Determination of confinement-deconfinement transition from Roberge-Weiss periodicity

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Takahiro M. Doi (RIKEN)

- **K.K.,** and A. Ohnishi, Phys. Lett. B750 (2015) 282
- **K.K.,** and A. Ohnishi, Phys. Rev D. 93 (2016) 116002
- **K.K.**, and A. Ohnishi, arXiv:1701.04953
- T. M. Doi, **K.K.**, arXiv:1706.00614

Introduction : QCD phase diagram

Purpose of this study

To determine

the confinement-deconfinement transition

in the system with dynamical quarks

Heavy quark-mass limit

Polyakov-loop describes the confinement-deconfinement transition

➡ The $\mathbb{Z}_{N_c}$ symmetry relates with the deconfinement transition via the free-energy

We can well determine the deconfinement temperature

Finite quark-mass case

Polyakov-loop is **no longer** the order-parameter

Important point

Finite quark-mass case :

Polyakov-loop is **no longer** the order-parameter

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Finite quark-mass case :

Polyakov-loop is **no longer** the order-parameter

Ordinary phase transition ⟺ Spontaneous symmetry breaking

Phase transition described by the **topological order**

X. G. Wen, Int. J. Mod. Phys. B4 (1990) 239.

⟺ Ground-state degeneracy

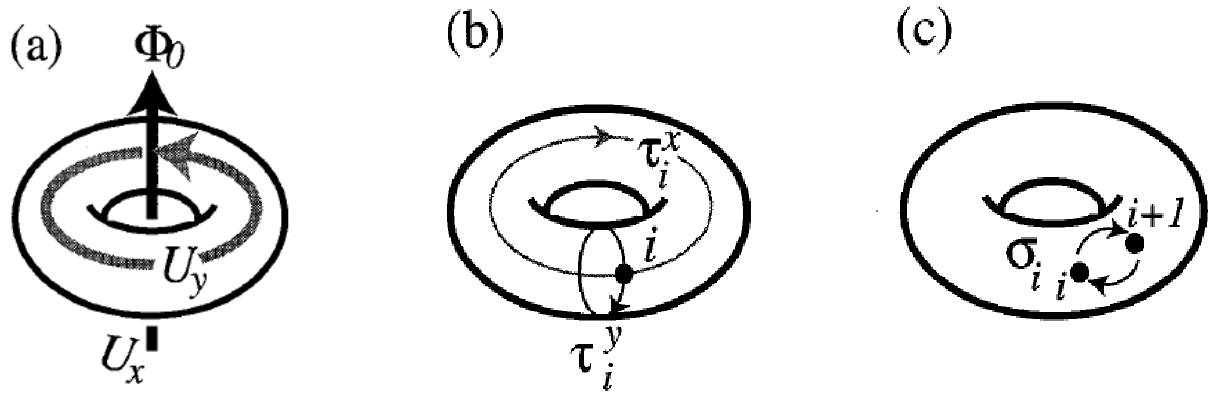
Question

How to see the topological order at T = 0 ?

Tree adiabatic operations

M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601.

M. Sato, PRD 77 (2008) 045013.



Introduction : Topological order in QCD

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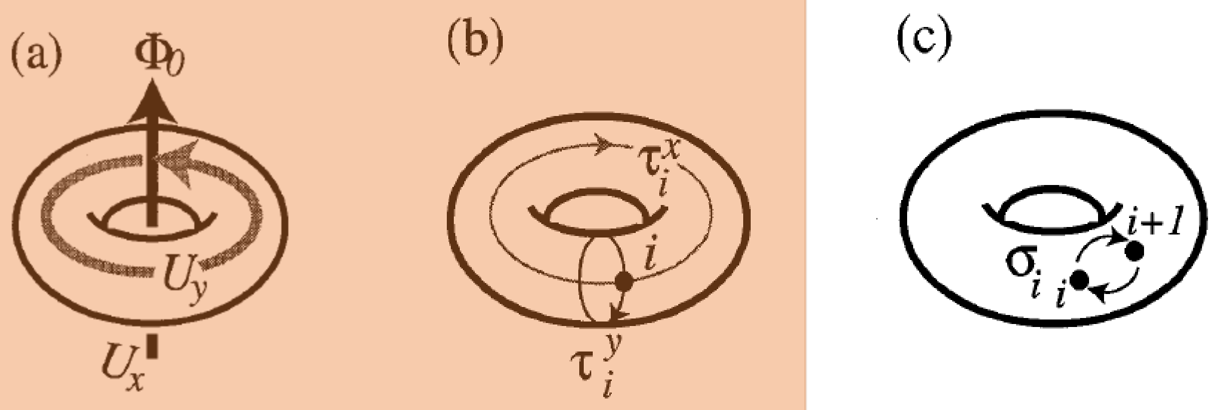

Aharonov-Bohm effect

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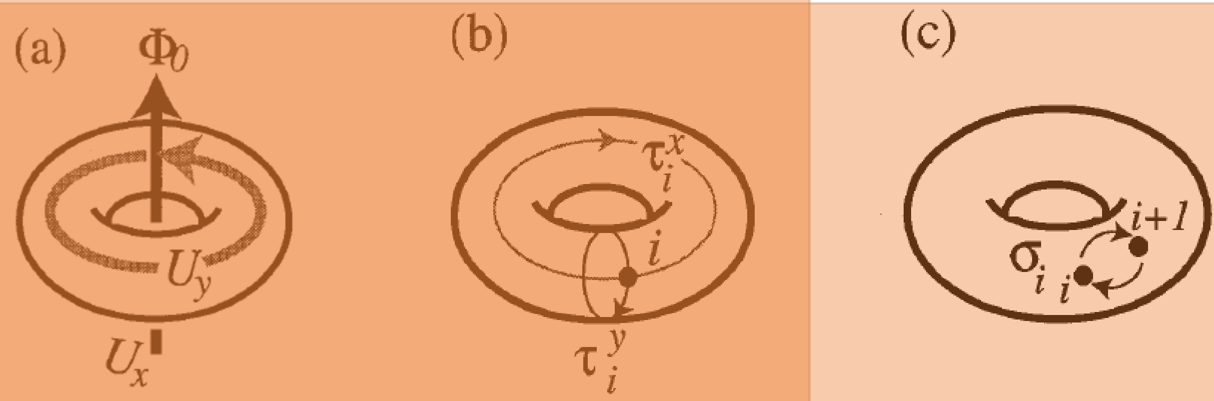

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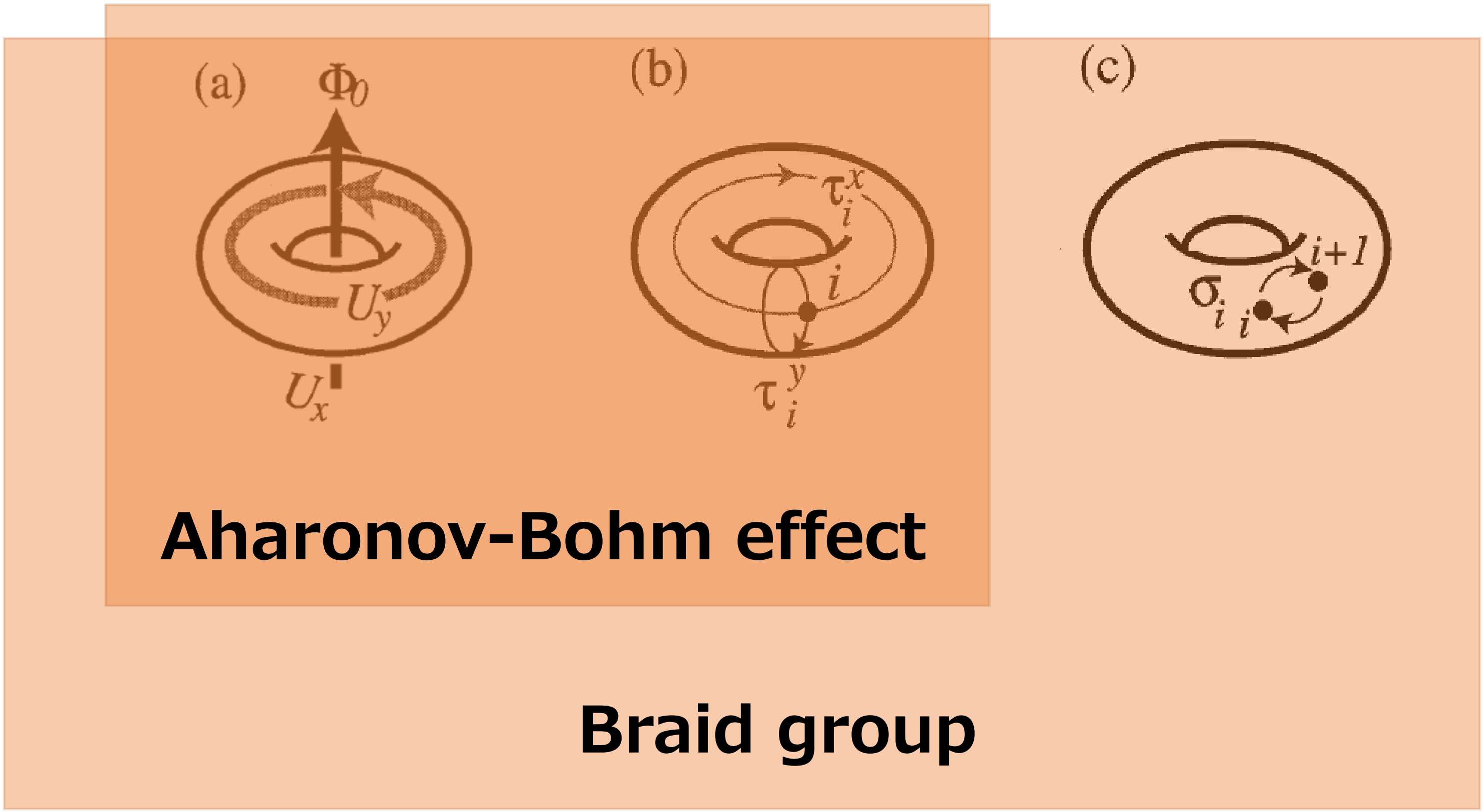

Without fractional charge : Commutable

With fractional charge : Non-commutable → Ground-state degeneracy

Tree adiabatic operations

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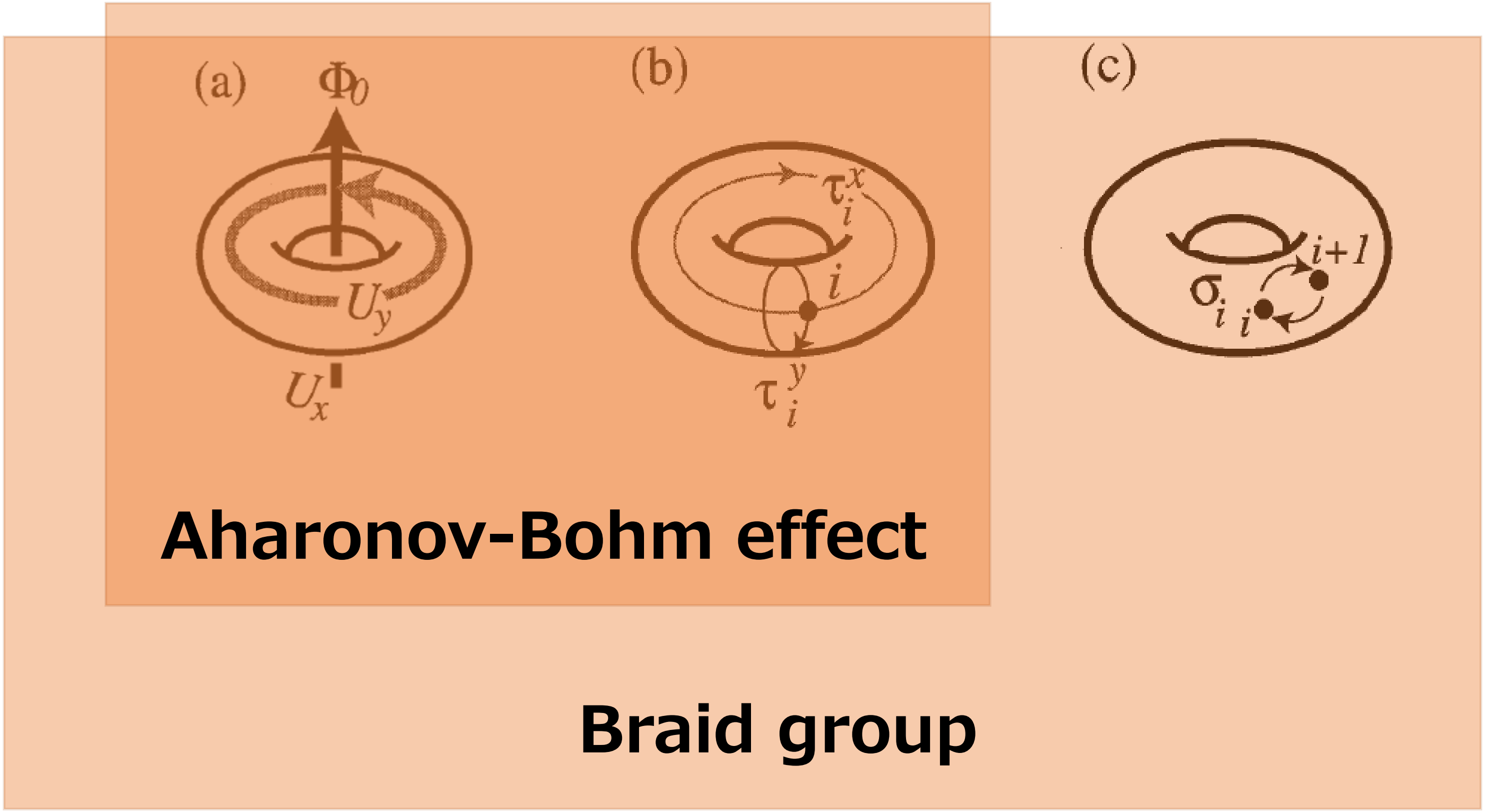


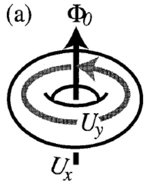
Confined state: Commutable

Deconfined state : Non-commutable → Ground-state degeneracy

We wish to **extend** it to **finite temperature QCD**

However, direct extension of the ground-state degeneracy is difficult⋯

We consider the **imaginary chemical potential** as



an external parameter to detect the deconfinement transition

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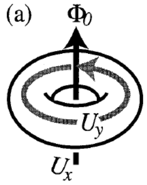


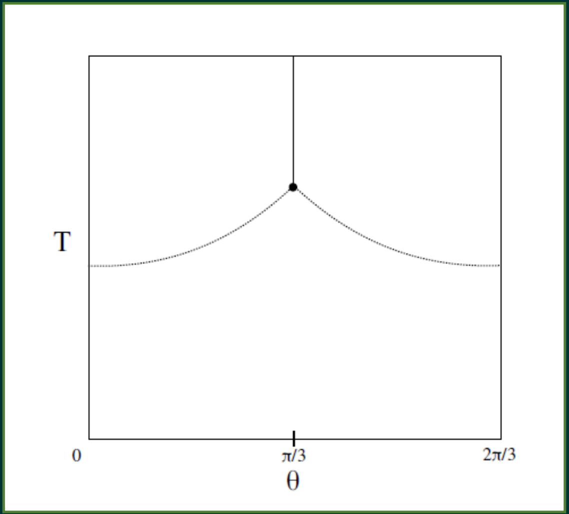
We consider the **imaginary chemical potential** as

an external parameter to detect the deconfinement transition

- There is no sign problem
- This region has all information of the region with finite μ_R
- There are topological differences between the low and high T regions
- We can consider similar special operations (Today, I do not explain this point)

Important point

A. Roberge and N. Weiss,
Nucl. Phys. B275 (1986) 734



Roberge-Weiss periodicity

Special π/N_c periodicity along θ-direction

It appears at low and high T

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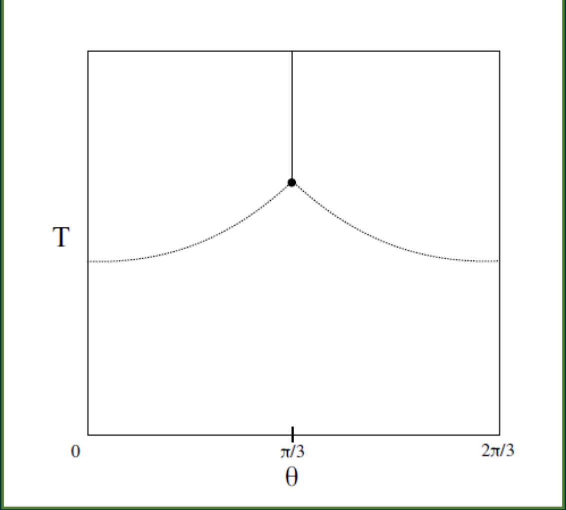
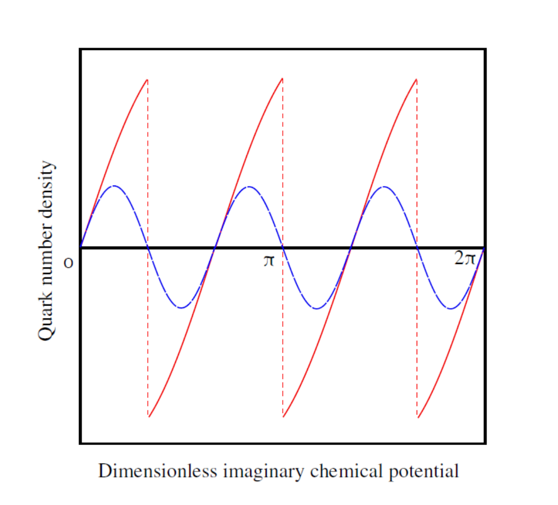


Roberge-Weiss transition

First-order transition along T-direction

It is characterized by the gap
of the quark number density

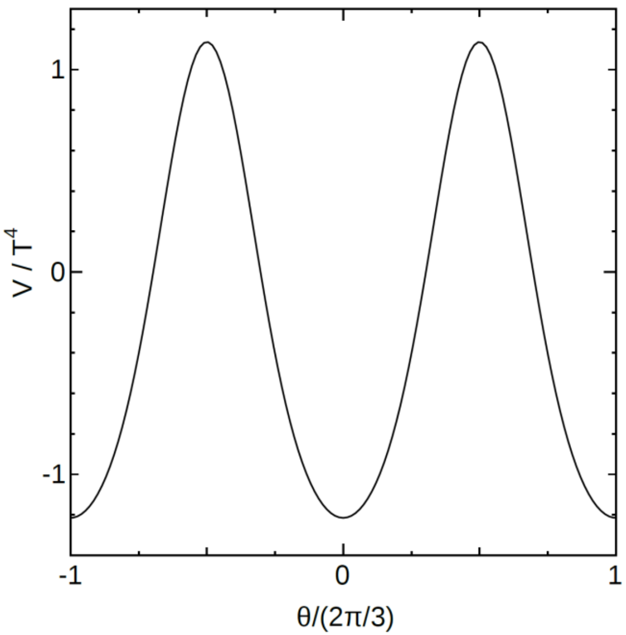


Question

How to use these properties to determine
the deconfinement transition ?

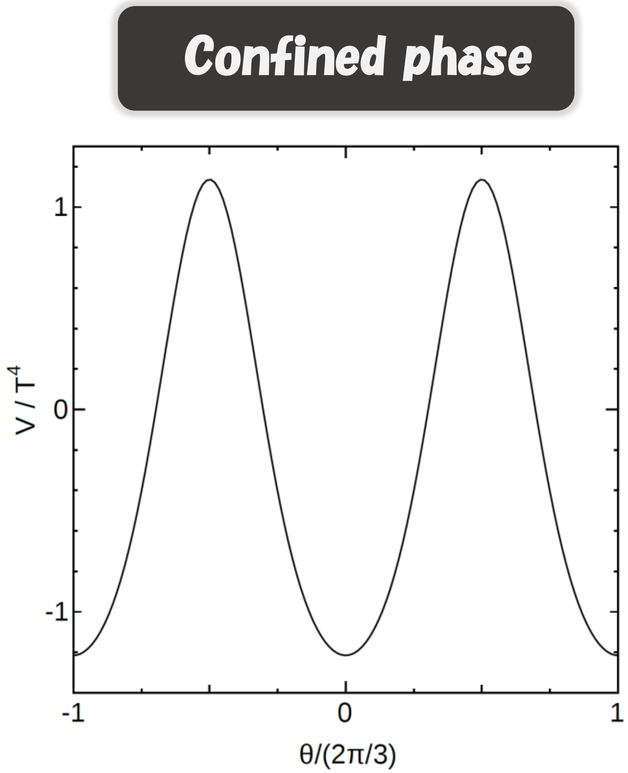
Result 1 : Free-energy degeneracy

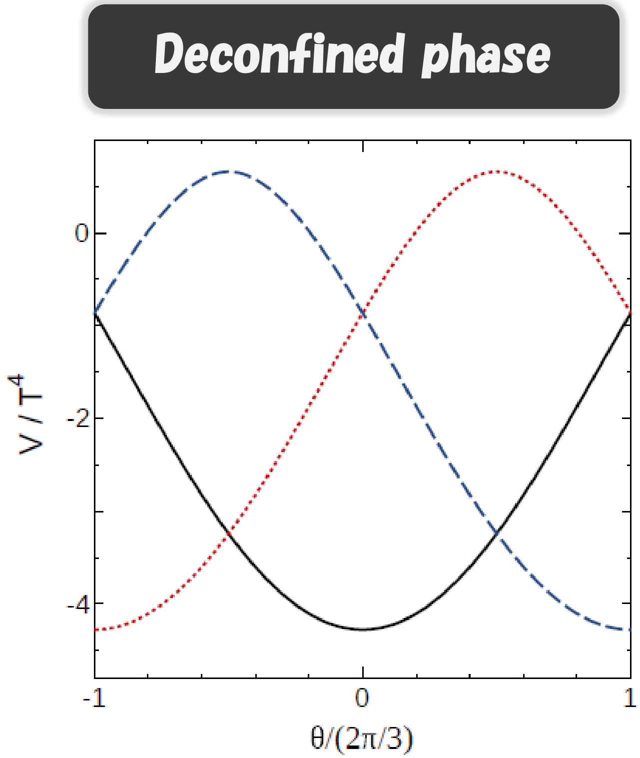
Confined phase



Confined phase : There is no non-trivial degeneracy

Result 1 : Free-energy degeneracy



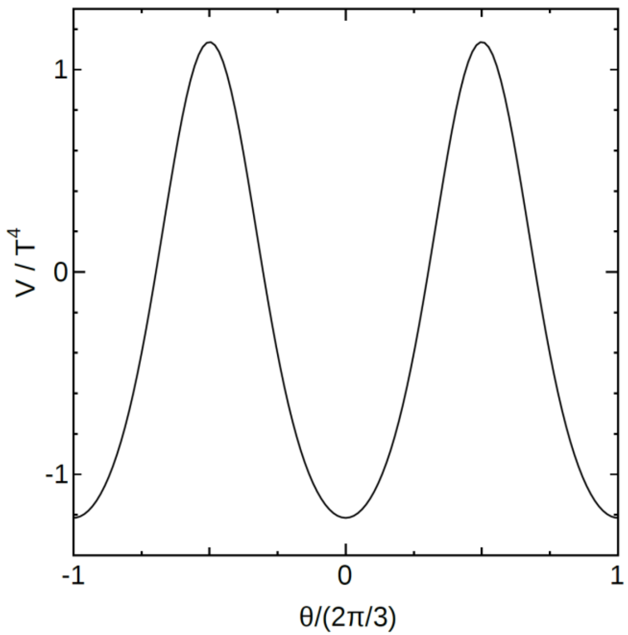


Confined phase : There is no non-trivial degeneracy

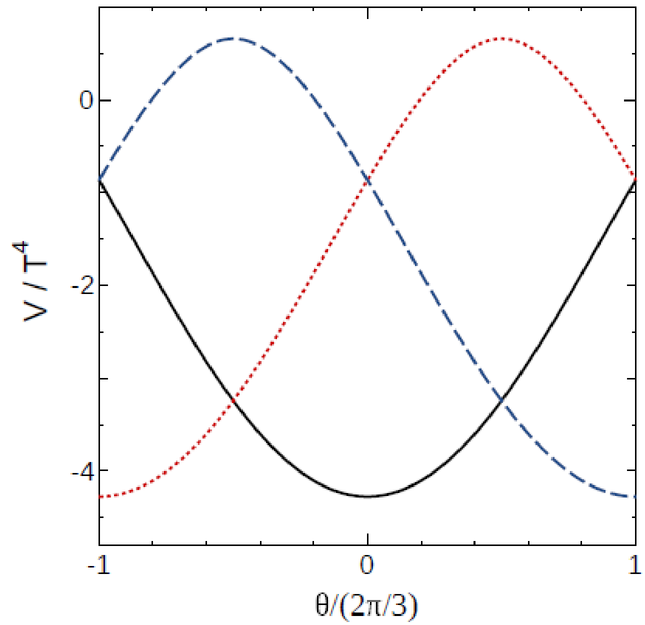
Deconfined phase : There is the non-trivial degeneracy

Qualitative differences are already known in A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734

Confined phase



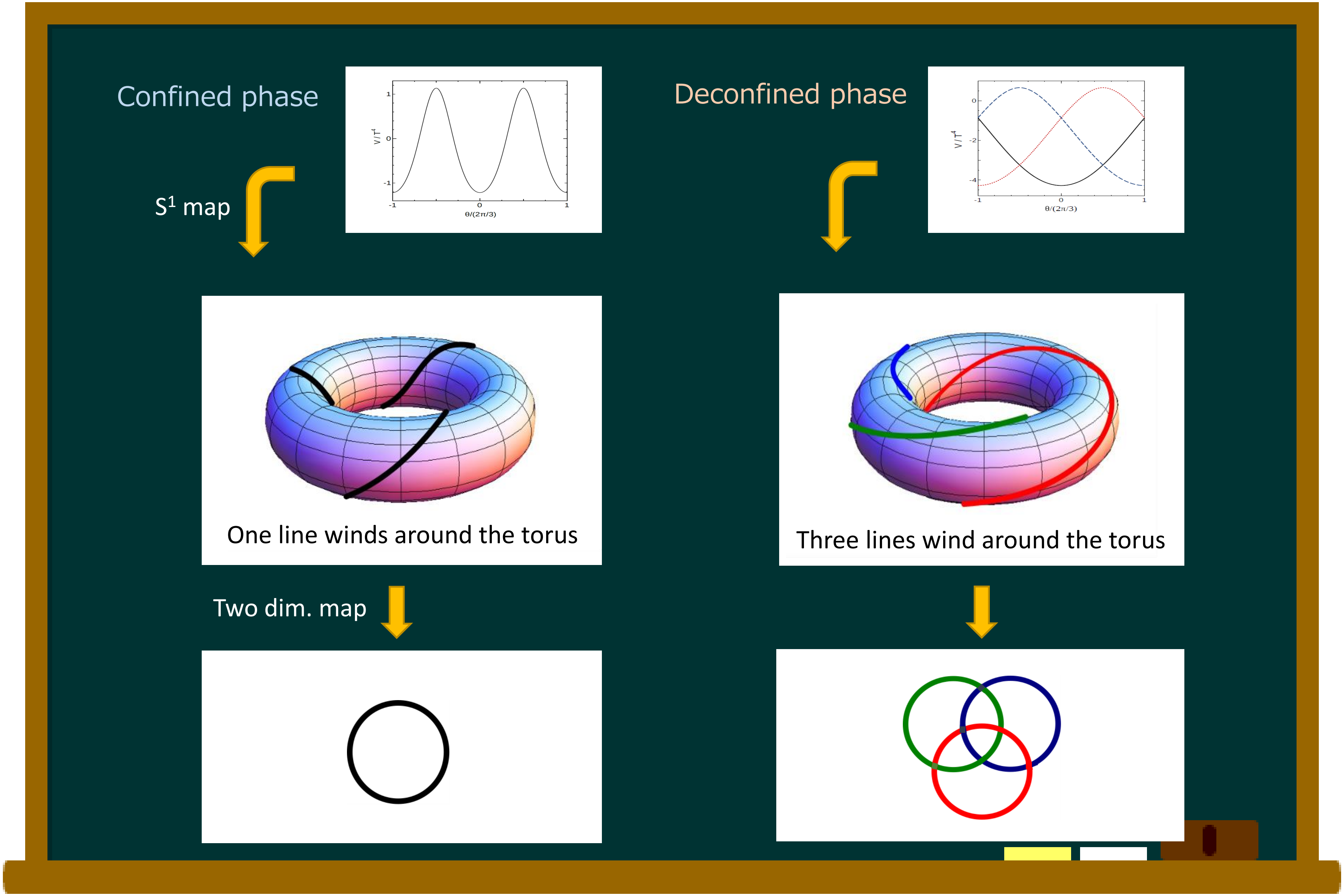
Deconfined phase



This difference relates to appearance of quark-gluon dynamics

Dominant degree of freedom : Hadrons → Quarks

Result 1 : Free-energy degeneracy



K.K., A. Ohnishi, Phys. Rev D. 93 (2016) 116002

Based on the topological difference,
we can construct the **quantum order-parameter**

Quark number holonomy

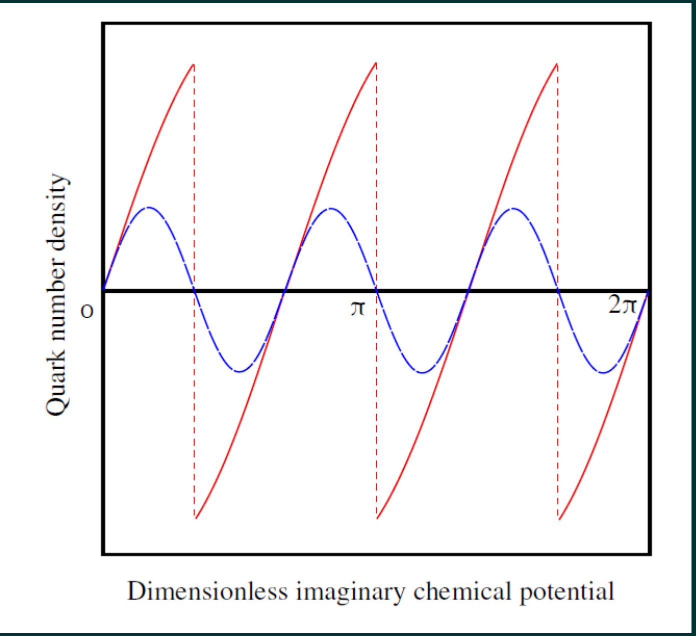
$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im} \left(\frac{d\tilde{n}_q}{d\theta} \bigg|_T \right) \right\} d\theta \right]$$

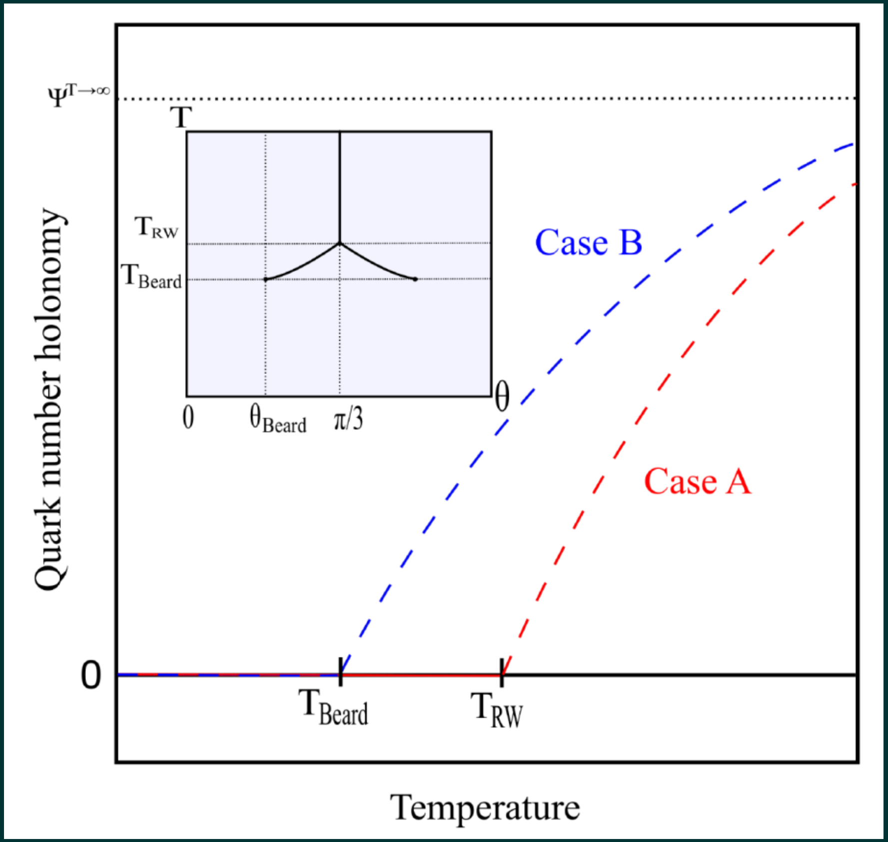
$$\tilde{n}_q \equiv C n_q$$

This quantity can count the number of gap in
the quark number density along θ-axis

Result 2 : Quantum order-parameter

Important point





We can well determine
the deconfinement transition

Lattice QCD data predict the
first-order RW endpoint

M. D' Elia and F. Sanfilippo, Phys. Rev. D80, 111501 (2009).
C. Bonati, G. Cossu, M. D' Elia, and F. Sanfilippo, Phys. Rev. D83 (2011) 054505.

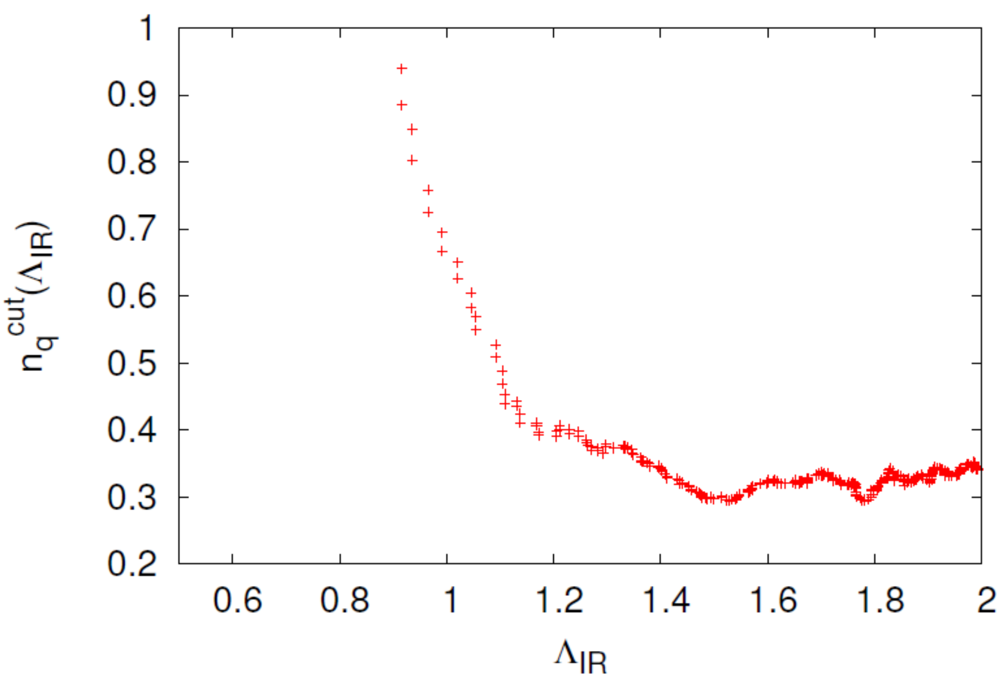
Result 2 : Quantum order-parameter

T. M. Doi, **K.K.**, arXiv:1706.00614

We have checked properties of the quark number holonomy via the quark number density

$$\langle n_q \rangle = \frac{1}{2V} \left\langle \mathrm{Tr}_{\gamma,c} \left[\frac{\partial D}{\partial \mu} \frac{1}{D+m} - \left(\frac{\partial D}{\partial \mu} \frac{\gamma_4}{D+m} \right)^{\dagger} \right] \right\rangle \simeq \frac{i}{V} \mathrm{Im} \left\langle \sum_n \left\langle n \left| \frac{\partial D}{\partial \mu} \right| n \right\rangle \frac{1}{\Lambda_n + m} \right\rangle$$

The **Dirac modes** may provide the important knowledge for the topologically determined confinement-deconfinement



The **absolute value depends on low-lying Dirac modes,** but the **sign does not**

It indicates that quark number holonomy **do not** have **dominant Dirac modes**

←Quenched approximation, θ ~ π/3

K.K., A. Ohnishi, arXiv:1701.04953

What happen at finite **real chemical potential**?

In our determination, we should consider

the **complex chemical potential**

and thus it is very difficult to discuss…

So, we consider the isospin chemical potential

- **Sign problem free**

$$\tau_2 \gamma_5 D \gamma_5 \tau_2 = D^\dagger \Rightarrow \det(D) \geq 0$$

- **Orbifold equivalence**

Outside of the pion condensed region,

the phase diagram at finite real μ and
the phase diagram at finite isospin μ

are identical to each other in the large N_c limit

In this study,
we employ **Polyakov-loop extended Nambu—Jona-Lasinio** model

PNJL Lagrangian density

$$\mathcal{L} = \bar{q}(\not{D} + m_0) - G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + \mathcal{V}_{\mathrm{g}}(\Phi, \bar{\Phi})$$

(Good point)

It can reproduce the RW periodicity and transition

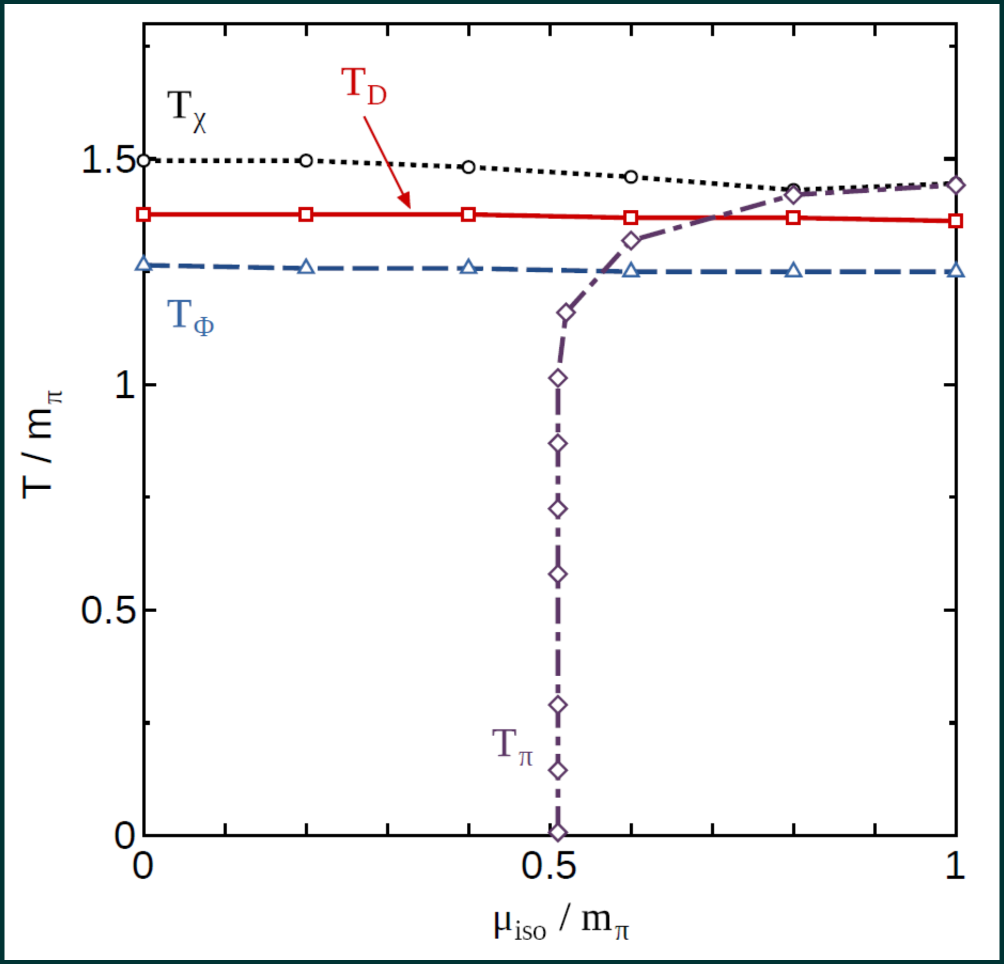
(Bad point)

Unfortunately,
this model has the model sign problem at finite real chemical potential

→ We need to extend our study Y. Tanizaki, H. Nishimura, **K.K.**, PRD 91 (2015) 101701

Result 3 : Isospin chemical potential

Phase diagram from PNJL model



How to investigate the deconfinement transition at finite real chemical potential?

We need better ways to handle the **sign problem**

Result 3 : Isospin chemical potential

How to investigate the deconfinement transition at finite real chemical potential?

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- Lefschetz-thimble path-integral method
- Complex Langevin method

How to investigate the deconfinement transition at finite real chemical potential?

We need better ways to handle the **sign problem**

- Lefschetz-thimble path-integral method
- Complex Langevin method
- **Path optimization method**

Yuto Mori, K.K., Akira Ohnishi, arXiv:1705.05605

We can use the deep neural network

Unfortunately, there is no talk about it here ･･･

We investigate the deconfinement transition from **topological viewpoints**

1. To discuss the deconfinement transition at finite temperature, we use the **nontrivial free-energy degeneracy**

2. We determine the **new order-parameter** of deconfinement transition

$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im}\left(\frac{d\tilde{n}_q}{d\theta}\Big|_T \right) \right\} d\theta \right]$$

3. The **density-dependence** of the deconfinement transition is shown by introducing the isospin chemical potential to the PNJL model

Result 2 : Estimation of deconfinement transition temperature

Important point

- 2+1 flavor lattice QCD — C. Bonati, M D'Elia, M. Mariti, M. Mesiti and F. Negro, Phys. Rev. D 93 (2016) 074504

T_{RW} = **208(5)** [MeV]

Quark number holonomy becomes nonzero above this temperature

- Recent 2+1 flavor effective model — A. Miyahara, Y. Torigoe, H. Kouno, M. Yahiro, Phys. Rev. D 94 (2016) 016003

T_d = **215** [MeV]

Using quantities to determine the deconfinement transition are different, but there is good agreement. (accidental?)