The curvature of the chiral pseudocritical line from lattice QCD

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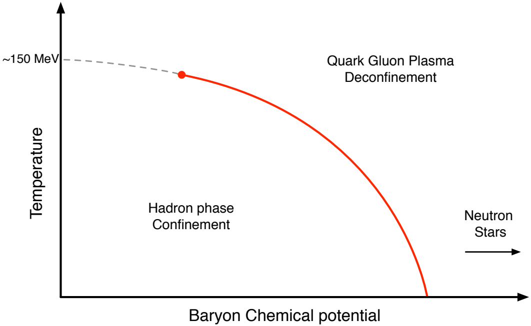
Outline

- Introduction and phenomenological motivations
- "Historical" motivation
- The curvature from analytic continuation
- The curvature from Taylor expansion (PRELIMINARY)
- Comparisons and conclusions

Introduction: QCD phase diagram

Quantum Chromo Dynamics (QCD): the quantum field theory describing quarks and gluons inside hadrons.

The QCD phase diagram is strongly related with the physics of HIC experiments. What do we know about the $T - \mu_B$ phase diagram? Not that much!



- Crossover transition to the QGP phase (at $\mu_B = 0$) located at $T_c \sim 150\ \mathrm{MeV}$
- Transition vacuum-ordinary matter (at $T = 0$)
- Critical endpoint (??)
- Other phases (??)

Due to the symmetry under charge conjugation, the pseudocritical line can be parametrized as:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + O(\mu_B^4)$$

Introduction: The Lattice QCD approach

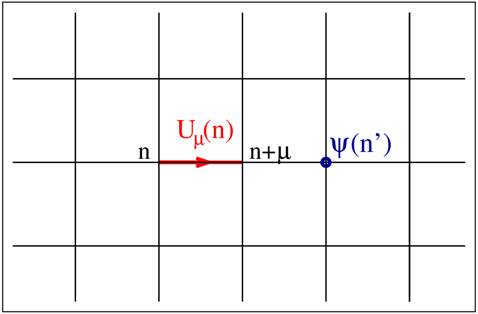
QCD is a Non Perturbative Theory ⟶ **LQCD** is the only reliable NP approach

Lattice QCD in 3 steps:

1– Feynman path integral formulation of the Euclidean theory

2– Regularization via a lattice:
UV cut-off → a IR cut-off → V

3– We get a well defined multidimensional integral
→ Monte Carlo approach



$$\langle O(\mu_B)\rangle = Z^{-1}\int \mathcal{D}[U] O[U,\mu_B] e^{-S_E[U,\mu_B]}$$

The Sign Problem

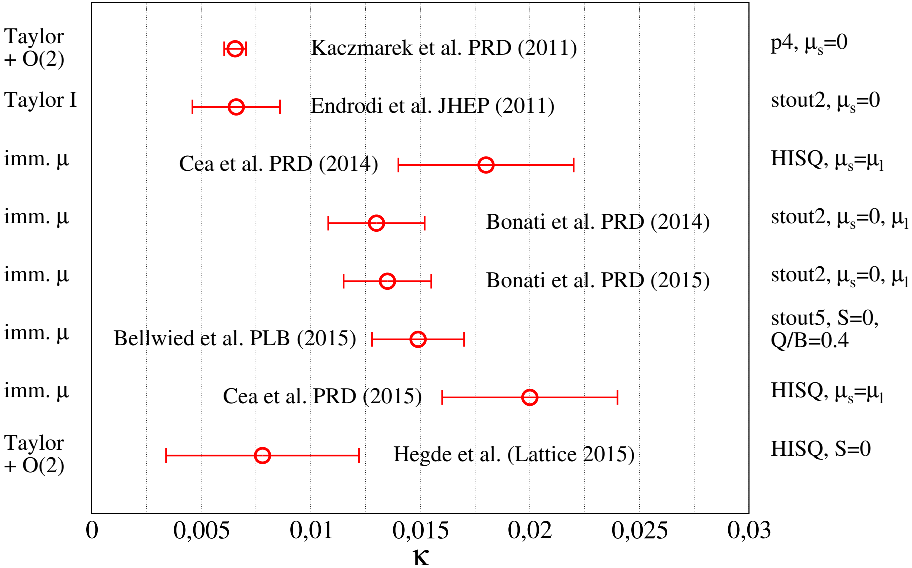
The region at $\mu_B \neq 0$ is anyhow unreachable!
The would be probability distribution $e^{-S_E[U,\mu_B]}$ gets complex . . .

Tackling the Sign Problem

Lattice QCD $\rightarrow$ first principle computations.
Anyhow, simulations are hindered by the Sign Problem at $\mu_B \neq 0$.

- Taylor expansion
  [Allton et al., 2002] [Kaczmarek et al., 2011] [Endrodi et al., 2011] [Borsanyi et al., 2012]
- Analytic continuation from Imaginary μ_B
  [de Forcrand et al., 2002] [D'Elia et al., 2003] [Azcoiti et al., 2005] [Wu, Luo and Chen, 2007] [Cea et al., 2008] [Nagata and Nakamura, 2011] [Laermann et al., 2013] [Bellwied et al., 2015]
- Reconstruction of the canonical partition function
  [Kratochvila and de Forcrand, 2005] [Alexandru et al., 2005]
- Reweighting [Fodor and Katz, 2002-2004]
- Density of state methods [Fodor et al., 2007] [Alexandru et al., 2015]
- Complex Langevin [Sexty, Aarts, Stamatescu, Jaeger, Seiler, ...]
- Dual variables [Gattringer, Bruckmann, Goschl, Marchis, ...]
- Lefschetz thimble [Di Renzo, Scorzato, Schmidt, ...]

Results for κ in the recent literature



Still no general consensus regarding κ

“Historical” motivation

The scattering of the data seems to indicate the presence of tensions, at least regarding Taylor expansion and analytic continuation.
The general question is: are systematics under control?

We considered different strategies and tried to take care of the systematics:

- Analytic continuation [Bonati et al. PRD 2014 & PRD 2015]
- Taylor expansion [PRELIMINARY]

We discretize $N_f = 2 + 1$ QCD at the physical point adopting:

- Tree level Symanzik improved gauge action
- Stout smearing improved staggered fermions (2 levels with $\rho = 0.15$)
- We adopt the line of constant physics determined in [Aoki *et al.*, 09] .

Simulations run on Fermi, Galileo and Marconi A2 at CINECA (Bologna, Italy), and on a GPU cluster in Pisa.

Analytic continuation from Imaginary μ_B

$$e^{-S_E[U,\mu_B]} \quad \text{is complex (Sign Problem)}$$

$$\text{BUT} \quad e^{-S_E[U,i\mu_B]} \equiv e^{-S_E[U,\mu_{B,I}]} \quad \text{is real and non} - \text{negative}$$

At $\mu_{B,I} \neq 0$ simulations are feasible!
By transforming $\mu_B \to i\mu_{B,I}$, the pseudocritical line parametrization becomes:

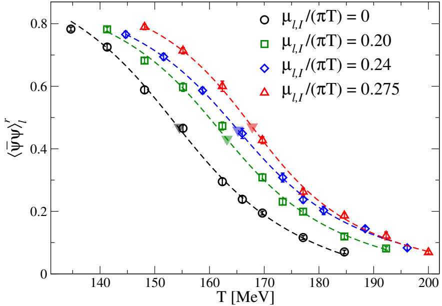
$$\frac{T_c(\mu_{B,I})}{T_c} = 1 + \kappa \left(\frac{\mu_{B,I}}{T_c}\right)^2 + O(\mu_{B,I}^4)$$

We simulated QCD at zero and non-zero $\mu_{B,I}$ and observed the behaviour of $T_c(\mu_{B,I})$.
We study the chiral symmetry restoration transition by computing chiral symmetry related observables:

1) Renormalized Chiral Condensate $\langle\bar{\psi}\psi\rangle_r$ (in two different ways)
2) Renormalized Chiral Susceptibility $\chi^r_{\bar{\psi}\psi}$

How can we determine $T_c(\mu_{B,I})$?

We fix $\mu_{\ell,I}/\pi T$ and we perform simulations at ~ 10 values of T. $\mu_{s,I} = 0$

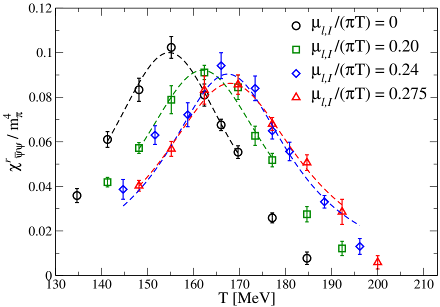


We fit the renormalized chiral condensates (I) and (II) with:

$$\langle\bar\psi\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\left(T - T_c\right)\right]$$

We fit the peak of the renormalized chiral susceptibility:

$$\chi^r_{\bar\psi\psi}(T) = \frac{A_2}{(T - T_c)^2 + B_2^2}$$

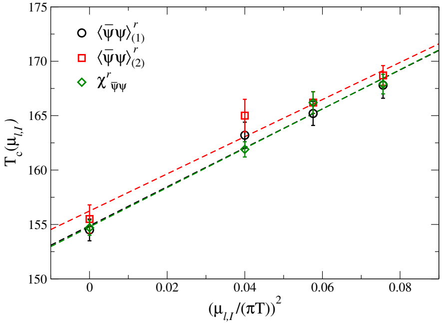


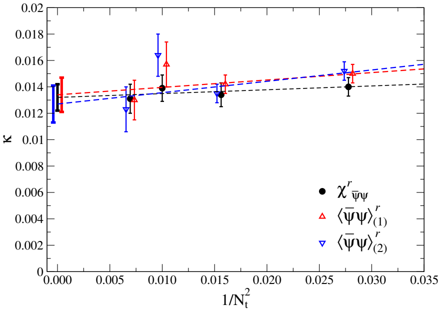
Data on $48^3 \times 12$ [Bonati et al., 15]

Continuum Limit I

T_c depends linearly on $(\mu_{\ell,I}/(\pi T))^2$. The slope is κ.

Repeat for $N_t = 6, 8, 10$ and 12. Take the continuum limit assuming $a^2 \propto N_t^{-2}$ corrections.





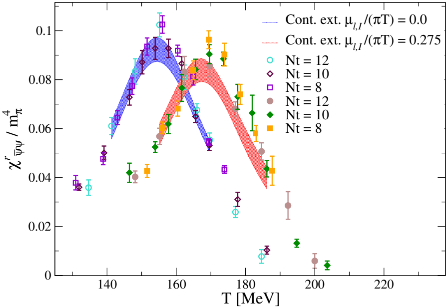
Curvatures in the Continuum Limit I

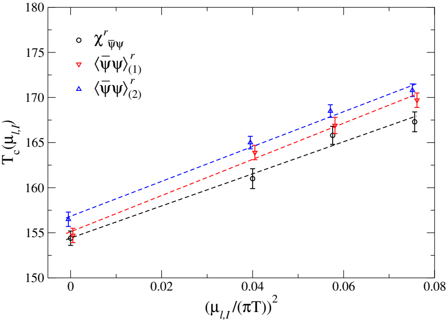
$$\kappa_{\bar{\psi}\psi,1} = 0.0127(14)$$
$$\kappa_{\bar{\psi}\psi,2} = 0.0134(13)$$
$$\kappa_{\chi} = 0.0132(10)$$

We explored also another possibility:

1- obtain the continuum limit extrapolated values of T_c at fixed $\mu_{\ell,I}/(\pi T)$

2- determine the curvature (which is already in the continuum)





The values of κ we get are compatible with C.L. I.

Considering the various systematics discussed,

our estimate for the curvature is $\kappa = 0.0135(20)$.

Taylor Expansion I

Taylor expansion: stay at $\mu = 0$ and compute derivatives with respect to μ.
Let $\mathcal{O}(T,\mu)$ be the chiral condensate. By expanding it close to $\mu = 0$ we get

$$\boxed{\mathcal{O}(T,\mu) \simeq \mathcal{O}(T,0) + \mu^2 \partial_{\mu^2}\mathcal{O}(T,0) \equiv A(T) + \mu^2 B(T)}$$

Given the value of $T_c = T_c(\mu = 0)$, we assume (following [Endrodi et al., 2011]) the pseudocritical line $T_c(\mu)$ to be defined by:

$$\begin{aligned}
\mathcal{O}(T_c, 0) &= \mathcal{O}(T_c(\mu), \mu) \\
A(T_c) &= A(T_c(\mu)) + \mu^2 B(T_c(\mu)) \\
\cancel{A(T_c)} &= \cancel{A(T_c)} + (T_c(\mu) - T_c)A'(T_c) + \mu^2 (B(T_c) + (T_c(\mu) - T_c)B'(T_c))
\end{aligned}$$

We adopt the standard choice $T_c(\mu) = T_c(1 - \kappa(\mu/T_c)^2)$. The previous conditions then reads

$$0 = -\kappa \frac{\mu^2}{T_c} A'(T_c) + \mu^2 B(T_c) - \kappa \frac{\mu^4}{T_c} B'(T_c) \quad \longrightarrow \quad \boxed{\kappa_{\mathrm{I}} = T_c \frac{B(T_c)}{A'(T_c)}}$$

$$\kappa_{\mathrm{I}} = T_c(0) \left(\left.\frac{\partial \mathcal{O}}{\partial \mu^2}\right|_{\mu=0,\, T=T_c(0)} \right) \Big/ \left(\left.\frac{\partial \mathcal{O}}{\partial T}\right|_{\mu=0,\, T=T_c(0)} \right)$$

Taylor Expansion II

Can we get rid of the assumption $\mathcal{O}(T_c, 0) = \mathcal{O}(T_c(\mu), \mu)$? Of course!

Since $\mathcal{O}$ is the chiral condensate we can locate $T_c(\mu)$ by imposing to be at an **inflection point**: $\mathcal{O}''(T, \mu) = 0$.

$\mathcal{O}''(T) = A''(T) + B''(T)\mu^2 = 0$

We further expand close to T_c
$0 = \cancel{A''(T_c)} + A'''(T_c)(T_c(\mu) - T_c) + \mu^2(B''(T_c) + B'''(T_c)(T_c(\mu) - T_c))$.

Solving for $T(= T_c(\mu))$ and taking the quadratic in μ part we get
$\kappa_{\mathrm{II}} = T_c \frac{B''(T_c)}{A'''(T_c)}$ (while $\kappa_{\mathrm{I}} = T_c \frac{B(T_c)}{A'(T_c)}$)

Hence, the ingredients to determine k_{II} are, apart from $T_c(\mu = 0)$,
$A'''(T_c) = \frac{\partial^3}{\partial T^3} \langle \overline{\psi}\psi \rangle \Big|_{T=T_c}$ and $B''(T_c) = \frac{\partial^2}{\partial T^2} \left(\frac{\partial \langle \overline{\psi}\psi \rangle}{\partial \mu^2} \right)\Big|_{\mu=0}\Bigg|_{T=T_c}$

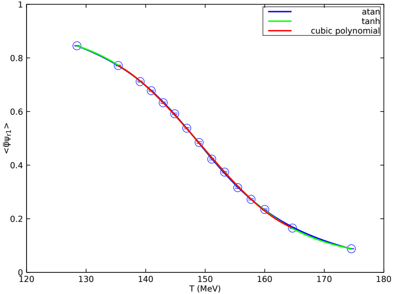
More derivatives $\longrightarrow$ Much more noisy . . .

Results from Taylor I and II (PRELIMINARY)

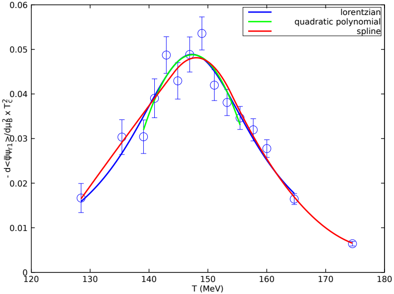
Observables contains many derivatives: we need higher statistics with respect to the analytic continuation approach. Noisy vectors $8 \longrightarrow 256$.
Numerical cost: 3 Millions core hours on Marconi A2 + GPU cluster in Pisa.

Results on the $24^3 \times 6$ lattice

$A(T)$



$B(T)T_c^2$



Ansatz	$A'(T_c)$	$A'''(T_c)$
atan	-0.0270(5)	$3.0(5)\cdot 10^{-4}$
tanh	-0.0269(5)	$2.7(4)\cdot 10^{-4}$
cubic	-0.0267(4)	$2.2(3)\cdot 10^{-4}$

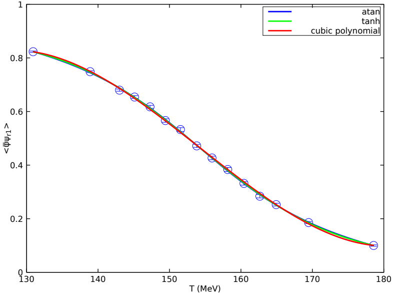
Ansatz	$B(T_c)T_c^2$	$B''(T_c)T_c^2$
Lorentzian	0.048(2)	$-5.1(8)\cdot 10^{-4}$
quadratic	0.048(2)	$-4.8(9)\cdot 10^{-4}$
spline	0.048(2)	$-4.6(8)\cdot 10^{-4}$

Results from Taylor I and II (PRELIMINARY)

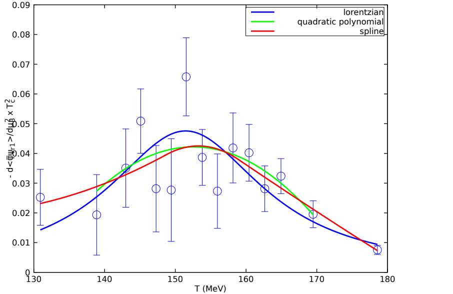
Observables contains many derivatives: we need higher statistics with respect to the analytic continuation approach. Noisy vectors $8 \longrightarrow 256$.
Numerical cost: 3 Millions core hours on Marconi A2 + GPU cluster in Pisa.

Results on the $32^3 \times 8$ lattice

$A(T)$



$B(T)T_c^2$



Ansatz	$A'(T_c)$	$A'''(T_c)$
atan	-0.0225(6)	$1.8(5) \cdot 10^{-4}$
tanh	-0.0223(6)	$1.5(4) \cdot 10^{-4}$
cubic	-0.0219(4)	$1.2(3) \cdot 10^{-4}$

Ansatz	$B(T_c)T_c^2$	$B''(T_c)T_c^2$
Lorentzian	0.046(5)	$-4(2) \cdot 10^{-4}$
quadratic	0.041(5)	$-1.6(6) \cdot 10^{-4}$
spline	0.042(6)	$-3(2) \cdot 10^{-4}$

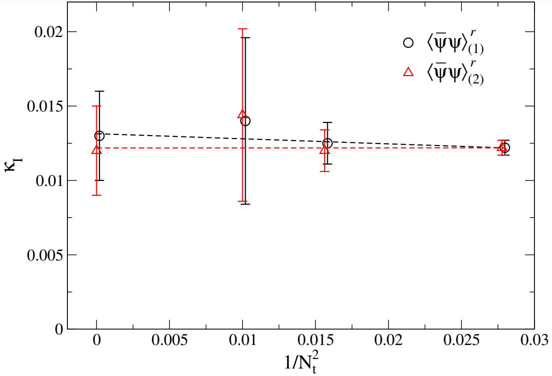
Results from Taylor I and II (PRELIMINARY)

$\langle\overline{\psi}\psi\rangle_1^r$

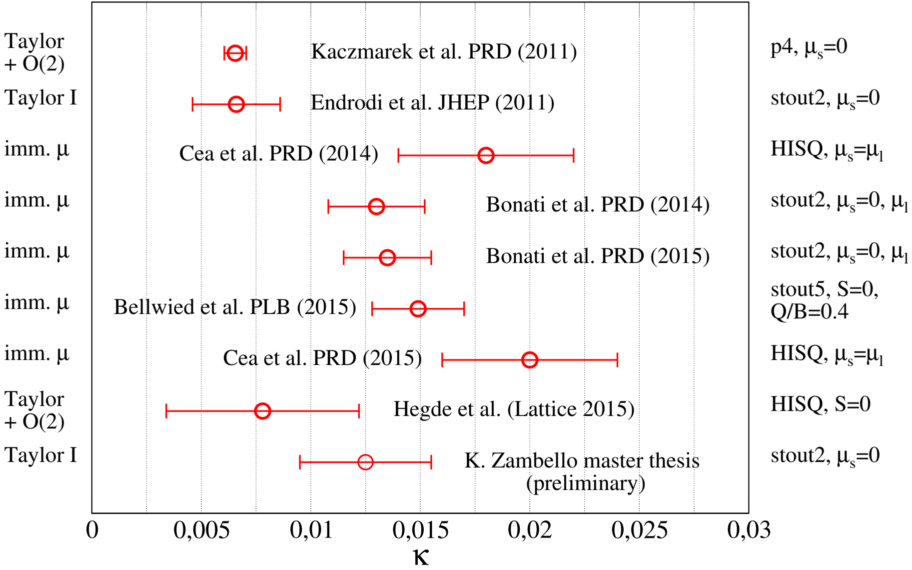
N_t	κ_{I}	κ_{II}
6	0.0122(5)	0.013(4)
8	0.0125(14)	0.014(10)
10	0.0140(56)	-
∞	0.013(3)	-

$\langle\overline{\psi}\psi\rangle_2^r$

N_t	κ_{I}	κ_{II}
6	0.0122(5)	0.013(4)
8	0.0120(14)	0.013(9)
8	0.0144(58)	-
∞	0.012(3)	-



Final comparisons



Summary and conclusions

- We determined the curvature κ of the chiral pseudocritical line in the continuum limit (via $N_t = 6,\ 8,\ 10$ and 12) by means of the analytic continuation method.

- We also discussed many systematics observing that systematic and statistical errors are comparable in size.
(finite volume, range of μ, chemical potential setup, choice of observable and renormalization, way to approach the continuum limit)

- We considerd Taylor expansion method I and proposed method II
  - Results on $N_t = 6,\ 8,\ 10$ lattices are still preliminary
  - We need to improve $N_t = 10$ to determine κ_{I} in the continuum limit.
  - Although independent on any assumption, κ_{II} has large errors.

- All the values of κ we get are compatible one each other.

THANK YOU
FOR YOUR ATTENTION!