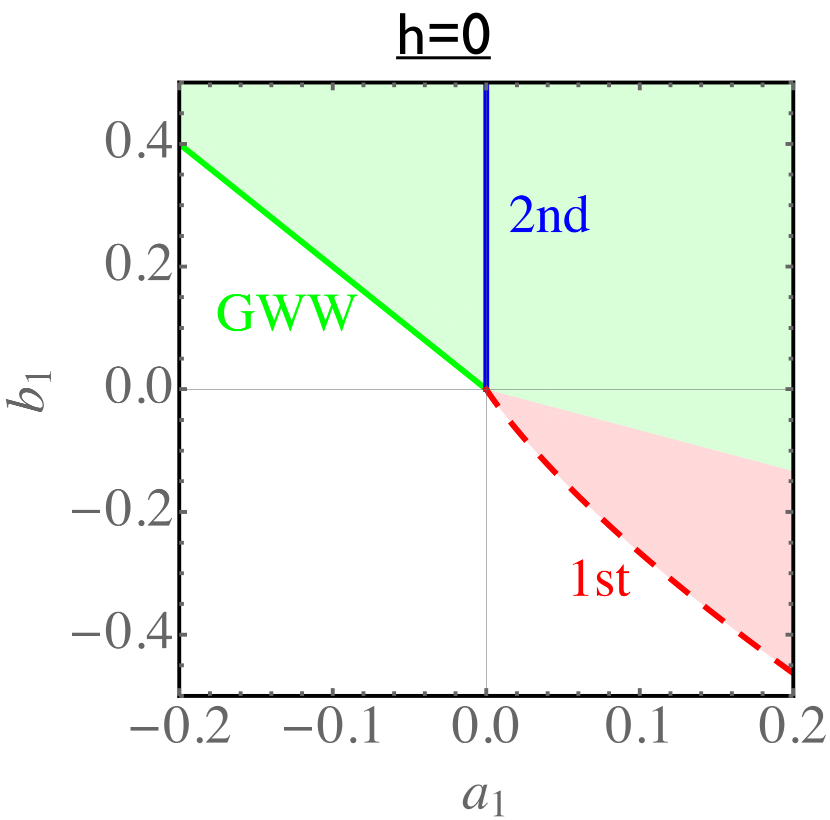
<A. Dumitru, J. Lenaghan and R. Pisarski, 2005>

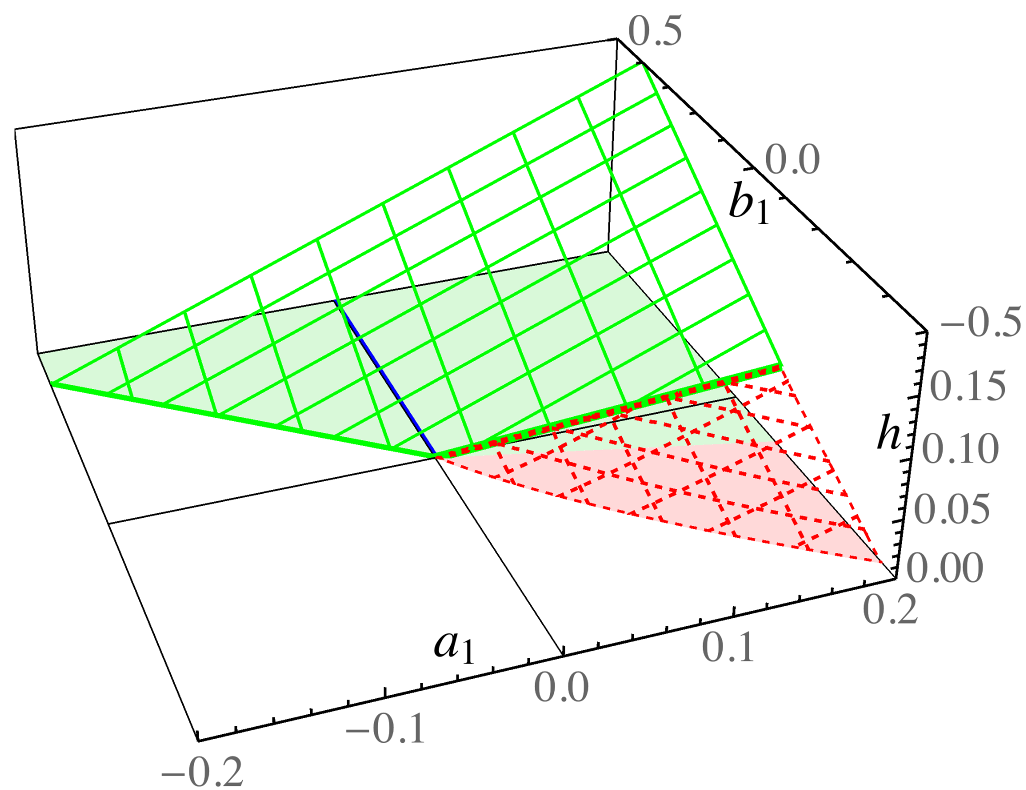
# General structure

- With the external field, h

$$V_1 = \frac{1}{16}(4a_1 + b_1 - 16h) + \left(a_1 + \frac{b_1}{2} - 2h\right)\delta\rho_1 + \left(a_1 + \frac{3b_1}{2}\right)\delta\rho_1^2 + 2b_1\delta\rho_1^3 + b_1\delta\rho_1^4$$

where $\rho_1 = \frac{1}{2} + \delta\rho_1$ with $\delta\rho_1 < 0$




- Higher-order phase transition at GWW

# Models

$$V_{eff} = V_{pert} + V_{nonpertur}$$

✦ One-loop gluonic effective potential, $V_{pert}$

$$V_{pert} = -\sum_{n=1}^{\infty} c_d \frac{T^d}{n^d} |\rho_n|^2$$

Z(N) maximally broken

✦ Nonperturbative confining potential, $V_{nonpert}$

$$V_{nonpert} = \sum_{n=1}^{\infty} c(T) \frac{1}{n^p} |\rho_n|^2$$

- p=1 and 2 are the Haar measure and mass deformation types, respectively.

<P. Meisinger, T. Miller, and M. Ogilvie, 2002>

- We consider N=∞ and p=1,2,3,4.

# Models

- Pert. + Nonpert.

$$a_n = c(T)\left(\frac{1}{n^p} - \frac{T}{T_c}\frac{1}{n^d}\right)$$

with p<d

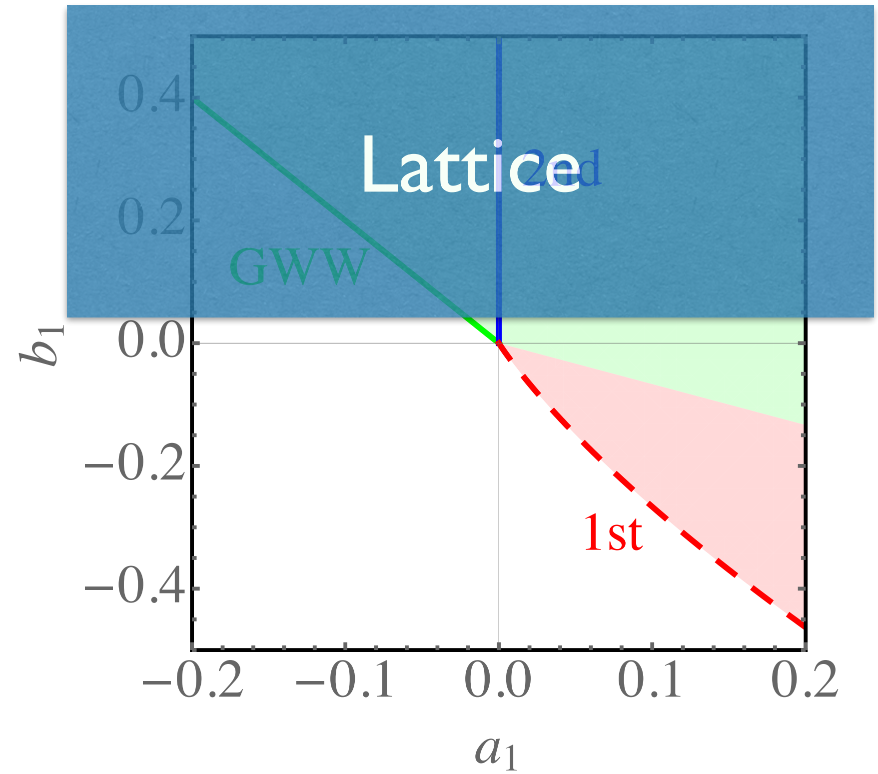
- Adding a quartic term

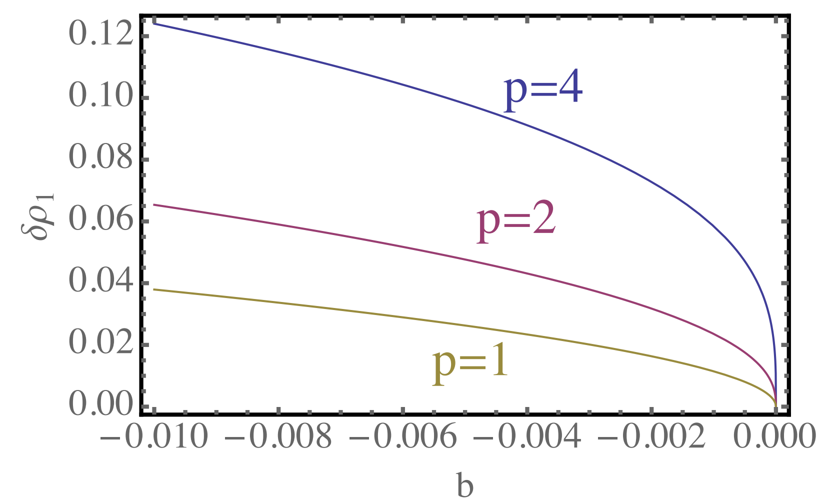
$$V_{eff} = \sum_{n=1}^{\infty} a_n \left|\rho_n\right|^2 + b_1 \left|\rho_1\right|^4$$

- Comes from three-loop?

- $b_1 \leq 0$ from lattice simulations at large N.

$|b_1|$ should be small.



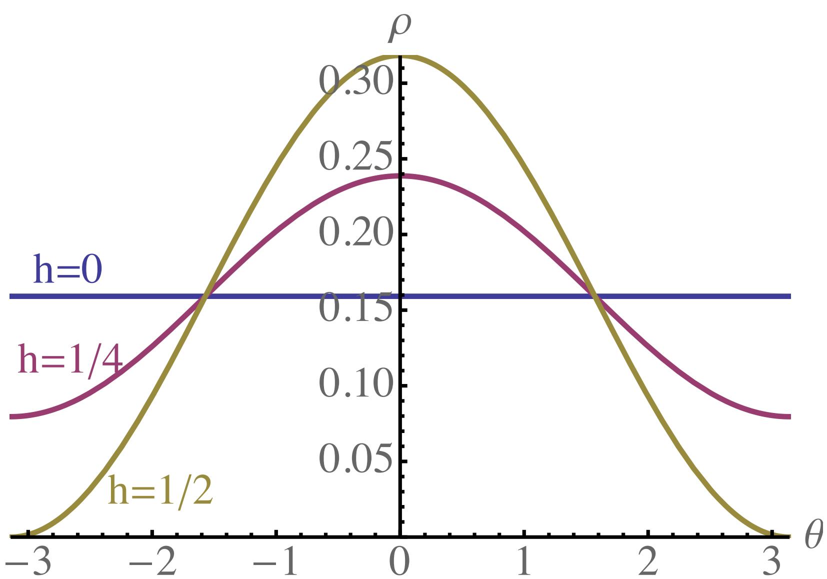
# Models

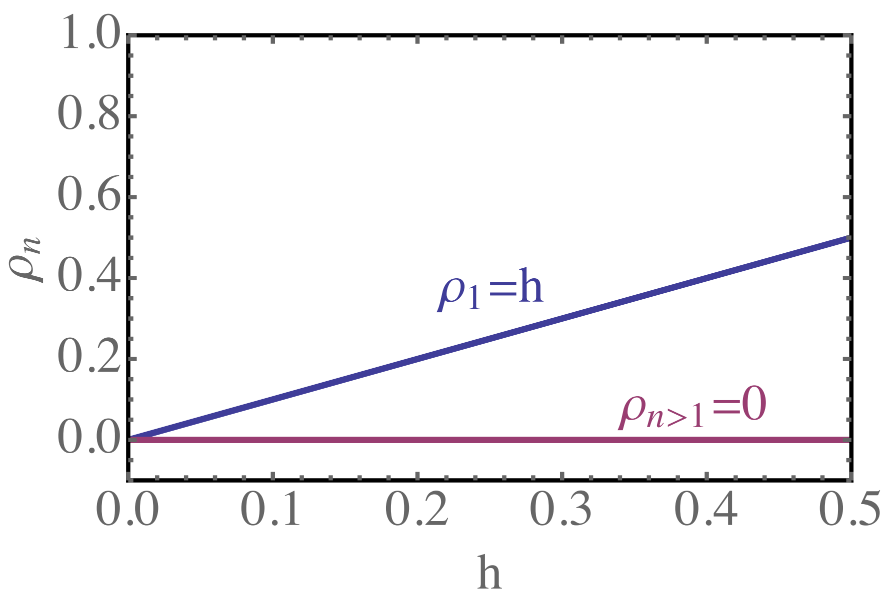
$$V_{eff} = \sum_{n=1}^{\infty} \frac{1}{n^p} |\rho_n|^2 - 2h\rho_1 \quad \xrightarrow{\text{EoM}} \quad h \sin\theta = \sum_{n=1}^{\infty} \frac{\rho_n}{n^{p-1}} \sin(n\theta)$$

where $\rho_n = \int d\theta \rho(\theta) \cos(n\theta)$

- Eigenvalue distributions

Below GWW (h<1/2) $\rho(\theta) = \frac{1}{2\pi}(1 + 2h\cos\theta)$ for any p



# Models

- Eigenvalue distributions Above GWW (h>1/2)

**p=1:**

$$\rho = \frac{2h}{\pi} \cos\frac{\theta}{2} \sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}$$

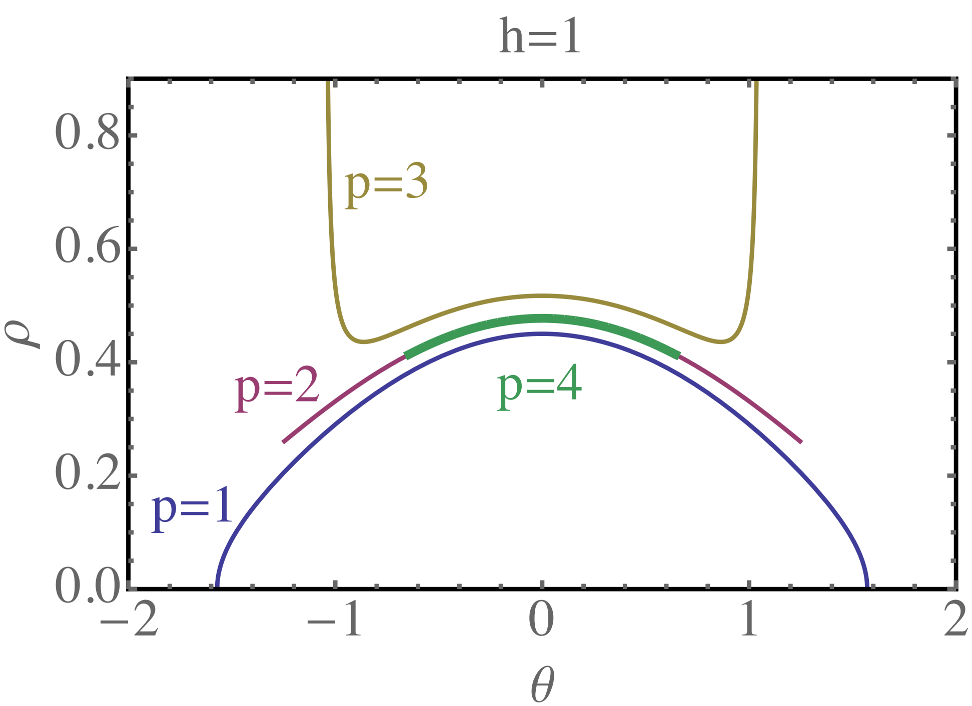
**p=3:**

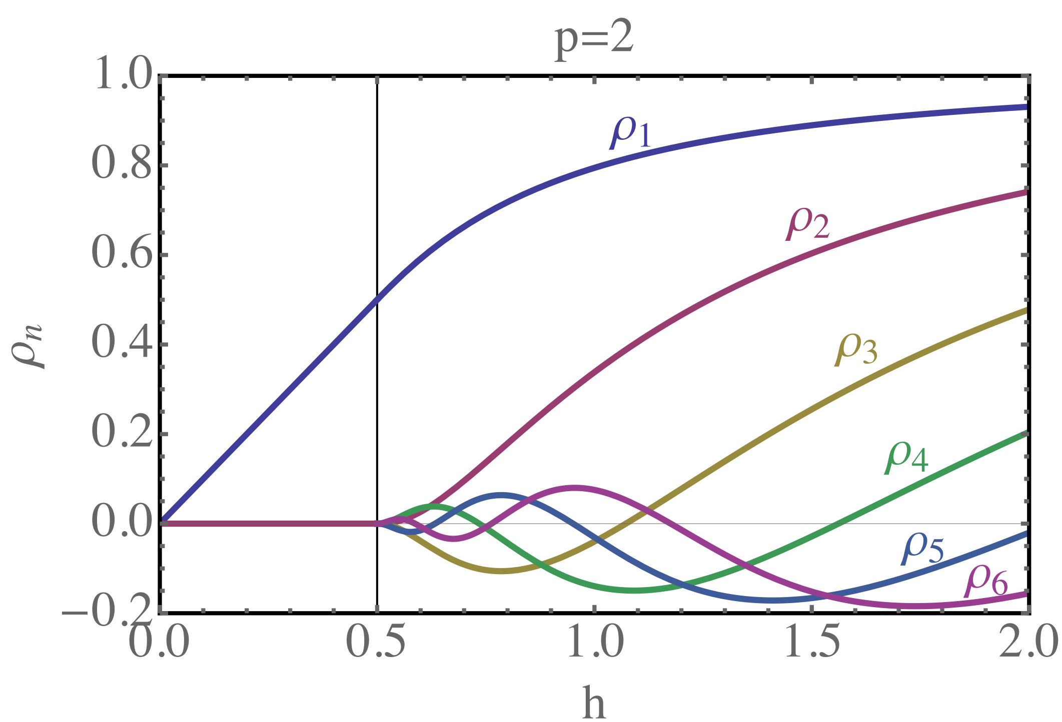
$$\rho = \frac{\cos\frac{\theta}{2}\left\{1 + 2h\left(-1 + \cos\theta + \sin^2\frac{\theta_0}{2}\right)\right\}}{\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}}$$

**p=2,4:**

$$\rho = \frac{1}{2\pi}\left(1 + 2h\cos\theta\right)$$

where $-\theta_0 < \theta < \theta_0$

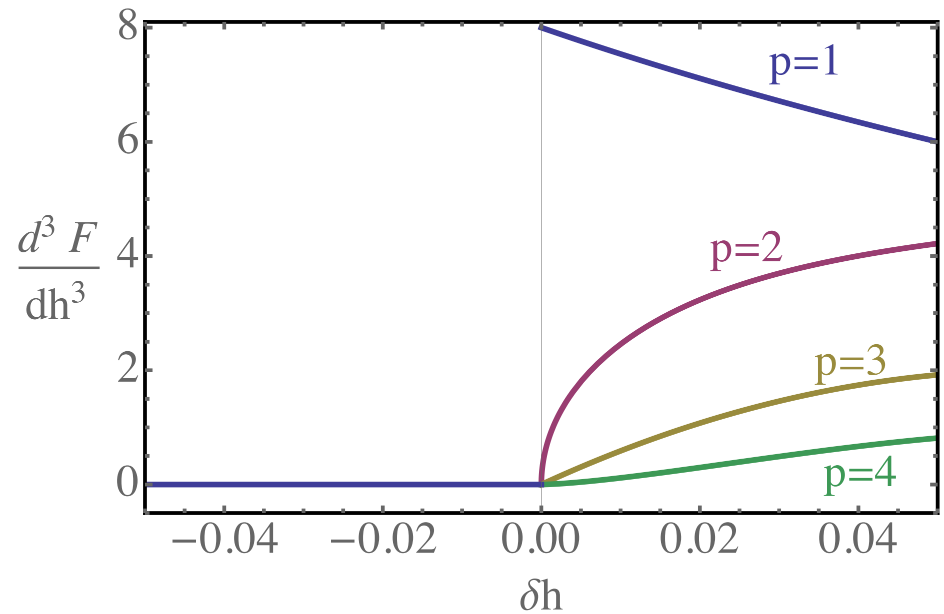


# Models

$$V_{eff} = \sum_{n=1}^{\infty} a_n |\rho_n|^2 - 2h\rho_1$$

- Free energy $\delta h = h - 1/2$




Below GWW

$$F = -\frac{1}{4} - \delta h - \delta h^2$$

Just above GWW

$$F = -\frac{1}{4} - \delta h - \delta h^2 + c_p \delta h^{(5+p)/2} + \cdots$$ for p=1,2,3,4

Higher-order phase transition in the Ehrenfest classification (3rd, 4th, 5th).

# Conclusions

- General structure of the phase diagram using the simple Landau theory in SU(∞) is discussed.

- Depending on the type of confining potential, we have a higher-order phase transition at the GWW point. This can be tested in lattice simulations at large N.