Chaotic dynamical phase induced by non-equilibrium quantum fluctuations

Alessio Lerose – 13th Dicember, 2017 – SM&FT 2017, Bari

Based on joint work with:

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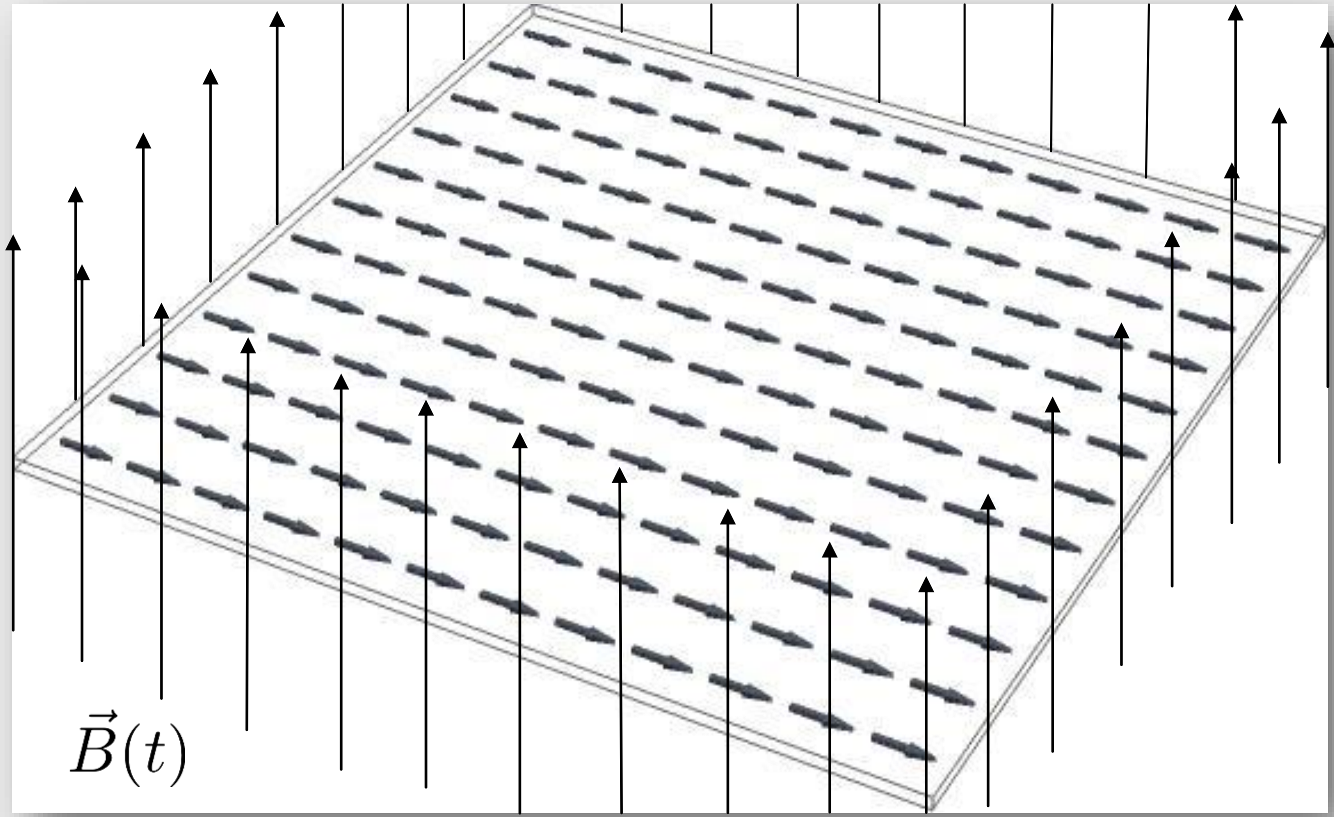
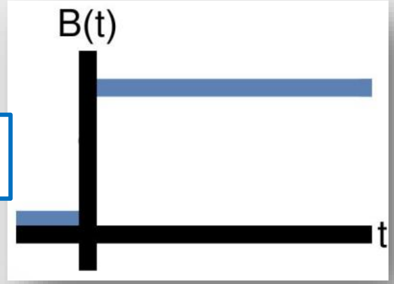
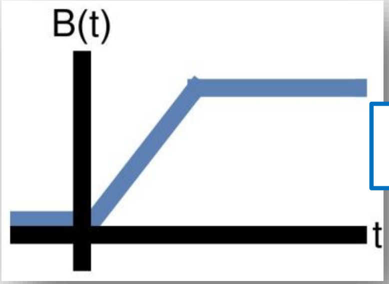
arXiv:1706.05062

THE PROBLEM



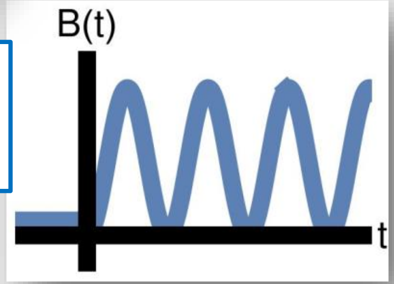
QUENCH





RAMP

PERIODIC DRIVING



A MEAN-FIELD LIMIT: INFINITE-RANGE ISING MAGNET

FERROMAGNETIC COUPLING

$$H = -\frac{\lambda}{N}\sum_{i,j}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

EXTERNAL TRANSVERSE FIELD

$$\propto \left(\widetilde{S}^x_{k=0}\right)^2 \qquad \propto \widetilde{S}^z_{k=0}$$

A.Gambassi, P.Calabrese, EPL 2011

B.Sciolla, G.Biroli, JSM 2011

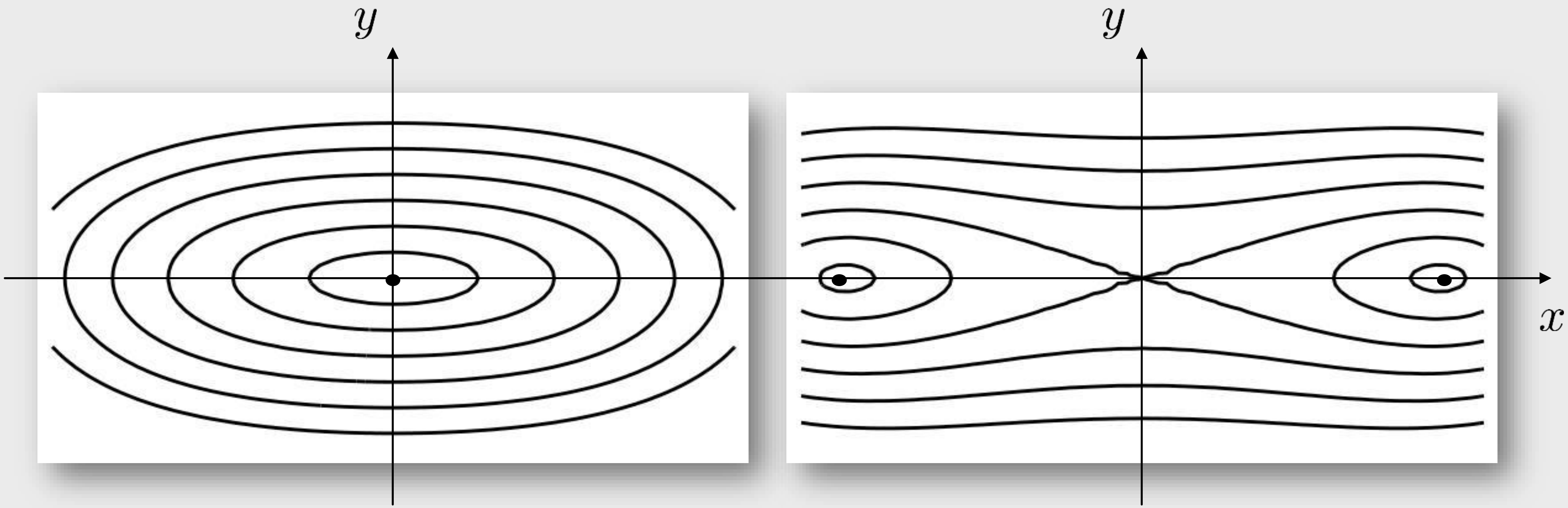
B.Zunkovic, A.Silva, M.Fabrizio, PTRS 2016

GROUND STATE → $\left|\vec{S}_{\text{tot}}\right| = \frac{N}{2}$ → CLASSICAL LIMIT

GROUND STATE = MINIMIZE CLASSICAL HAMILTONIAN ON THE SPHERE

NON-EQUILIBRIUM DYNAMICS = CLASSICAL DYNAMICS ON THE SPHERE

STATICS (T=0): SPONTANEOUS BREAKING OF Z_2

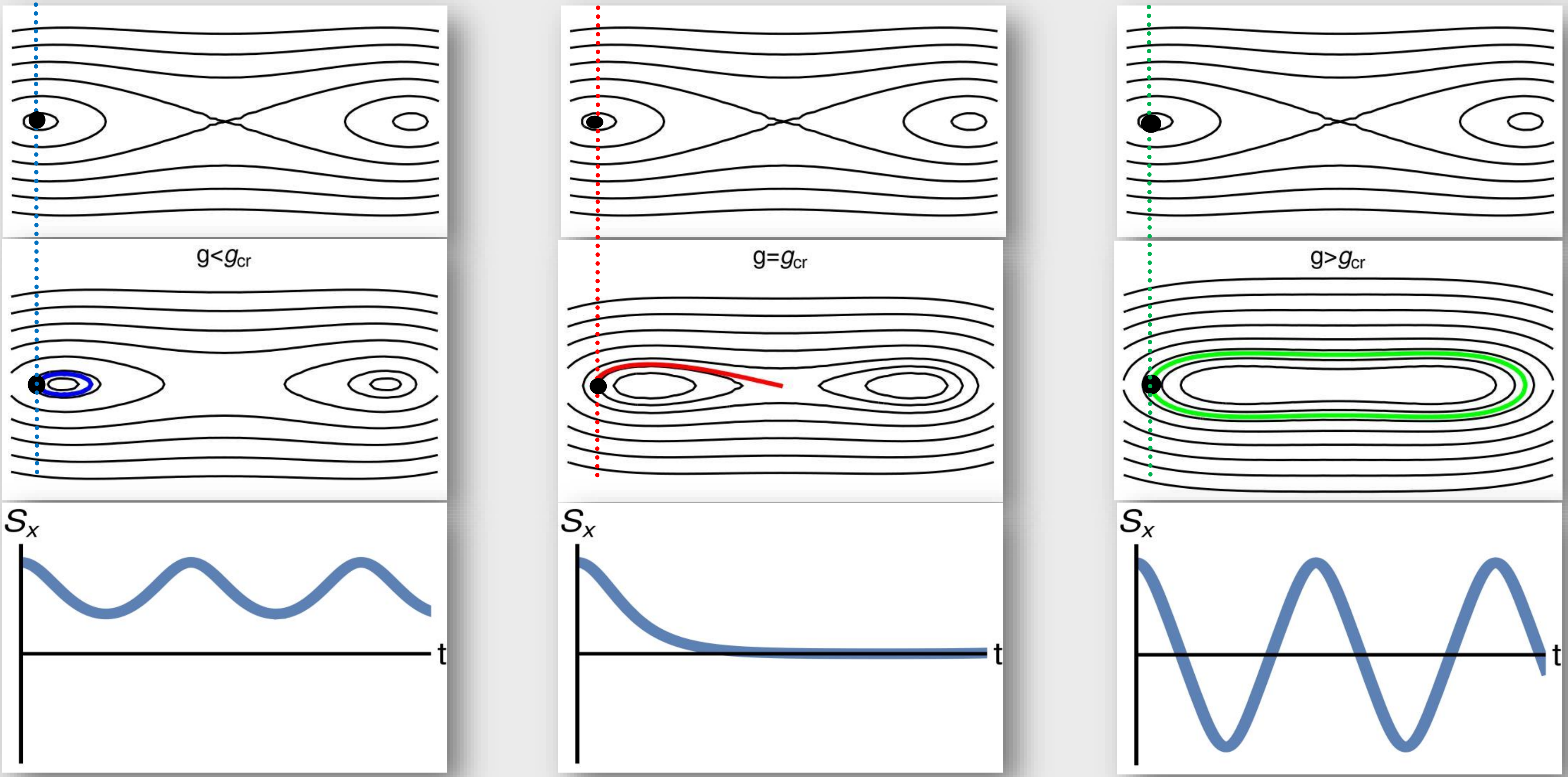


$g > 2\lambda$

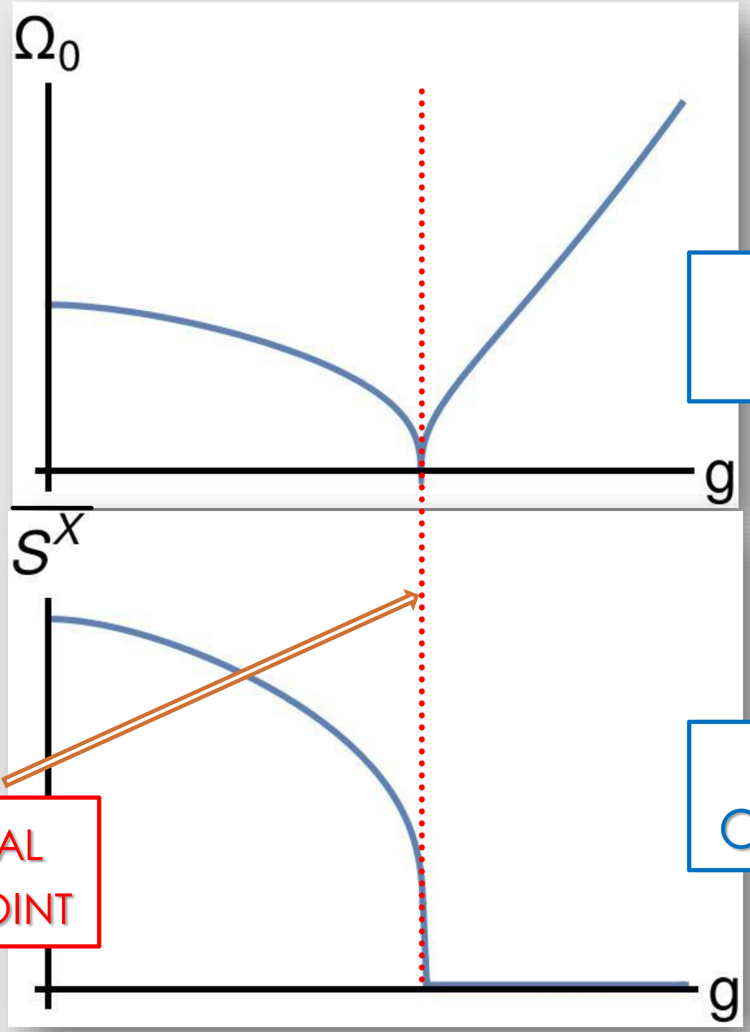
$g < 2\lambda$

(''QUANTUM'' PHASE TRANSITION)

POST-QUENCH DYNAMICS



DYNAMICAL CRITICALITY



CLASSICAL FREQUENCY

TIME-AVERAGED ORDER PARAMETER

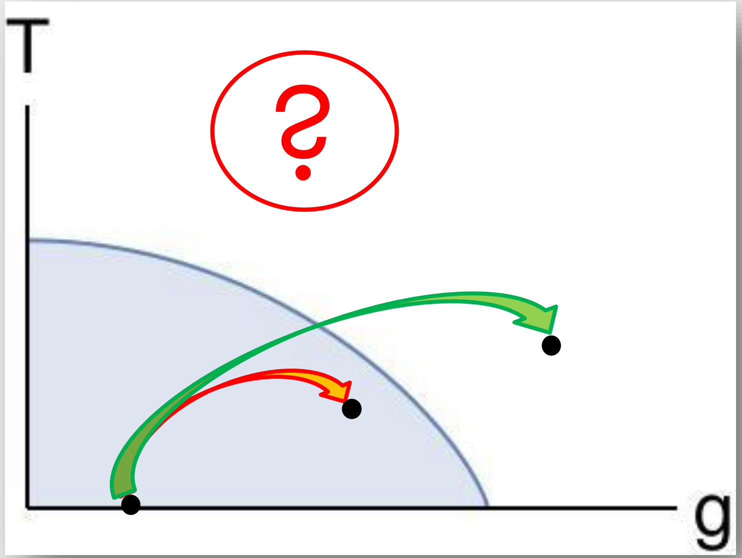
DYNAMICAL CRITICAL POINT

WHAT ABOUT FLUCTUATIONS?

- $d = \infty$ → DYNAMICAL CRITICALITY

  UNDAMPED COLLECTIVE MOTION

  → NO THERMALIZATION AT ALL!



- $1 \ll d < \infty$ → DYNAMICAL OR EQUILIBRIUM CRITICALITY?

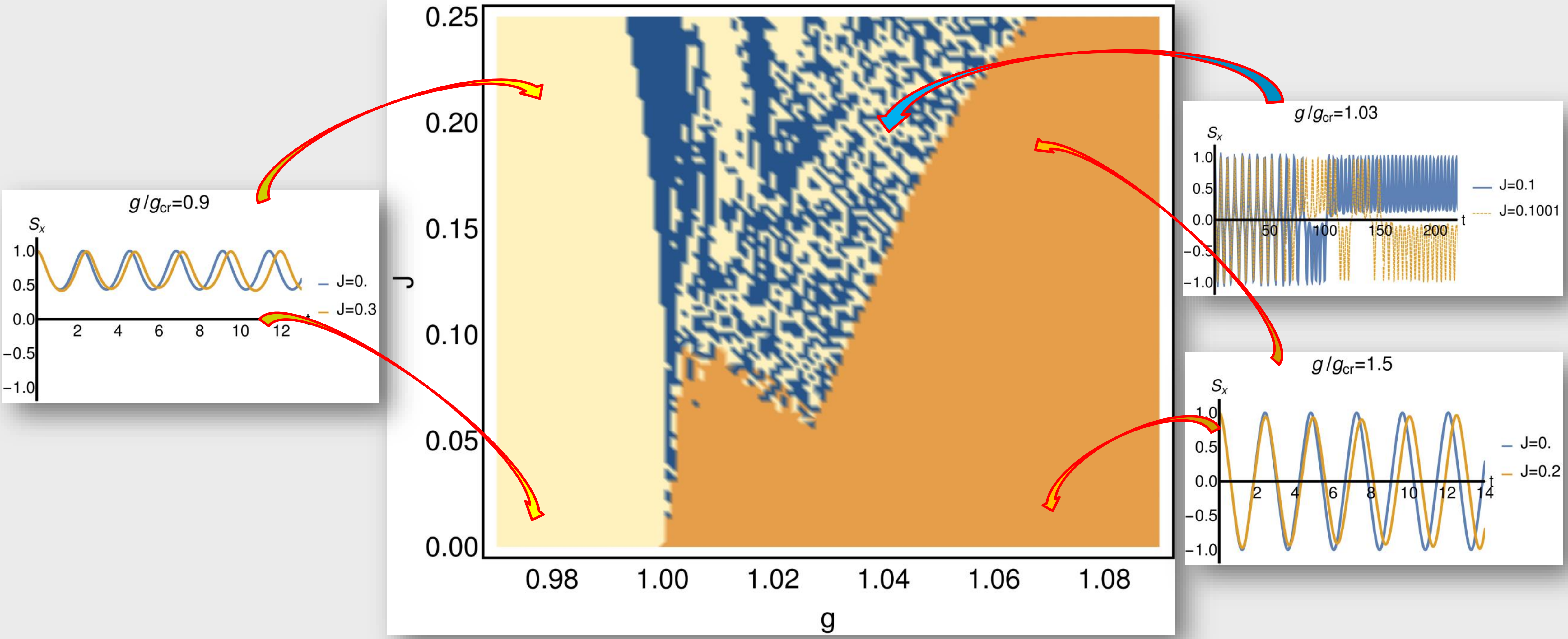
IMPACT OF FLUCTUATIONS: THE MODEL

$1 \ll d < \infty$ → IMPACT OF QUANTUM FLUCTUATIONS CONTROLLED BY $\frac{1}{d}$

$$H = -\frac{\lambda}{N} \sum_{i,j} \sigma_i^x \sigma_j^x - g(t) \sum_i \sigma_i^z - J \sum_i \sigma_i^x \sigma_{i+1}^x$$

- ANALYTICAL APPROXIMATION SCHEME: TIME-DEPENDENT SPIN-WAVE THEORY
- MPS-TDVP NUMERICAL STUDY OF THE FULL MANY-BODY QUANTUM EVOLUTION

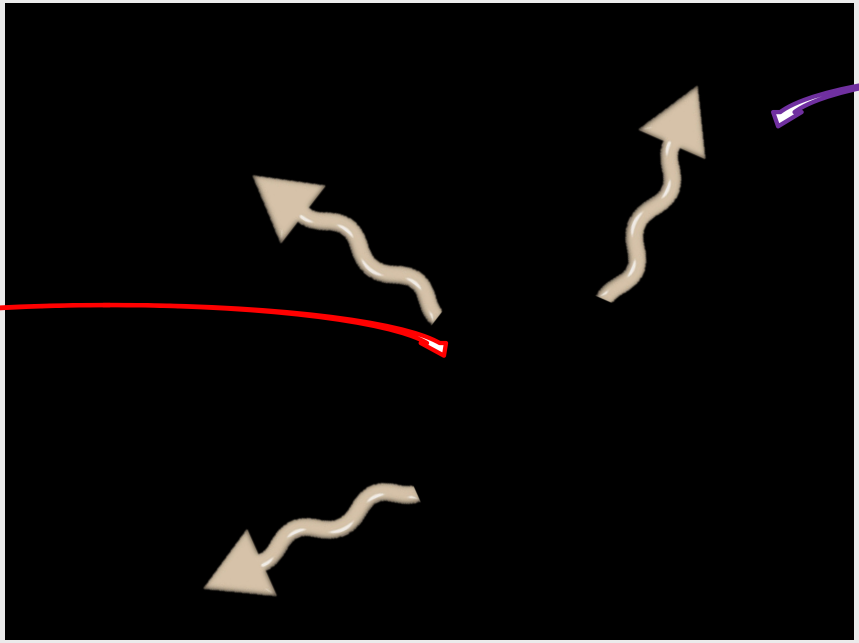
RESULT: QUENCH PHASE DIAGRAM $(g_0 = 0,\ \lambda + J \equiv 1)$



MOTION IN THE CHAOTIC DYNAMICAL PHASE

Animation showing the evolution of two trajectories corresponding to two very close parameters here

ANALOGY: THE "COIN" PHASE

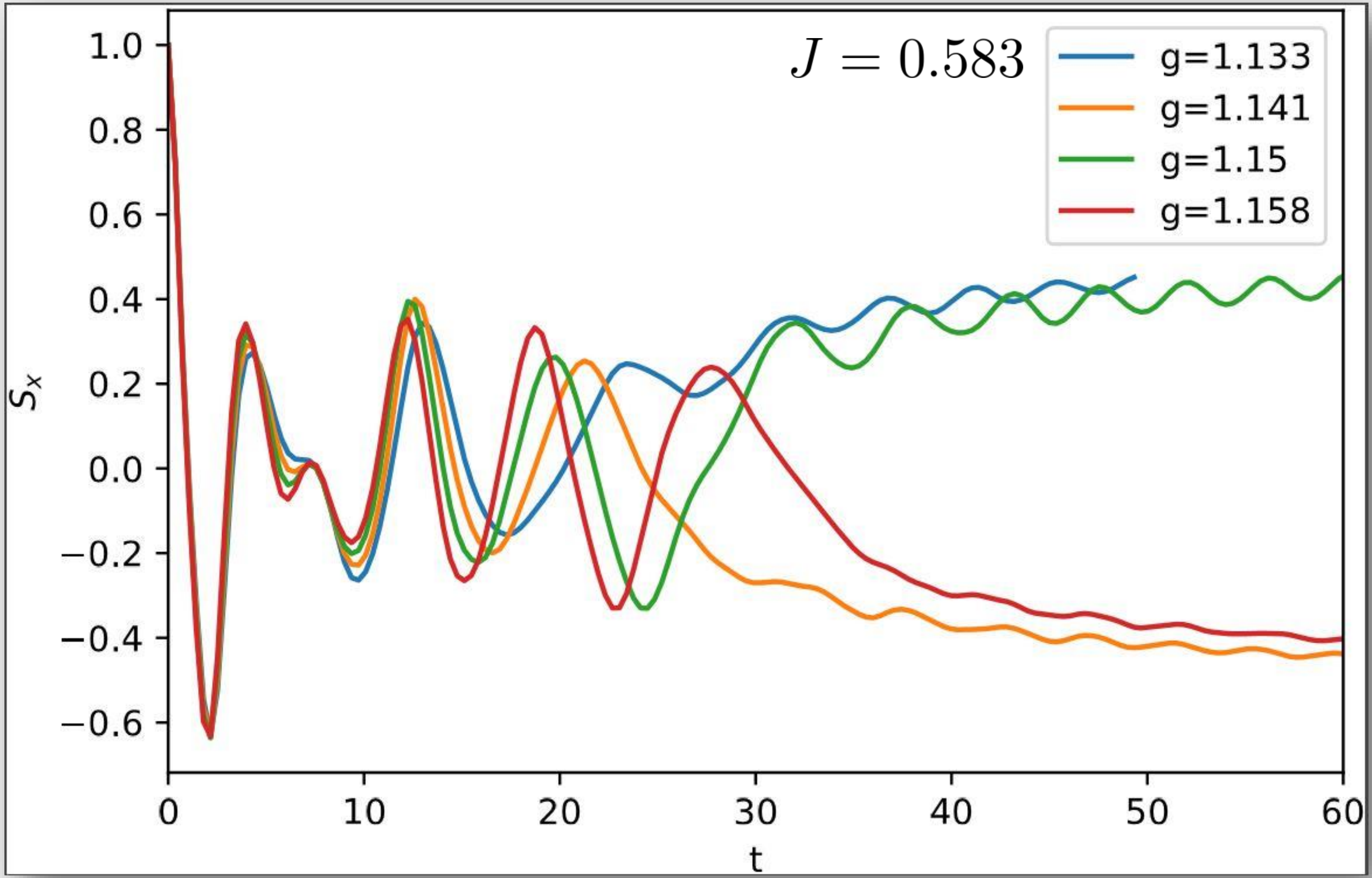


SUDDEN QUENCH

" RANDOMLY" CHOSEN STABLE STATE

MICROSCOPIC FLUCTUATION MODES

MPS-TDVP NUMERICS: A CONFIRMATION



TIME-DEPENDENT VACUUM AND EXCITATIONS

- MF DYNAMICS: $$|\Psi(t)\rangle = |\nwarrow\nwarrow\cdots\nwarrow\rangle \equiv \hat{U}\big(\theta(t),\phi(t)\big)\ |\uparrow\uparrow\ldots\uparrow\rangle$$

- WHAT DOES THE J-TERM DO? $$\propto \quad J\sum_k \cos k\ \big(\hat{U}\widetilde{S}_k^-\hat{U}^\dagger\big)\ \big(\hat{U}\widetilde{S}_{-k}^-\hat{U}^\dagger\big)$$

→ COUPLED EVOLUTION OF THE EXCITATIONS AND OF THE VACUUM!

- EXCITATIONS ARE QUANTUM, VACUUM IS CLASSICAL

- IDEA:

  WORK IN A MOBILE REFERENCE FRAME THAT ABSORBS THE VACUUM DYNAMICS

→ FIXED BACKGROUND MANY-BODY HILBERT SPACE FOR EXCITATIONS' DYNAMICS

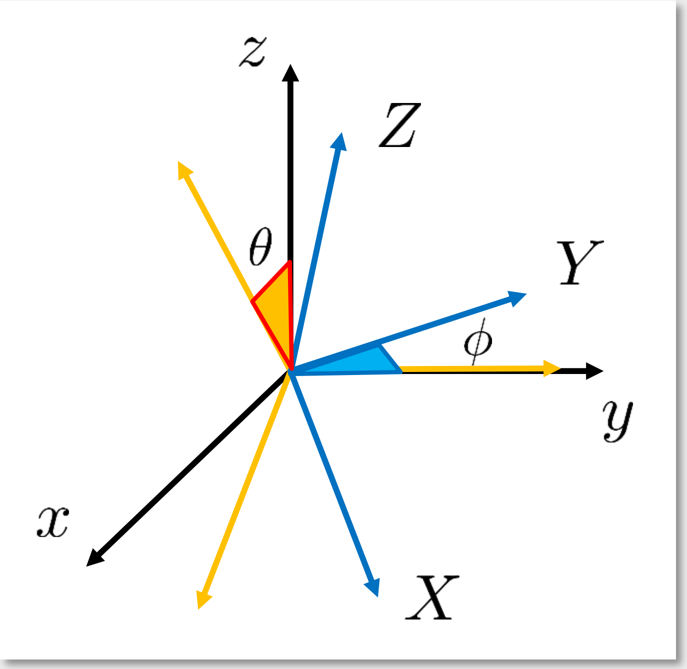
STEP 1 : TIME-DEPENDENT REFERENCE FRAME

$$\hat{V}(t) := \exp\left(-i\phi(t)\, S^z_{\text{tot}}\right)\, \exp\left(-i\theta(t)\, S^y_{\text{tot}}\right)$$

$$\Longrightarrow \qquad H \mapsto \widetilde{H}(t) := H + i\hat{V}\dot{\hat{V}}^\dagger$$

"INERTIAL FORCES"

$$-\vec{\omega}(t) \cdot \vec{S}_{\text{tot}}$$



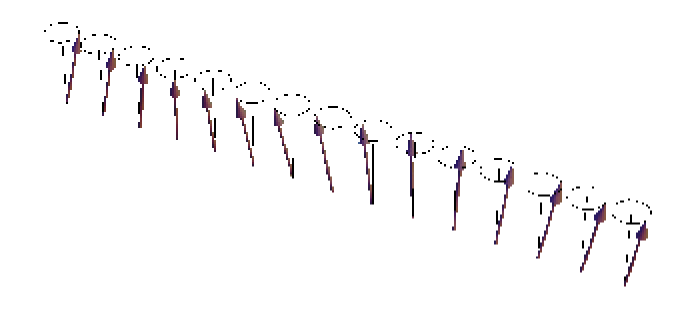
$$\omega^X = -\sin\theta\, \dot{\phi}, \qquad \omega^Y = \dot{\theta}, \qquad \omega^Z = \cos\theta\, \dot{\phi}$$

$$\frac{\widetilde{H}}{N} = -\lambda\left(\frac{\tilde{S}^x_0}{Ns}\right)^2 - g(t)\,\frac{\tilde{S}^z_0}{Ns} - J\sum_k \cos k\, \frac{\tilde{S}^x_k}{Ns}\,\frac{\tilde{S}^x_{-k}}{Ns} - s\vec{\omega}(t)\cdot\frac{\tilde{\vec{S}}_0}{Ns}$$

STEP 2 : TIME-DEPENDENT SPIN-WAVES

HOLSTEIN-PRIMAKOFF TRANSFORMATION TO CANONICAL VARIABLES:

$$\begin{cases} \frac{S_j^X}{s} \approx \frac{q_j}{\sqrt{s}} \\ \frac{S_j^Y}{s} \approx \frac{p_j}{\sqrt{s}} \\ \frac{S_j^Z}{s} = 1 - \frac{q_j^2 + p_j^2 - 1}{2s} \end{cases} \quad \rightsquigarrow \quad \begin{cases} \frac{\tilde{S}_k^X}{Ns} \approx \frac{\tilde{q}_k}{\sqrt{Ns}} \\ \frac{\tilde{S}_k^Y}{Ns} \approx \frac{\tilde{p}_k}{\sqrt{Ns}} \\ \frac{\tilde{S}_k^Z}{Ns} = \delta_{k,0} - \sum_{k'} \frac{\tilde{q}_{k'}\tilde{q}_{k-k'} + \tilde{p}_{k'}\tilde{p}_{k-k'} - \delta_{k,0}}{2Ns} \end{cases}$$



STEP 3 : LOW-DENSITY EXPANSION

- IMPOSE FRAME = VACUUM :

$$\langle S^X_{\text{tot}} \rangle (t) \equiv \langle S^Y_{\text{tot}} \rangle (t) \equiv 0$$

→ EVOLUTION EQ.S FOR QUANTUM EXCITATIONS IN A FIXED MANY-BODY HILBERT SPACE

- NATURAL APPROXIMATION: LOW-DENSITY

$$\epsilon(t) := \frac{\sum_{k\neq 0} \langle \tilde{b}^\dagger_k \tilde{b}_k \rangle}{Ns}, \qquad \langle S^Z_{\text{tot}} \rangle (t) = Ns - \sum_k \langle \tilde{b}^\dagger_k \tilde{b}_k \rangle \equiv Ns\big(1 - \epsilon(t)\big)$$

AS LONG AS $\epsilon(t) \ll 1$ THE TRUNCATION AT QUADRATIC ORDER IS SELF-CONSISTENT

(''TIME-DEPENDENT BOGOLUBOV-LIKE SCHEME'')

SELF-CONSISTENT COUPLED EVOLUTION OF VACUUM AND FLUCTUATIONS

$$\frac{d}{dt}\theta = +\,4\bar{\lambda}(1-\epsilon)\sin\theta\cos\phi\sin\phi$$
$$-\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{pp}\right)\sin\theta\cos\phi\sin\phi$$
$$+\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\cos\theta\sin\theta\cos^2\phi,$$

$$\frac{d}{dt}\phi = -\,2g + 4\bar{\lambda}(1-\epsilon)\cos\theta\cos^2\phi$$
$$-\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qq}\right)\cos\theta\cos^2\phi$$
$$+\,4J\left(\frac{1}{Ns}\sum_{k\neq 0}\cos k\ \Delta_k^{qp}\right)\sin\phi\cos\phi$$

$$\frac{d}{dt}\Delta_k^{qq} = +\left(+\,8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{qq} + \left(+\,8\bar{\lambda}\cos^2\phi - 8J\cos k\ \sin^2\phi\right)\Delta_k^{qp}$$

$$\frac{d}{dt}\Delta_k^{qp} = +\left(-\,4\bar{\lambda}\cos^2\phi + 4J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qq} + \left(+\,4\bar{\lambda}\cos^2\phi - 4J\cos k\ \sin^2\phi\right)\Delta_k^{pp}$$

$$\frac{d}{dt}\Delta_k^{pp} = +\left(-\,8\bar{\lambda}\cos^2\phi + 8J\cos k\ \cos^2\theta\cos^2\phi\right)\Delta_k^{qp} + \left(-\,8J\cos k\ \cos\theta\cos\phi\sin\phi\right)\Delta_k^{pp}$$

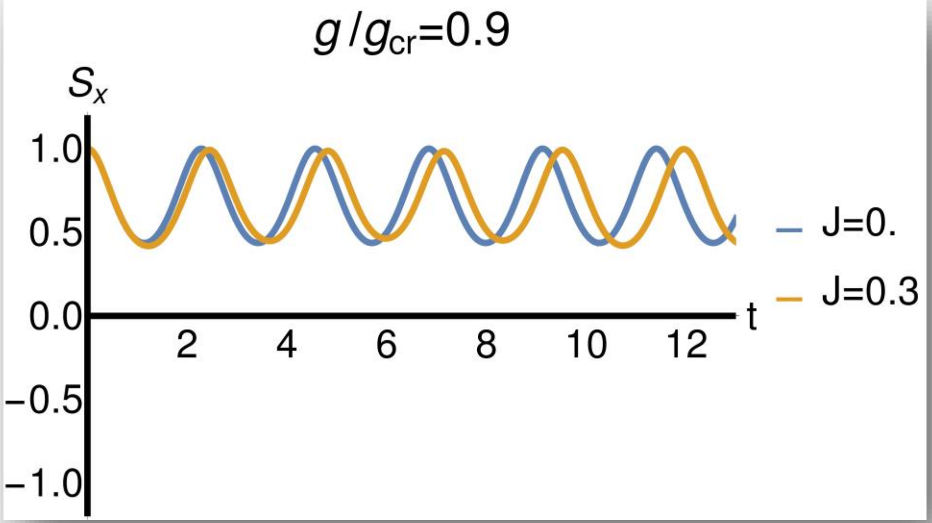
SUMMARY

1. *The dynamical phases are robust to the impact of a short-range interaction*
2. *The critical point reshapes into an extended crossover region*
3. *Macroscopic chaos appears as a consequence of quantum fluctuations*

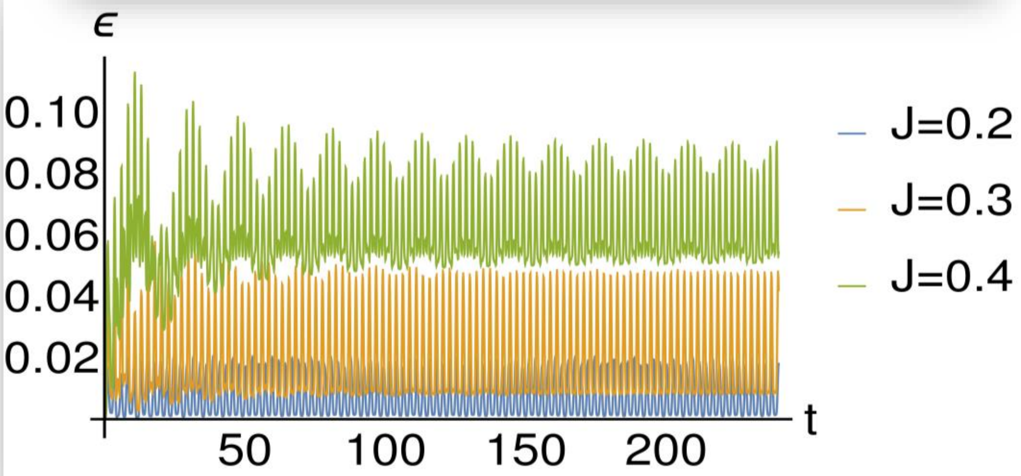
THANK YOU
FOR YOUR ATTENTION

arXiv:1706.05062

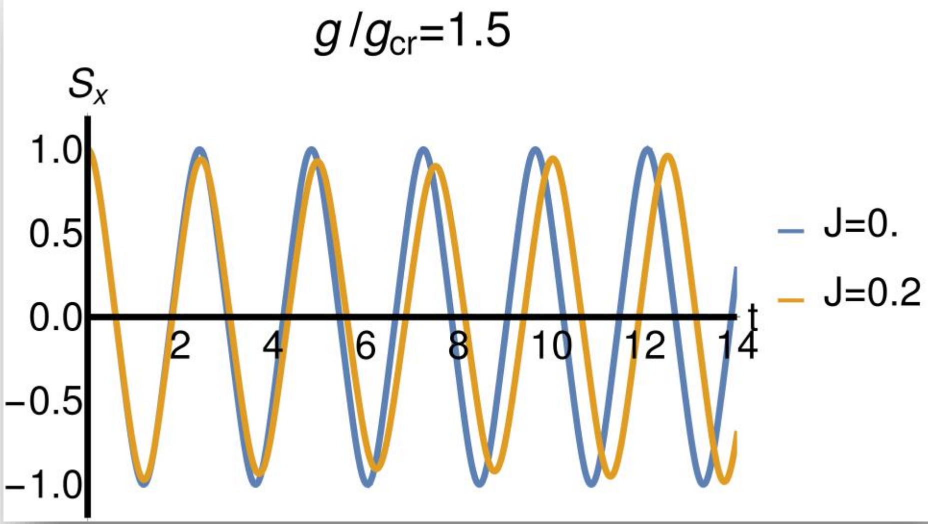
$g \ll g_{cr}$


g/gcr=0.9
Sx
1.0
0.5
0.0
−0.5
−1.0
2
4
6
8
10
12
t
J=0.
J=0.3

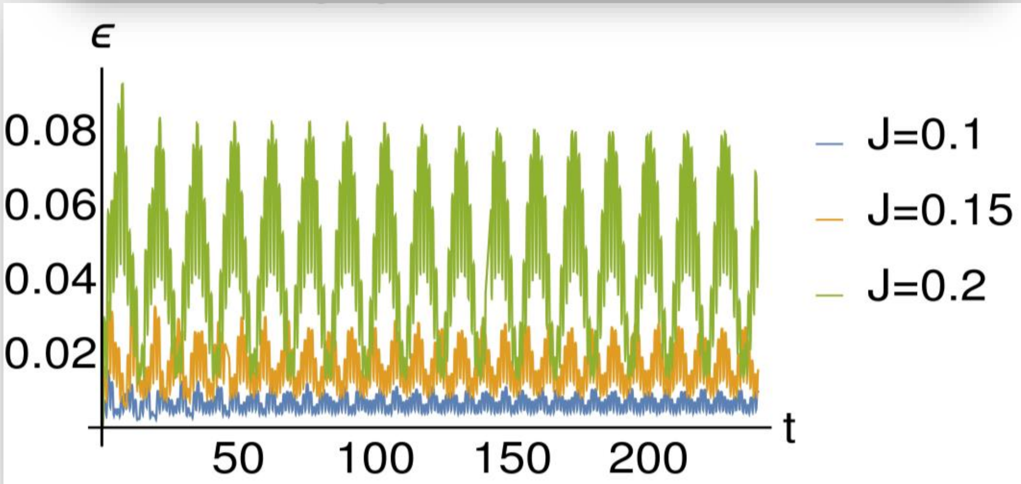

J=0.
J=0.3


ϵ
0.10
0.08
0.06
0.04
0.02
50
100
150
200
t
J=0.2
J=0.3
J=0.4

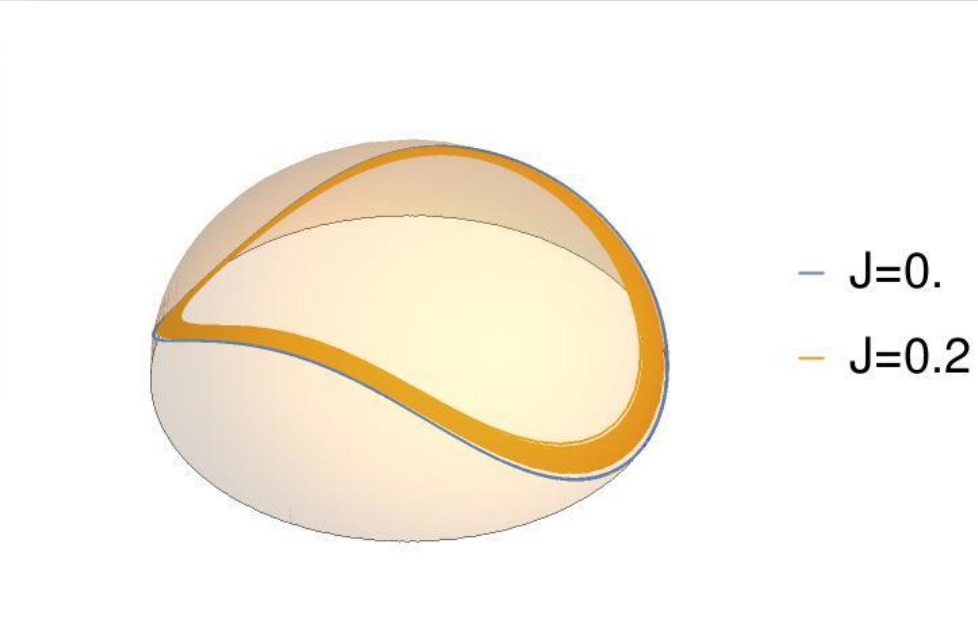
$g \gg g_{cr}$



g/g_{cr}=1.5
S_x
1.0
0.5
0.0
−0.5
−1.0
2
4
6
8
10
12
14
t
J=0.
J=0.2

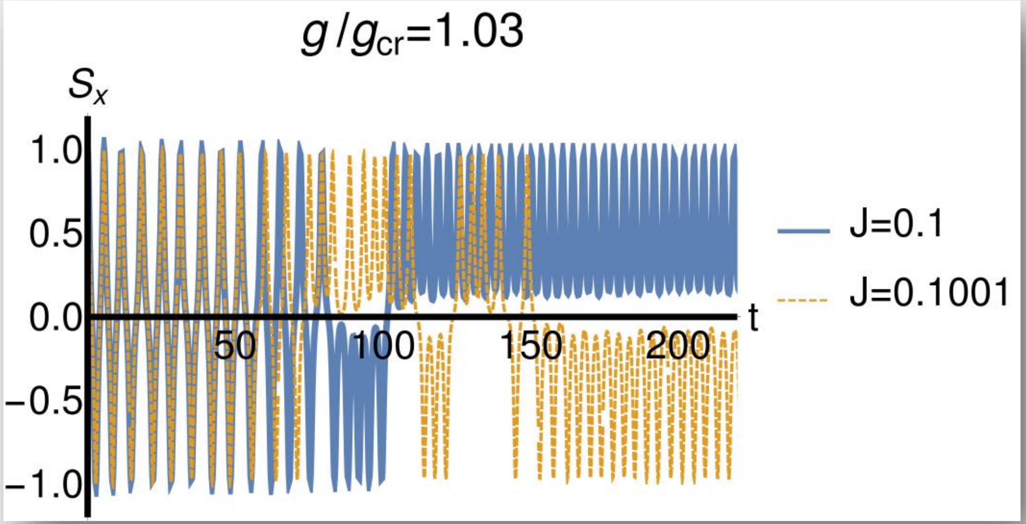


ϵ
0.08
0.06
0.04
0.02
50
100
150
200
t
J=0.1
J=0.15
J=0.2

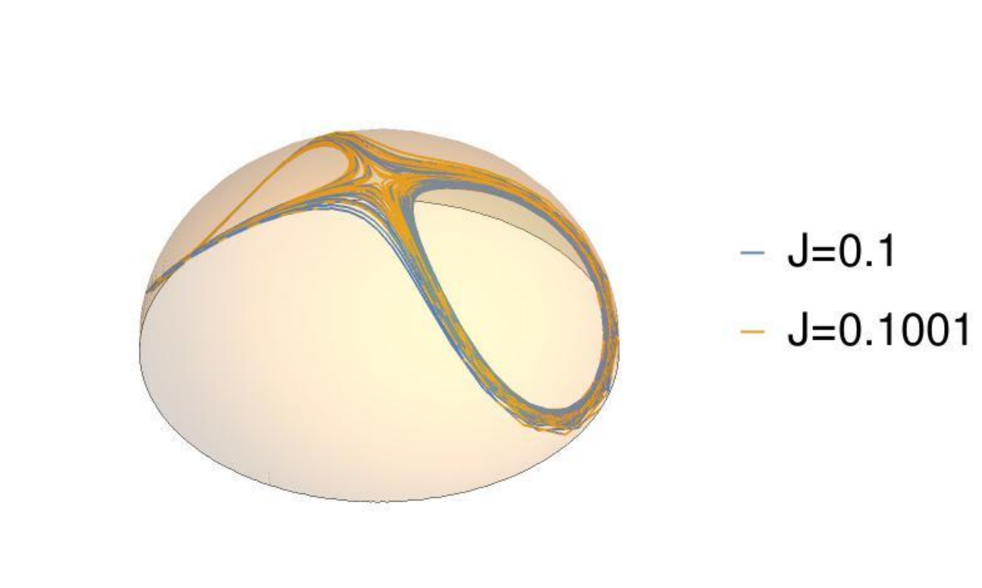


J=0.
J=0.2

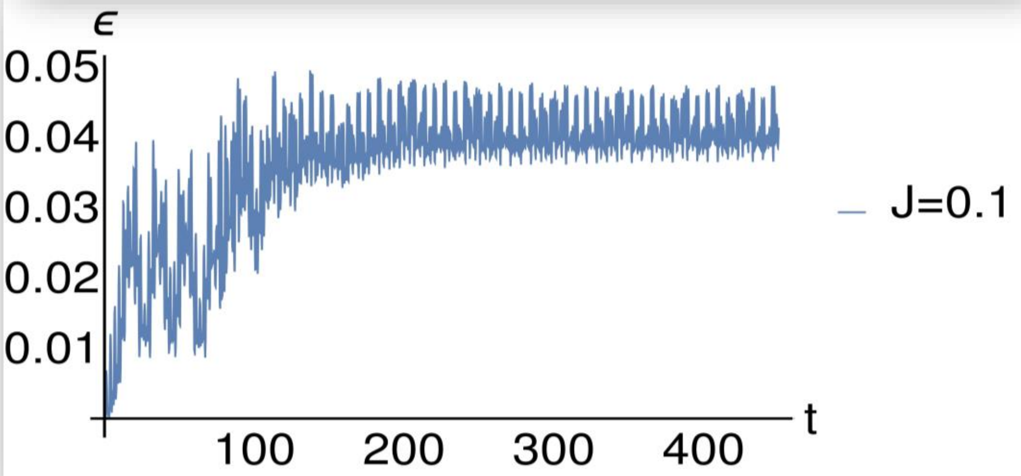
$g \approx g_{cr}$



g/g_{cr}=1.03
S_x
1.0
0.5
0.0
−0.5
−1.0
50
100
150
200
t
J=0.1
J=0.1001



J=0.1
J=0.1001



ϵ
0.05
0.04
0.03
0.02
0.01
100
200
300
400
t
J=0.1

ILLUSTRATION: ISING MAGNET IN TRANSVERSE FIELD

FERROMAGNETIC COUPLING

$$H = -\sum_{i,j} J_{ij}\sigma_i^x\sigma_j^x - g(t)\sum_i \sigma_i^z$$

EXTERNAL TRANSVERSE FIELD