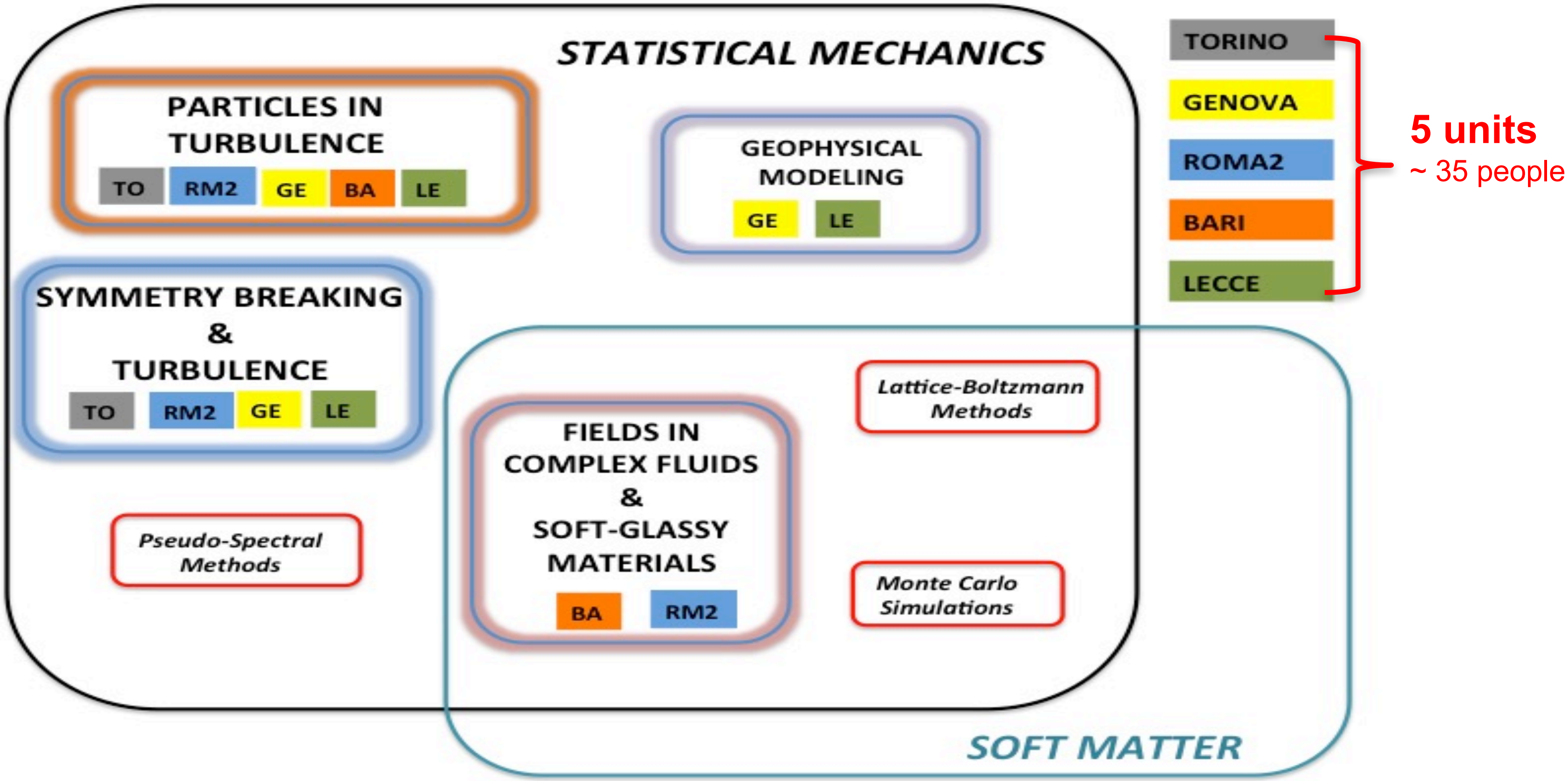
# *FIELDTURB*: PARTICLES AND FIELDS IN TURBULENCE AND IN COMPLEX FLOWS




**Guido Boffetta - Torino**
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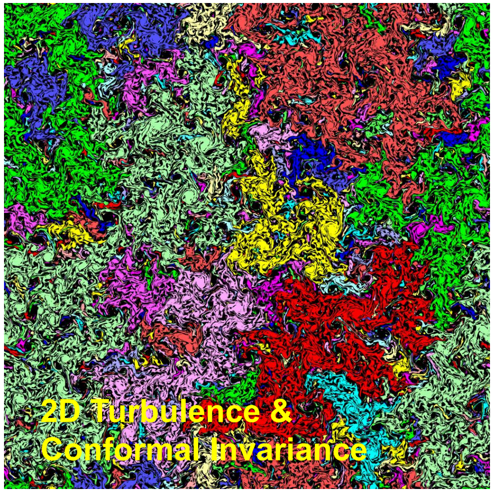
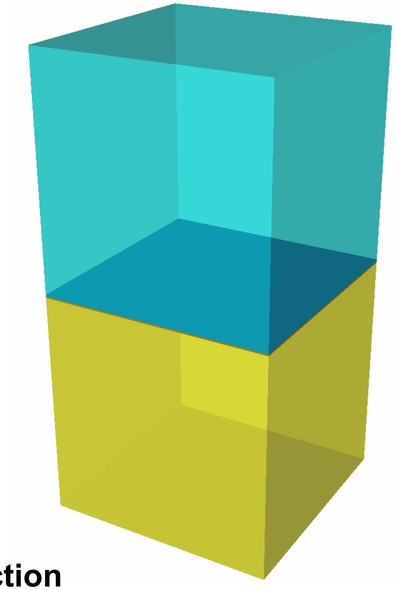
**Luca Biferale - Roma 2**
**Giuseppe Gonnella - Bari**
**Alessandra Lanotte - Lecce**

# HPC in turbulence

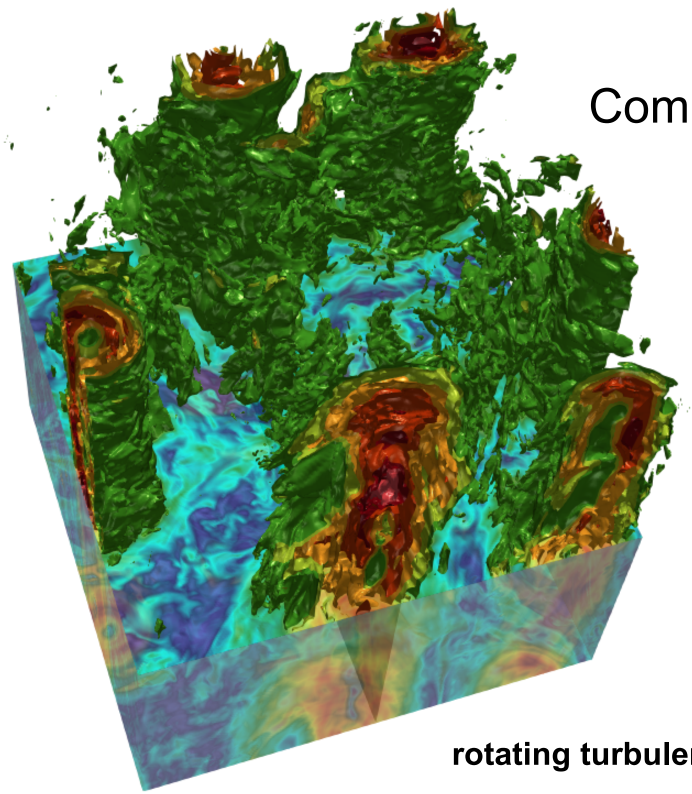



Pseudo-spectral method
Lattice-Boltzmann method
Immerse-boundary method

MPI-openMP decompositions



turbulent convection



rotating turbulence

Complementary to experiments (lower Re, full statistics)

A tool for investigating conditions non accessible to experiments:

- the effect of dimensionality (2D, quasi-2D, nD)
- role of conservation laws (more or less than the physical ones)
- role of boundary conditions (and absence)

# HPC projects and related activities

EUROPEAN JOINT DOCTORATE STIMULATE 2018-2022

**HPC-LEAP High Performance Computing in Life sciences, Engineering and Physics**

SUPERCOMPUTING GRANTS

**PRACE 2012** (22 Mh) No.04-806 *Eulerian and Lagrangian Turbulence over a reduced fractal skeleton*

**PRACE 2014** (55 Mh) No.09-2256 *Effect of Helicity and Rotation in Turbulent flows: Eulerian and Lagrangian statistics*

**PRACE 2015** (22 Mh) *Anisotropic Homogeneous Turbulence*

**PRACE 2016** (25 Mh) *How stratification, rotation and confinement impact on the turbulent mixing*

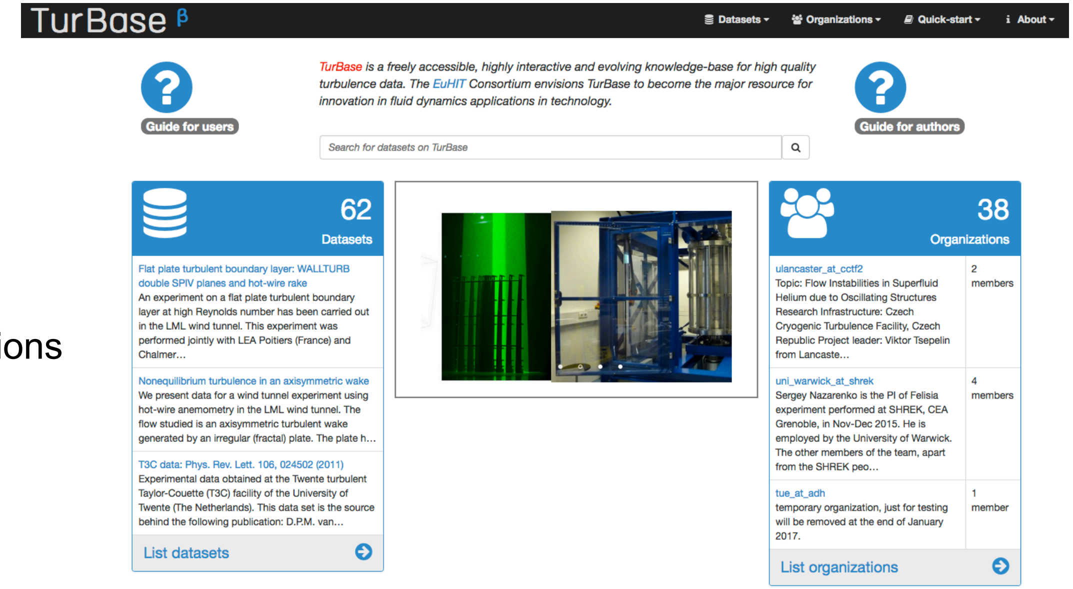
**PRACE 2018** (30 Mh) *Rayleigh-Taylor tubulence in complex fluids*

**ISCRA @ CINECA**

ISCRA A : 2010, 2011, 2013

ISCRA B : 2011, 2013, 2016

ISCRA C : 2015




OPEN ACCESS DATABASE

**TURBASE**

62 different datasets produced by 38 organizations

Developed within the EuHIT project

COMMUNITY SERVICES

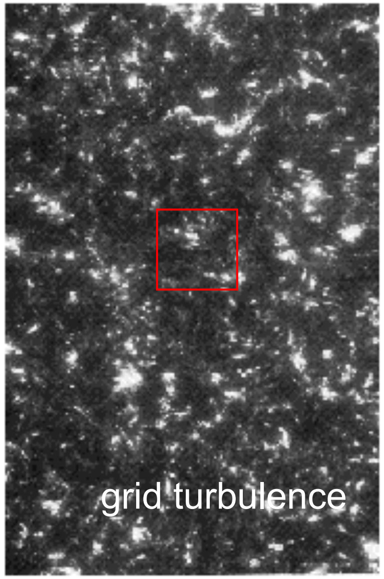
**EUDAT**

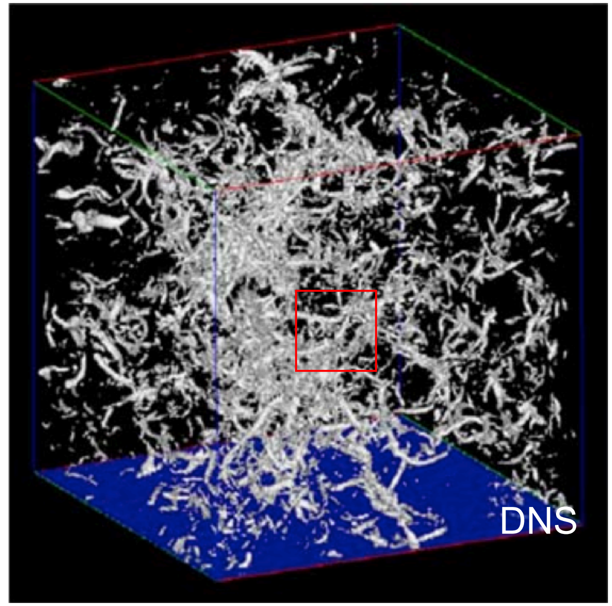
Collaborative Data Infrastructure for "*sharing data across borders and disciplines*"

funded by EU FP7 and H2020

# Why turbulence ?

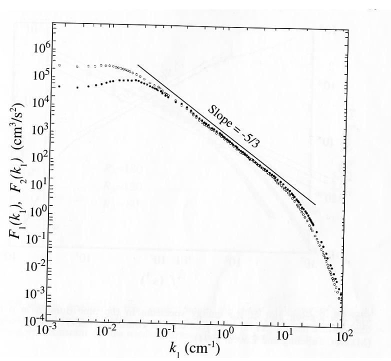
Classical non-linear field theory, out of equilibrium, non-perturbative with non Gaussian, anomalous fluctuations

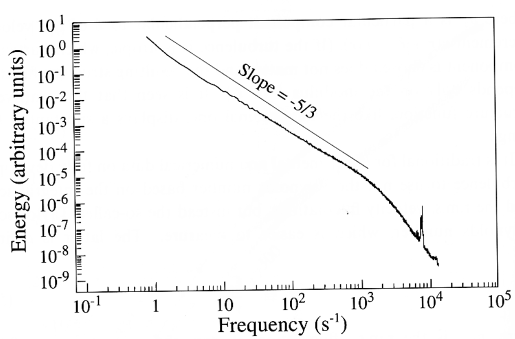




**Universality** of small scale statistics (for $Re \to \infty$)

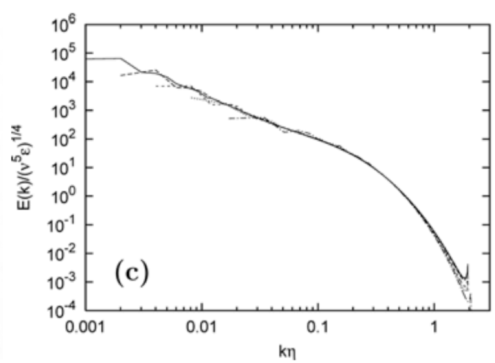
Kolmogorov spectrum $E(k) \simeq \varepsilon^{2/3} k^{-5/3}$



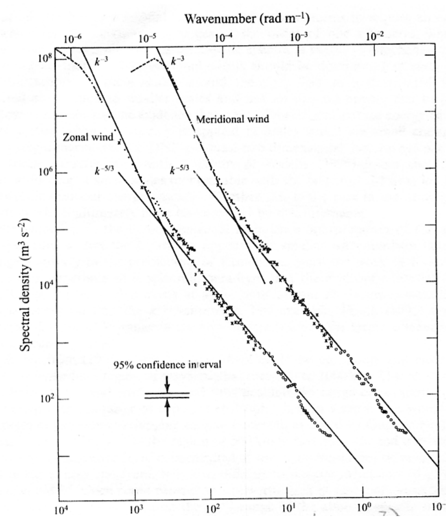

L=10 cm




L=10 m (Onera)



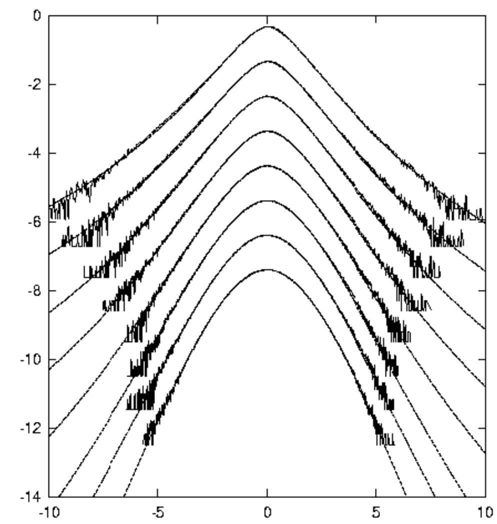

DNS




L=100 km

Main prediction of Kolmogorov theory

*4/5 law* $\langle (\delta_r u)^3 \rangle = -\frac{4}{5}\varepsilon r$ $\delta_r u = u_2 - u_1$




assuming self-similarity:

$\delta_r u \sim (\varepsilon r)^{1/3}$ h=1/3 global scaling exponent

and $E(k) \simeq \varepsilon^{2/3} k^{-5/3}$

# Anomalous scaling

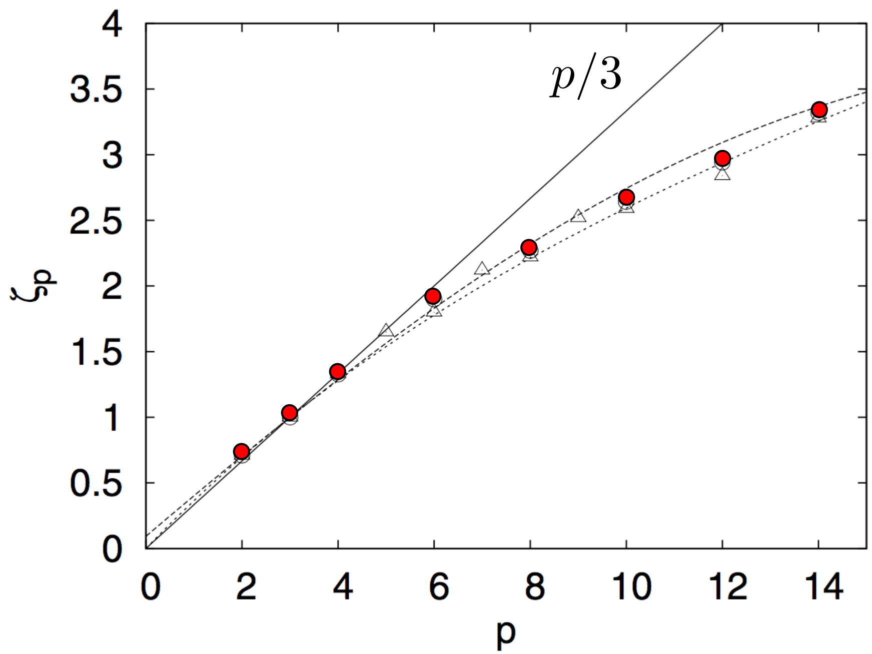
Empirical evidence that PDFs of velocity increments $\delta_r u$ are not self-similar in the inertial range of scales

No single exponent to characterize velocity structure functions

$$\langle (\delta_r u)^p \rangle \simeq (\varepsilon L)^{p/3} \left( \frac{r}{L} \right)^{\zeta(p)}$$

with

$$\zeta_p \neq p/3$$




Experimental data at $R_\lambda=850$
From G.Boffetta, A.Mazzino, A.Vulpiani,
J. Phys. A. Math. Theor. **41** 363001 (2008)

# Multifractal model [G. Parisi & U. Frisch, 1983]

$$\delta_r u \equiv (\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})) \cdot \frac{\mathbf{r}}{r}$$

**Idea**: replace global scaling exponent h=1/3 with a local exponent h distributed with probability P(h)

$$\delta_r u \sim r^h \qquad P(h) \sim r^{3-D(h)}$$

D(h): dimension of the set on which h is realized

Structure functions $$\langle (\delta_r u)^p \rangle \sim \int dh r^{ph+3-D(h)} \sim r^{\zeta(p)}$$

by saddle point $$\zeta(p) = \min_h [ph + 3 - D(h)]$$

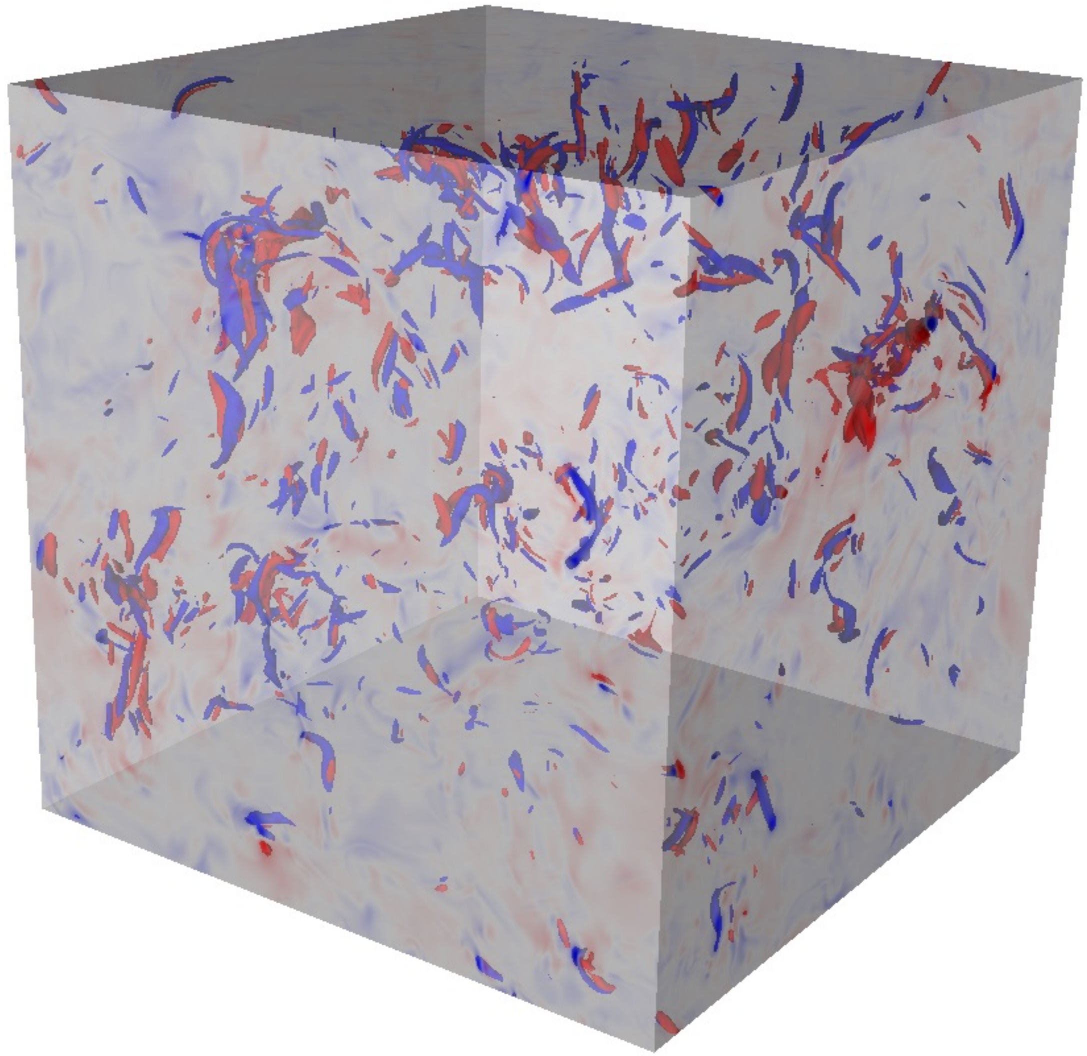
with $\zeta(3) = 1$ i.e. $D(h) \leq 3h + 2$

D(h) is not obtained by NS, but several phenomenological models for D(h)

R. Benzi, G. Paladin, G. Parisi, A. Vulpiani, *J. Phys. A: Math.Gen.***17**, 3521 (1984)
Z.S. She, E. Leveque, *Phys. Rev. Lett.* **72**, 336 (1994)

The MF model is a consistent framework for computing different statistical quantities in fully developed turbulence (one example follows)

# Single point time irreversibility in turbulence



M. Cencini, L. Biferale, G. Boffetta, M. De Pietro, *Phys. Rev. Fluids* **2**, 104604 (2017)

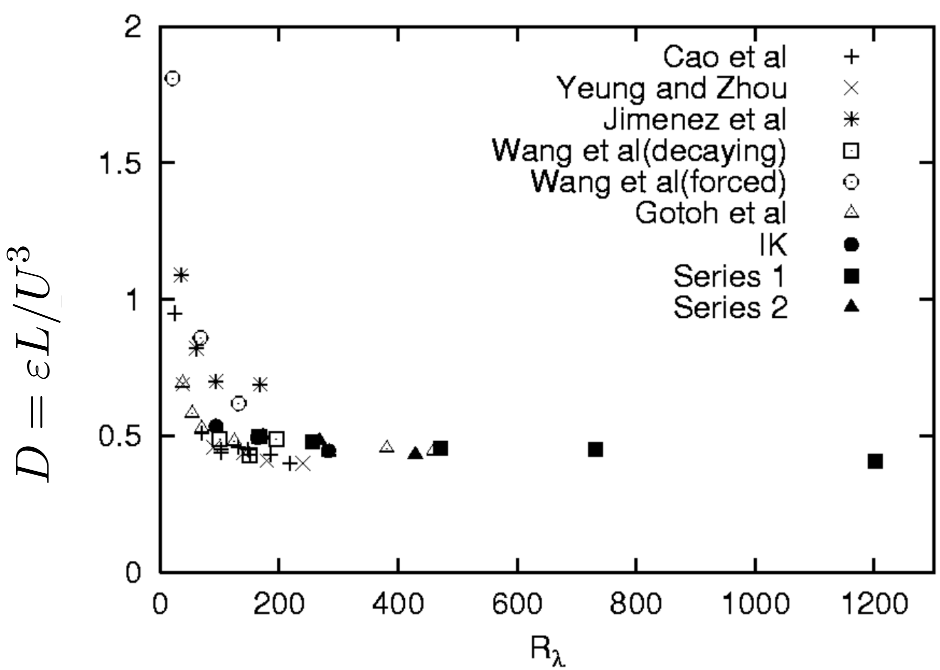
# Navier-Stokes and time-irreversibility

Euler $\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla \mathbf{p}$ time reversible $t \to -t$ $\mathbf{u} \to -\mathbf{u}$

NS $\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}$ time irreversible

## Turbulence at high Reynolds numbers is time irreversible

$$Re = \frac{UL}{\nu} \to \infty$$




**Dissipative anomaly**:
Viscous dissipation rate remains finite in the inviscid limit

$$\lim_{\nu\to 0} \nu\langle(\nabla\mathbf{u})^2\rangle = \varepsilon \neq 0$$

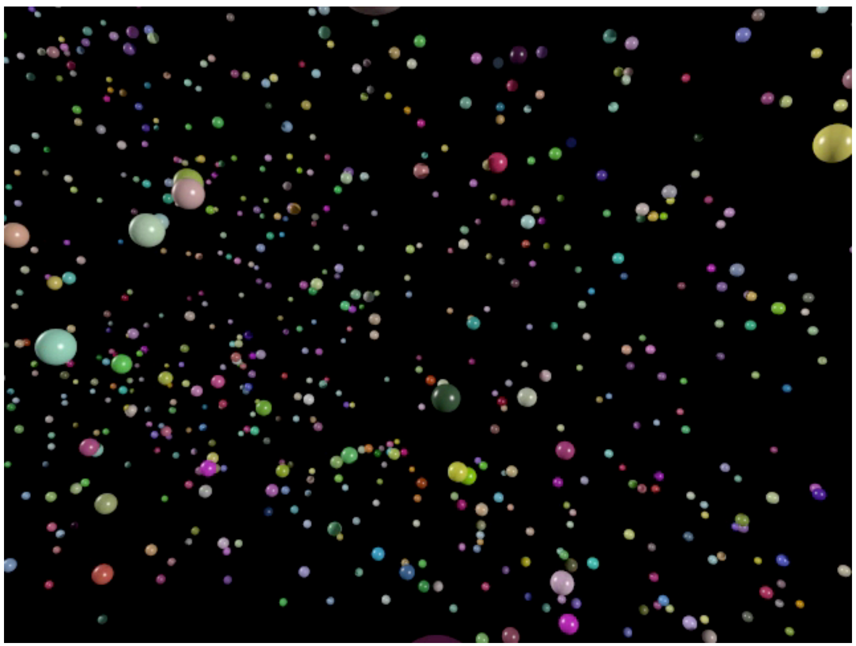
Y. Kaneda *et al.*, *Phys. Fluids* **15**, L21 (2003)

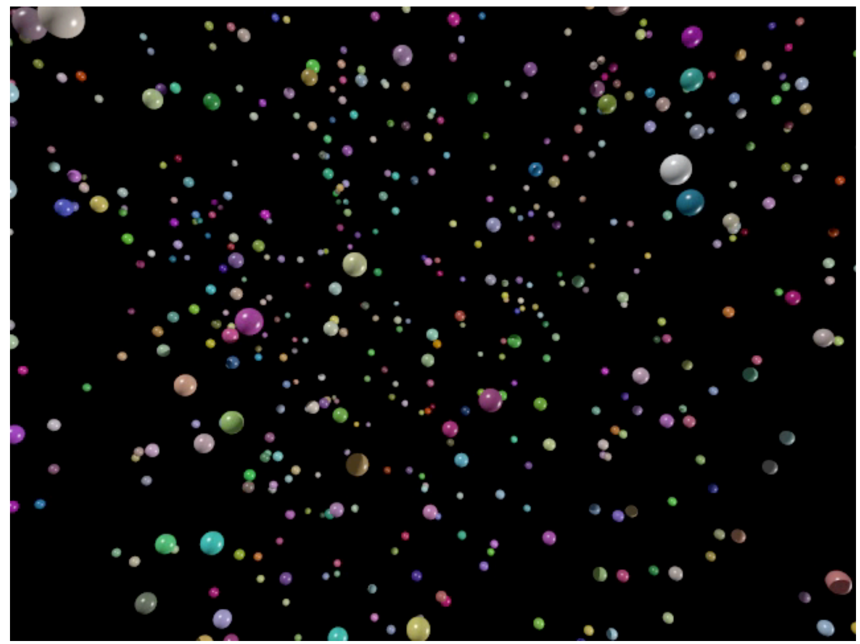
# Navier-Stokes and time-irreversibility

Dissipative anomaly $$\lim_{\nu \to 0} \nu \langle (\nabla \mathbf{u})^2 \rangle = \varepsilon \neq 0$$

"An inviscid-equation symmetry—in this case, time-reversal invariance—remains broken even as the symmetry- breaking viscosity becomes vanishingly small.
*A trained eye viewing a movie of steady turbulence run backwards can tell that something is indeed wrong!"*

G. Falkovich, K.R. Sreenivasan, *Phys. Today* **59**, 43 (2006)



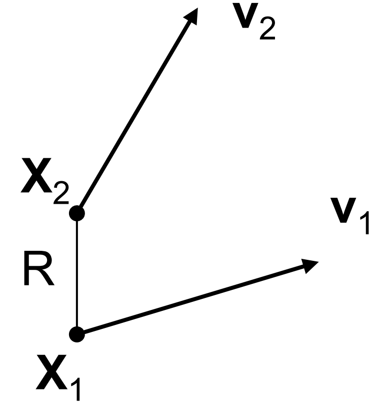


# Time irreversibility in the Lagrangian frame

Two point, one (**initial**) time

$$\frac{d}{dt}\langle|\mathbf{v}_2(t)-\mathbf{v}_1(t)|^2\rangle = -4\varepsilon$$

for particles at separation R in the inertial range




$$\mathbf{R}(t) = \mathbf{x}_2(t) - \mathbf{x}_1(t)$$

$$\delta\mathbf{v}(t) = \mathbf{v}_2(t) - \mathbf{v}_1(t)$$

Taylor expansion of particle separation

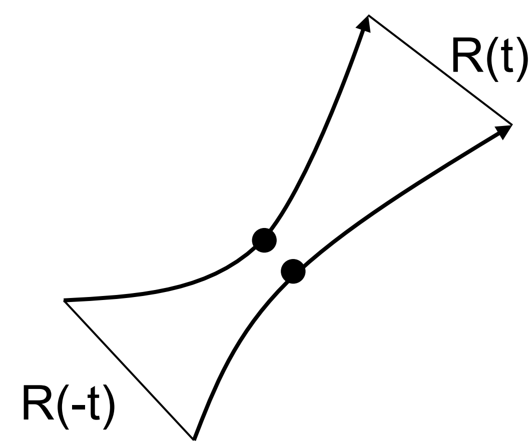
$$\begin{aligned}\langle R^2(t)\rangle &= R^2(0) + \langle\delta v(0)^2\rangle t^2 + \langle\delta\mathbf{v}(0)\cdot\delta\mathbf{a}(t)\rangle t^3 + \mathcal{O}(t^4)\\ &= R^2(0) + \langle\delta v(0)^2\rangle t^2 - 2\varepsilon t^3 + \mathcal{O}(t^4)\end{aligned}$$

(*anti-Richardson* dispersion)

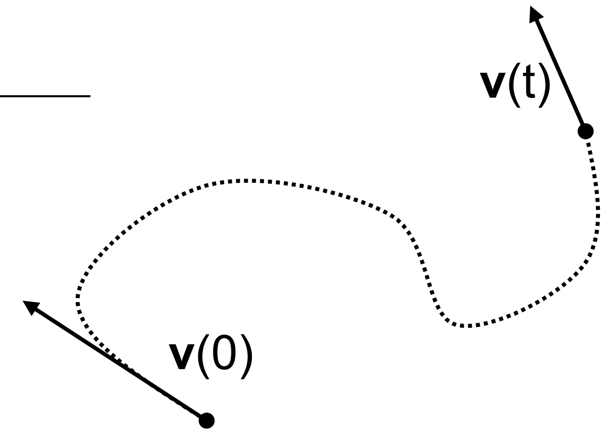
asymmetric relative dispersion at short time

$$\langle R^2(-t)\rangle - \langle R^2(t)\rangle = 4\varepsilon t^3 + \mathcal{O}(t^5)$$

BIT dispersion is faster than FIT



# Single particle statistics



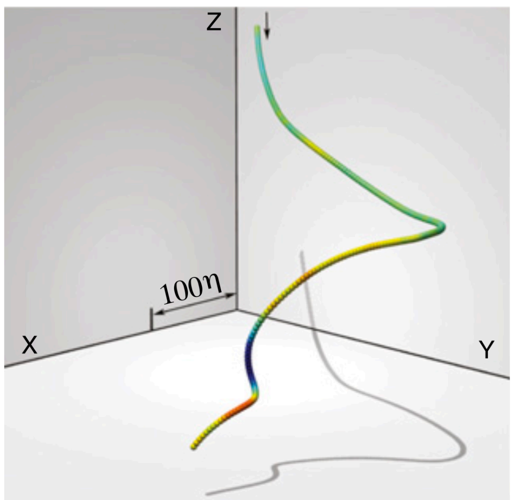

Velocity increments along a Lagrangian trajectory are time reversible: for $t \to -t$

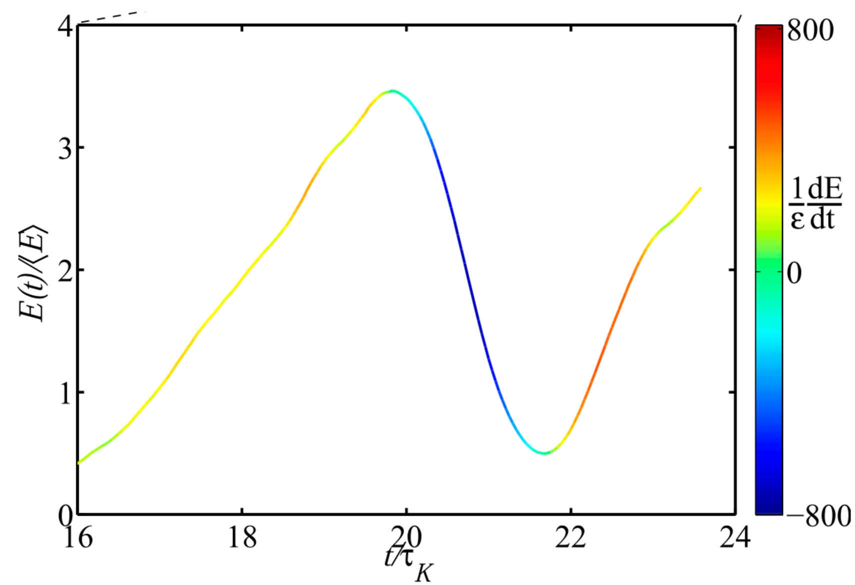
$$\mathbf{v}(t) - \mathbf{v}(0) \to -\mathbf{v}(-t) + \mathbf{v}(0) = \mathbf{v}(t) - \mathbf{v}(0)$$

in stationary turbulence

This is not true for increments of energy $E(t) = \frac{1}{2}|\mathbf{v}(t)|^2$ along a Lagrangian trajectory

$$W(t) = E(t) - E(0)$$




Experimental and numerical data show that energy of a Lagrangian tracer increases slower than decreases: PDF of W(t) is negatively skewed

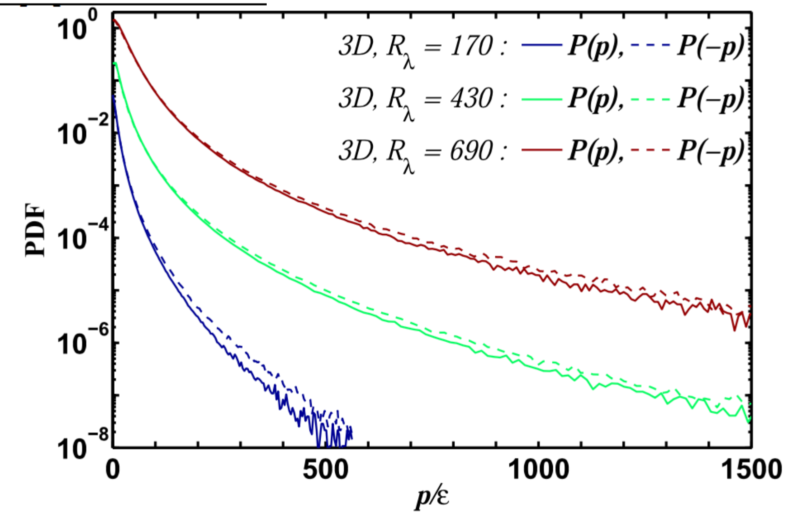
H. Xu, A. Pumir, G. Falkovich, E. Bodenschatz, M. Shats, H. Xia, N. Francois, G. Boffetta, PNAS **111**, 7558 (2014)

# Statistics of Lagrangian power

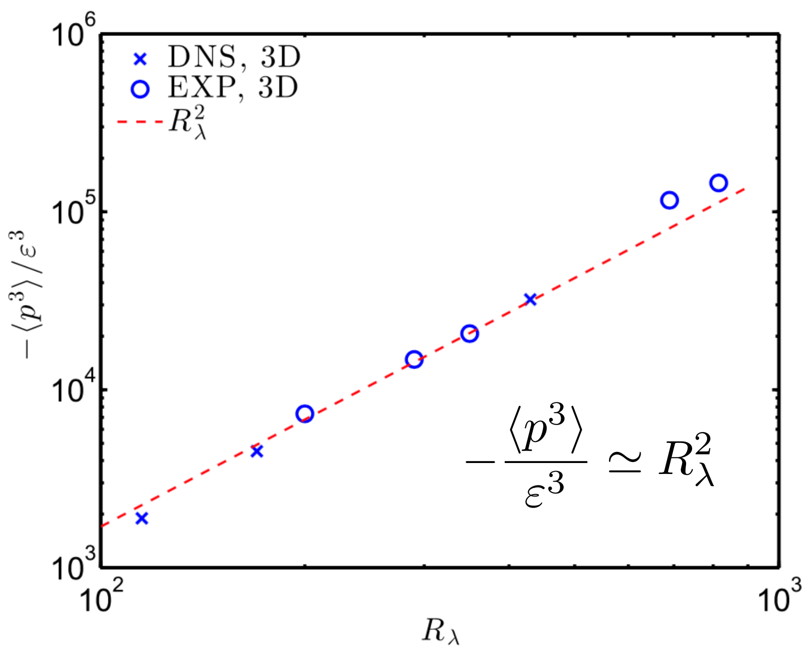
Change of kinetic energy along a trajectory (single particle, single time)

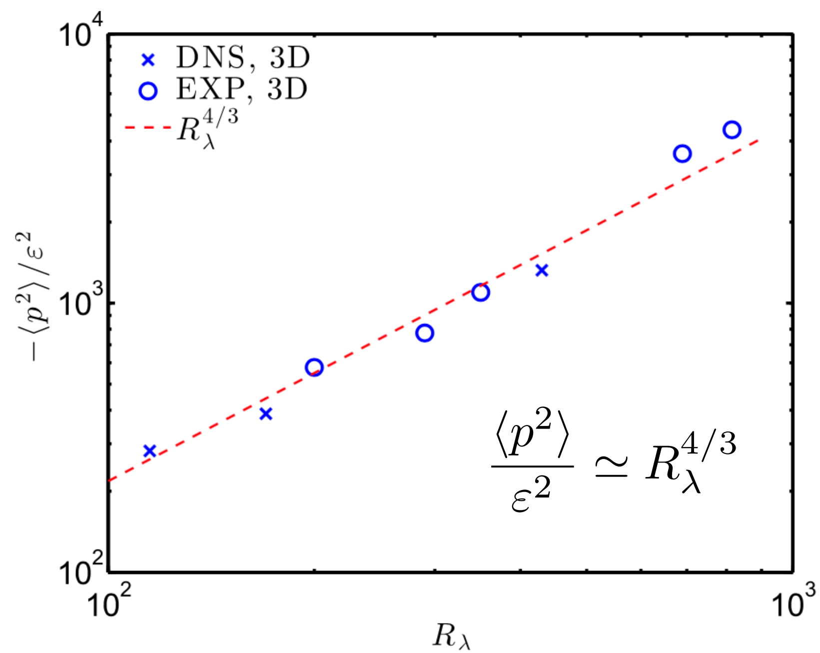
$$p = \lim_{t \to 0} \frac{W(t)}{t} = \frac{dE}{dt} = \mathbf{v} \cdot \mathbf{a}$$

$$\langle p \rangle = 0$$




Asimmetry grows with Reynolds




Skewness: $S = \frac{\langle p^3 \rangle}{\langle p^2 \rangle^{3/2}} \simeq const < 0$

H. Xu, A. Pumir, G. Falkovich, E. Bodenschatz, M. Shats, H. Xia, N. Francois, G. Boffetta, PNAS **111**, 7558 (2014)

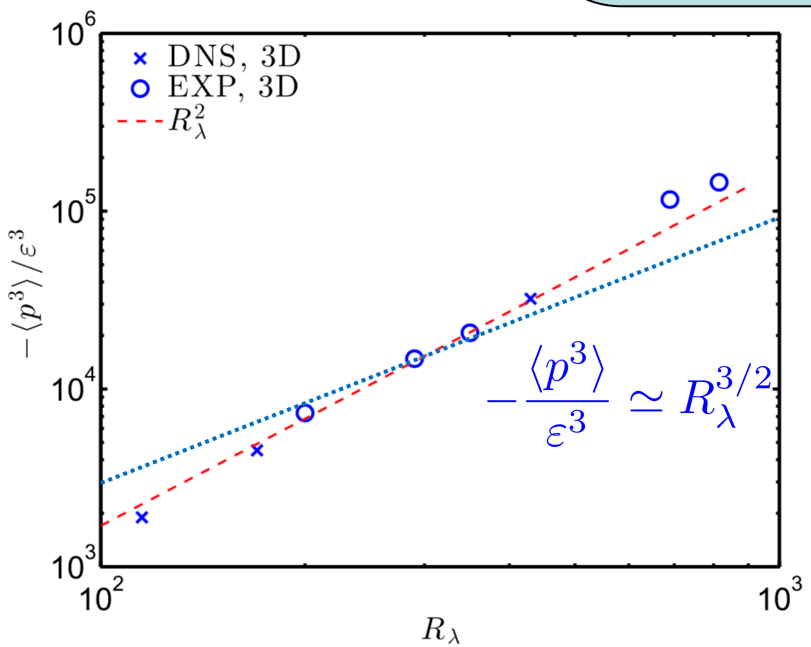
# Anomalous scaling of Lagrangian power

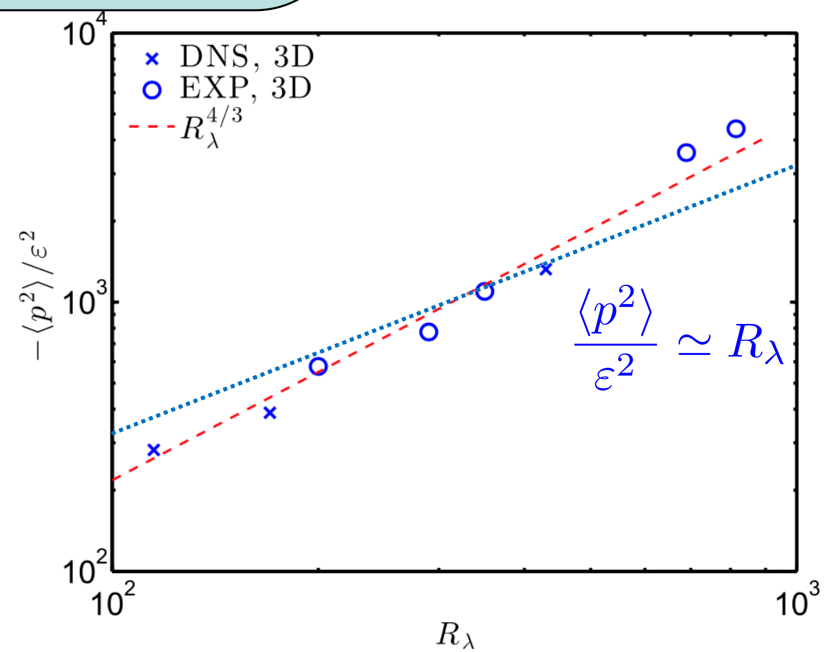
**Dimensional prediction**

$$p = \mathbf{v} \cdot \mathbf{a}$$

$$v \simeq u_{rms} \qquad a \simeq \frac{u_\eta}{\tau_\eta} \simeq \frac{u_{rms}}{T} R_\lambda^{1/2}$$

$$p \simeq \frac{u_{rms}^2}{T} R_\lambda^{1/2} \simeq \varepsilon R_\lambda^{1/2}$$




Anomalous scaling due to “flight-crash” events

# Prediction by the Multifractal Model

## Lagrangian multifractal model

M.Borgas, *Proc. R. Soc. London A* **342**, 379 (1993)
G.Boffetta, F.De Lillo, S.Musacchio., *Phys. Rev. E* **66**, 066307 (2002)

$$\delta v(\tau) \simeq \delta u(r) \quad \text{with} \quad \tau \simeq \frac{r}{\delta u(r)} \simeq T\left(\frac{r}{L}\right)^{1-h}$$

Prediction for acceleration statistics

L.Biferale, G.Boffetta, A.Celani, B.J.Devenish, A.Lanotte, F.Toschi
*Phys. Rev. Lett.* **93**, 064502 (2004).

$$a \simeq \frac{\delta u(\tau_\eta)}{\tau_\eta} \quad \text{with} \quad \tau_\eta \simeq T\left(\frac{\nu}{LU}\right)^{\frac{1-h}{1+h}} \quad \text{gives} \quad a \simeq \frac{\delta v(\tau_\eta)}{\tau_\eta} \simeq \nu^{\frac{2h-1}{1+h}} U^{\frac{3}{1+h}} L^{-\frac{3h}{1+h}}$$

By integrating over *P(h)* one obtains $\langle p^q \rangle \simeq \varepsilon^q R_\lambda^{\alpha(q)}$

$$\alpha(q) = \sup_h \left[ 2\frac{(1-2h)q - 3 + D(h)}{1+h} \right]$$

where *D(h)* is given from Eulerian statistics

MF model has no information on asymmetry (no sign)

# Numerical simulations

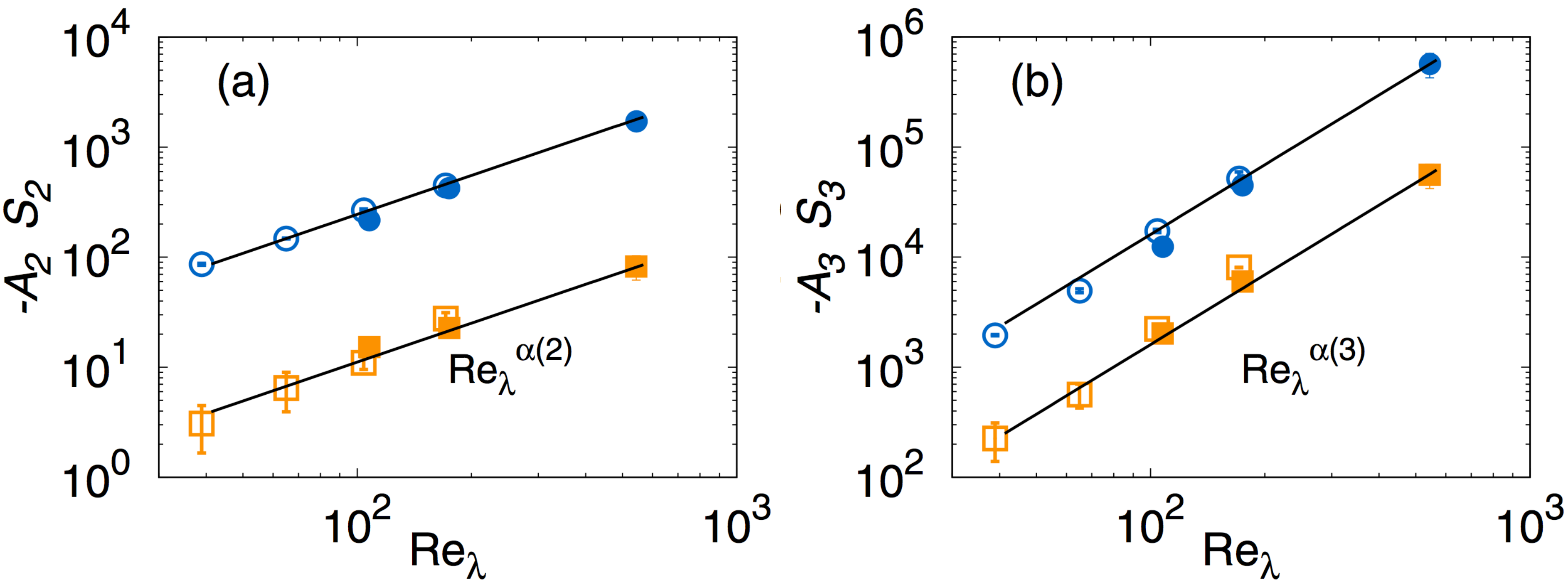
Symmetric and asymmetric power statistics

$$S_q = \frac{\langle |p|^q \rangle}{\varepsilon^q}$$

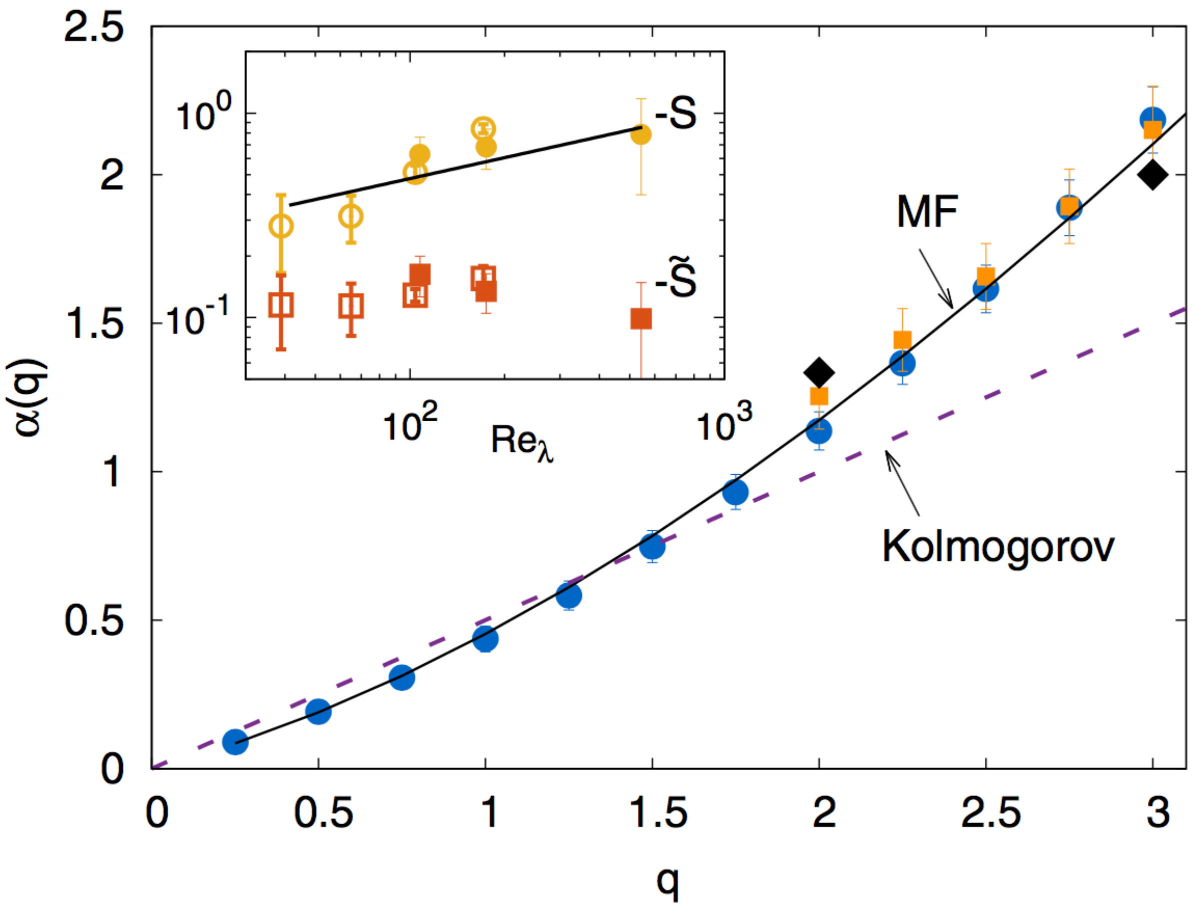
$$A_q = \frac{\langle |p|^{q-1} p \rangle}{\varepsilon^q} \qquad A_1 = 0$$

| Set | N | $R_\lambda$ |
|---|---|---|
| DNS1 | 2048 | 544 |
| DNS1 | 1024 | 176 |
| DNS1 | 512 | 115 |
| DNS2 | 1024 | 171 |
| DNS2 | 512 | 104 |
| DNS2 | 256 | 65 |
| DNS2 | 128 | 39 |

$$A_2 \sim S_2 \sim R_\lambda^{1.17} \qquad -A_3 \sim S_3 \sim R_\lambda^{2.10}$$



# Anomalous scaling




$$S_q = \frac{\langle |p|^q \rangle}{\varepsilon^q}$$

$$A_q = \frac{\langle |p|^{q-1} p \rangle}{\varepsilon^q}$$

◆ H. Xu, et al, PNAS **111**, 7558 (2014)

$$S = \frac{\langle p^3 \rangle}{\langle p^2 \rangle^{3/2}}$$

$$\tilde{S} = \frac{\langle p^3 \rangle}{\langle |p|^3 \rangle}$$

Symmetric and asymmetric moments have same scaling exponents (in the explored range of Reynolds numbers)

# Conclusions and open questions

Multifractal model is able to predict the (anomalous) scaling exponents of the power as a function of $R_\lambda$ as observed in experiments and simulations.

What about 2D ? Simulations indicate same scaling in $R_\lambda$ but here there is no intermittency.

Numerical simulations in Shell Models (much higher Re) show that symmetric and asymmetric moments have different scaling.
Is this true also for NS equation ?
Are there other mechanisms which become dominant at high Re ?

References
H.Xu, A.Pumir, G.Falkovich, E.Bodenschatz, M.Shats, H.Xia, N.Francois, G.Boffetta, *PNAS* **111**, 7558 (2014)
A. Pumir, H. Xu, G. Boffetta, G. Falkovich, E. Bodenschatz, *Phys. Rev. X* **4**, 041006 (2014)
M. Cencini, L. Biferale, G. Boffetta, M. De Pietro, *Phys. Rev. Fluids* **2**, 104604 (2017)