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INSTABILITY OF NON-ABELIAN GAUGE THEORIES
AND IMPOSSIBILITY OF CHOICE OF COULOMB GAUGE*

V. N. Gribov

ABSTRACT

In this lecture it is demonstrated that by virtue of the impossibility of introducing Coulomb gauge for large fields and of the growth of the invariant charge at large distances, non-Abelian gauge theories may not be formulated as a theory of interacting massless particles. This assertion appears as a strong argument in favor of the idea that the spectrum of states in non-Abelian theories is substantially different from the spectrum of states in perturbation theory.

Confinement and the Coulomb gauge

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Outline

- 1 Motivation
 - 2 Kugo-Ojima vs. Gribov-Zwanziger
 - 3 Hamiltonian approach in Coulomb gauge
 - 4 Lattice Coulomb gauge
 - 5 Conclusions



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Motivation

Confinement is a non-perturbative, gauge invariant phenomenon:

Lattice path integral → gauge invariant

- Wilson-Polyakov confinement criterion
- String tension, spectrum, role of topology etc...

Continuum path integral → gauge fixing

- Where is confinement encoded?
- Can we describe it through Green's functions?
- Long history of approaches: Casher, Kogut, Susskind, Gribov, Mandelstam, Cornwall, Kugo, Ojima...
- Mostly in covariant gauges...



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Covariant gauges: Kugo-Ojima confinement

Mimicks Gupta-Bleuler in QED to eliminate “wrong” norm states.

- Assume that:
 - FP ghost dressing function diverges as $p \rightarrow 0$;
 - $Q_{\text{BRST}}|\Psi\rangle_{\text{ph}} = 0$;
- then for S-matrix states:
 - no ghosts;
 - $Q_c^a|\Psi\rangle_{\text{ph}} = 0$;
- Basis for most DSE approaches, however...
 - ...can a global Q_{BRST} be defined beyond perturbation theory?
[e.g. Neuberger 1986, Dudal et al. 2008-2017]
 - Physical quantities not straightforward to construct from correlators.



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Coulomb gauge: Gribov-Zwanzinger Confinement

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to either:
 - $-\nabla \cdot D > 0$: Gribov Region Ω , local maxima $F(A)$ easy
 - absolute maxima $F(A)$: Fundamental Modular Region Λ hard
- $(-D \cdot \nabla)^{-1}$ singular at $\partial\Omega$ ($\partial\Lambda$)!
- Singularity restricts functional integral to Ω (Λ)
- Radius of Ω (Λ) introduces an IR scale (Gribov mass M_G)
- $M_G \simeq O(\Lambda_{QCD}, \sigma_W) \Rightarrow$ confinement?



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GZ confinement scenario in CG

"Natural" tests:

- Confinement:
 - quasi-particle dispersion relations ω_A, ω_ψ
 - static Coulomb potential V_C .
- χ -symmetry breaking:
 - quark running mass $M(\mathbf{p})$
- Ghost form factor $d(\mathbf{p})$ IR divergent $\Rightarrow \omega_{A,\psi}$ should grow at large distances - no free quarks, gluons! [Gribov NPB 1978]
- $d(\mathbf{p}) \propto \epsilon(\mathbf{p})^{-1}$ vacuum dielectric function \Rightarrow
Dual superconductor! [Reinhardt PRL 2008]
- $V_C \geq V_{phys} \Rightarrow V_C(r) \simeq \sigma_C r$ $V_C(\mathbf{p}) = 8\pi\sigma_C |\mathbf{p}|^{-4}$, $\sigma_C \geq \sigma_W$
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QCD Hamilton in Coulomb gauge [Christ-Lee PRD 1980]

Steps

- Canonical quantization in Weyl gauge $A_0 = 0$
- Residual gauge freedom for A^\perp
- "Fix" $\nabla \cdot \mathbf{A}^a = 0$ through Gauss's law

$$H = \frac{1}{2} \left[-\mathcal{J}_A^{-1} \frac{\delta}{\delta A} \mathcal{J}_A \frac{\delta}{\delta A} + B^2 \right] + \psi^\dagger (-i\alpha \cdot \nabla + \beta m) \psi - g\psi^\dagger \alpha \cdot \mathbf{A} \psi + \frac{g^2}{2} \mathcal{J}_A^{-1} \rho \mathcal{J}_A F_A \rho$$

- $\frac{\delta}{\delta A^c} (B^c)$: non-abelian electric (magnetic) field
- $\mathcal{J}_A = \text{Det}[-\partial_i D_i^{ab}]$: FP-determinant ($D = i\nabla + \mathbf{A}$)
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- $\frac{\delta}{\delta A^c} (B^c)$: non-abelian electric (magnetic) field
- $\mathcal{J}_A = \text{Det}[-\partial_i D_i^{ab}]$: FP-determinant ($D = i\nabla + \mathbf{A}$)
- $\rho^a = \psi^\dagger T^a \psi - if^{abc} A^b \frac{\delta}{\delta A^c}$: colour charge density
- $F_A = (-\mathbf{D} \cdot \nabla)^{-1} (-\nabla^2) (-\mathbf{D} \cdot \nabla)^{-1}$: Coulomb kernel



QCD Hamilton in Coulomb gauge [Christ-Lee PRD 1980]

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From QCD-Hamiltonian to variational approach in CG

The good...

- Hamiltonian and states well defined, at least on the lattice
- analogy to standard quantum mechanics
- appeals to physical intuition

... the bad...

- non-covariant
- continuum renormalization not fully worked out

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Static Green's functions and QCD-vacuum

$$\text{V.e.v.: } \langle K \rangle = \int \mathcal{D}A \mathcal{J}_A \mathcal{D}\xi^\dagger \mathcal{D}\xi \Psi^*[A, \xi, \xi^\dagger] K \Psi[A, \xi, \xi^\dagger]$$

- integration over A^\perp and Grassmann fields ξ, ξ^\dagger
- Ψ is the vacuum wave functional
- Static Green functions $\rightarrow \langle AA \rangle, \langle \xi \xi^\dagger \rangle, \langle \xi \xi^\dagger A \rangle, \dots$

Writing now the vacuum wave functional as

$$|\Psi[A, \xi, \xi^\dagger]|^2 =: \exp\left\{-S[A, \xi, \xi^\dagger]\right\}$$

we can define a QFT with the “action” $S[A, \xi, \xi^\dagger]$, where:

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Canonical Dyson–Schwinger equations

Gluon and quark cDSEs are derived from the identity

$$0 = \int \mathcal{D}A \mathcal{D}\xi^\dagger \mathcal{D}\xi \frac{\delta}{\delta\phi} \left\{ \mathcal{J}_A e^{-S[A, \xi, \xi^\dagger]} K[A, \xi, \xi^\dagger] \right\}$$

where $\phi \in \{A, \xi, \xi^\dagger\}$.

Ghost cDSEs follow from the operator identity

$$G_A = G_0 - G_0 \tilde{\Gamma}_0 A G_A$$

where $G_A^{-1} = -\partial \cdot D$, and $\tilde{\Gamma}_0$ is the bare ghost-gluon vertex.

Not quite equations of motion, rather relations between Green functions and the—so far undetermined—kernels of $S[A, \xi, \xi^\dagger]$.



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Kernels of the vacuum wave functional

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The **coefficients** in the vacuum wave functional play the role of the bare vertices, but are still unknown... \Rightarrow **variational kernels**

- are non-local functions
- have a non-trivial expansion in powers of the coupling

- evaluate the energy in the state defined by the kernel-Ansatz
- use the cDSEs to express the energy through the kernels
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Gives a set of **gap equations**, combined with the cDSEs.



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Results from variational approach

Easiest Ansatz: Gaussian! γ_2 "a la Gribov", everything else $\rightarrow 0$

- Asymptotics for $\omega_A(\mathbf{p})$, $d(\mathbf{p})$
 - IR power laws κ_{gl} , κ_{gh}
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1 Motivation

2 Kugo-Ojima vs. Gribov-Zwanziger

3 Hamiltonian approach in Coulomb gauge

4 Lattice Coulomb gauge

5 Conclusions



Goals of CG lattice investigation

Extract $d, V_C, \omega_A, \omega_\psi, M(\mathbf{p})$ to test:

- GZ confinement scenario
- Results from variational approach

PBC: Weyl gauge not viable! Time (A_4) dependence either

① Explicit:

- $D(\mathbf{p}, p_4) = \delta^{ab} \delta_{ij} \langle A_i^a(\mathbf{p}, p_4) A_j^b(-\mathbf{p}, -p_4) \rangle$
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② Implicit, i.e. hidden in the dynamics

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Goals of CG lattice investigation

Extract $d, V_C, \omega_A, \omega_\psi, M(\mathbf{p})$ to test:

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Minimizing lattice artifacts I (FP sector)

Anisotropic action. Closer to the Hamiltonian limit $a_t \rightarrow 0!$

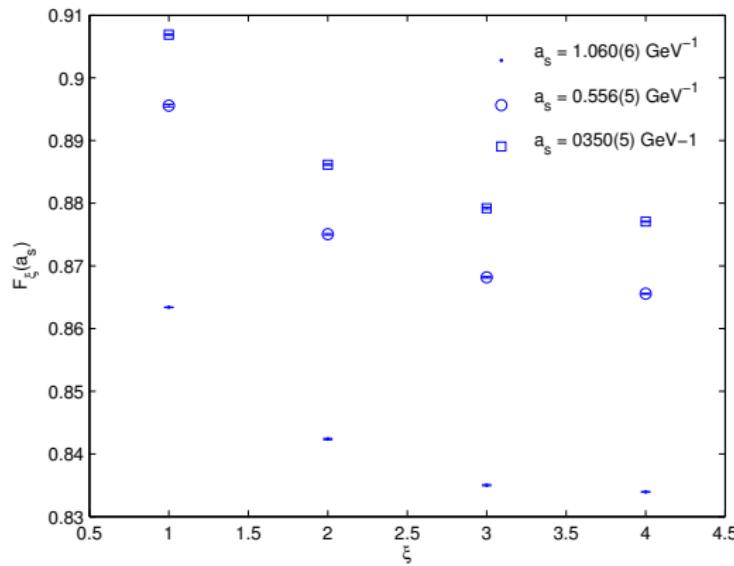
$$\begin{aligned} S = & \beta \sum_x \left\{ \gamma \sum_{j>i=1}^d \left(1 - \frac{1}{N_c} \Re \mathfrak{e} [\text{Tr}(P_{ij}(x))] \right) \right. \\ & \left. + \frac{1}{\gamma} \sum_{i=1}^d \left(1 - \frac{1}{N_c} \Re \mathfrak{e} [\text{Tr}(P_{i,d+1}(x))] \right) \right\} \end{aligned}$$

γ bare anisotropy. Must fix $\xi = \frac{a_s}{a_t}$ non-perturbatively!

$T = 0$ simulations on a $L^3 \times (\xi L)$ lattice. Need $\xi \gg 1!$



Strong effect, e.g. when minimizing gauge functional



Large corrections, scale with a_t^2 , a_t^4 (glueball spectrum!)



Minimizing lattice artifacts II (gauge sector)

Equal-time component \Leftrightarrow integrate over p_4 . Cutoff effects scale with a_t^{-1}

Scaling violations in static gluon $\sum_{p_4} D(\mathbf{p}, p_4)$:

- model explicit p_4 dependence
- calculate $\int dp_4$ “analytically” to extract cutoff dependence

One finds:

$$\begin{aligned} D_\beta(|\mathbf{p}|, p_4) &= \frac{f_\beta(|\mathbf{p}|)}{|\mathbf{p}|} g\left(\frac{p_4}{|\mathbf{p}|}\right) \\ g(z) &\sim \frac{f_\beta(|\mathbf{p}|)}{|\mathbf{p}|} (1 + z^2)^{\alpha - 1} \end{aligned}$$



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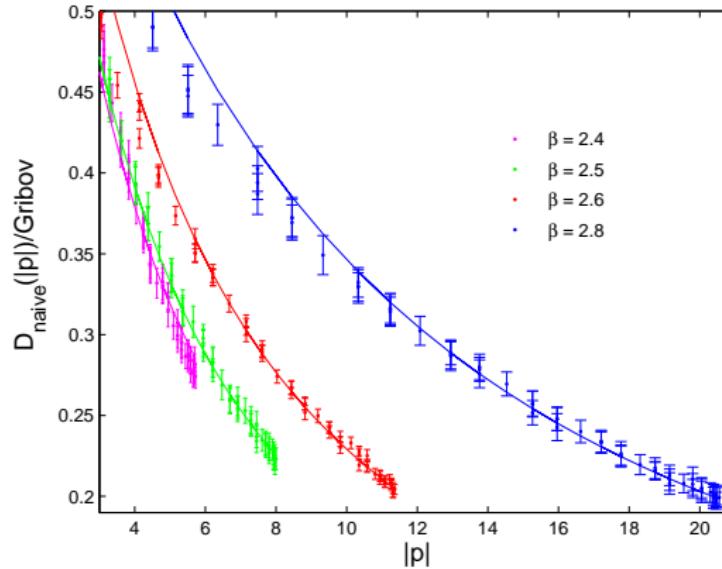
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Gluon propagator



$$D_{\text{naive}}(|\mathbf{p}|) \simeq \int_{-\frac{2}{a_t}}^{\frac{2}{a_t}} \frac{dp_4}{2\pi} D_\beta(|\mathbf{p}|, p_4) = \frac{f_\beta(|\mathbf{p}|)}{|\mathbf{p}|} \frac{1}{2\pi} B\left(\frac{4\xi^2}{4\xi^2 + \hat{p}^2}, \frac{1}{2}, -\alpha + \frac{1}{2}\right)$$



Minimizing lattice artifacts III (full QCD)

- Lesson from Landau gauge:

full QCD exhibits strong scaling violations

- Generate them yourself: *expensive...*
- Download configurations from ILDG, e.g. Asqtad (MILC) ☺

◦ Parameters rarely test the “deep” IR ☺

“Dynamical” differences to Hamiltonian approach are hard to control!



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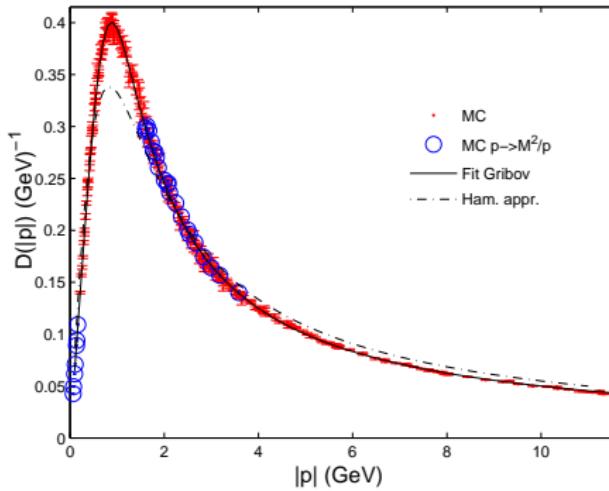
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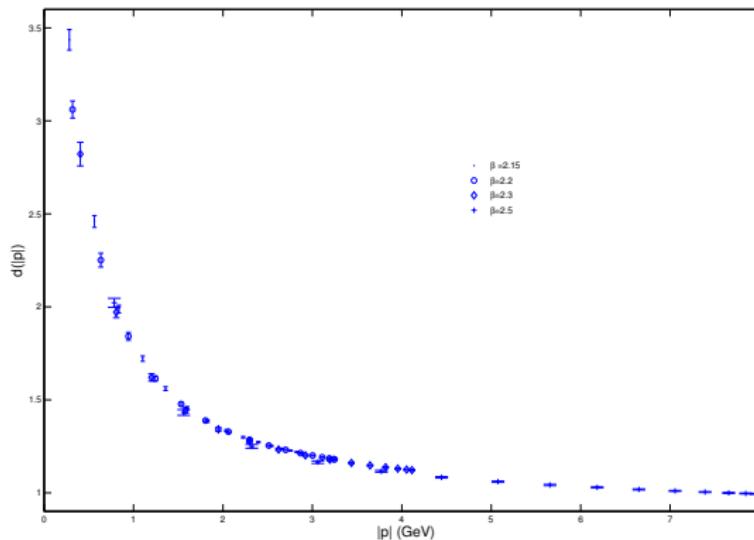


Static $D(\mathbf{p})$ renormalizable $\omega_A(|\mathbf{p}|) \propto \sqrt{|\mathbf{p}|^2 + \frac{M_G^4}{|\mathbf{p}|^2}}$

$$M_G = 0.856(8) \text{ GeV}.$$



Ghost form factor d



Extrapolating in ξ : UV $\gamma_{gh} = 1/2$; IR $\kappa_{gh} \gtrsim 0.5$

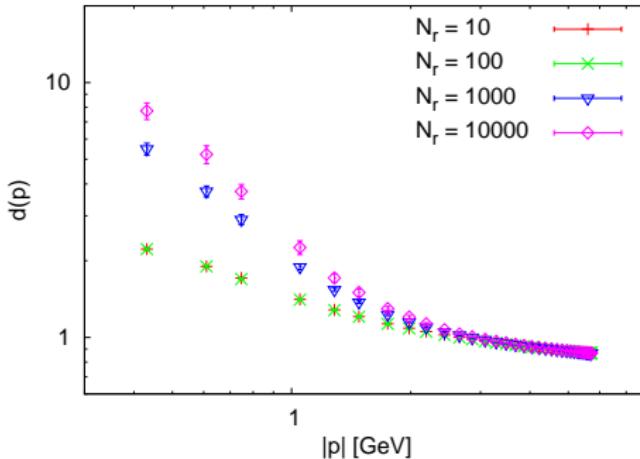
Agrees with GZ scenario!

IR sum rule violated?



Alternative gauge fixing [Cooper, Zwanziger PRD 2016]

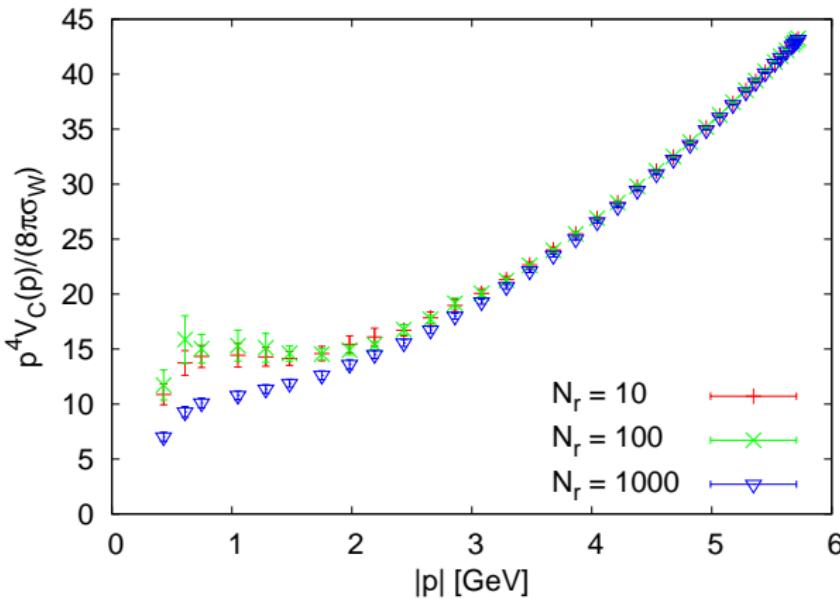
Choose configurations with smallest λ_1 for FP-operator
Closer to $\partial\Omega$, relevant in thermodynamical limit



κ_{gh} indeed grows... Too much?



Coulomb potential I



$$|\mathbf{p}|^4 V_C(\mathbf{p}) = 8\pi\sigma_C + 4\pi\lambda|\mathbf{p}|^2 + \mathcal{O}(|\mathbf{p}|^3).$$

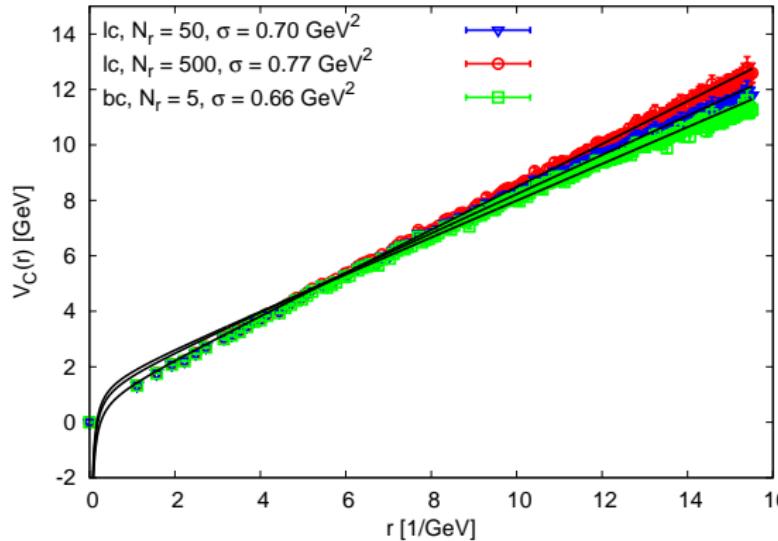
Hard to extrapolate, unstable with alternative gauge fixing.



Coulomb potential II

Alternative definition [Marinari et al. PLB 1993]:

$$-aV_C(|\mathbf{x} - \mathbf{y}|) = \lim_{t \rightarrow 0} \frac{d}{dt} \ln \langle \text{tr } P_t(\mathbf{x}) P_t^\dagger(\mathbf{y}) \rangle = \ln \langle \text{tr } U_0(\mathbf{x}) U_0^\dagger(\mathbf{y}) \rangle$$





Quark

- From the Dirac operator in CG we get

$$S^{-1}(\mathbf{p}, p_4) = i \not{\mathbf{p}} A_s(\mathbf{p}) + i \not{p}_4 A_t(\mathbf{p}) + B_m(\mathbf{p})$$

- If renormalizable:

$$S^{-1}(\mathbf{p}, p_4) = Z^{-1}(\mathbf{p})[i \not{\mathbf{p}} + i \not{p}_4 \alpha(\mathbf{p}) + M(\mathbf{p})]$$

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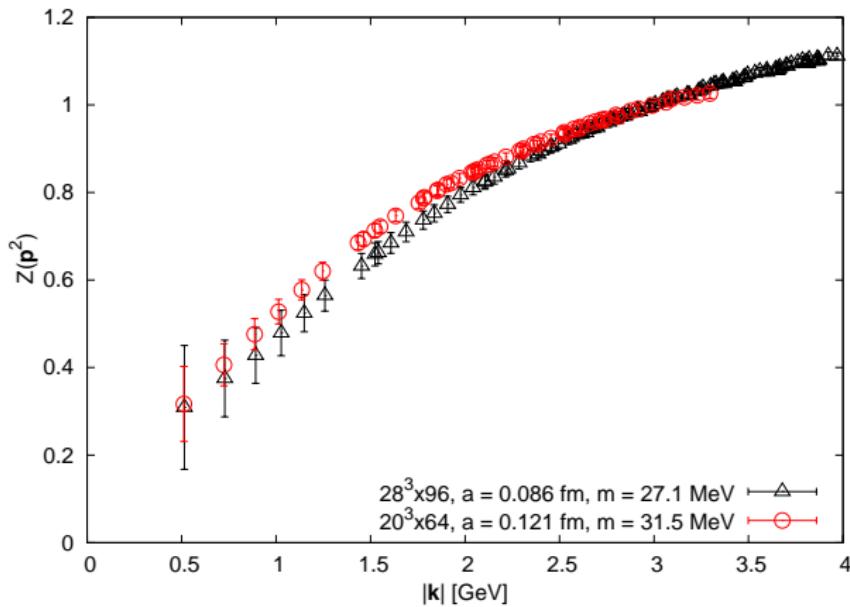
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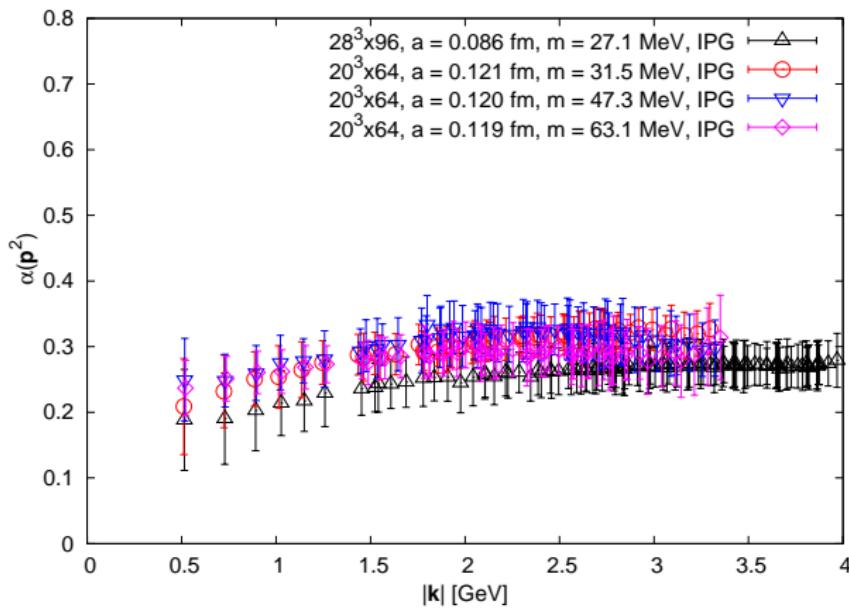


$Z(p)$ renormalizable



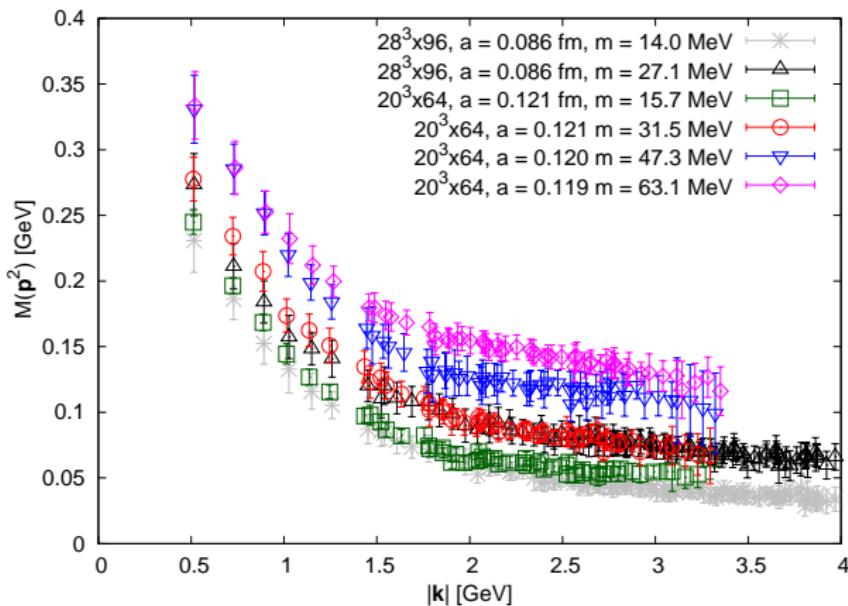


$\alpha(p)$ scale invariant



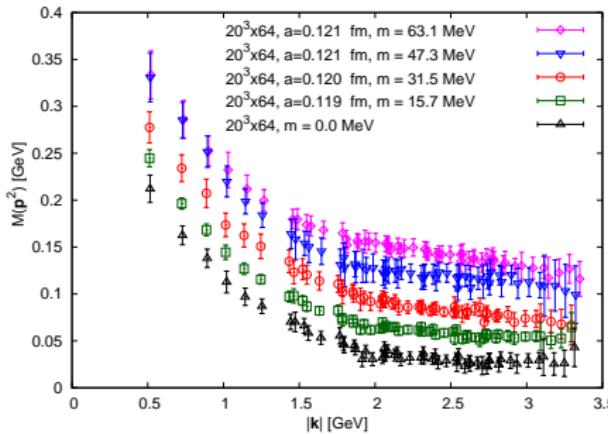


$M(\mathbf{p})$ scale invariant (for fixed quark mass!)





Chiral limit for $M(\mathbf{p})$



$$M(|\mathbf{p}|, m_b) = \frac{m_\chi(m_b)}{1 + b \frac{|\mathbf{p}|^2}{\Lambda^2} \log \left(e + \frac{|\mathbf{p}|^2}{\Lambda^2} \right)^{-\gamma}} + \frac{m_r(m_b)}{\log \left(e + \frac{|\mathbf{p}|^2}{\Lambda^2} \right)^\gamma}$$

$b = 2.9(1)$, $\gamma = 0.84(2)$, $\Lambda = 1.22(6)$ GeV, $m_\chi(0) = 0.31(1)$ GeV,
 $\chi^2/\text{d.o.f.} = 1.06$



Define $S(\mathbf{p})$, $\omega_\psi(|\mathbf{p}|)$ |

Analogy with free fermion: Hamiltonian

$$S^H(\mathbf{p}) = \int dp_4 S(\mathbf{p}, p_4) \propto H$$

$$\bullet S^H(\mathbf{p}) = \frac{Z(\mathbf{p})}{\alpha(\mathbf{p})} \frac{\sqrt{\mathbf{p}^2 + M^2(|\mathbf{p}|)}}{i\cancel{\mathbf{p}} + M(\mathbf{p})} = \frac{Z(\mathbf{p})}{\alpha(\mathbf{p})} \frac{-i\cancel{\mathbf{p}} + M(\mathbf{p})}{\sqrt{\mathbf{p}^2 + M^2(|\mathbf{p}|)}}$$

No divergences ☺

- Coefficient $^{-1}$ of $-i\cancel{\mathbf{p}} + M(\mathbf{p})$ eigenvalue of H : quark effective energy!

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Analogy with free fermion: Euclidean

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- $S^E(\mathbf{p}) = \Lambda \frac{Z(\mathbf{p})}{i \not{\mathbf{p}} + M(\mathbf{p})}$

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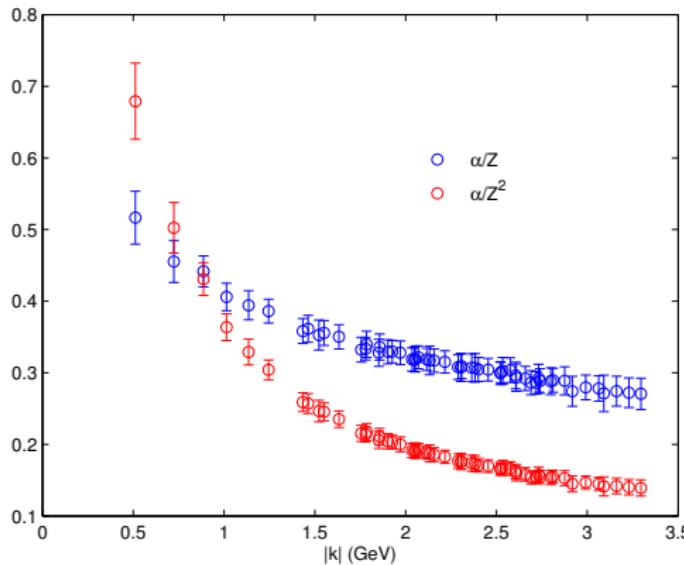
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$M(|\mathbf{p}|) \rightarrow m_\chi$. Only $\frac{\alpha}{Z^{(2)}}$ relevant for IR...



Both IR enhanced! What happens at lower momenta?



1 Motivation

2 Kugo-Ojima vs. Gribov-Zwanziger

3 Hamiltonian approach in Coulomb gauge

4 Lattice Coulomb gauge

5 Conclusions



Conclusions

Joint efforts from variational approach and lattice CG

- Static propagators in CG renormalizable
- GZ works!
 - $d(\mathbf{p}) \gtrsim |\mathbf{p}|^{-1/2}$ IR divergent
 - $\sigma_C \simeq 2\sigma > \sigma$
 - ω_A, ω_B IR divergent
- $M(\mathbf{p})$ consistent χ -symmetry breaking

Open issues

- On the lattice $A_4 \neq 0$. Effects not fully understood...
 - Smallest FP eigenvalue can be made arbitrarily small
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- "Correct" V_C ? Clean extrapolation of σ_C hard



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 - $\sigma_C \simeq 2\sigma > \sigma$
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Open issues

- On the lattice $A_4 \neq 0$. Effects not fully understood...
 - Smallest FP eigenvalue can be made arbitrarily small
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Conclusions

Joint efforts from variational approach and lattice CG

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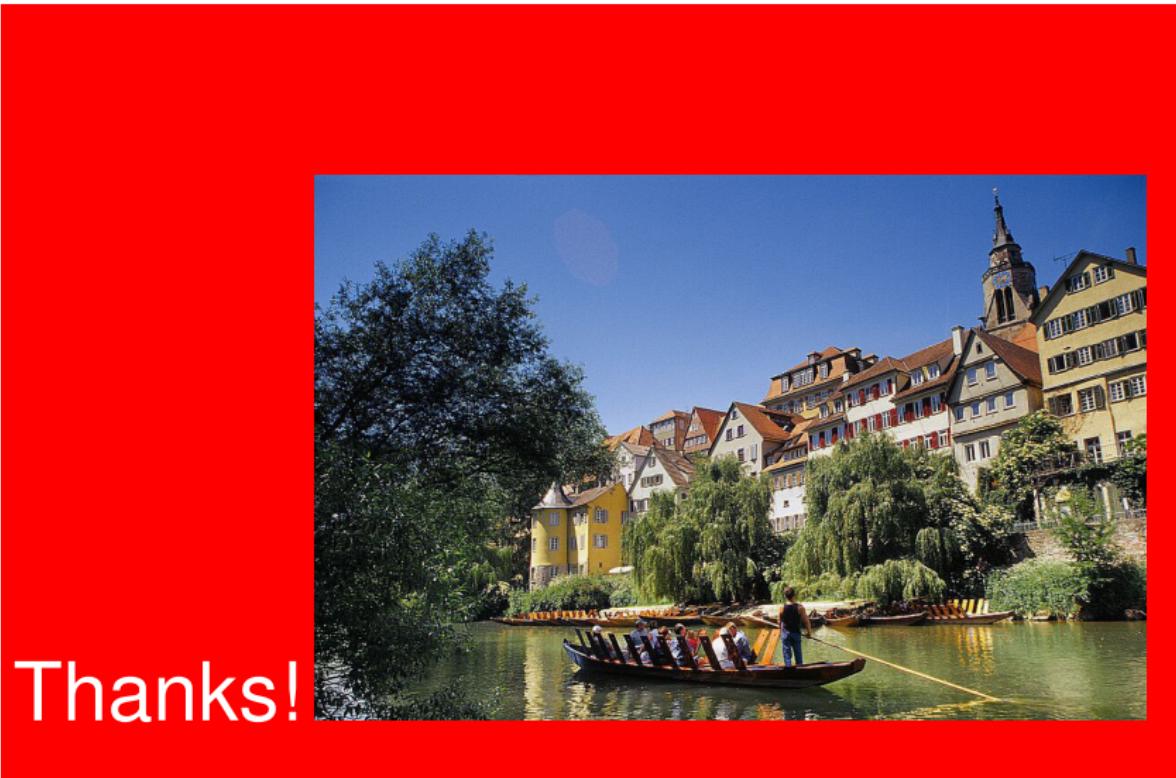
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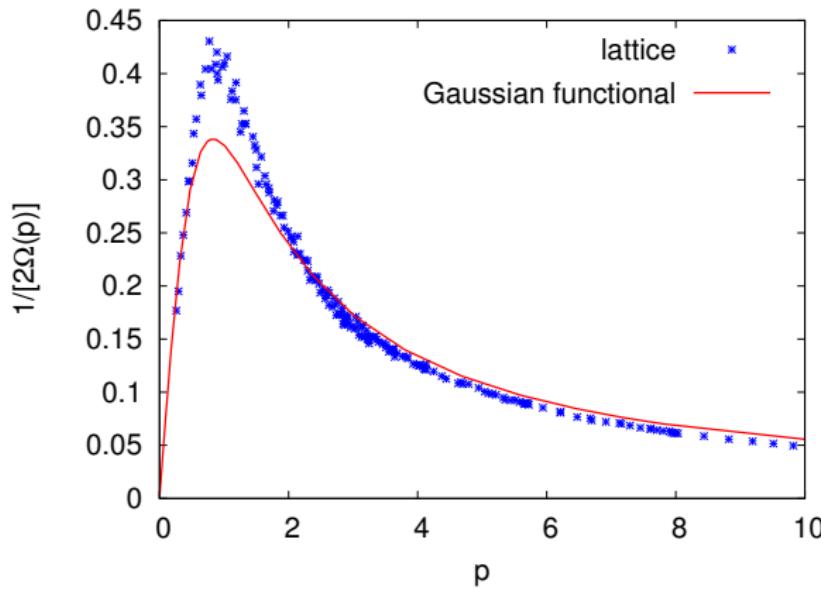
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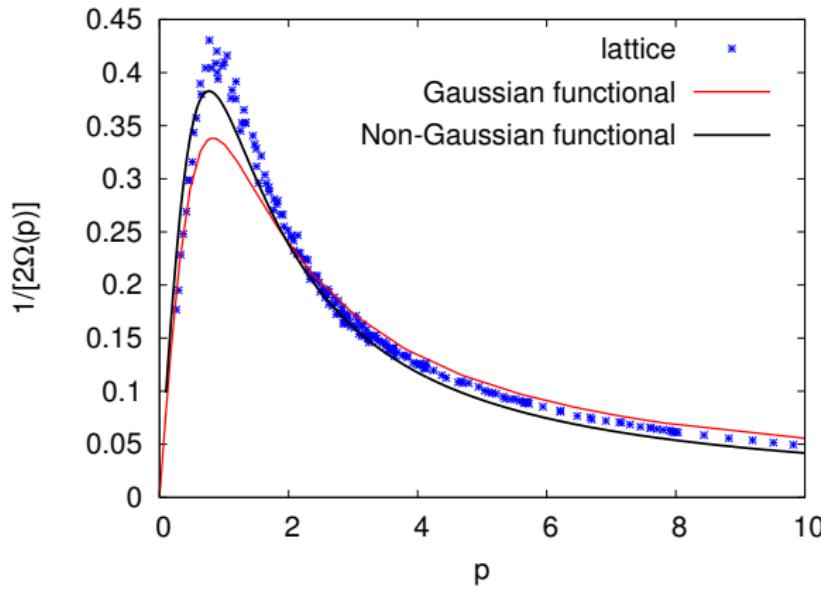


Gluon propagator with non-Gaussian functional



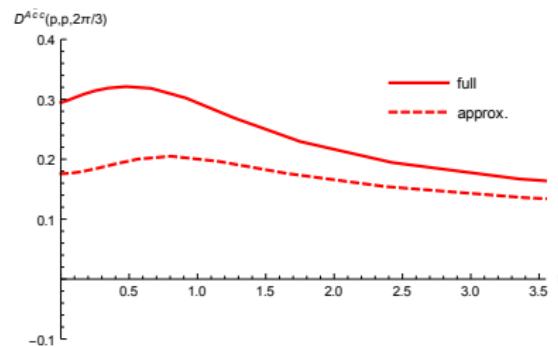
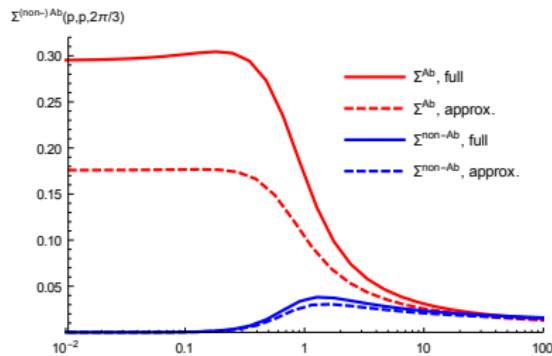


Gluon propagator with non-Gaussian functional



Ghost-gluon vertex

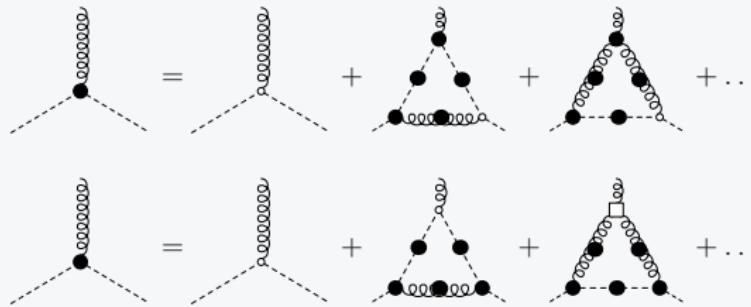
Truncated cDSE





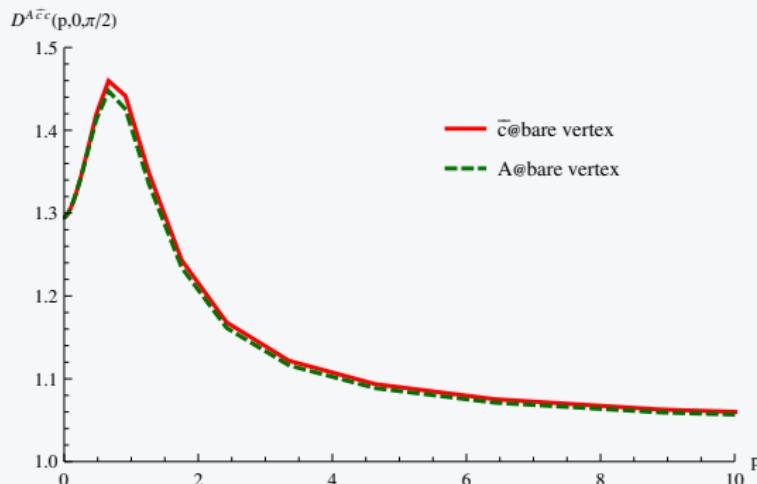
Ghost-gluon vertex

Two different cDSEs



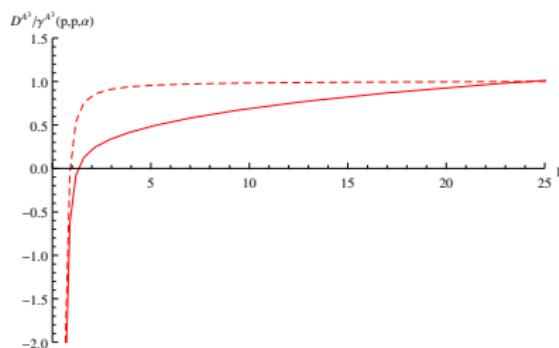


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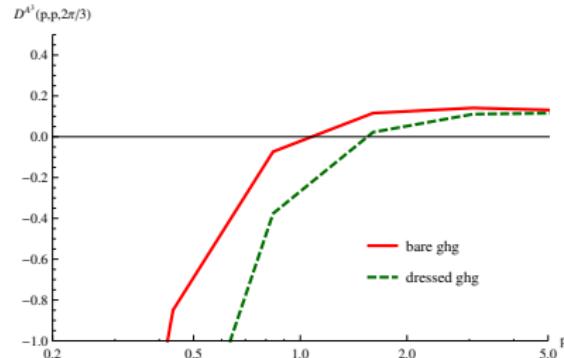


The three-gluon vertex

(Truncated) Three-gluon vertex cDSE



Dashed line: Ghost triangle only
 Full line: Full cDSE



Full red line: Bare ghost-gluon vertex
 Dashed green line: Full ghost-gluon vertex



The two-quark and quark-gluon kernel
In the exponent of the wave functional

$$S[A, \xi, \xi^\dagger] = \frac{1}{2} \gamma_2 A^2 + \frac{1}{3!} \gamma_3 A^3 + \frac{1}{4!} \gamma_4 A^4 + \xi^\dagger (\bar{\gamma} + \bar{\Gamma}_0 A) \xi$$

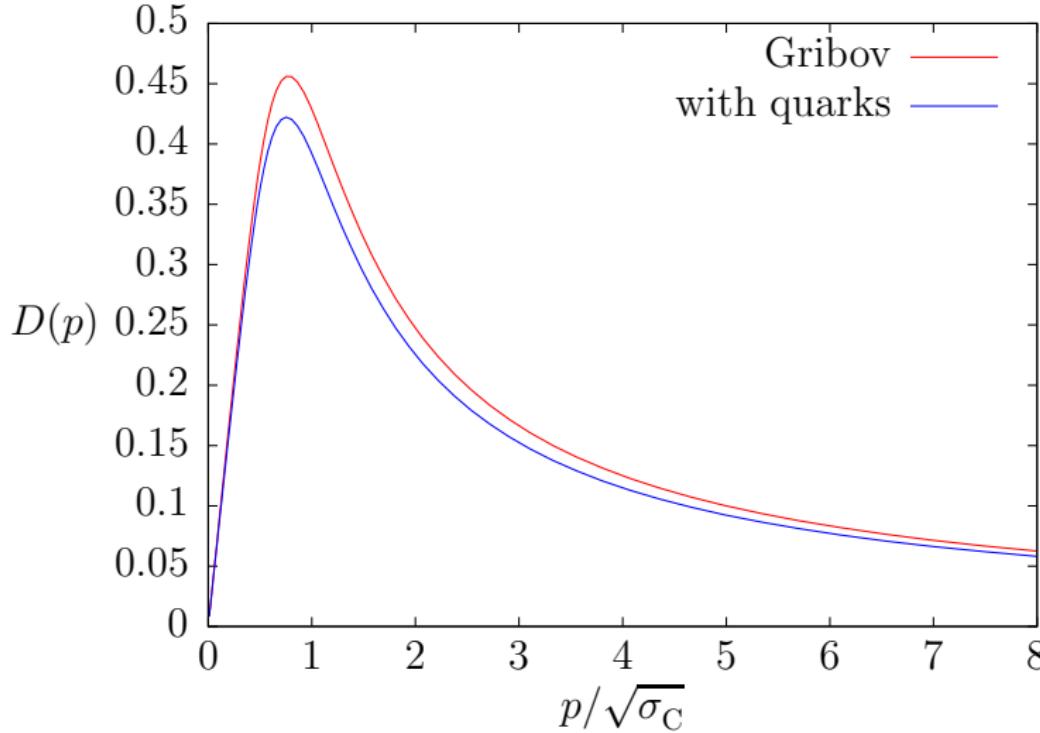
take the most simple Dirac and colour structure

$$\bar{\gamma} \sim \beta s(\mathbf{p}), \quad \bar{\Gamma}_0 \sim \alpha_i t^a v(\mathbf{p}, \mathbf{q})$$

with s and v being scalar variational kernels.



Gluon propagator with quark loop





The quark gap equation

Variation of the energy density fixes the quark-gluon vector kernel, which allows to write the quark gap equation

$$\begin{aligned} M(\mathbf{p}) = & \frac{g^2 C_F}{2} \int \frac{d^3 q}{(2\pi)^3} \frac{F(\mathbf{p} - \mathbf{q})}{\sqrt{\mathbf{q}^2 + M^2(\mathbf{q})}} \left[M(\mathbf{q}) - \frac{\mathbf{p} \cdot \mathbf{q}}{\mathbf{p}^2} M(\mathbf{p}) \right] \\ & + \frac{g^2 C_F}{2} \int \frac{d^3 q}{(2\pi)^3} \frac{X(\mathbf{p}, \mathbf{q}) v^2(\mathbf{p}, \mathbf{q})}{\Omega(\mathbf{p} + \mathbf{q}) \sqrt{\mathbf{q}^2 + M^2(\mathbf{q})}} \\ & \quad \times \{ \text{stuff arising from the Dirac trace} \} \end{aligned}$$

Looks harmless (maybe even nice), but

- bad, bad linear divergences
- chiral condensate and constituent quark mass way too small

Are we missing something?



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