

# MANYBODY

## Theory of nuclear quantum many-body systems

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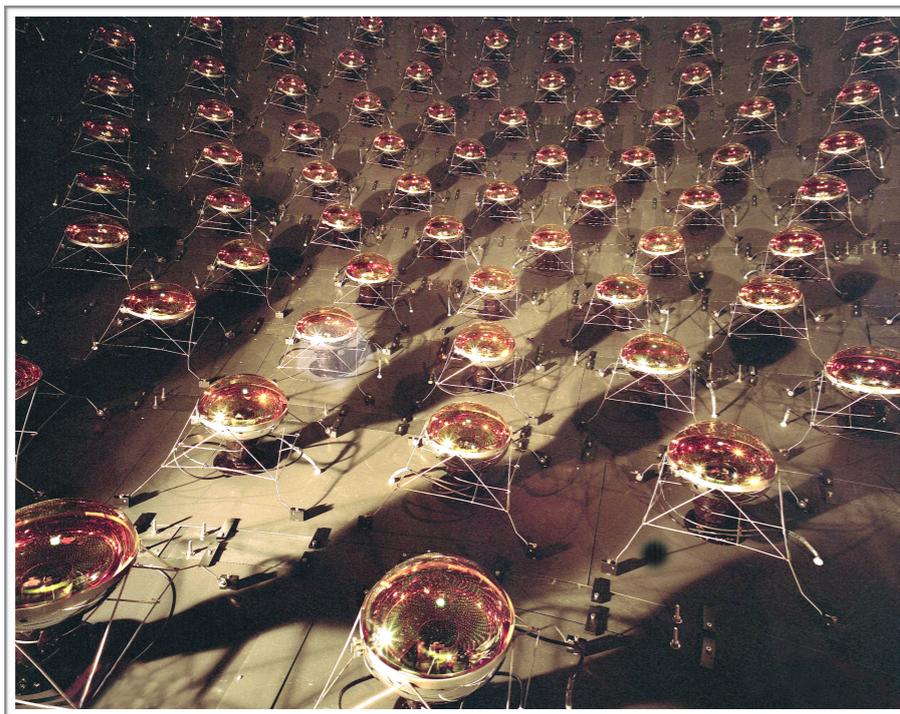
SM&FT 2017

Alessandro Lovato



# Introduction

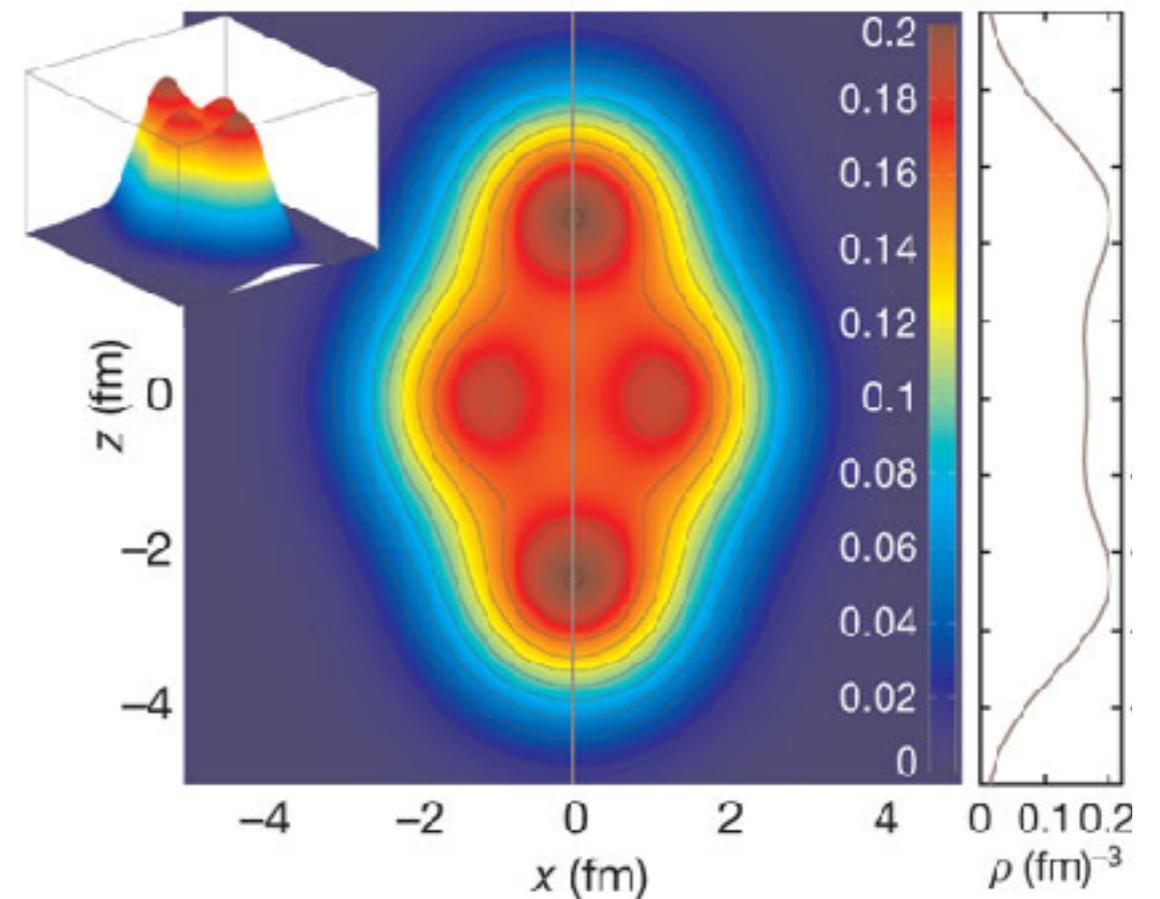
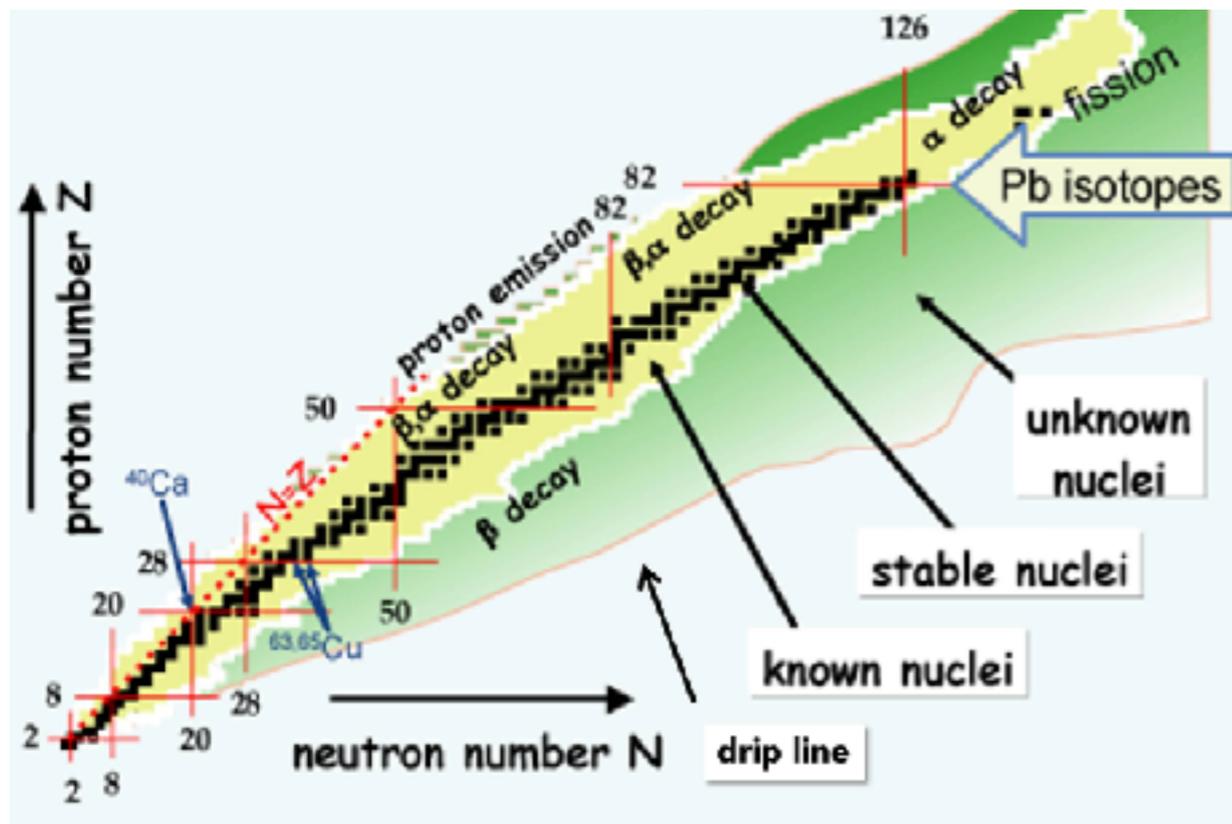
- Multi-messenger era for nuclear astrophysics
  - \* Gravitational waves have (just) been detected!
  - \* Supernovae neutrino will be detected by the current and next generation neutrino experiments
  - \* Nuclear dynamics determine the structure and the cooling of neutron stars (hyperons?)



- Ton-scale neutrino-oscillation and  $0\nu\beta\beta$  experiments
  - \* Charge-parity (CP) violating phase and the mass hierarchy will be measured
  - \* Determine whether the neutrino is a Majorana or a Dirac particle
  - \* Need for including nuclear dynamics; mean-field models are inadequate to describe neutrino-nucleus interaction

# Modern nuclear physics

- Atomic nuclei are strongly interacting many-body systems exhibiting fascinating properties including: shell structure, pairing and superfluidity, deformation, and self-emerging clustering.



- The nuclear chart is fully determined by only five parameters: the up-, down- and strange-quark masses, the overall scale of the strong interactions and the electromagnetic coupling constant

# The MANYBODY collaboration

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## Goals of the collaboration

- Electron and neutrino interactions with nuclei
- Equation of state of dense nuclear matter and neutrino propagation in nuclear matter
- Monte Carlo techniques to compute ground- and excited-state properties of many-body systems

## Nodes:

- Bologna
- Lecce
- Pavia → ~50 Kcore/hr
- Roma 1 → ~100 Kcore/hr
- Torino
- Trento-TIFPA → ~120 Mcore/hr



## National PI:

Francesco Pederiva (TIFPA)

## Current computing time awards include:

- PRACE: *A unified computational protocol for QCD nuclei*, 37M core hours on at CINECA
- ALCC: *Nuclear Spectra with chiral forces*, 35M core hours of computing time on Theta at ANL
- INCITE: *Nuclear structure and dynamics*, 60M core hours of computing time on Mira at ANL

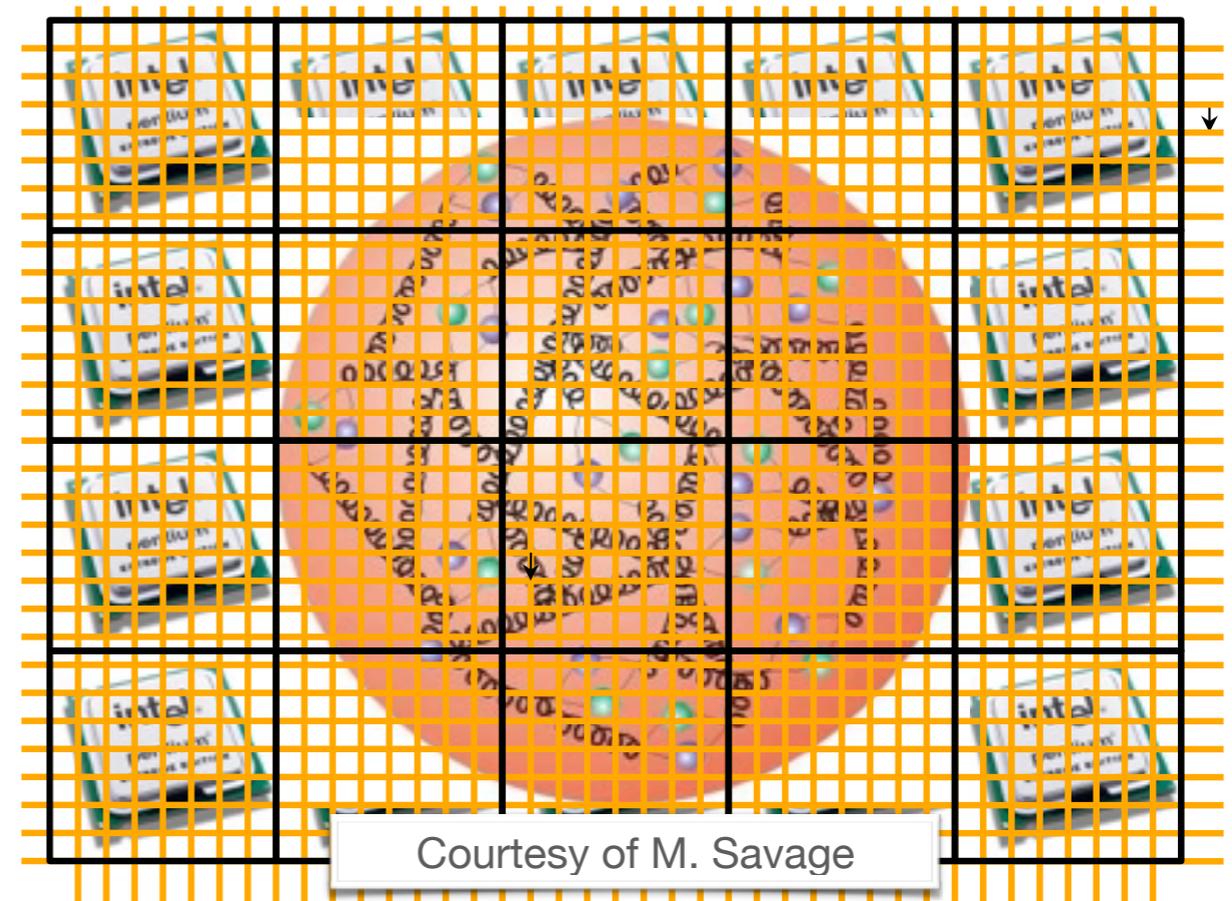
# From QCD to nuclear physics

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- Owing to its non-abelian character, QCD is strongly non-perturbative in its coupling constant at “large” distances.

- Lattice-QCD is the most reliable way of “solving” QCD in the low-energy regime, and it promises to provide a solid foundation for the structure of nuclei directly from QCD

- Lattice-QCD calculations of light nuclei are currently carried out at large values of the pion masses

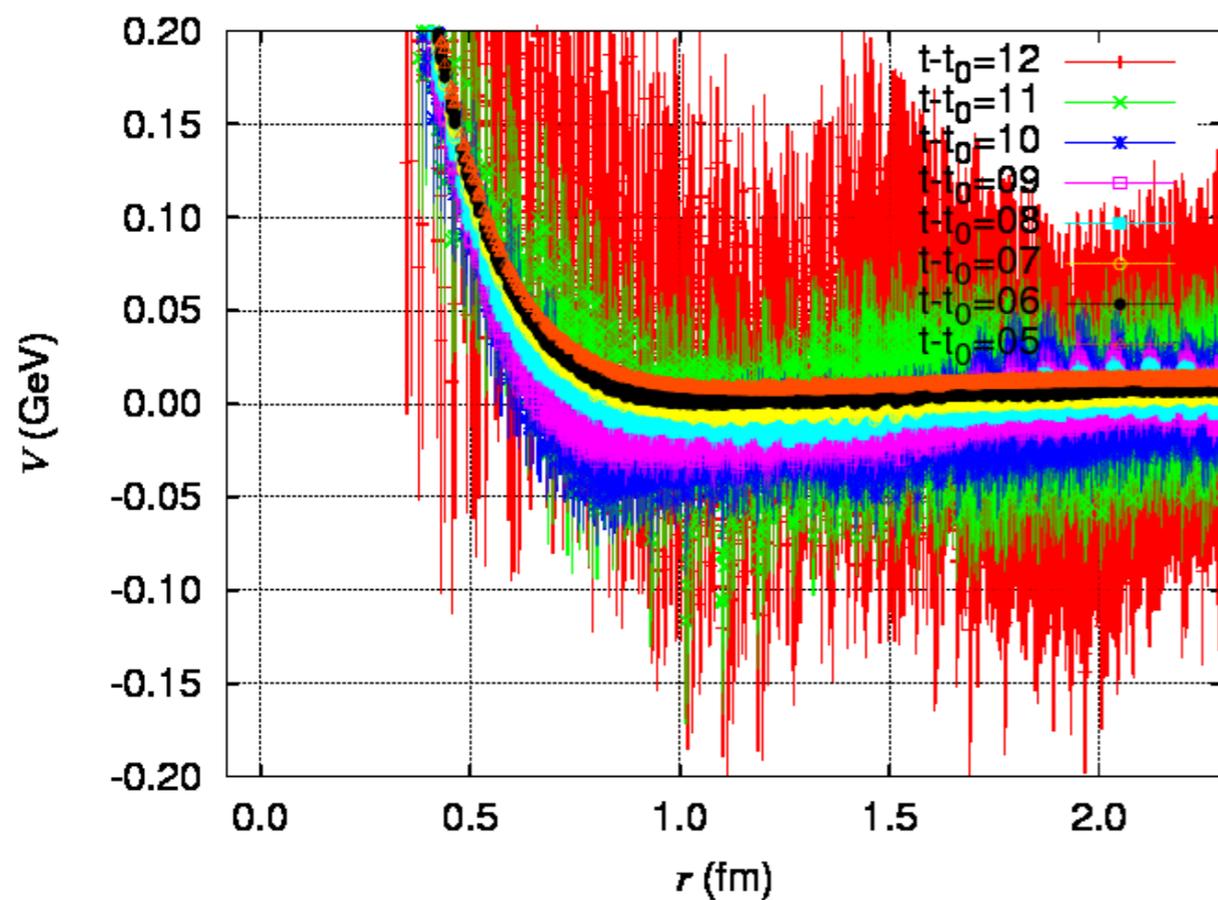


- This allows to understand whether Standard Model parameters might have to be finely tuned for nuclei to be stable

# From QCD to nuclear physics

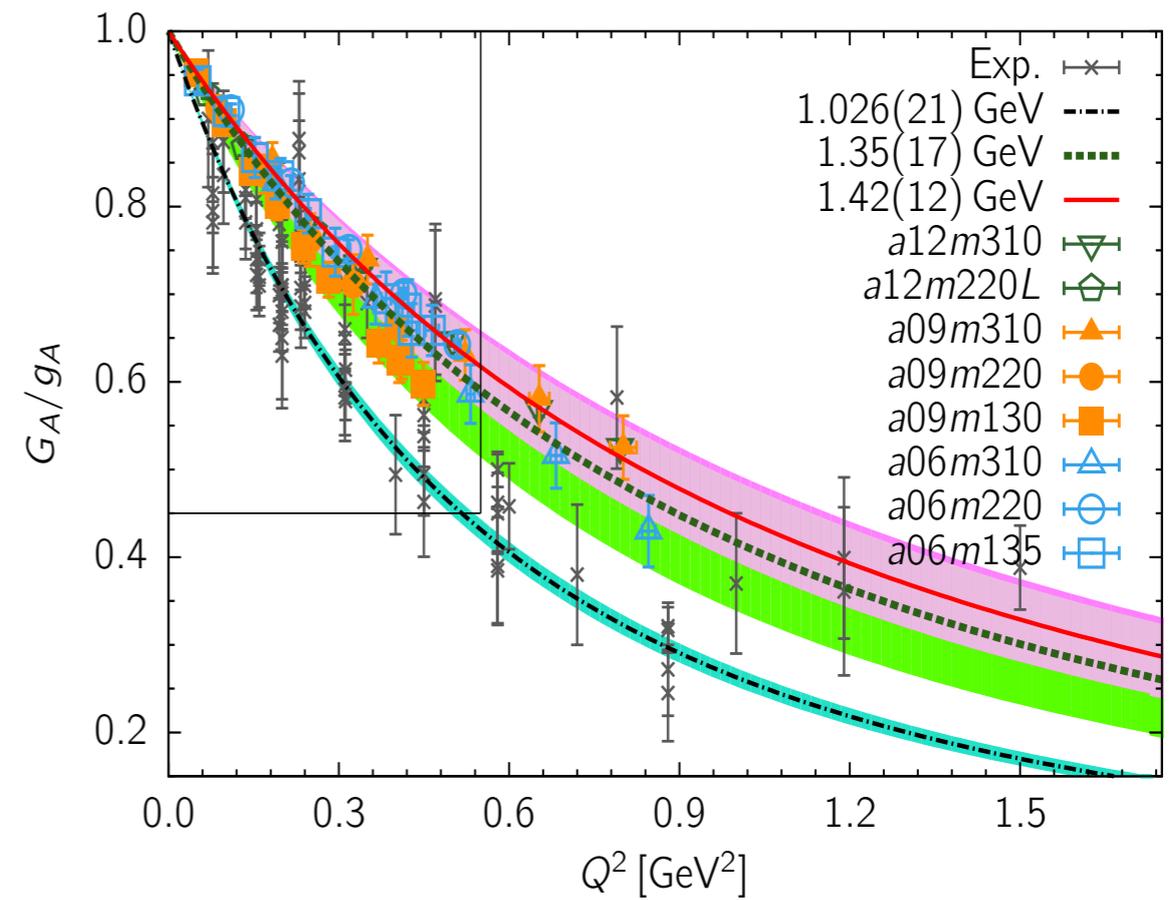
- Lattice-QCD inputs are essential when experimental data are scarce

## Nucleon-hyperon interactions



H. Nemura arXiv:1702.00734 (2017)

## Nucleon axial form factor



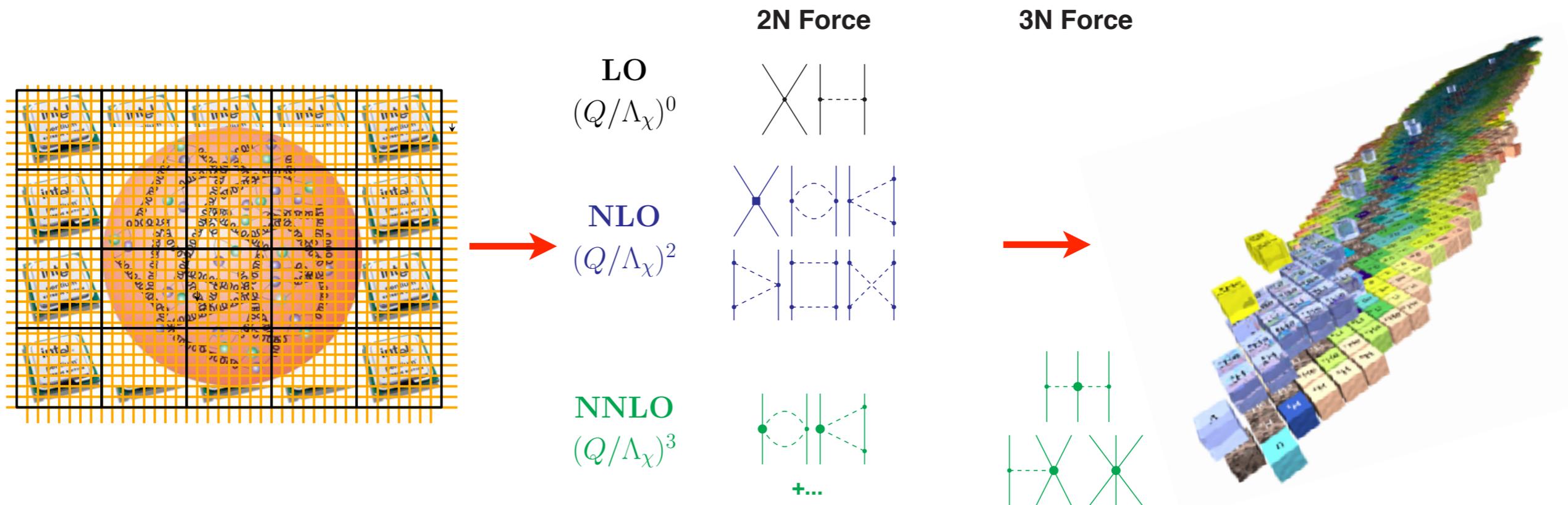
R. Gupta et al. PRD **96**,114503 (2017)

# From QCD to nuclear physics

- At the energy regime relevant for the description of nuclei, quark and gluons are confined inside hadrons. Nucleons can be treated as point-like particles interacting through the Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

- Effective field theories are the bridge between QCD and nuclear observables. They exploit the separation between the “hard” ( $M \sim$  nucleon mass) and “soft” ( $Q \sim$  exchanged momentum) scales



# Diffusion Monte Carlo

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- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.
- Any trial wave function can be expanded in the complete set of eigenstates of the the hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n |\Psi_n\rangle$$

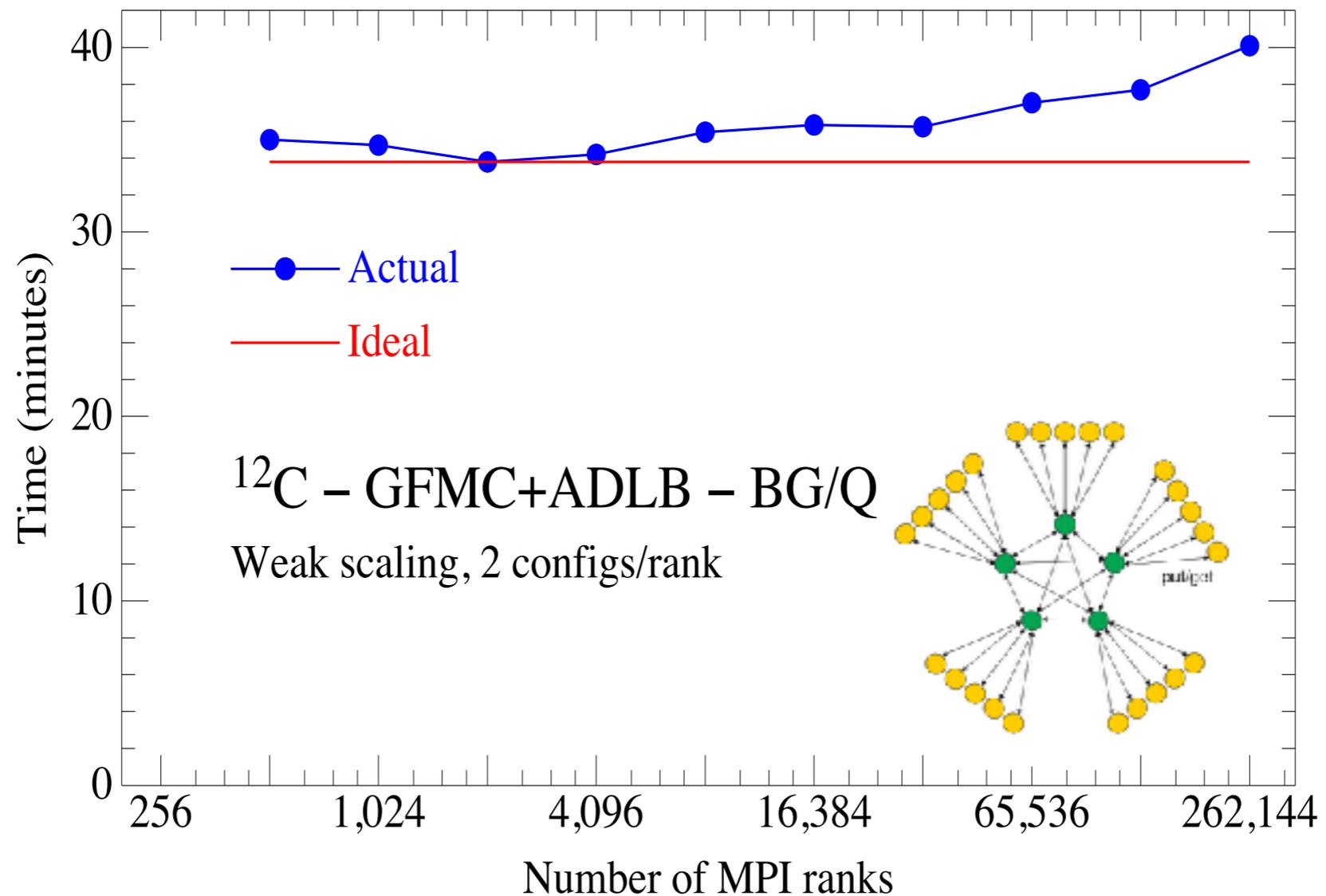
which implies

$$\lim_{\tau \rightarrow \infty} e^{-(H-E_0)\tau} |\Psi_T\rangle = \lim_{\tau \rightarrow \infty} \sum_n c_n e^{-(E_n-E_0)\tau} |\Psi_n\rangle = c_0 |\Psi_0\rangle$$

where  $\tau$  is the imaginary time. Hence, GFMC and AFDMC project out the exact lowest-energy state, provided the trial wave function it is not orthogonal to the ground state.

# Exploiting supercomputers

- Our QMC codes have steadily undergone development to take advantage of each new generation of parallel machine and was one of the first to deliver new scientific results each time.

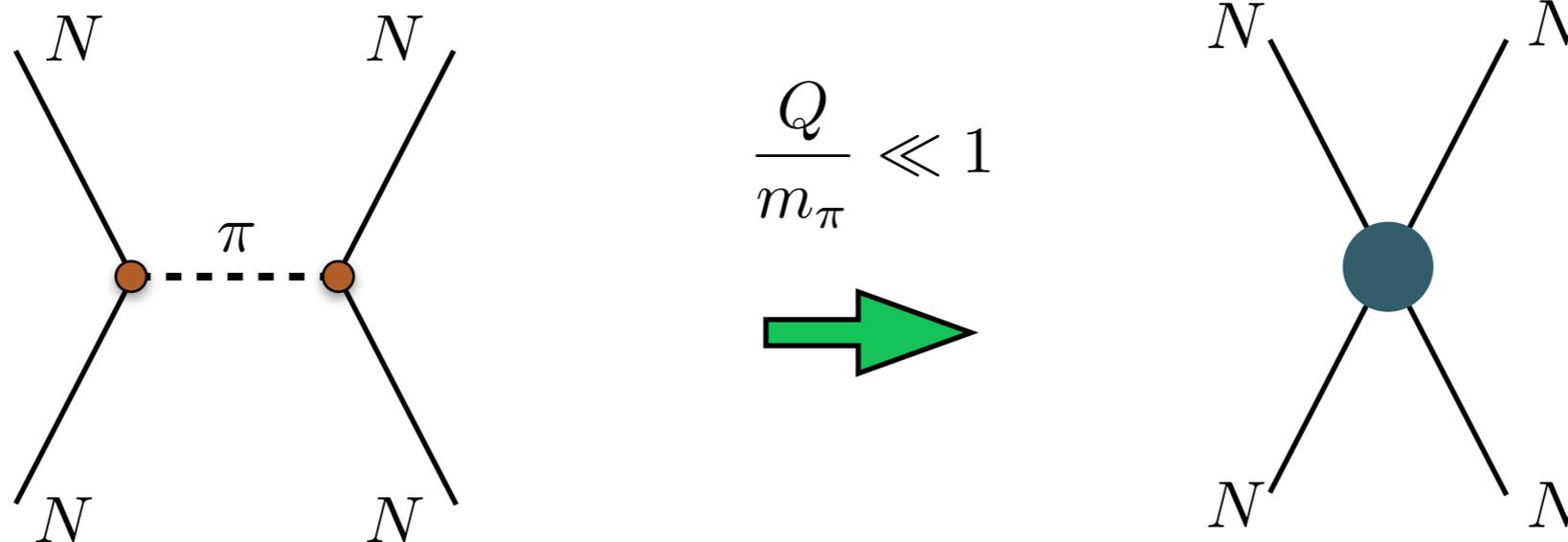


# Chapter I

What binds nuclei

# Pionless EFT

- We aim to study the evolution of the nuclear chart as a function of the quark-masses, which are input parameters of the Standard Model.
- Since pions are very massive, the range of applicability of Pionless effective field theory (EFT) is larger than in the “real” world



The pionless EFT Hamiltonian is naturally formulated in momentum space

	LO	NLO
$v_{12}$ →	$C_0 + C_1 \sigma_{12}$	$C_2(k^2 + q^2) + C_3(k^2 + q^2) \sigma_{12}$
$V_{123}$ →	$D_0$	/

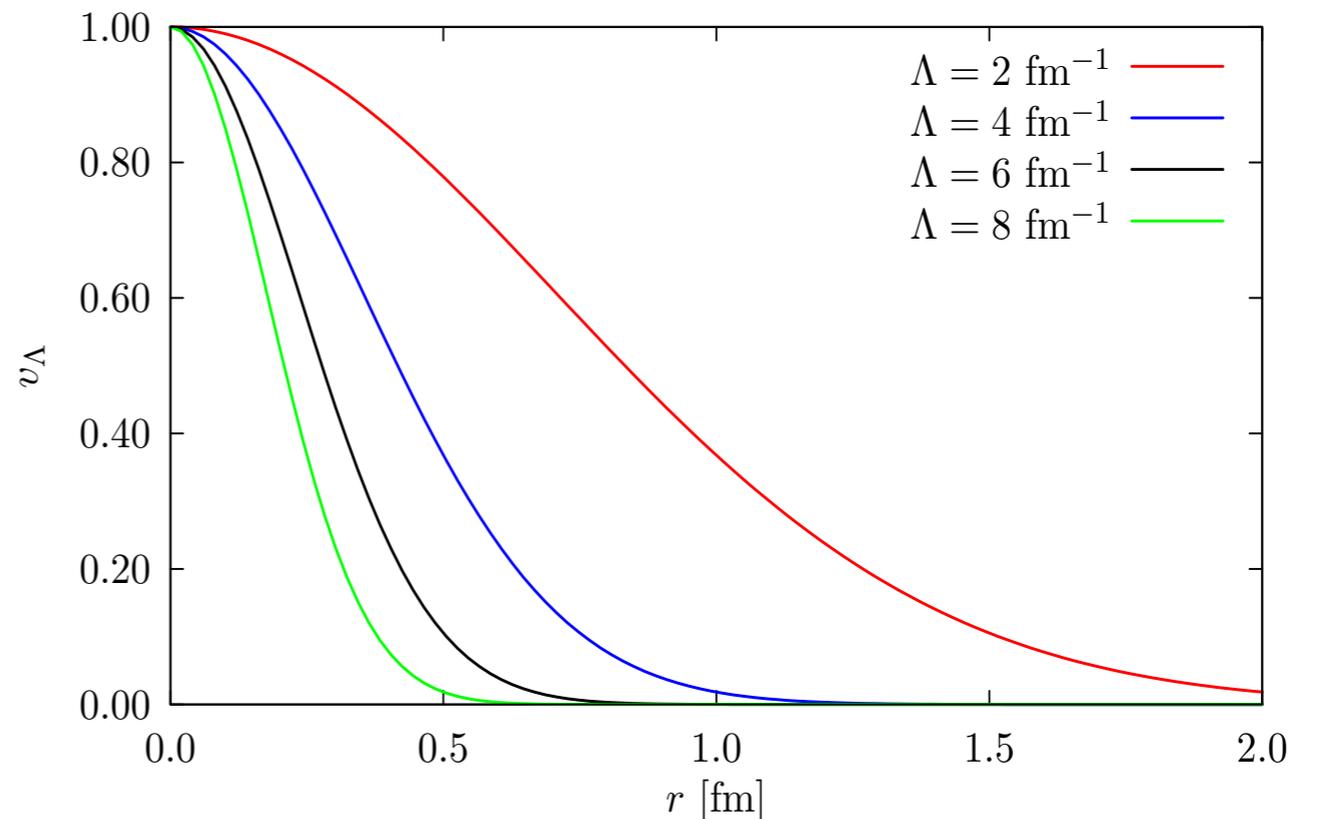
# Pionless EFT potential

The coordinate-space version of the potential needs is regularized as

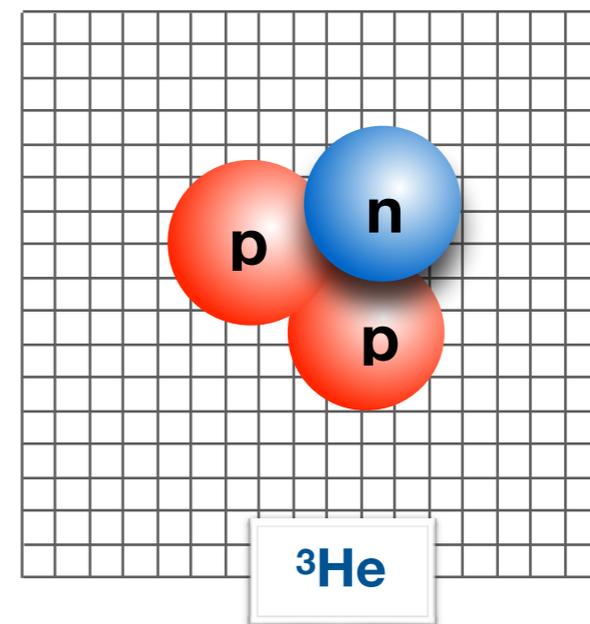
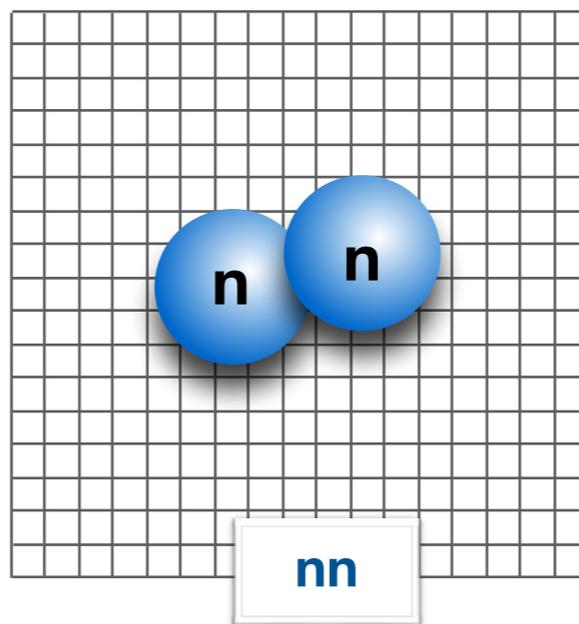
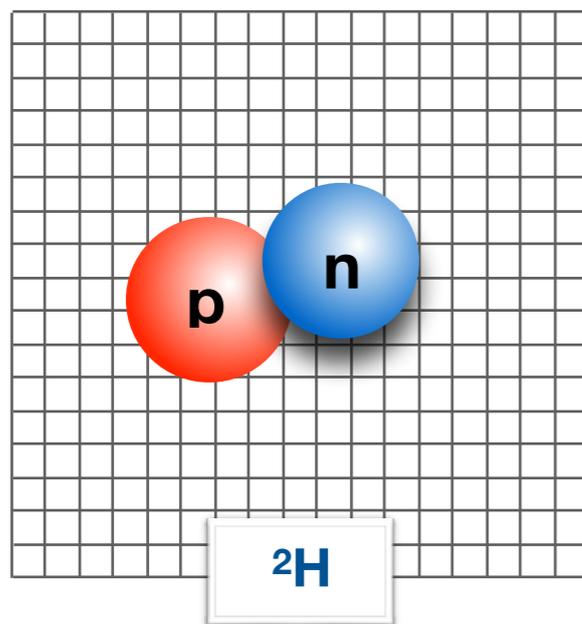
$$v_{12} = C_1 v_\Lambda(r_{12}) + C_2 v_\Lambda(r_{12}) \sigma_{12}$$

$$V_{123} = D_0 \sum_{\text{cyc}} v_\Lambda(r_{12}) v_\Lambda(r_{13})$$

$$v_\Lambda(r_{12}) = e^{-r_{12}^2 \Lambda^2 / 4}$$



The low-energy constants are fixed on Lattice-QCD results for  $A < 3$  nuclear systems



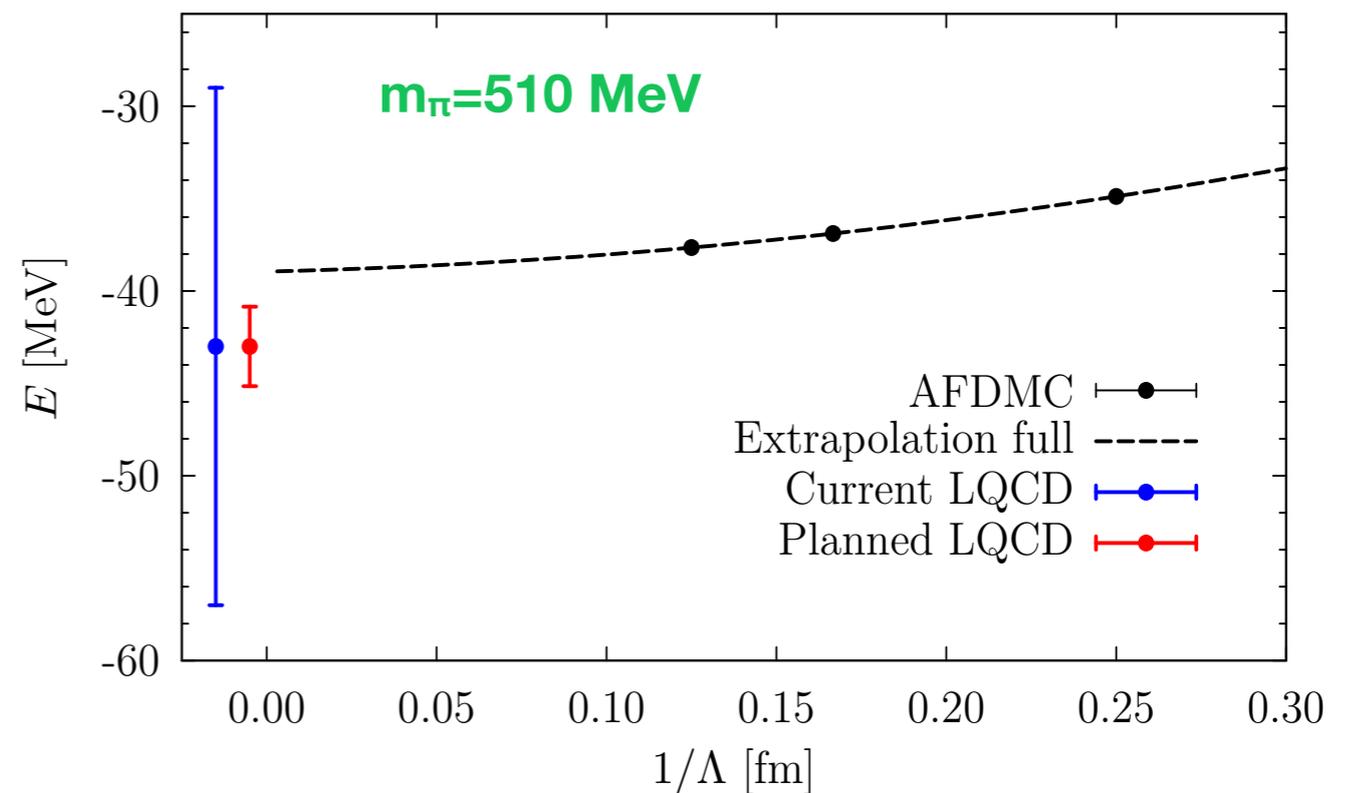
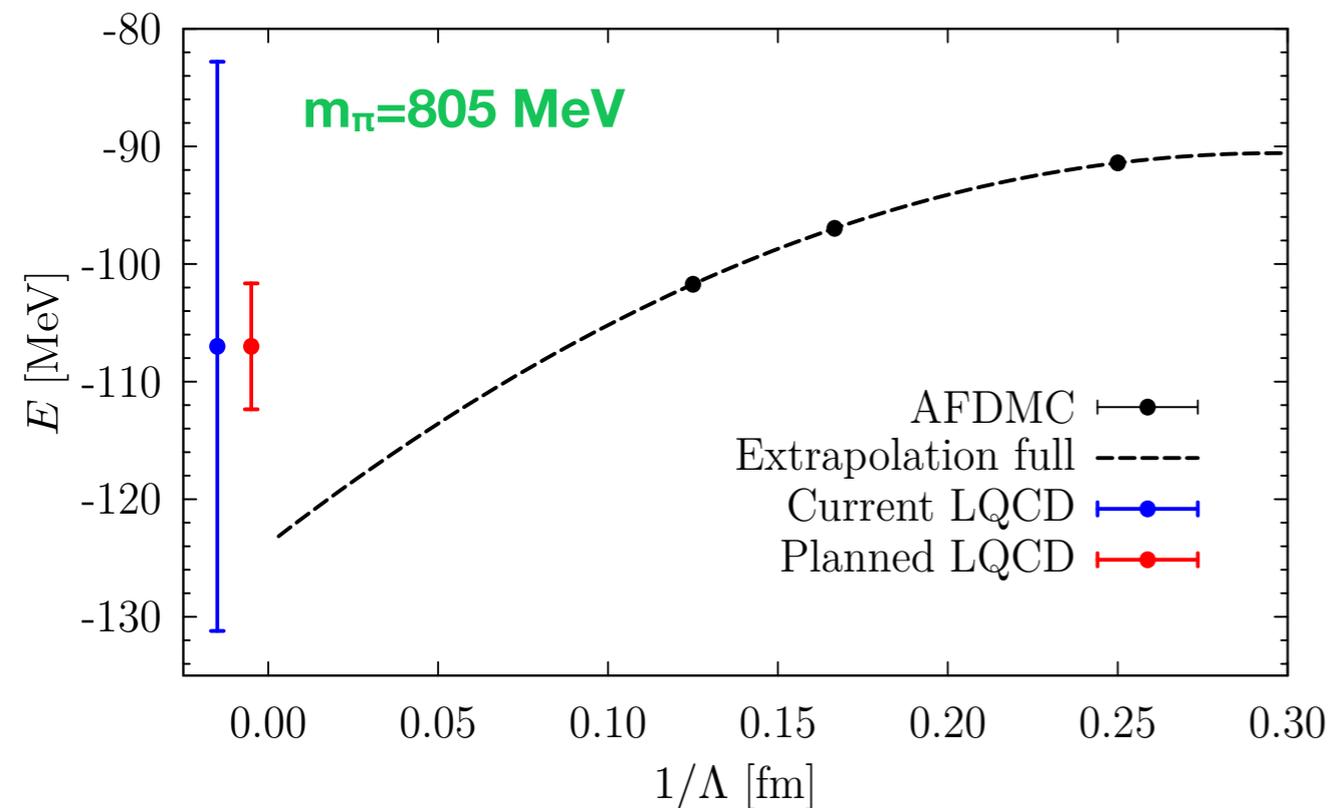
# $^4\text{He}$ results

- To minimize the regularization error, we fit finite-cutoff results with

$$O_\Lambda = O + \frac{C_0}{\Lambda} + \frac{C_1}{\Lambda^2} + \dots$$

- We found that an expansion up to  $1/\Lambda^2$  suffices to extrapolate the  $^4\text{He}$  energies
- The truncation error dominates over the statistical and extrapolation errors

$\Lambda$	$m_\pi = 140 \text{ MeV}$	$m_\pi = 510 \text{ MeV}$	$m_\pi = 805 \text{ MeV}$
$2 \text{ fm}^{-1}$	$-23.17 \pm 0.02$	$-31.15 \pm 0.02$	$-88.09 \pm 0.01$
$4 \text{ fm}^{-1}$	$-23.63 \pm 0.03$	$-34.88 \pm 0.03$	$-91.40 \pm 0.03$
$6 \text{ fm}^{-1}$	$-25.06 \pm 0.02$	$-36.89 \pm 0.02$	$-96.97 \pm 0.01$
$8 \text{ fm}^{-1}$	$-26.04 \pm 0.05$	$-37.65 \pm 0.03$	$-101.72 \pm 0.03$
$\rightarrow \infty$	$-30^{+0.3(\text{sys})}_{\pm 2(\text{stat})}$	$-39^{+1(\text{sys})}_{\pm 2(\text{stat})}$	$-124^{+3(\text{sys})}_{\pm 1(\text{stat})}$
Exp.	$-28.30$	–	–
LQCD	–	$-43.0 \pm 14.4$	$-107.0 \pm 24.2$



# $^{16}\text{O}$ results

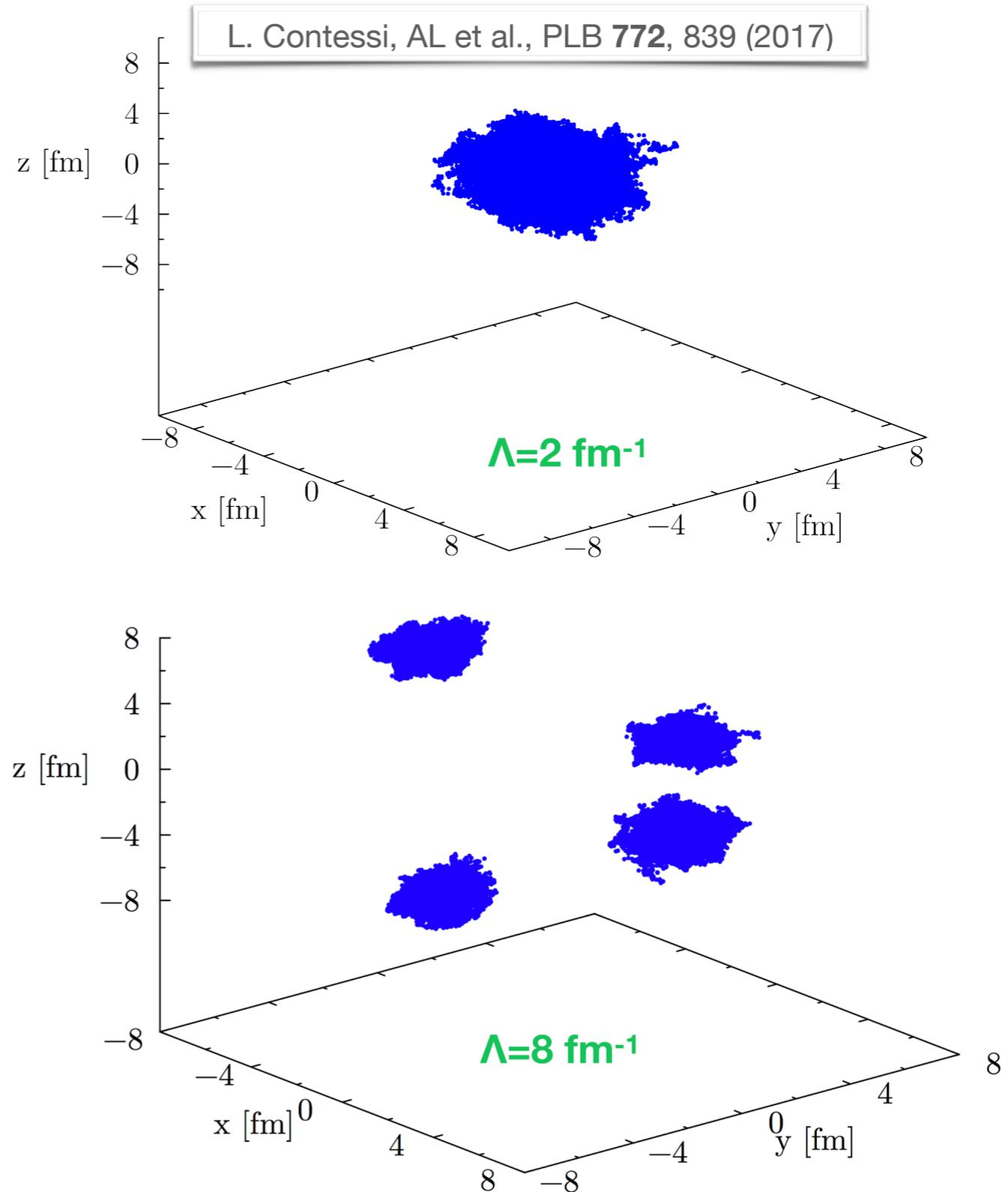
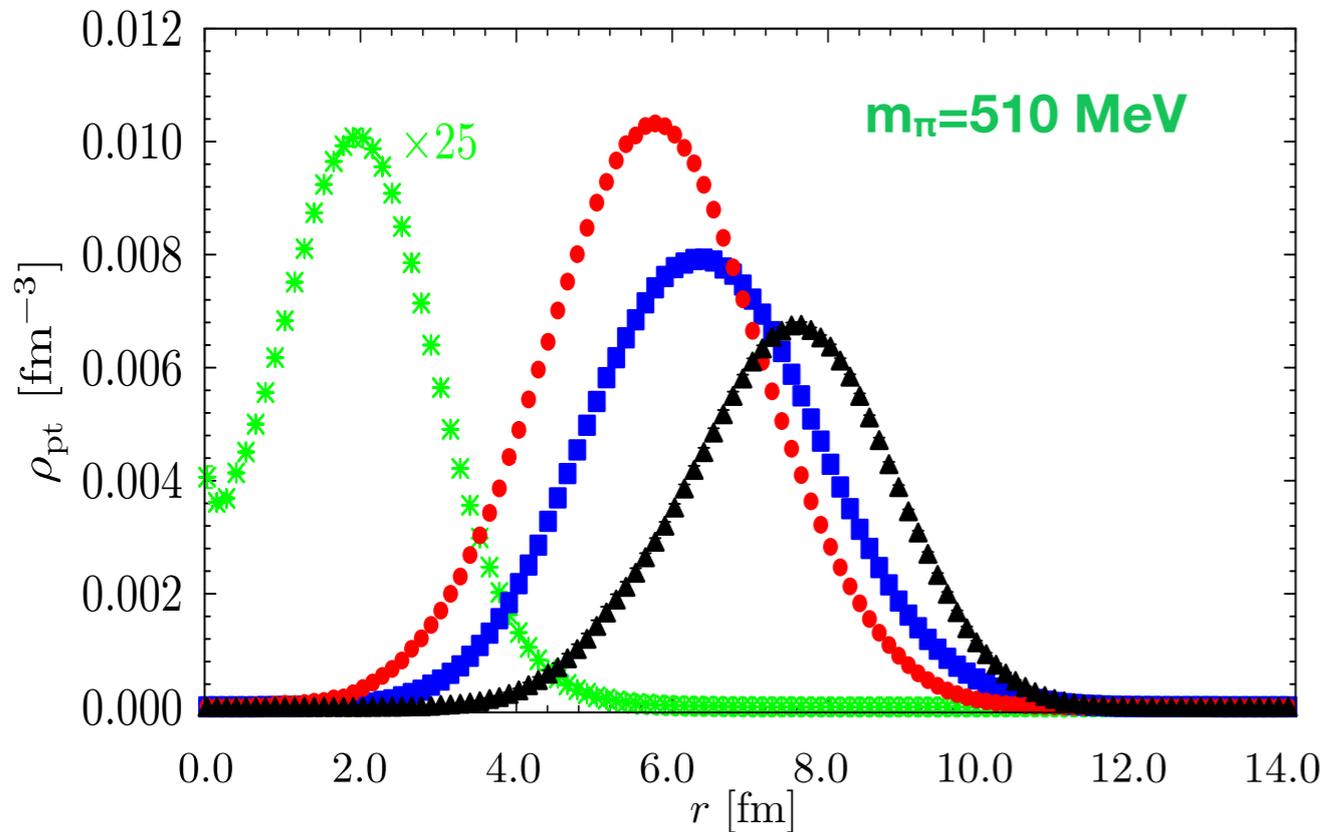
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- $^{16}\text{O}$  is not stable against breakup into four  $^4\text{He}$  clusters in almost all the cases
- The only exception occurring for  $m_\pi=140$  MeV and  $\Lambda =2$  fm $^{-1}$ , where  $^{16}\text{O}$  is 4.5 MeV more bound than four  $^4\text{He}$  nuclei. This might be related to the range of the interaction.
- Even considering only statistical and extrapolation errors the asymptotic values of the  $^{16}\text{O}$  energy cannot be separated from the four- $^4\text{He}$  threshold.
- In the other cases we miss the four- $^4\text{He}$  threshold by about 5 MeV, which is beyond our statistical errors and reveals a lower bound on the systematic error of our QMC method.

$\Lambda$	$m_\pi = 140$ MeV	$m_\pi = 510$ MeV	$m_\pi = 805$ MeV
2 fm $^{-1}$	$-97.19 \pm 0.06$	$-116.59 \pm 0.08$	$-350.69 \pm 0.05$
4 fm $^{-1}$	$-92.23 \pm 0.14$	$-137.15 \pm 0.15$	$-362.92 \pm 0.07$
6 fm $^{-1}$	$-97.51 \pm 0.14$	$-143.84 \pm 0.17$	$-382.17 \pm 0.25$
8 fm $^{-1}$	$-100.97 \pm 0.20$	$-146.37 \pm 0.27$	$-402.24 \pm 0.39$
$\rightarrow \infty$	$-115^{+1(\text{sys})}_{\pm 8(\text{stat})}$	$-151^{+2(\text{sys})}_{\pm 10(\text{stat})}$	$-504^{+20(\text{sys})}_{\pm 12(\text{stat})}$
Exp.	$-127.62$	–	–
<b>4-<math>^4\text{He}</math> threshold</b>	$-120^{+1.2}_{\pm 8}$	$-156^{+4}_{\pm 8}$	$-496^{+12}_{\pm 4}$

# $^{16}\text{O}$ results at $m_\pi=510$ MeV

- **Bound  $^{16}\text{O}$ :** during the imaginary-time propagation nucleons diffuse in the region where the single-nucleon density is large
- **Unbound  $^{16}\text{O}$ :** the nucleons forming the four  $^4\text{He}$  clusters remain close to the corresponding centers of mass

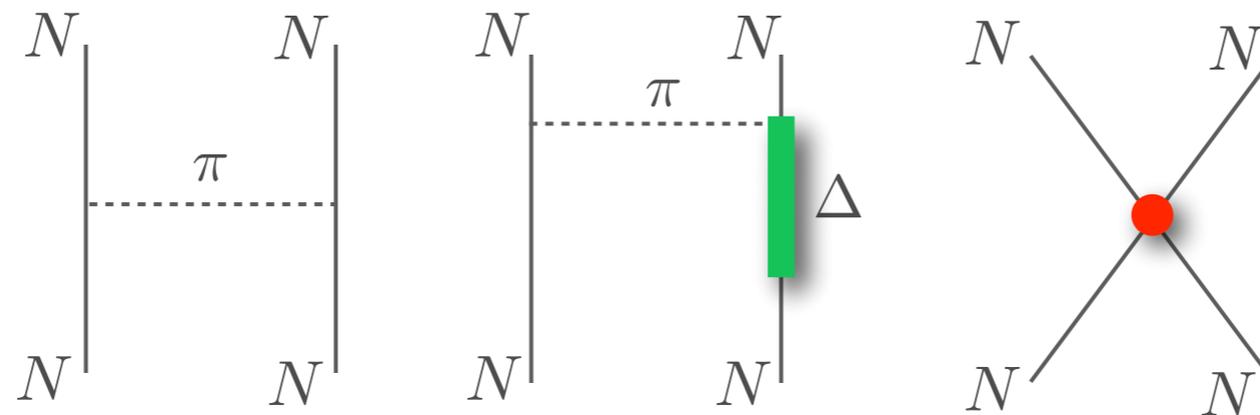


# Chapter II

## Neutrino-nucleus scattering

# Phenomenological Hamiltonian and currents

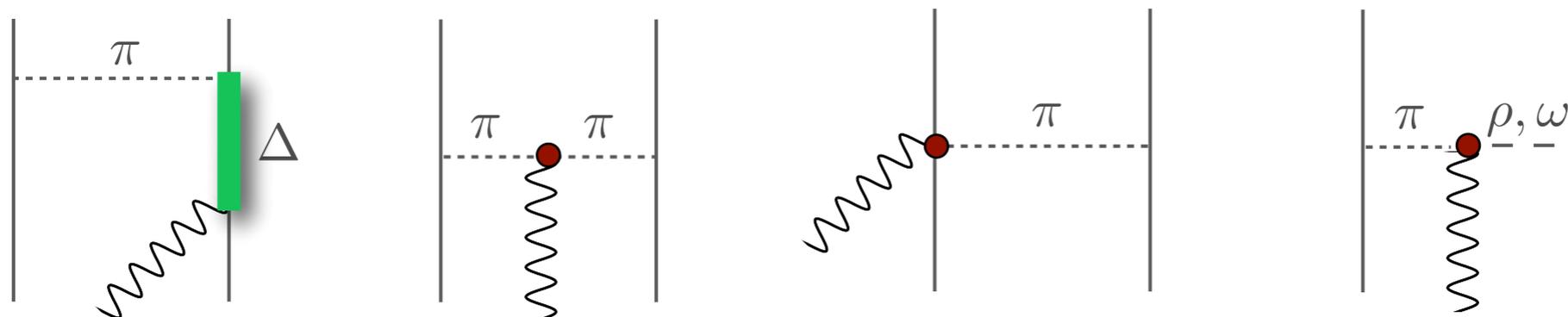
The Argonne  $v_{18}$  is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database



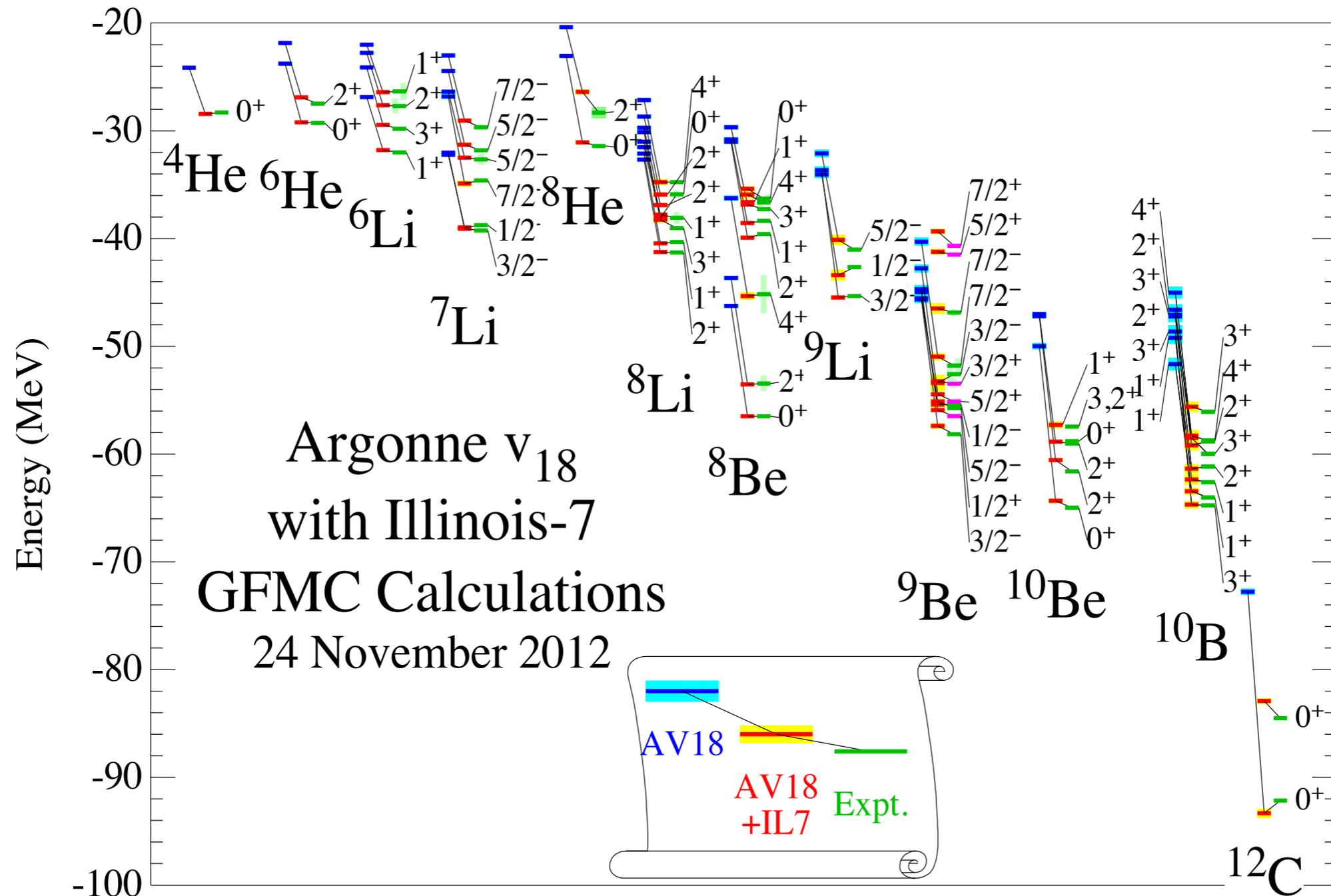
Three-nucleon interactions effectively include the lowest nucleon excitation, the  $\Delta(1232)$  resonance, and other nuclear effects

The nuclear electromagnetic current is constrained by the continuity equation

$$\nabla \cdot \mathbf{J}_{\text{EM}} + i[H, J_{\text{EM}}^0] = 0$$



# Quantum Monte Carlo methods

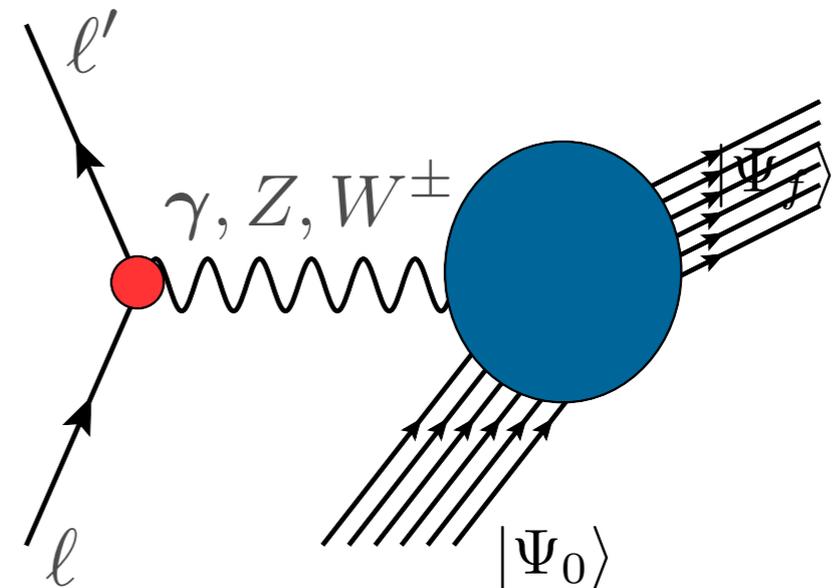


# Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'} d\Omega_{\ell}} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

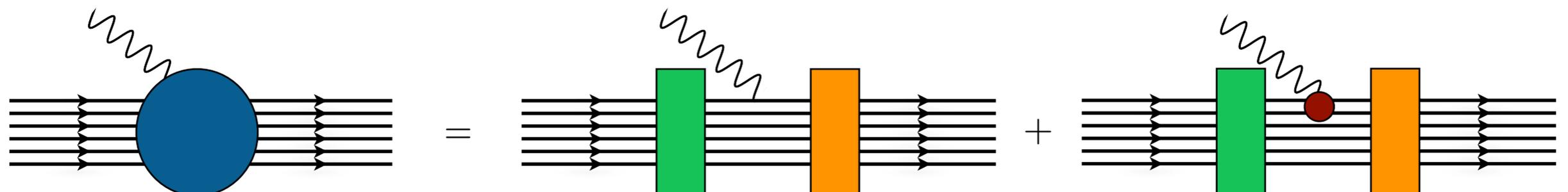
- In the electromagnetic case only the longitudinal and the transverse response functions contribute



- The response functions contain all the information on target structure and dynamics

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle \Psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_{\beta}(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

- They account for initial state correlations, final state correlations and two-body currents

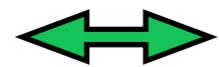
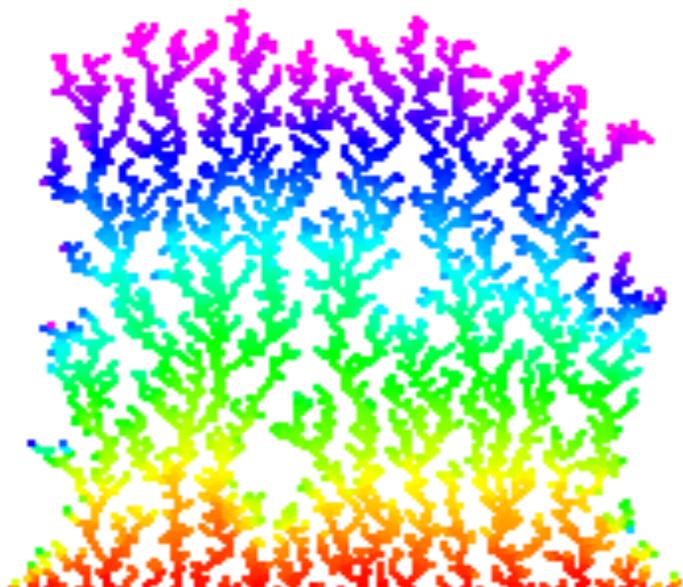
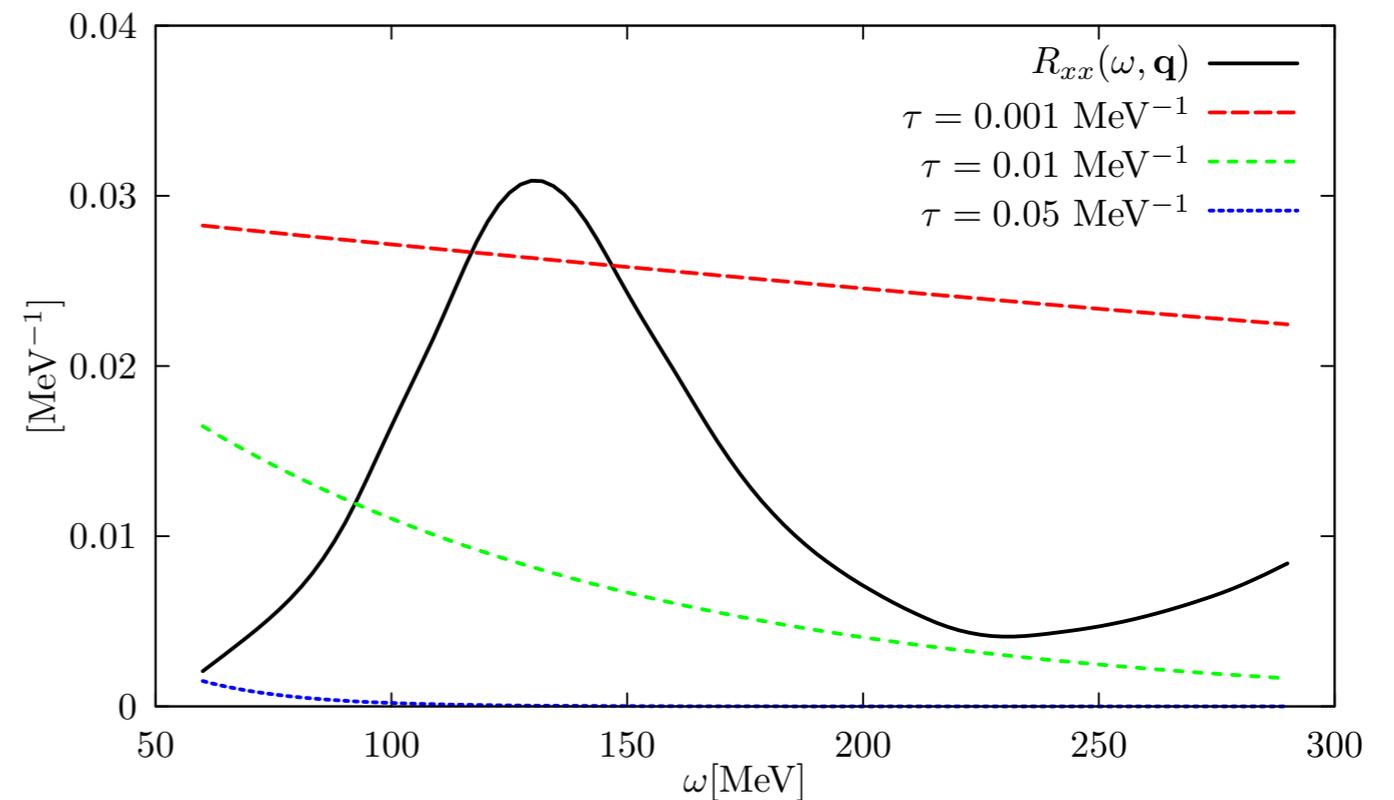


# Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed



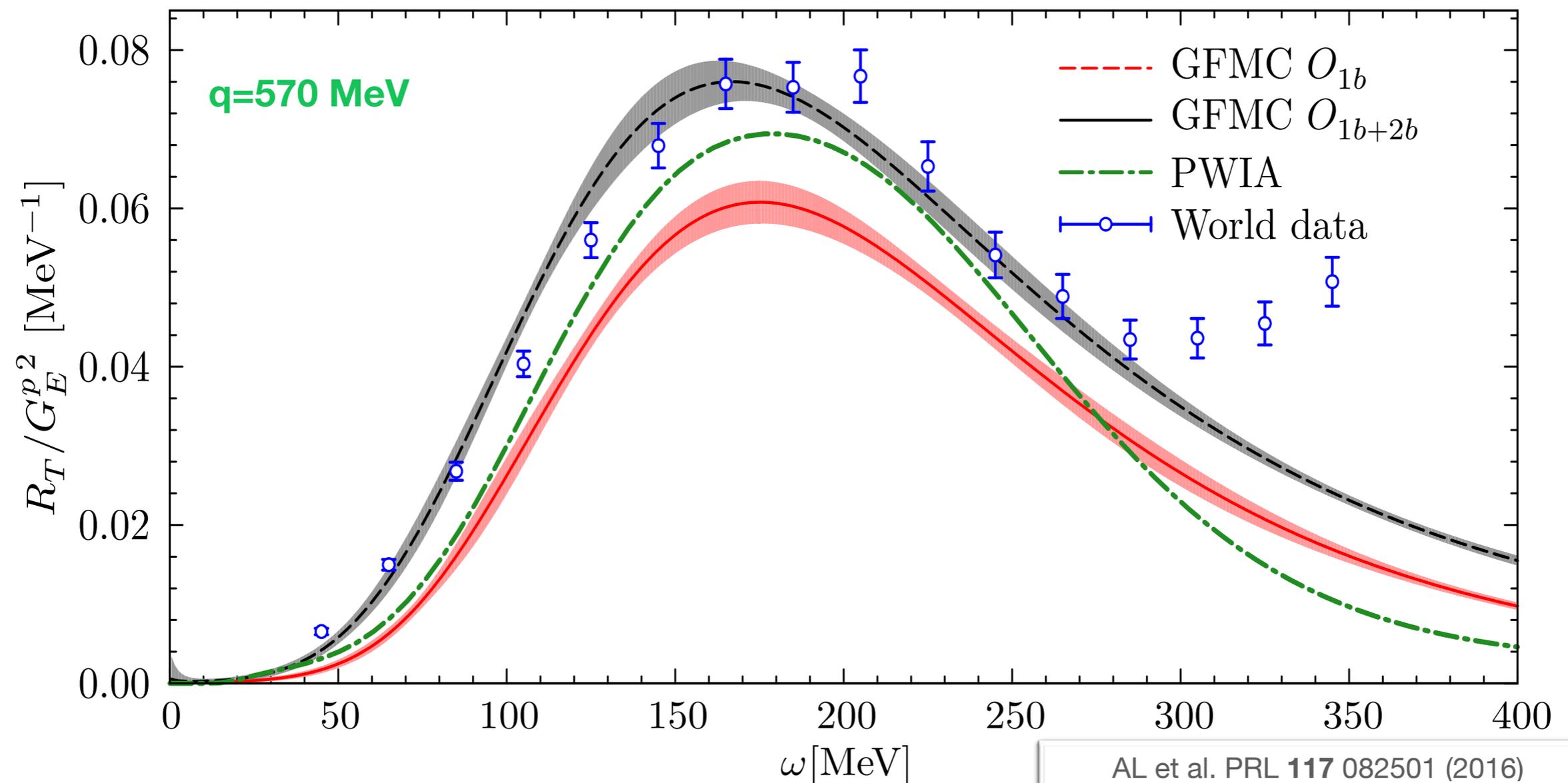
The system is first heated up by the transition operator. Its cooling determines the Euclidean response of the system

$$E_{\alpha\beta}(\tau, \mathbf{q}) = \langle \Psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) e^{-(H-E_0)\tau} J_{\beta}(\mathbf{q}) | \Psi_0 \rangle$$

Same technique used in Lattice QCD, condensed matter physics...

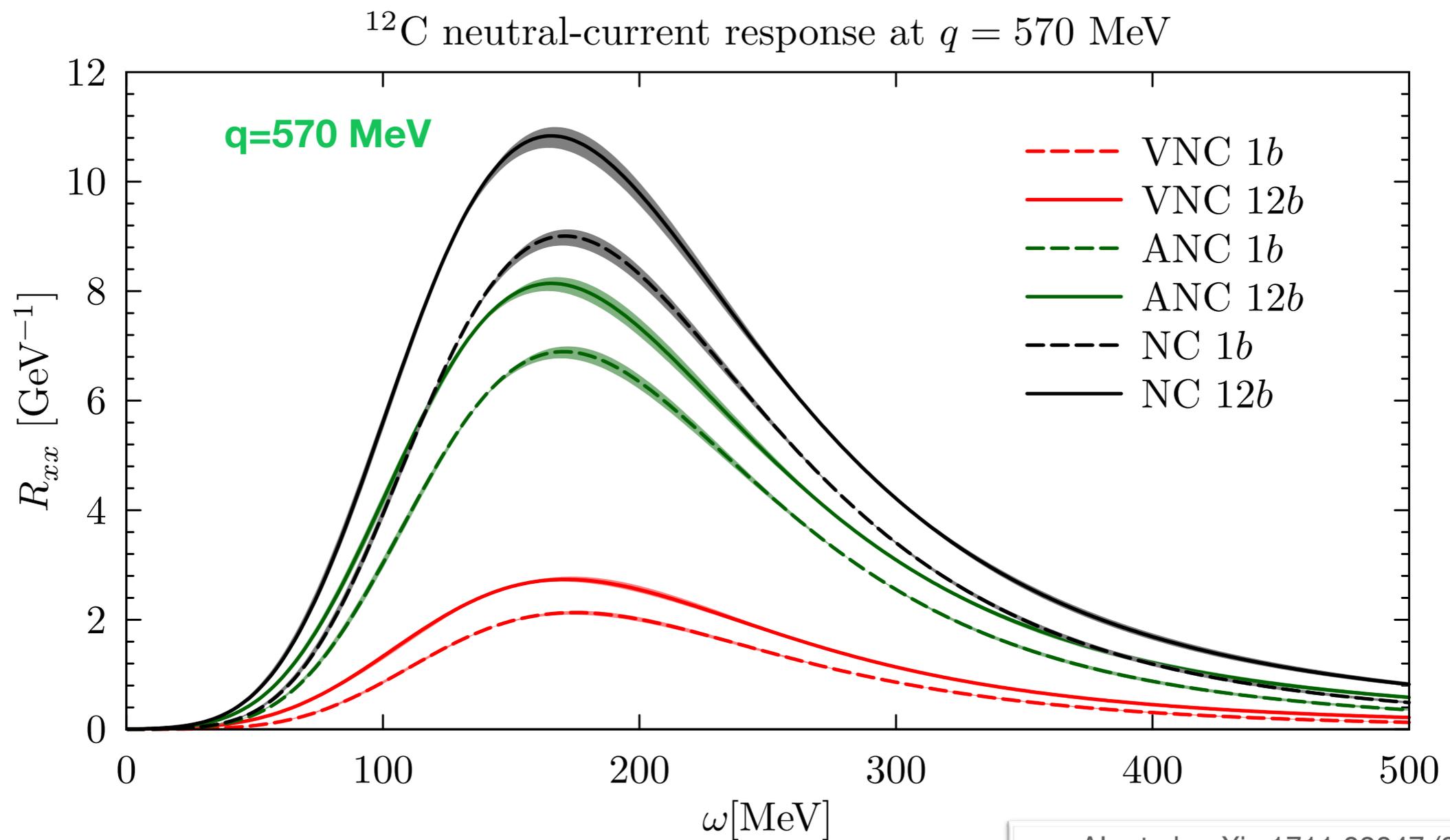
# $^{12}\text{C}$ electromagnetic response

- We inverted the electromagnetic Euclidean response of  $^{12}\text{C}$
- Very good agreement with the experimental data once two-body currents are accounted for



# $^{12}\text{C}$ neutral-current response

- Recently, we were able to invert the neutral-current Euclidean responses of  $^{12}\text{C}$

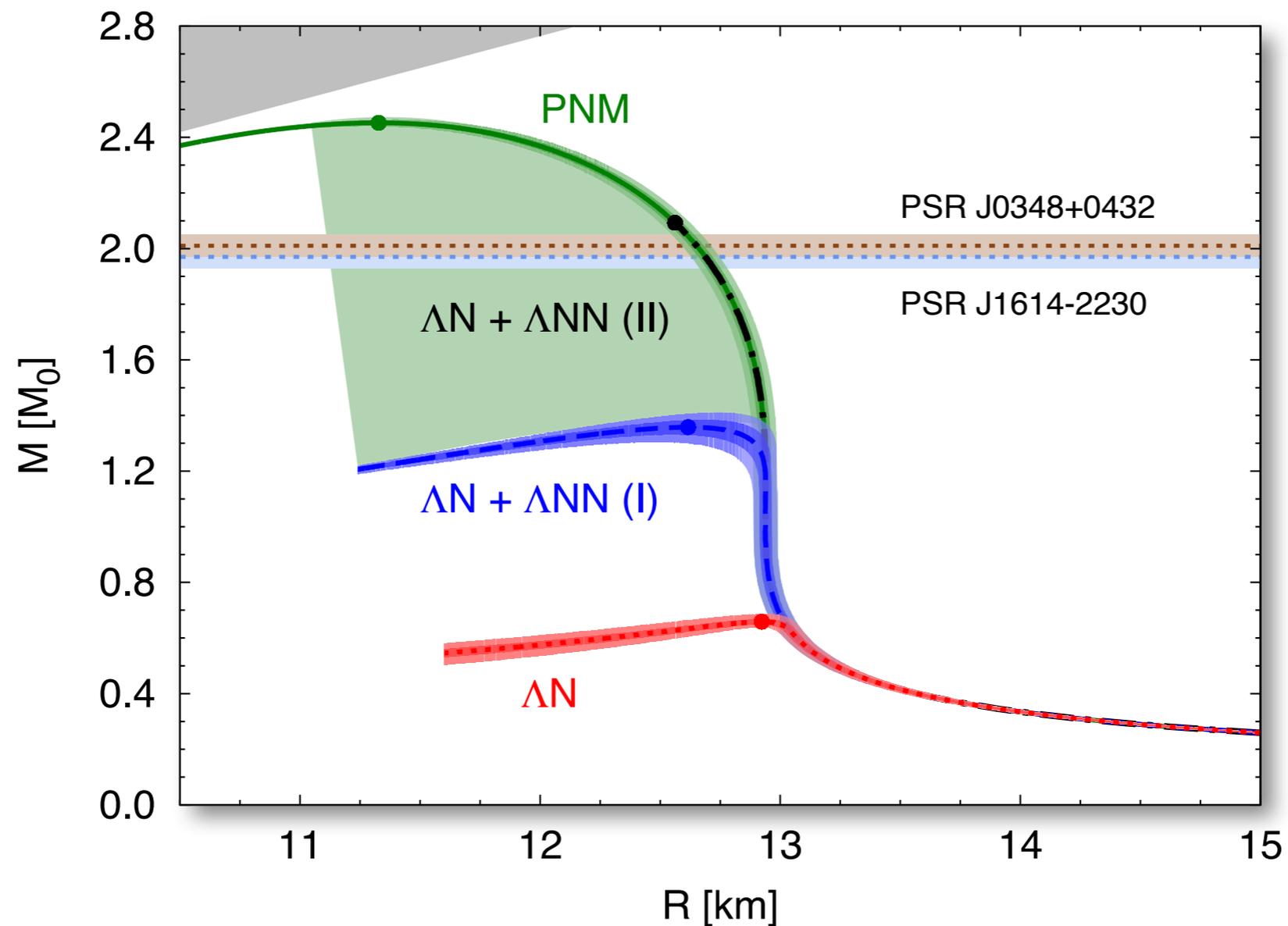


# Chapter III

## Hyperons in neutron stars

# Diffusion Monte Carlo: hyperons

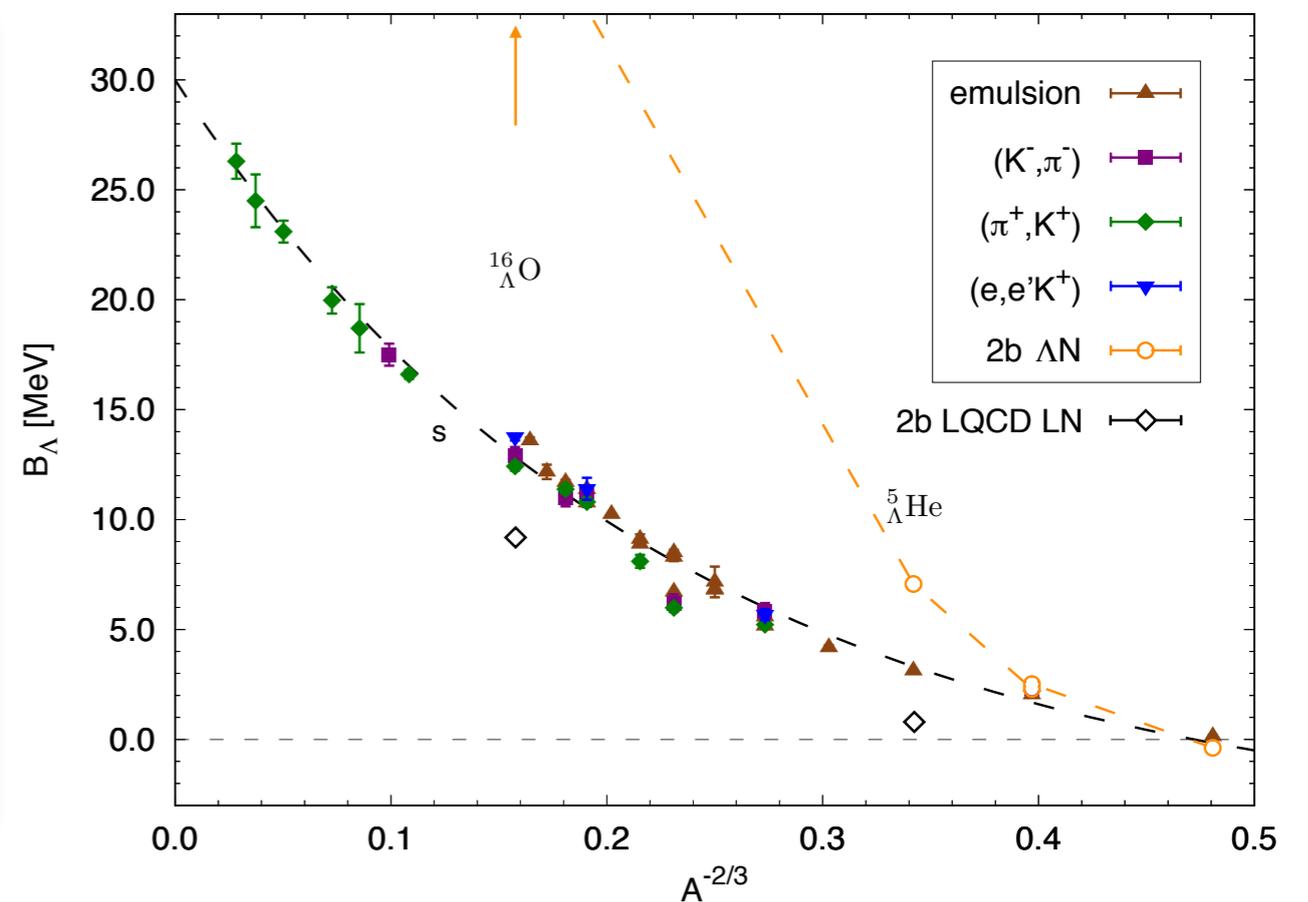
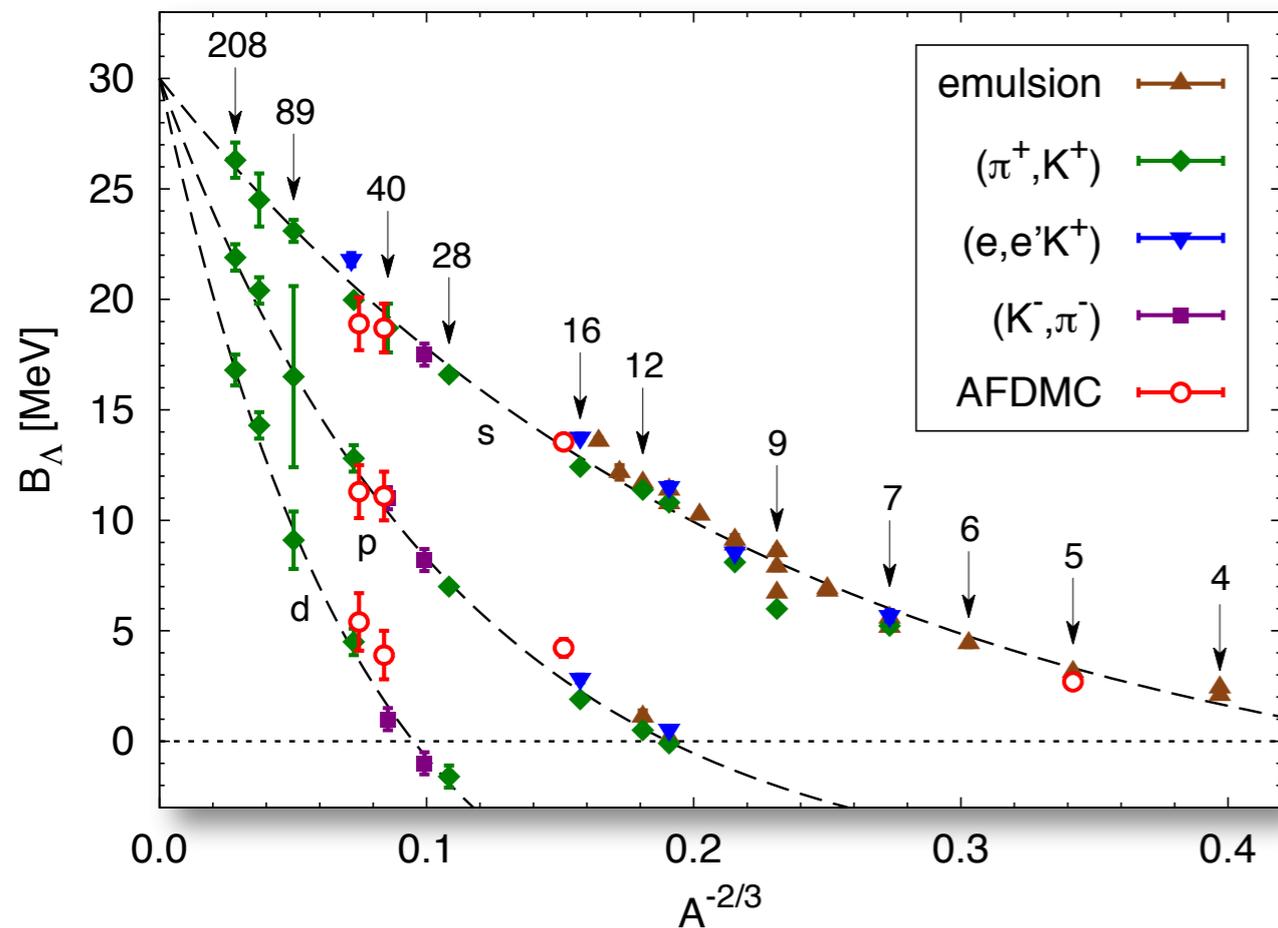
- The interactions between hyperons and nucleons play a fundamental role in the softening of the Equation of State and the consequent reduction of the neutron star maximum mass.



# Diffusion Monte Carlo: hyperons

- By fitting the hyperon-nucleon-nucleon force on the s-wave lambda-separation energy of  ${}^{17}_{\Lambda}\text{O}$  and  ${}^3_{\Lambda}\text{He}$  we can reproduce the experimental results over a wide-mass range.

- We have recently implemented Lattice-QCD hyperon-nucleon interactions. Our preliminary results are promising



# Conclusions

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## **Nuclei from Lattice-QCD**

- Our results for the  ${}^4\text{He}$  binding energy are in agreement with LQCD calculations
- At large pion mass,  ${}^{16}\text{O}$  is unstable with respect to break-up into four  ${}^4\text{He}$  nuclei.
- The long-range structure of the interaction is deficient

## **Neutrino-nucleus scattering**

- The two-body currents enhancement is effective in the entire energy transfer domain.
- Two-body current contributions enhance the longitudinal and transverse axial responses, plausible solution of the “axial mass puzzle”.

## **Hyperons in neutron stars**

- The onset of hyperons in neutron stars largely depends upon the hyperon-nucleon interaction
- Lattice-QCD data supplement the scarce experimental inputs that is currently available

Thank you

Backup slides

# Chiral EFT

In chiral-EFT, the symmetries of quantum chromodynamics, in particular its approximate chiral symmetry, are employed to systematically constrain classes of Lagrangians describing the interactions of baryons with pions and the interactions of these hadrons with electroweak fields

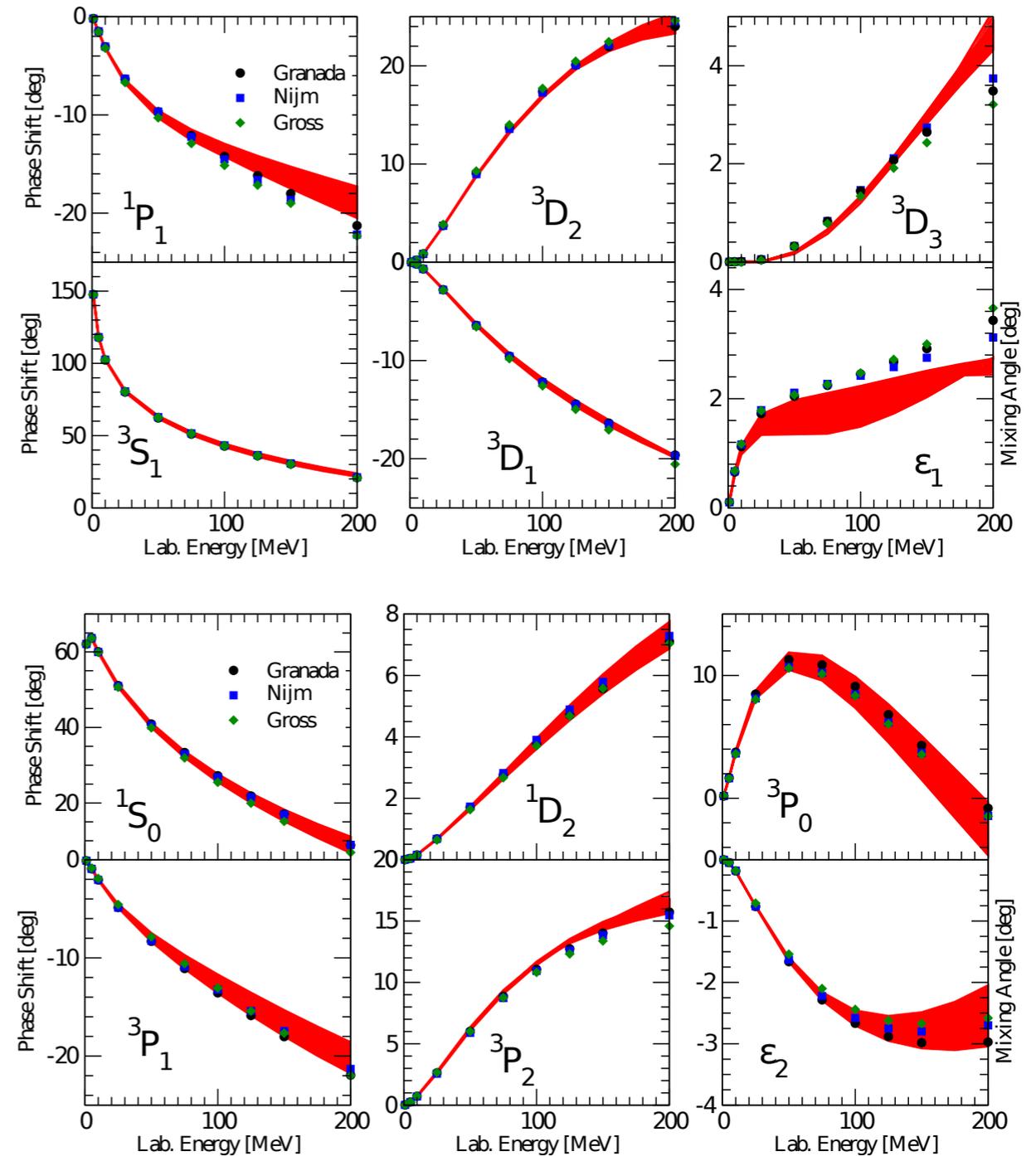
	NN potential	NNN potential	NNNN potential
LO		—	—
NLO		—	—
N <sup>2</sup> LO			—
N <sup>3</sup> LO			

# $\Delta$ -full local chiral potential

We have complemented the historical “Argonne” approach by considering a local chiral  $\Delta$ -full potential giving an excellent fit to the NN scattering data that can be readily used in QMC.

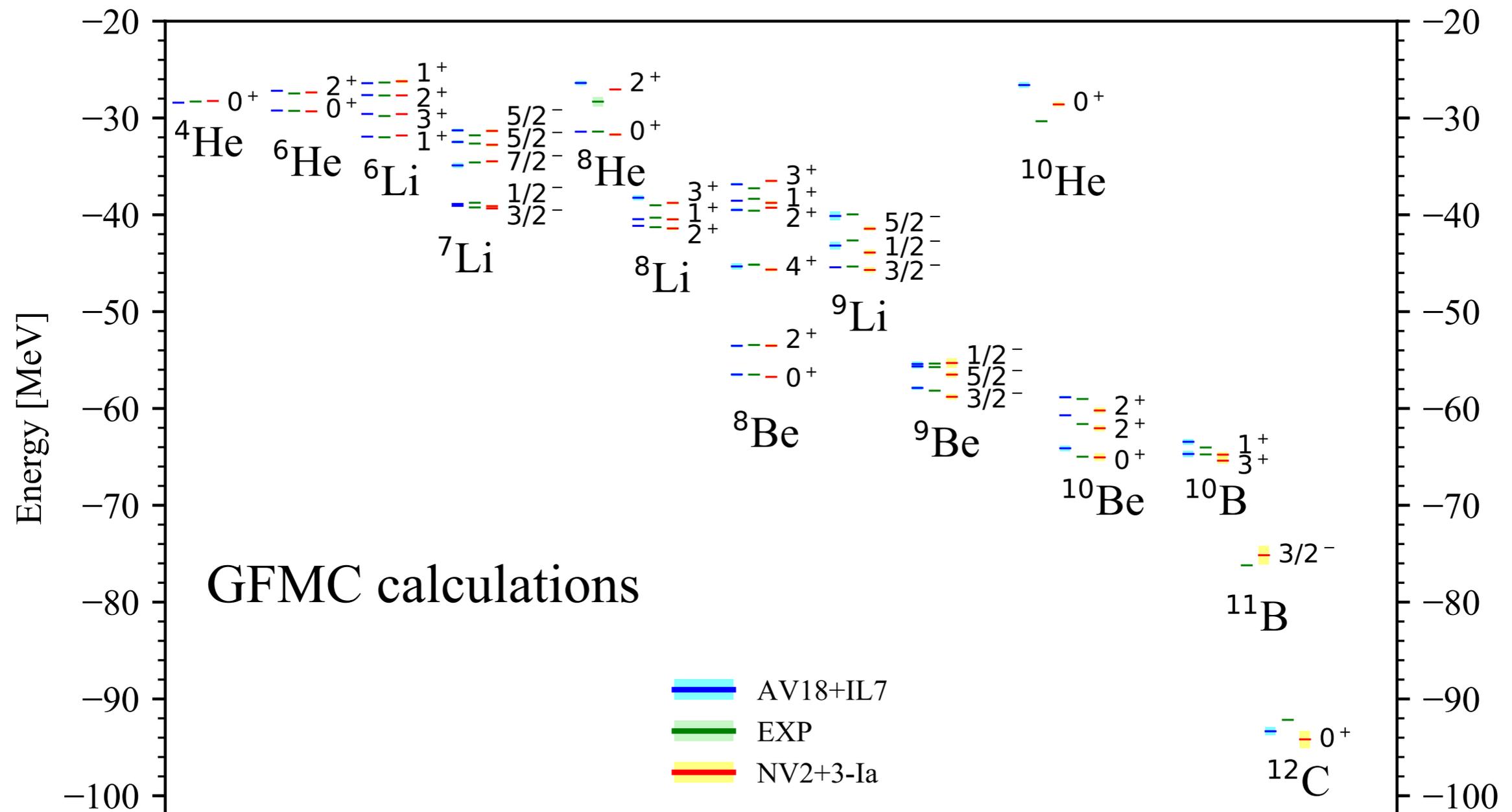
- Closer connection with QCD
- Consistent MEC being constructed
- Reliable theoretical uncertainty estimation

model	order	$E_{\text{Lab}}$ (MeV)	$N_{pp+np}$	$\chi^2/\text{datum}$
$b$	LO	0–125	2558	59.88
$b$	NLO	0–125	2648	2.18
$b$	N2LO	0–125	2641	2.32
$b$	N3LO	0–125	2665	1.07
$a$	N3LO	0–125	2668	1.05
$c$	N3LO	0–125	2666	1.11
$\tilde{a}$	N3LO	0–200	3698	1.37
$\tilde{b}$	N3LO	0–200	3695	1.37
$\tilde{c}$	N3LO	0–200	3693	1.40



# $\Delta$ -full local chiral potential

The experimental  $A \leq 12$  ground- and excited state energies are very well reproduced by the local  $\Delta$ -full NN+NNN chiral interaction



# The linear method

- Automatic optimization techniques have been introduced in VMC and CVMC by Maria Piarulli and Diego Lonardonì
- In recent AFDMC calculations the stochastic reconfiguration (SR) method has been adopted.
- The linear method allows us to deal with a much larger number of variational parameters

