



Modeling gravitational-waves from binary neutron stars



Note: [LVC] = Plots/figures from LIGO&Virgo collaboration

GW150914 : GW astronomy has started

September, 14th 2015, 09:50:45 UT







- 1st GW detected on Earth
- 1st GW from BHs
- 1st BBH merger
- Massive stellar-mass BBH population
- Test of GR in strong/dynamical regime
- 5 BBH events since 2015

["Observation of Gravitational Waves from a Binary Black Hole Merger" LIGO&Virgo collab. Phys. Rev. Lett. 116, 061102 (2016)]

GW170817 : Multimessenger astronomy !

["GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral" LIGO&Virgo collab. Phys. Rev. Lett. 119, 161101 (2017)]



GW170817: Gravitational-waves & light from (likely) binary neutron star inspiral

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GW170817: Gravitational-waves & light from (likely) binary neutron star inspiral (cont.)



What did we learn from GW170817 ?

• GW → masses

Source is most likely a binary neutron star system (BNS)

• GW → tidal parameters

Constraints on NS matter

• GW+sGRB

Connection between BNS and sGRB

• UV/O/IR → Kilonova

Radioactive decay of r-process nuclei

Main channel of formation of heavy elements

Matter ejecta from BNS mergers are sites for r-process nucleosinthesis (ID from GW)

• EM+GW \rightarrow Hubble constant

Cosmography, independent measure

[LVC] GW170817 FACTSHEET							
LIGO-Hanford	LIGO-Livingston	Virgo					
observed by	H, L, V	inferred duration from 30	~ 60 s				
source type	binary neutron star (NS)	Hz to 2040 Hz					
date	17 August 2017	from 30 Hz to 2048 Hz**	~ 3000				
time of merger	12:41:04 UTC	initial astronomer alert	27 min				
signal-to-noise ratio	32.4	latency*					
false alarm rate	< 1 in 80 000 years	HLV sky map alert latency*	5 hrs 14 min				
d'alana a	85 to 160 million	HLV sky area†	28 deg ²				
distance	light-years	# of EM observatories that	~ 70				
total mass	2.73 to 3.29 M	followed the trigger					
primary NS mass	1.36 to 2.26 M,	also observed in	gamma-ray, X-ray, ultraviolet, optical, infrared, radio				
secondary NS mass	0.86 to 1.36 M _*						
mass ratio	0.4 to 1.0	host galaxy	NGC 4993				
radiated GW energy	> 0.025 M _* c ²	source RA, Dec	13'09"48", -23"22'53"				
radius of a 1.4 M _a NS	likely ≈ 14 km	sky location	in Hydra constellation				
effective spin parameter	-0.01 to 0.17	viewing angle (without and with host galaxy identification)	≤ 56° and ≤ 28°				
spin parameter	unconstrained	Justify constant informed					
GW speed deviation from speed of light	< few parts in 10 ¹⁵	from host galaxy identification	62 to 107 km s ⁻¹ Mpc ⁻¹				
0° 15b 12b -30°	96 30° 75	Images: time frequency tra (left, HL = light blue, improved HL\ optical source locat GW=gravitational wave, t M_=1 solar mas H/L=LIGO Hanford/Li Parameter ranges are 90 *referenced to the **maximum likelit 790% credib	ces (top), GW sky map HLV = dark blue, V = green, ion = cross-hair) EM = electromagnetic, s=2x10 ³⁰ kg, ivingston, V=Virgo 0% credible intervals. time of merger hood estimate le region				

GWs: Tiny signatures of extreme events

Collision of neutron stars [Mass~1.4 Msun, Radius~10 km]:



First numerical relativity simulation of neutron star merger with precessing spins: the double pulsar case



Viz by T.Dietrich

Baryon mass density

First numerical relativity simulation of neutron star merger with precessing spins: the double pulsar case



Viz by T.Dietrich

Weyl curvature scalar

The GW spectrum of binary neutron stars







- Faithful and **complete waveform model** (*inspiral+merger+postmerger*)
- Coverage of the **parameter space** (mass, spins, EOS, ...)
- Precise prediction of the merger remnant (e.g. collapse, black hole)

First waveform model for inspiral → merger

[SB,Nagar,Dietrich,Damour PRL 114 (2015)]



- Effective-one-body model with tides, GSF Resummed approach [Bini+ 2014]
- Valid from low frequencies to merger, PREDICT the merger waveform
- Accuracy: uncertainties of the numerical data (improve simulations!)

See [Hinderer+ PRL 116 (2016)] for an alternative approach

Methods for the GR 2-body problem



$$\begin{split} \partial_t \tilde{\Gamma}^i &= -2\,\tilde{A}^{ij}\,\partial_j \alpha + 2\,\alpha \left[\tilde{\Gamma}^i{}_{jk}\,\tilde{A}^{jk} - \frac{3}{2}\,\tilde{A}^{ij}\,\partial_j \ln(\chi) \right. \\ &\left. -\frac{1}{3}\,\tilde{\gamma}^{ij}\,\partial_j(2\,\hat{K} + \Theta) - 8\,\pi\,\tilde{\gamma}^{ij}\,S_j \right] + \tilde{\gamma}^{jk}\,\partial_j\partial_k\beta \\ &\left. + \frac{1}{3}\,\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\,\partial_j\tilde{\Gamma}^i - (\tilde{\Gamma}_d)^j\,\partial_j\beta^i \right. \\ &\left. + \frac{2}{3}\,(\tilde{\Gamma}_d)^i\,\partial_j\beta^j - 2\,\alpha\,\kappa_1\,\left[\tilde{\Gamma}^i - (\tilde{\Gamma}_d)^i\right], \right. \\ \left. \partial_t\Theta &= \frac{1}{2}\,\alpha\left[R - \tilde{A}_{ij}\,\tilde{A}^{ij} + \frac{2}{3}\,(\hat{K} + 2\,\Theta)^2\right] \\ &\left. - \alpha\left[8\,\pi\,\rho + \kappa_1\,(2 + \kappa_2)\,\Theta\right] + \beta^i\partial_i\Theta\,, \end{split}$$

GR Formulation and Cauchy problem + GR hydrodynamics



Coordinates and Singularities

Numerical relativity in a nutshell

Numerical methods for PDEs on adaptive grids



High-performance-computing (HPC)



Numerical relativity: Cauchy problem in GR





- 3+1 formulation (hyperboloidal slices?)
- Initial data (Lichnerowic, York, ...)
- Evolution schemes (GHG; ADM \rightarrow BSSN, Z4c)
- Well posedness (Choquet-Bruhat; Friedrich; Gundlach&Martin-Garcia) [*need gauge fix*]

$$\begin{split} \partial_t \chi &= \frac{2}{3} \chi [\alpha(\hat{K} + 2\Theta) - D_i \beta^i], \\ \partial_t \tilde{\gamma}_{ij} &= -2\alpha \tilde{A}_{ij} + \beta^k \tilde{\gamma}_{ij,k} + 2\tilde{\gamma}_{k(i} \beta^k_{,j)} - \frac{2}{3} \tilde{\gamma}_{ij} \beta^k_{,k} , \\ \partial_t \hat{K} &= -D^i D_i \alpha + \alpha [\tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} (\hat{K} + 2\Theta)^2] \\ &+ 4\pi \alpha [S + \rho_{\rm ADM}] + \alpha \kappa_1 (1 - \kappa_2) \Theta + \beta^i \hat{K}_{,i} \\ \partial_t \tilde{A}_{ij} &= \chi [-D_i D_j \alpha + \alpha (R_{ij} - 8\pi S_{ij})]^{\rm tf} \\ &+ \alpha [(\hat{K} + 2\Theta) \tilde{A}_{ij} - 2\tilde{A}^k_i \tilde{A}_{kj}] \\ &+ \beta^k \tilde{A}_{ij,k} + 2\tilde{A}_{k(i} \beta^k_{,j)} - \frac{2}{3} \tilde{A}_{ij} \beta^k_{,k} \\ \partial_t \tilde{\Gamma}^i &= -2 \tilde{A}^{ij} \alpha_{,j} + 2\alpha [\tilde{\Gamma}^i_{jk} \tilde{A}^{jk} - \frac{3}{2} \tilde{A}^{ij} \ln(\chi)_{,j} \\ &- \frac{1}{3} \tilde{\gamma}^{ij} (2\hat{K} + \Theta)_{,j} - 8\pi \tilde{\gamma}^{ij} S_j] + \tilde{\gamma}^{jk} \beta^i_{,jk} \\ &+ \frac{1}{3} \tilde{\gamma}^{ij} \beta^k_{,kj} + \beta^j \tilde{\Gamma}^i_{,j} - \tilde{\Gamma}_d{}^j \beta^i_{,j} + \frac{2}{3} \tilde{\Gamma}_d{}^i \beta^j_{,j} \\ &- 2\alpha \kappa_1 (\tilde{\Gamma}^i - \tilde{\Gamma}_d{}^i), \\ \partial_t \Theta &= \alpha [\frac{1}{2} R - \frac{1}{2} \tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} (\hat{K} + 2\Theta)^2 \\ &- 8\pi \rho_{\rm ADM} - \kappa_1 (2 + \kappa_2) \Theta] + \mathcal{L}_\beta \Theta. \end{split}$$

[SB, Hilditch arXiv:0912.2920]

Numerical relativity: singularities & crash tests





- Crash at tau=pi (geodesic slicing)
- Lapse collapse, slice stretching (1+log, shift=0)
 e.g. [Bruegmann arXiv:9912009]

Numerical relativity: singularities & coordinates



[Thierfelder, SB, Hilditch, Bruegmann, Rezzolla arXiv:1012.3703]

Numerical relativity: numerical methods (some)

Adaptive mesh refinement (AMR) \rightarrow resolve multiple scales





Grid based, AMR Berger-Oliger Method of line w\ Runge-Kutta (Subcycling) Finite differencing and finite volumes Numerical relativity specs

- R.H.S. complexity (derivatives and contractions)
- \rightarrow stencil ("horizontal") + pointwise ("vertical") ops
- High-order operators (large 3D stencils > 5 pts/direction)
- \rightarrow communication overhead for distributed computations
- Memory: >~ O(100) 3D grid function per time level

Improved NR GW with high-order WENO schemes

[SB,Dietrich PRD94 064062 (2016)]



- Robust convergence assessment (although not 5th order)
- Large resolution span (64³-192³), no alignment
- Error budget: significant improvement wrt FV schemes

See also [SB+ arXiv:1205.3403] [Radice+ arxiv:1306.6052]

Spins & tides during merger: phasing

[Dietrich, SB, Ujevic, Tichy PRD 95 (2017)]



Closed-form tidal approximants from NR

[Dietrich, SB, Tichy arXiv:1706.02969]

First NR-based tidal approximant Fast, flexible, accurate



Exploring the BNS parameter space





Effective-One-Body

[Buonanno&Damour PRD 1999,2000]



- Includes test-mass limit (i.e. particle on Schwarzschild)
- Includes post-Newtonian and self-force results
- Uses resummation techniques \rightarrow predictive strong-field regime
- Includes tidal interactions (→ BNS) [Damour&Nagar PRD 2010]
- Flexible framework, can include NR results ("NR-informed")
- Most accurate framework to describe compact binary waveforms

See e.g. [Taracchini+ PRD 2014][SB+ PRL 2015][Nagar+ PRD 2015][Hinderer+ 2016]

Relativistic Tides





[Hinderer arXiv:0711.2420, Damour&Nagar arXiv:0906.0096, Binnington&Poisson arXiv:0906.1366]

$$k_2^T = 2\left[\frac{X_A}{X_B}\left(\frac{X_A}{C_A}\right)^5 k_2^A + \frac{X_B}{X_A}\left(\frac{X_B}{C_B}\right)^5 k_2^B\right]$$

[Damour&Nagar arXiv:0911.5041]

Tidal contribution to (post-) Newtonian dynamics and waveform:

Hamiltonian (Newtonian limit):

$$\begin{array}{l} H_{\rm EOB} \approx Mc^2 + \frac{\mu}{2} \left({\bf p}^2 + A(r) - 1 \right) \\ A(r) = 1 - 2/r - \kappa_2^T(\lambda_2)/r^6 \\ & \text{Tides are attractive and "act" at small separations} \\ & \text{Tidal coupling constant} \\ \end{array}$$
Waveform:

$$h \sim Af^{-7/6}e^{-i\Psi(f)} \approx Af^{-7/6}e^{-i\Psi_{PP}(f) + i39/4\kappa_2^T x(f)^{5/2}}$$

Key point: No other binary parameter (mass, radii, etc) enter separately the formalism

One parameter to characterize merger dynamics

[SB,Nagar,Balmelli,Dietrich,Ujevic PRL 112 (2014)]

Predict energy emitted in GW for all binaries, range 1-2% M (all possible EOS, masses, mas-ratios)

Predict energy emitted for given binary by specifying solely the kappa value



Tidal polarizability coef. (I=2)

Inspiral - merger → postmerger





Postmerger models spectrum



- Various models associate f₂ to isolated equil. star properties
- Possibility to extract "EOS-related info" (R_x, M_{max},...)
- Conceptually indepedent on inspiral-merger models

[Bauswein+ arXiv:1106.1616, Hotokezaka+ arXiv:1307.5888, Takami+ arXiv:1403.5672, Clark+ arXiv:1509.08522, ...]

Peak frequency correlates to tidal parameter

[SB, Dietrich, Nagar PRL 115 (2015)]



Large NR dataset (~100, 3 codes) [+ Hotokezaka+ arXiv:1307.5888, Takami+ arXiv:1403.5672]

- Postmerger frequencies essentially determined by *merger* physics
- Conceptually "compatible" with inspiral-merger → Unified model !

Remnant HMNS is the loudest GW phase

[SB, Radice, Ott, Roberts, Moesta, Galeazzi PRD94 024023 (2016)]



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Merger remnant reaches extreme densities

Can GW observations inform us about EOS changes at those densities?



- Baryon number density n ~ 3-5 n_{nuc}
- Extra DOF/phase transitions?
- Specific model: Λ-hyperons

[Banik+ arxiv:1404.6173]

Microphysical EOS compatibile with astro and nuclear phys constraints

In general: "softness" effects

GWs could probe such "softness effects"

[Radice, SB, Del Pozzo, Ott, Roberts ApJL (2017)]



log(Bayes factor) vs. Source distance

- Postmerger GW morfology contains unique info
- Detailed and generic models are necessary for DA studies
- High-freq. GW challenging to detect (\rightarrow Einstein telescope)

GW170817: A binary neutron star system

[LIGO&Virgo collab. Phys. Rev. Lett. 119, 161101 (2017)]



_VC]	GW17	70817	FACTSHEET
GO-Hanford		LIGO-Livingston	Virgo

60 s

3000

7 min

4 min deg² ~ 70

X-ray, ptical, radio

4993 22'53"

lation

≤ 28°

Mpc⁻¹

nap

observed by	H, L, V	inferred duration from 30 Hz to 2048 Hz**	
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Fundamental physics

Constraining the Equation of State of matter at supranuclear densities



Different EOS → different star's structure





Binary neutron star mergers

Example: observing tidal effects in GWs tells us about the neutron star matter



Tides determine the wave's phase during merger

Example: observing tidal effects in GWs tells us about the neutron star matter



Example: observing tidal effects in GWs tells us about the neutron star matter









Joint constraint on the neutron star equation of state from multimessenger observations



Summary

- Binary neutron stars key sources for GW astronomy
- Unique info about extreme matter
- GW measurements <u>require</u> precise waveform models
- Building GW models: interface analytical and numerical relativity method
- Strong-field GR-dynamics crucial input for electromagnetic emission models

