HPQCD

Charm and beauty: Heavy quark physics from lattice QCD

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VIA VERITAS VITA

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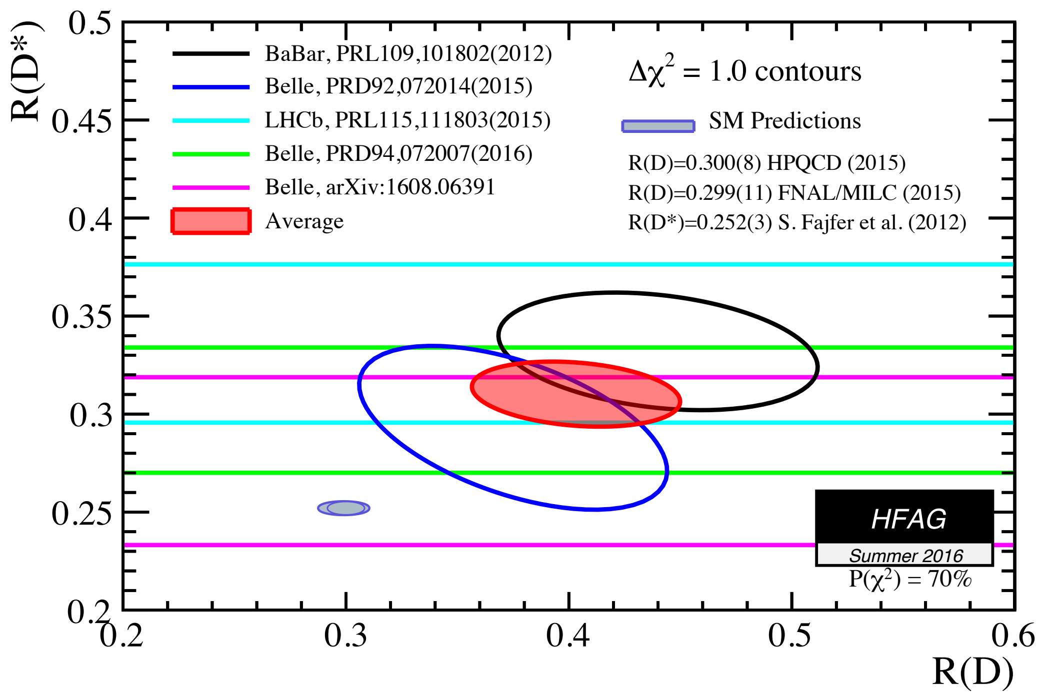
Outline

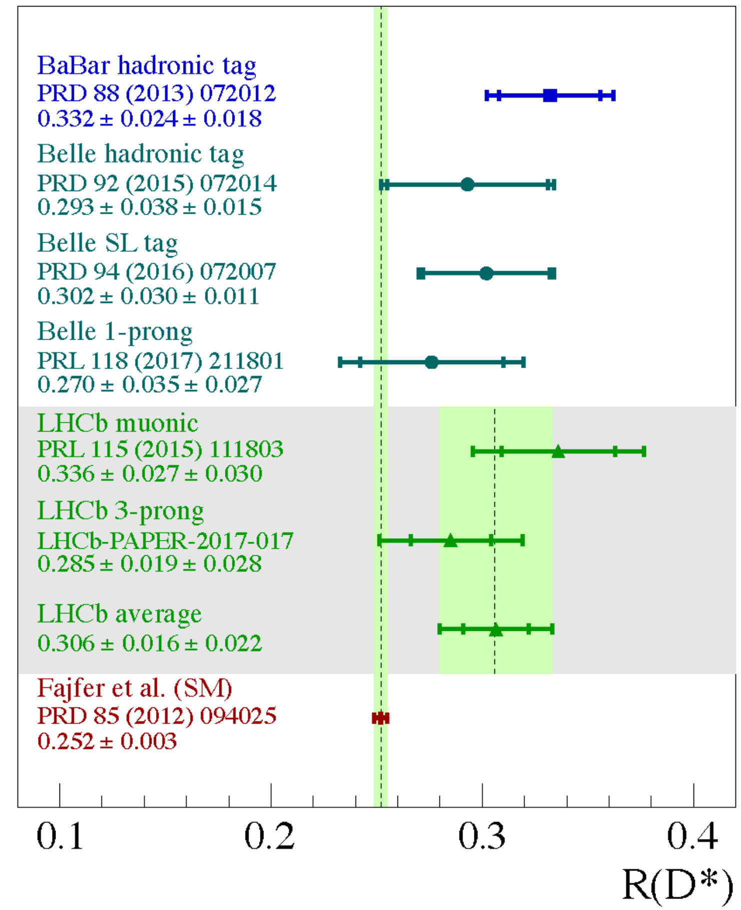
- Motivation: discrepancies with SM in $b \to c$ semileptonic observables
- b quarks on the lattice
  - Highly Improved Staggered Quarks
  - Non-relativistic QCD
- $B_c \to J/\psi$ semileptonic decay, form factors
- Summary and outlook

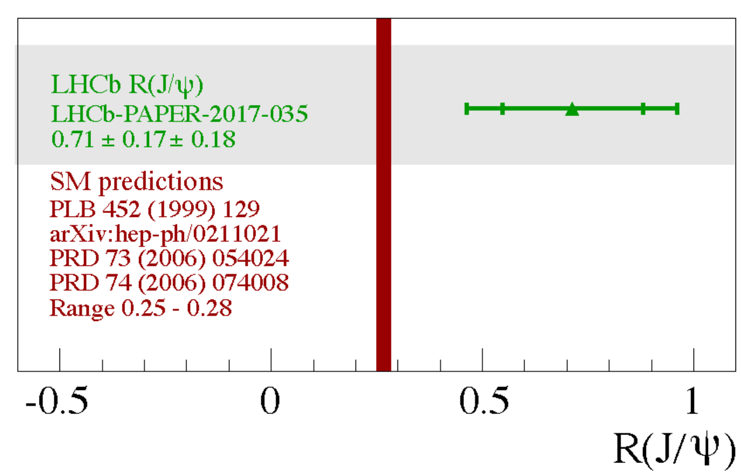
Motivation

- Persistent discrepancy with SM in $b \to c$ semileptonic observables (in both $B \to D^*$ and $B_c \to J/\psi$)

$$R(B \to D^*) = \frac{\mathcal{B}(B \to D^* \tau \nu)}{\mathcal{B}(B \to D^* \ell \nu)}, \quad \ell = e, \mu$$



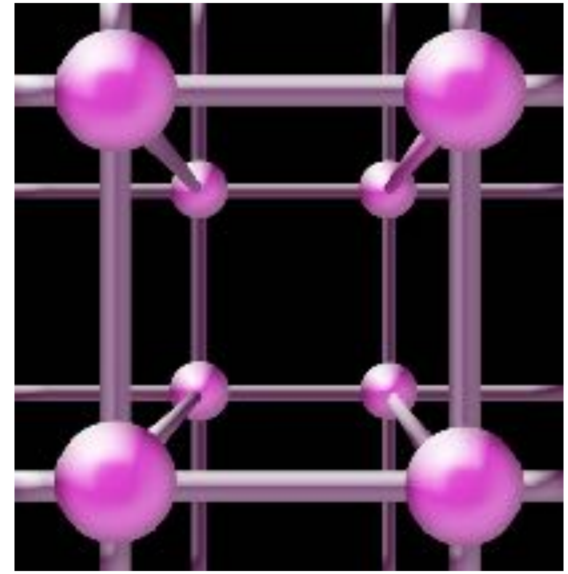


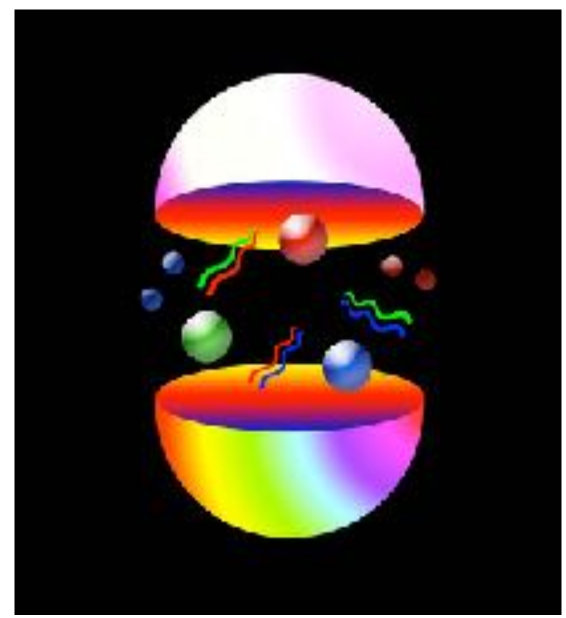


Reliable SM predictions are needed!

Lattice QCD

- Study mesons and their leptonic and semileptonic decays using state-of-the-art computer clusters
  - fully nonperturbative QCD calculation
  - high precision SM predictions





b-quark on the lattice

- Highly Improved Staggered Quarks (HISQ)
  - small discretisation errors, very good for c
  - typically discretisation errors grow with growing quark mass: $(ma)^2$, $\alpha_s(ma)^2$, $(ma)^4$
  - need $ma<1$ to control discretisation effects
  - go up from charm quark mass as high as possible, can almost reach m_b on the finest lattices
- Same action for heavy and light quarks
- Small a, physical pions, u/d, s and c quarks in the sea, multiple lattice spacings…

b-quark on the lattice

- NRQCD (Non-relativistic effective theory on the lattice, perturbative matching to QCD)
  - accurate through $\mathcal{O}(\alpha_s v^4)$
  - the scale of discretisation errors set by internal momenta pa
  - good for heavy quarks like b, can not be used for lighter quarks (e.g. charm)
  - need $ma>1$ to control coefficients of relativistic corrections

These two approaches are complementary. Ideally there is a range of overlap in applicability to check the approaches are mutually consistent.

NRQCD Hamiltonian

$$e^{-aH} = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)U_t^\dagger\left(1 - \frac{aH_0}{2n}\right)\left(1 - \frac{a\delta H}{2}\right)$$

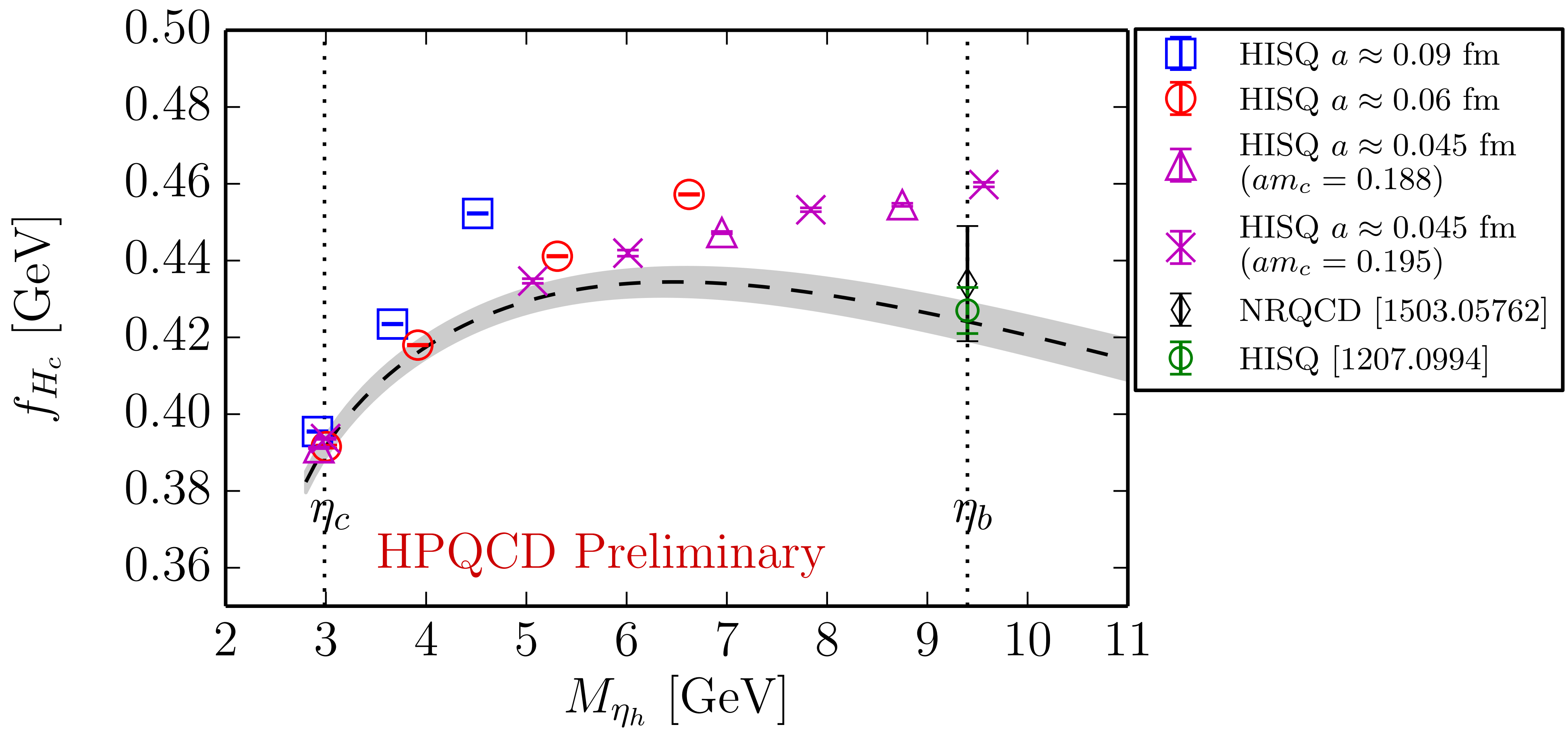
$$aH_0 = -\frac{\Delta^{(2)}}{2am_b}$$

$$a\delta H = -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2}(\nabla\cdot\tilde{E} - \tilde{E}\cdot\nabla) - c_3\frac{1}{8(am_b)^2}\sigma\cdot(\tilde{\nabla}\times\tilde{E} - \tilde{E}\times\tilde{\nabla})$$
$$- c_4\frac{1}{2am_b}\sigma\cdot\tilde{B} + c_5\frac{\Delta^{(4)}}{24am_b} - c_6\frac{(\Delta^{(2)})^2}{16n(am_b)^2}$$

Set	c_1	c_5	c_4	c_6
very coarse	1.36	1.21	1.22	1.36
coarse	1.31	1.16	1.20	1.31
fine	1.21	1.12	1.16	1.21

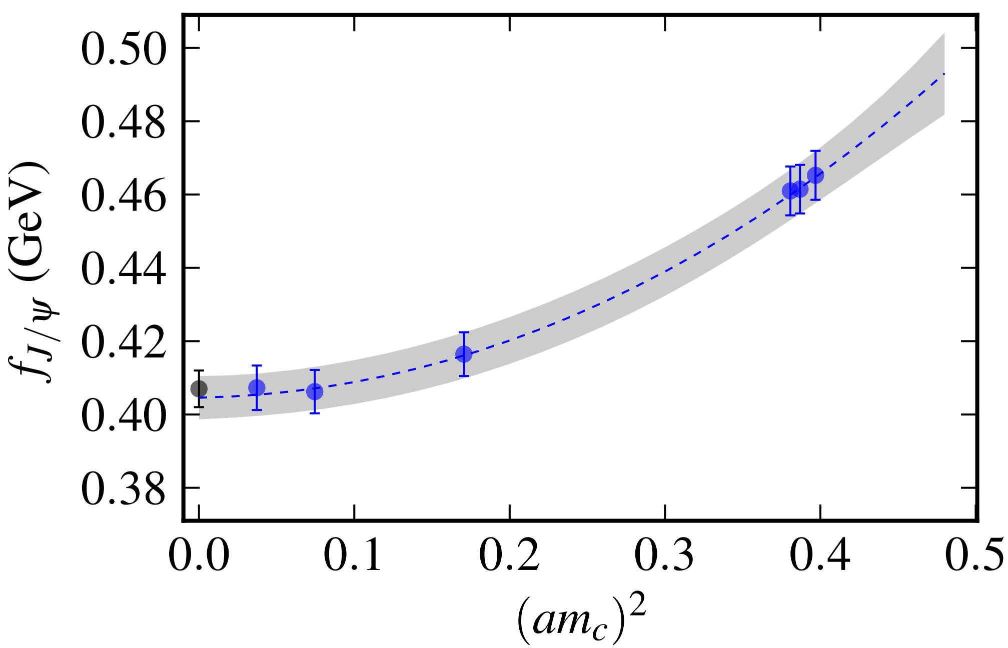
Spanning c to b with HISQ: meson decay constant

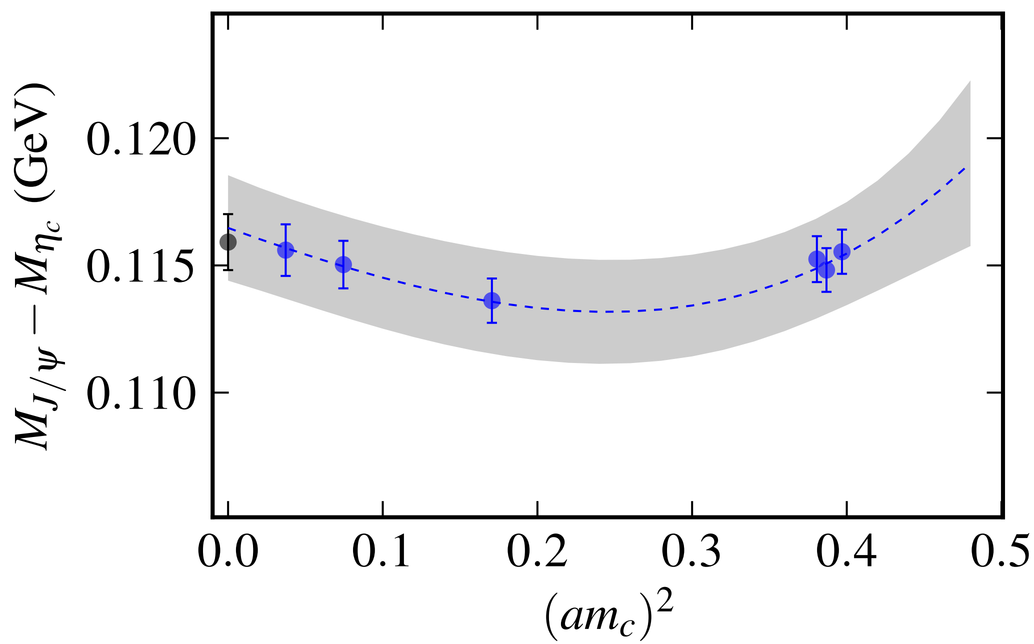
- Probe mass from m_c towards m_b and extrapolate



J/ψ mass and decay constant

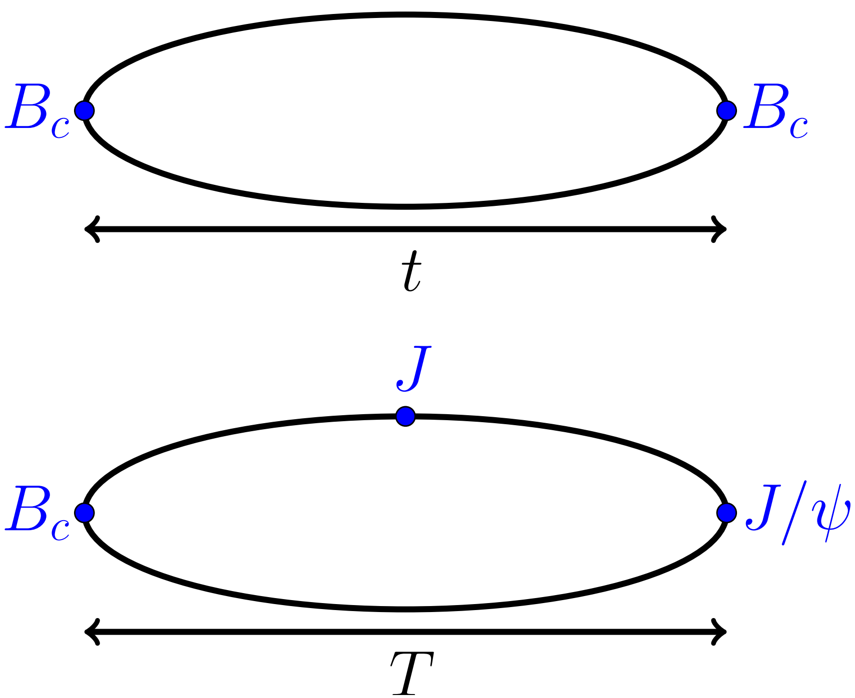
- Tune the charm quark mass accurately
- Use multiple lattice spacings, extrapolate to *a=0*
- Look at mass difference $M_{J/\psi}$-M_η instead of $M_{J/\psi}$

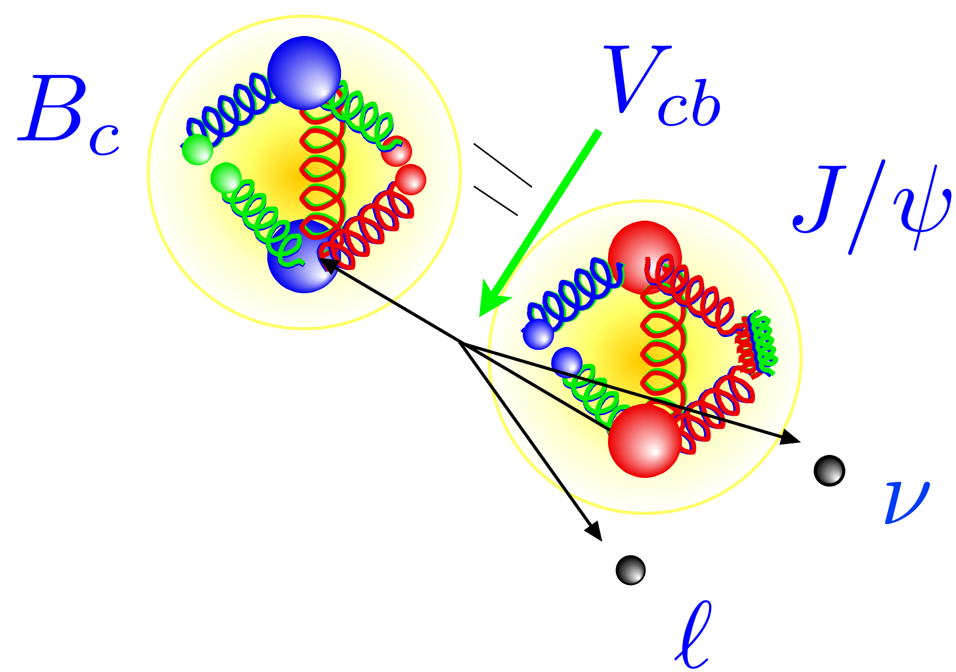




Semileptonic decays

- Study of $B_c \to \eta_c$, $B_c \to J/\psi$ decay matrix elements
- We work in the frame where the B_c is at rest
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions





$B_c \rightarrow J/\psi$ form factors

$$\langle J/\psi(p',\epsilon)|V^\mu - A^\mu|B_c(p)\rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M_{B_c}+M_{J/\psi}}\epsilon^*_\nu p'_\rho p_\sigma V(q^2)$$

$$-(M_{B_c}+M_{J/\psi})\epsilon^{*\mu}A_1(q^2) + \frac{\epsilon^*\cdot q}{M_{B_c}+M_{J/\psi}}(p'+p)^\mu A_2(q^2)$$

$$+2M_{J/\psi}\frac{\epsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2M_{J/\psi}\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2),$$

$$\text{where } A_3(q^2) = \frac{M_{B_c}+M_{J/\psi}}{2M_{J/\psi}}A_1(q^2) - \frac{M_{B_c}-M_{J/\psi}}{2M_{J/\psi}}A_2(q^2)$$

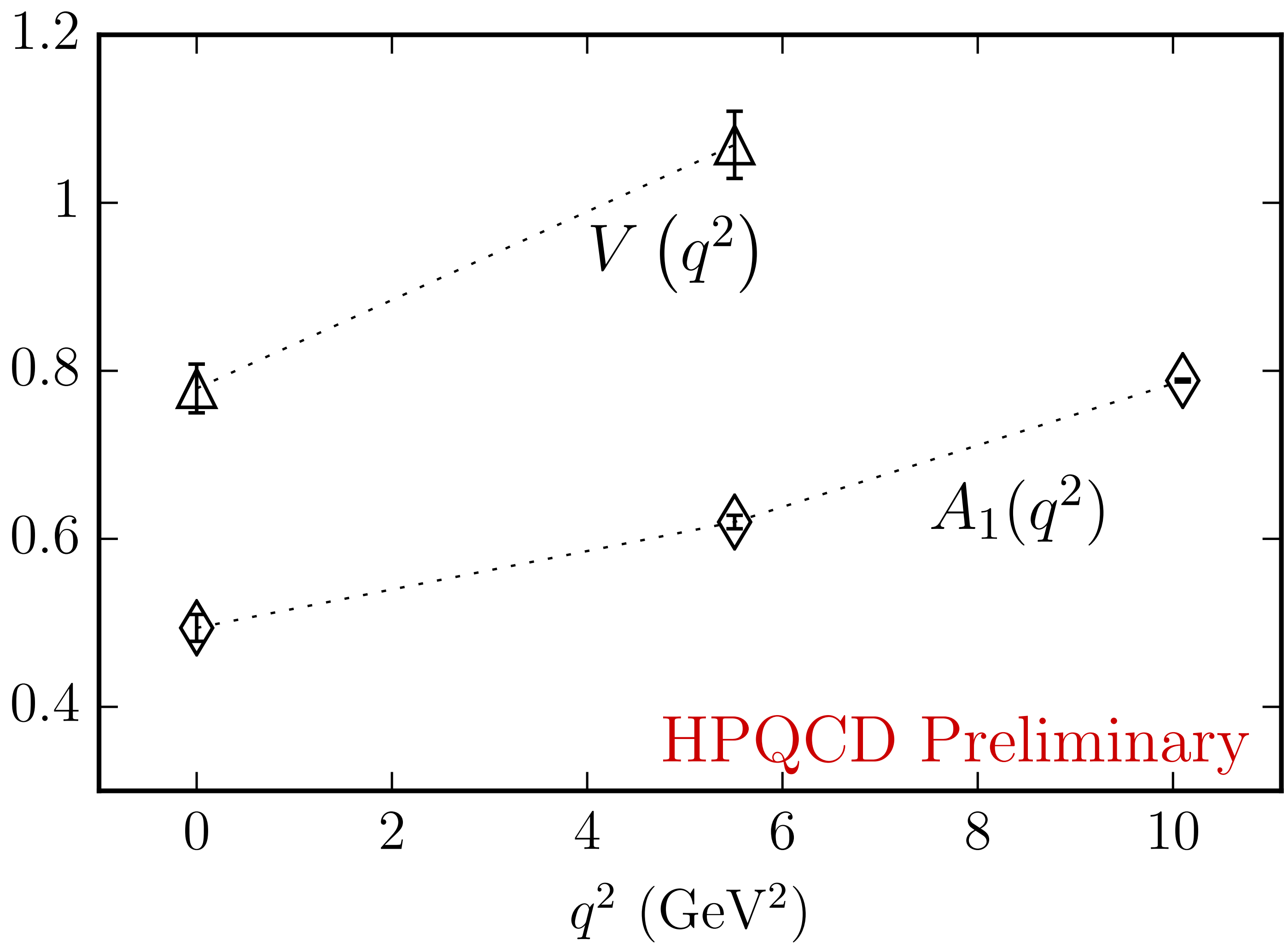
$$\text{and } A_3(0) = A_0(0)$$

The form factors which parametrise the matrix elements are functions of q^2, where q is the four-momentum transferred to the leptons

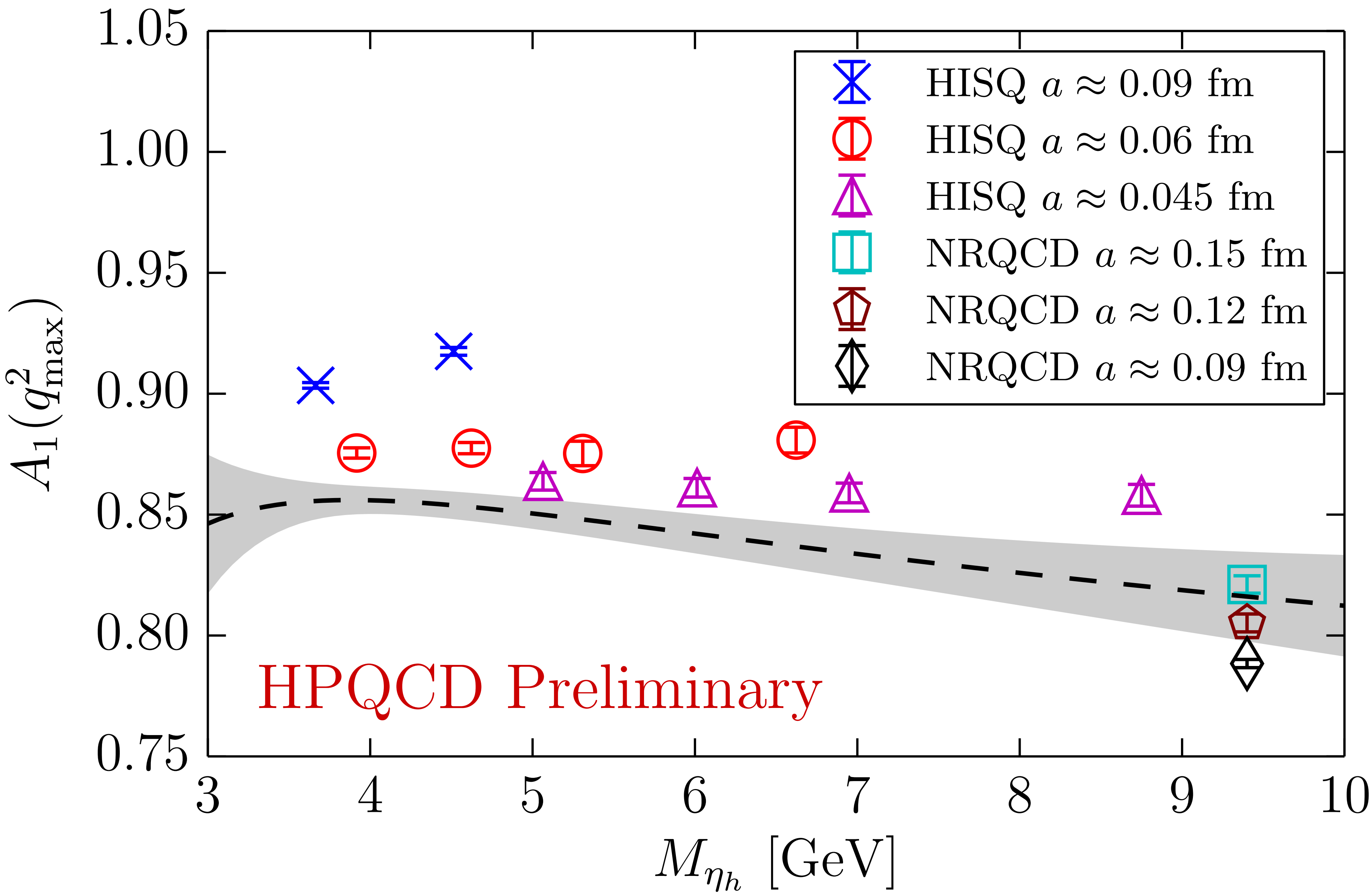
- $q^2 = (M_{B_c} - M_{J/\psi})^2$, zero recoil of decay hadron
- $q^2 = 0$, maximum recoil of decay hadron

NRQCD $B_c \to J/\psi$ form factors

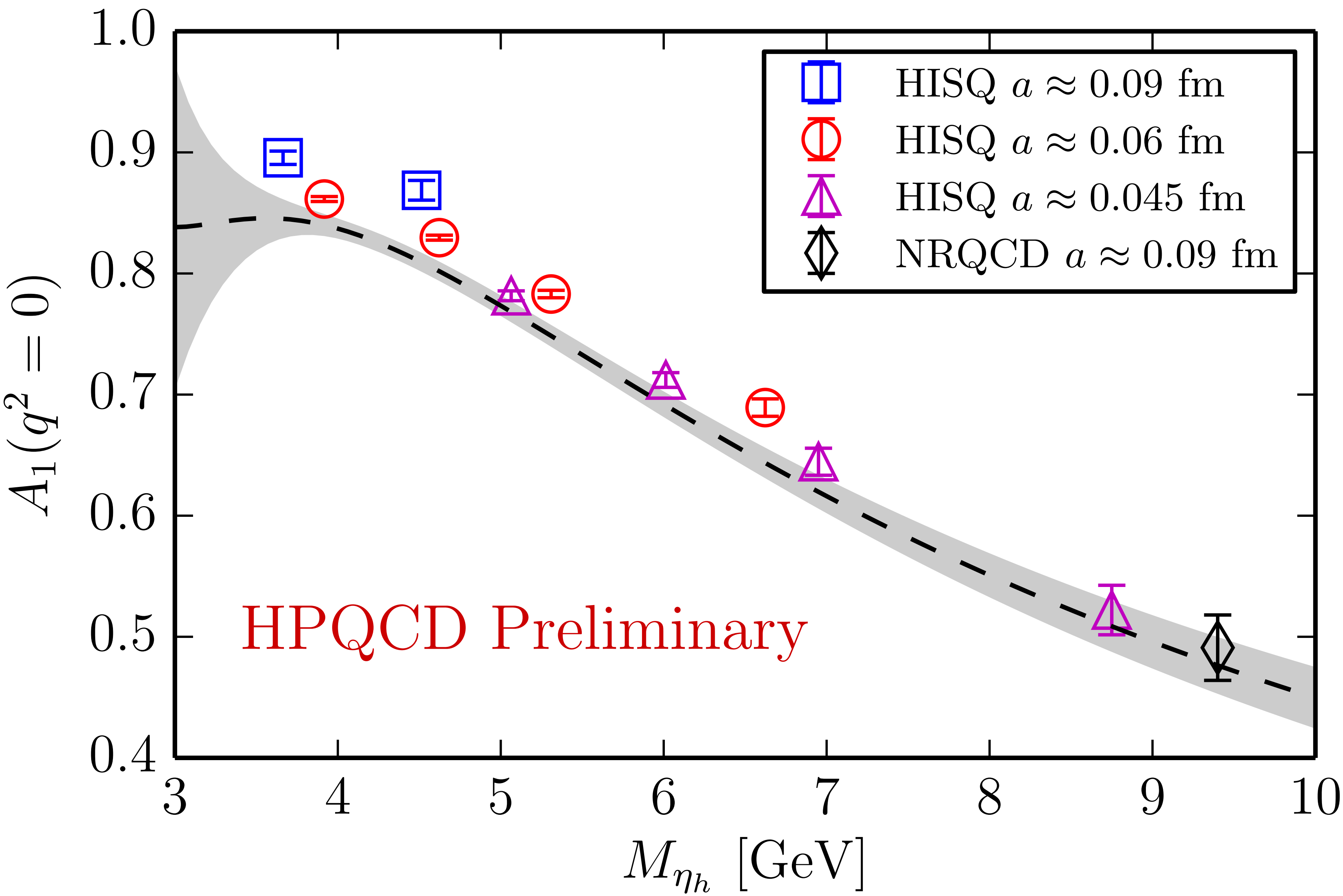
- Cover the full q^2 range
- Physical b quark mass



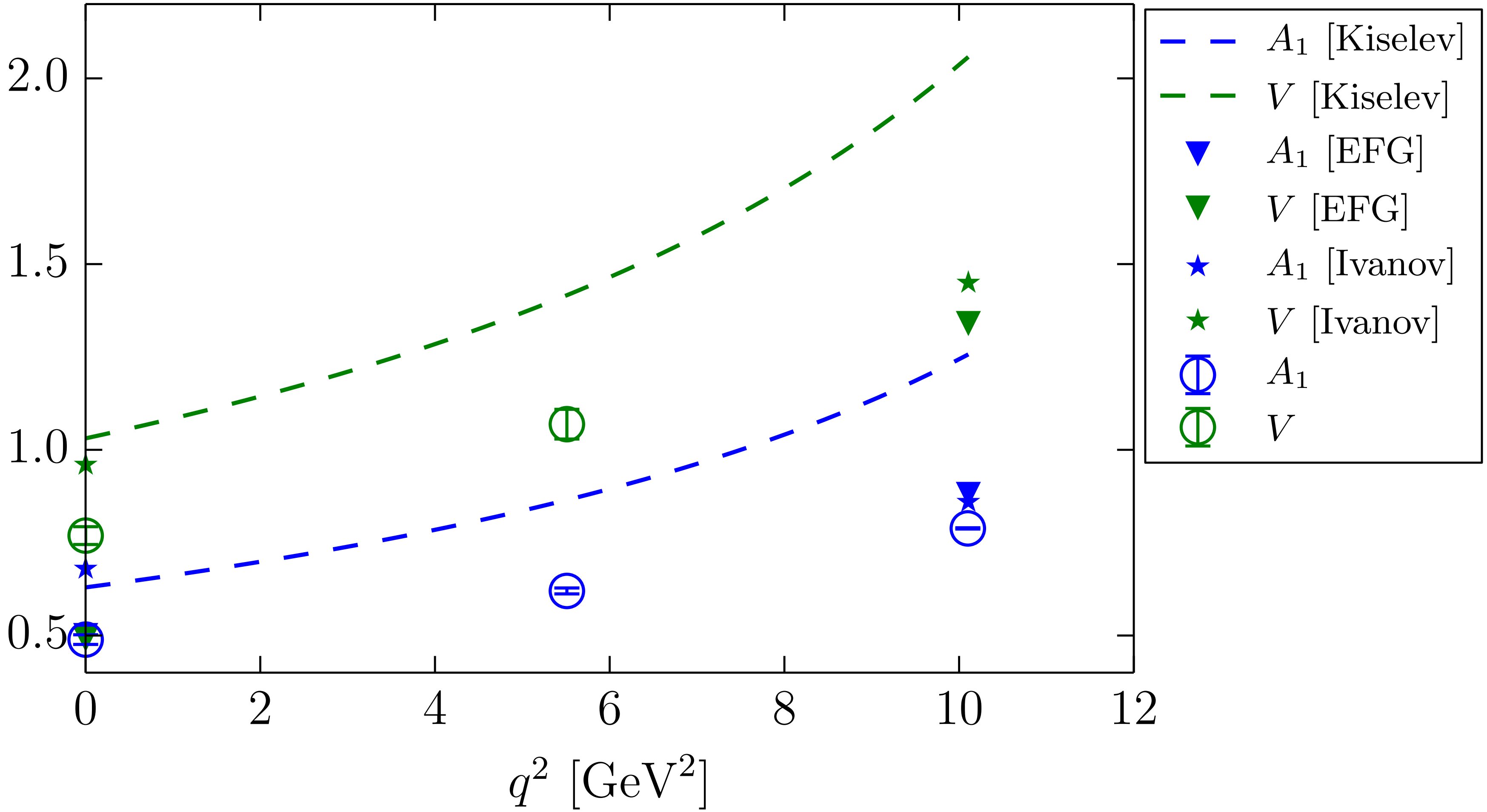
$B_c \to J/\psi$ form factors: $A_1(q^2{}_{\max})$



$B_c \to J/\psi$ form factors: $A_1(q^2{=}0)$



Comparisons $B_c \to J/\psi$



hep-ph/0007169,0211021,0306306
(relativistic quark model, QCD sum rules)

R-ratios

$$R(B_c \to J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi\tau\nu)}{\mathcal{B}(B_c \to J/\psi\ell\nu)}, \quad \ell = e, \mu$$

- Test lepton flavour universality
- There are persistent few-sigma anomalies in the ratios $R(B \to D^*)$ and $R(B \to D)$ involving the same $b \to c$ transition
- The current work will provide reliable SM determination for $R(B_c \to J/\psi)$, to be compared with recent measurement by LHCb

Summary

- A promising approach to study of $b \to c$ transitions:
  - Lattice NRQCD with HISQ quarks, plus
  - Fully relativistic formulation, extrapolate m_h to m_b
- Proof-of-principle demonstrated for B_c semileptonic decay
  - Controlled calculation over full q^2 range
  - Good agreement seen with NRQCD results

Outlook

- $B_c \to J/\psi$ - new possible determination of $|V_{cb}|$
- Reliable SM prediction for $R(B_c \to J/\psi)$
- Improved understanding of NRQCD currents feeds into additional calculations ($B \to D, B \to D^*, \ldots$)
- Expand relativistic formalism e.g. to $B_s \to D_s{}^*$ at zero recoil

Thank you!

Backup slides

Continuum extrapolation

- Generic HQET-inspired fit form given by

$$F(q^2, M_{\eta_h}, a^2) = A(q^2)\Big(\frac{M_{\eta_h}}{M_0}\Big)^b \times$$
$$\Big[\sum_{ijkl} c_{ijkl}(q^2)\Big(\frac{M_0}{M_{\eta_h}}\Big)^i \Big(\frac{am_c}{\pi}\Big)^{2j} \Big(\frac{am_h}{\pi}\Big)^{2k} \Big(\frac{a\Lambda_{\mathrm{QCD}}}{\pi}\Big)^{2l}\Big]$$

- Continuum result evaluated at $m_h = m_b$:

$$F(q^2, M_{\eta_b}, 0) = A(q^2)\Big(\frac{M_{\eta_b}}{M_0}\Big)^b \Big[\sum_{i000} c_{i000}(q^2)\Big(\frac{M_0}{M_{\eta_b}}\Big)^i\Big]$$

Normalising the currents

- Look at the pseudoscalar semileptonic decay and the scalar current (no normalisation factor needed)

$$\langle \eta_c(p')|S|B_c(p)\rangle = \frac{M_{B_c}^2 - M_{\eta_c}^2}{M_{b0} - m_{c0}} f_0(q^2)$$

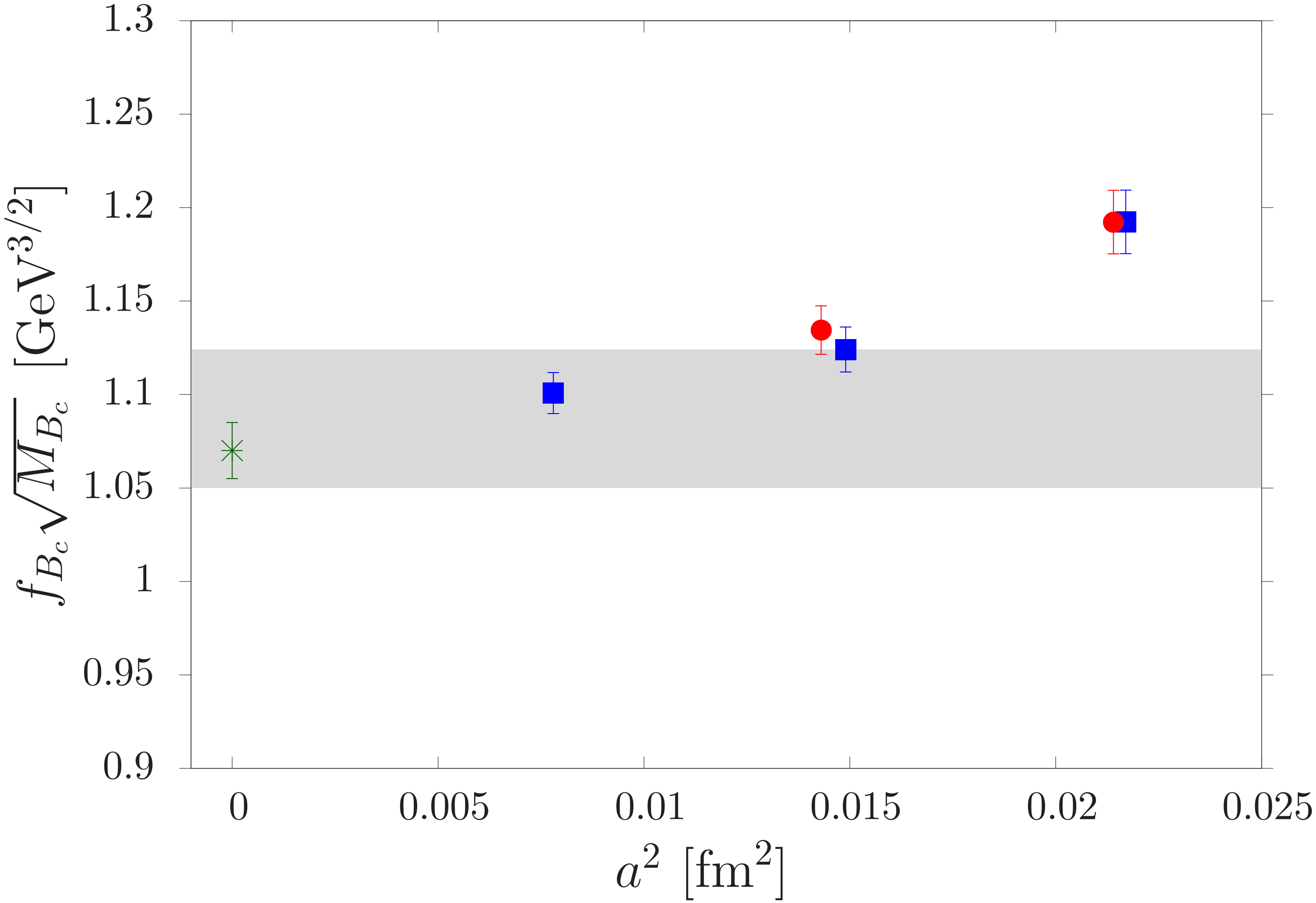
  and fix the vector current normalisation at $q^2{}_{\mathrm{max}}$

$$\begin{aligned}\langle \eta_c(p')|V^\mu|B_c(p)\rangle =& f_+\left[p^\mu + p'^\mu - \frac{M_{B_c}^2 - M_{\eta_c}^2}{q^2} q^\mu\right] \\ & + f_0(q^2)\frac{M_{B_c}^2 - M_{\eta_c}^2}{q^2} q^\mu\end{aligned}$$

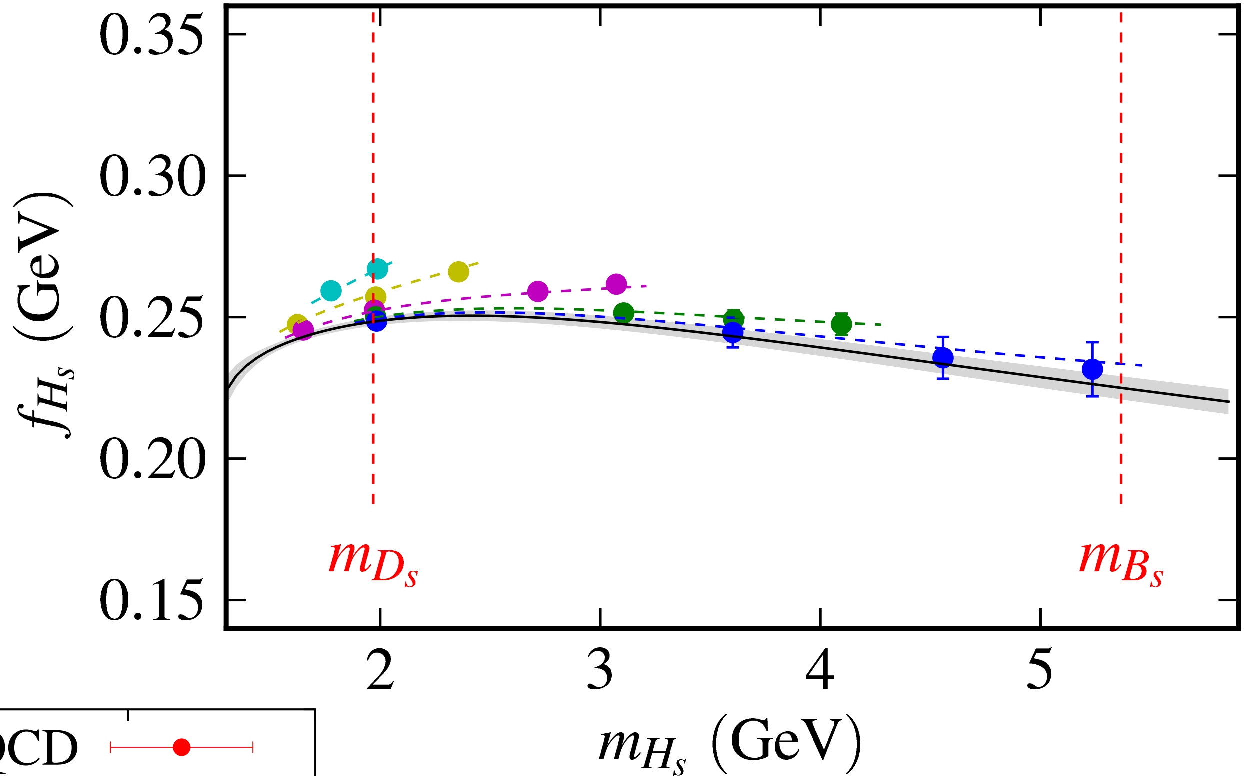
- Normalise the axial current using PCAC relation

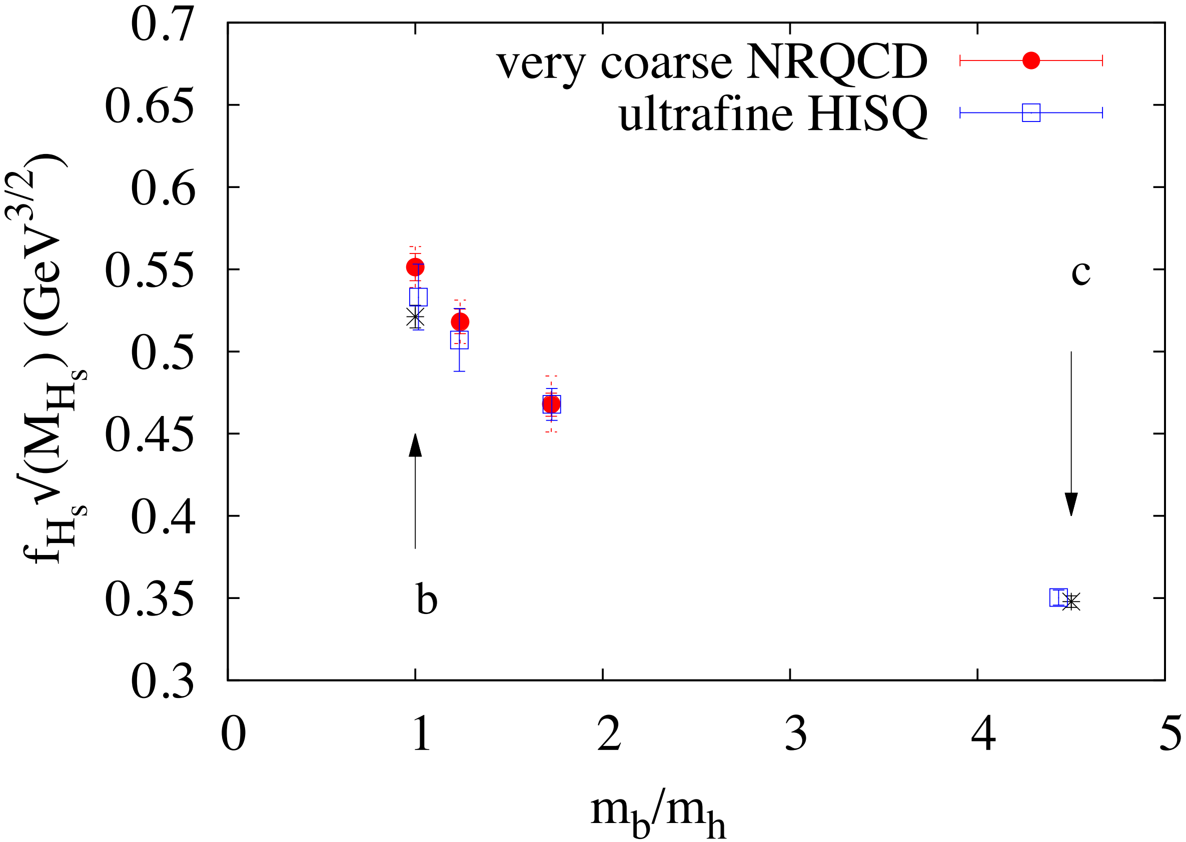
$$p_\mu \langle 0|A^\mu|B_c\rangle = (m_{c0} + m_{b0})\langle 0|P|B_c\rangle$$

B_c decay constant



Decay constant vs heavy quark mass

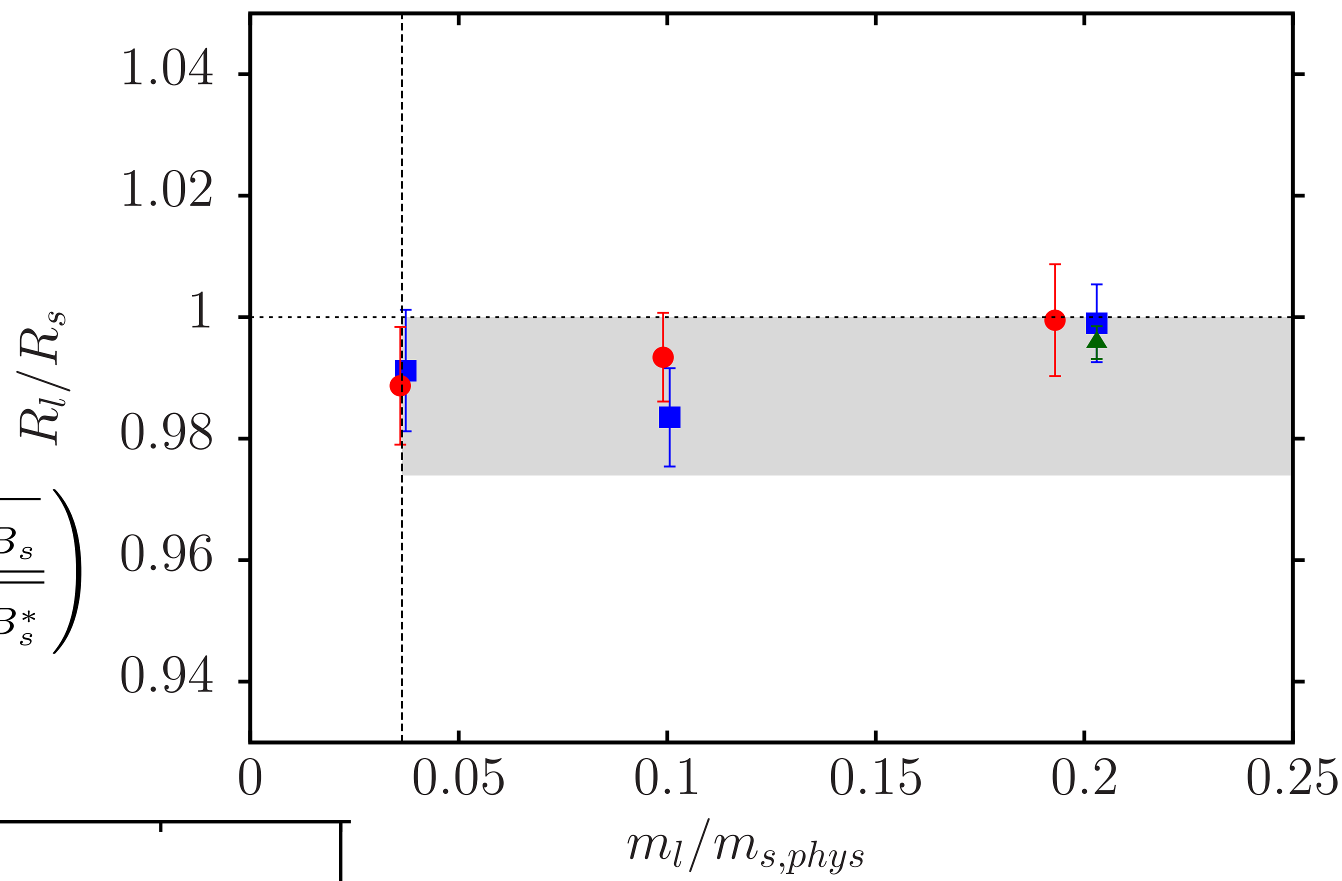


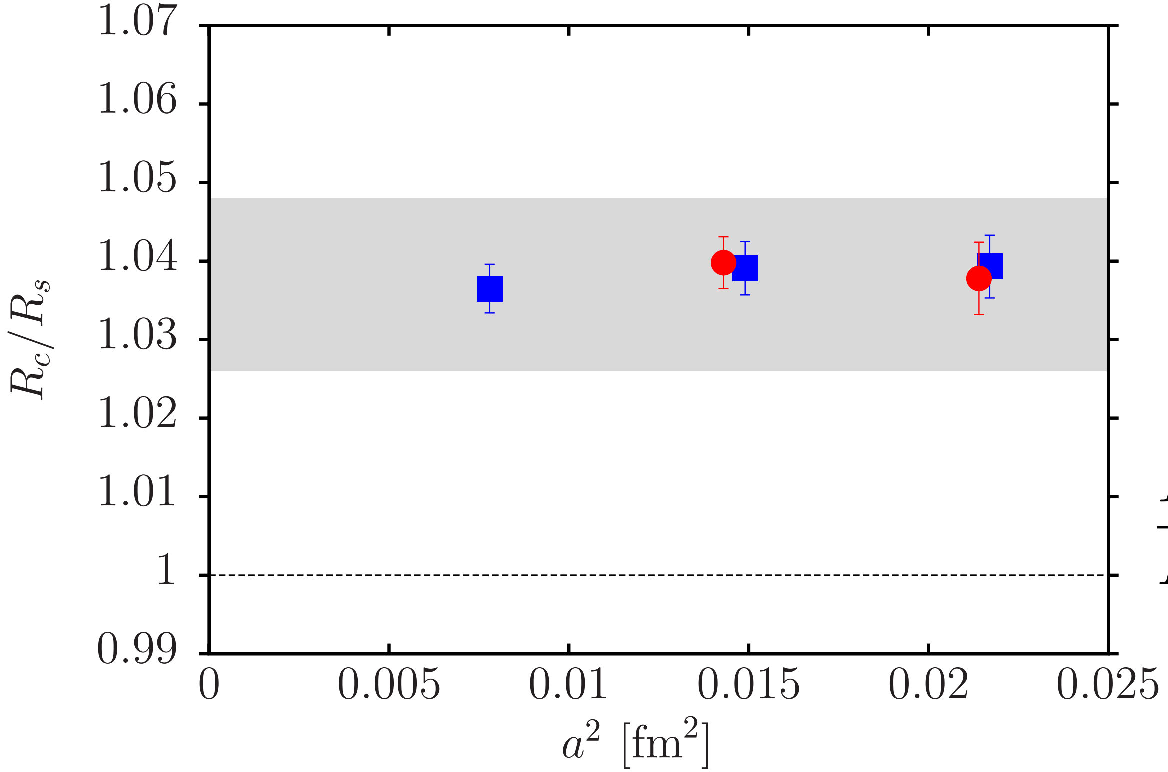


- covering the mass range from m_c to m_b
- comparing relativistic (HISQ) and non-relativistic (NRQCD) formalism

SU(3) breaking ratio

$$\frac{R_l}{R_s} \equiv \left(\frac{f_{B^*}\sqrt{M_{B^*}}}{f_B\sqrt{M_B}}\right)\left(\frac{f_{B_s}\sqrt{M_{B_s}}}{f_{B_s^*}\sqrt{M_{B_s^*}}}\right)$$



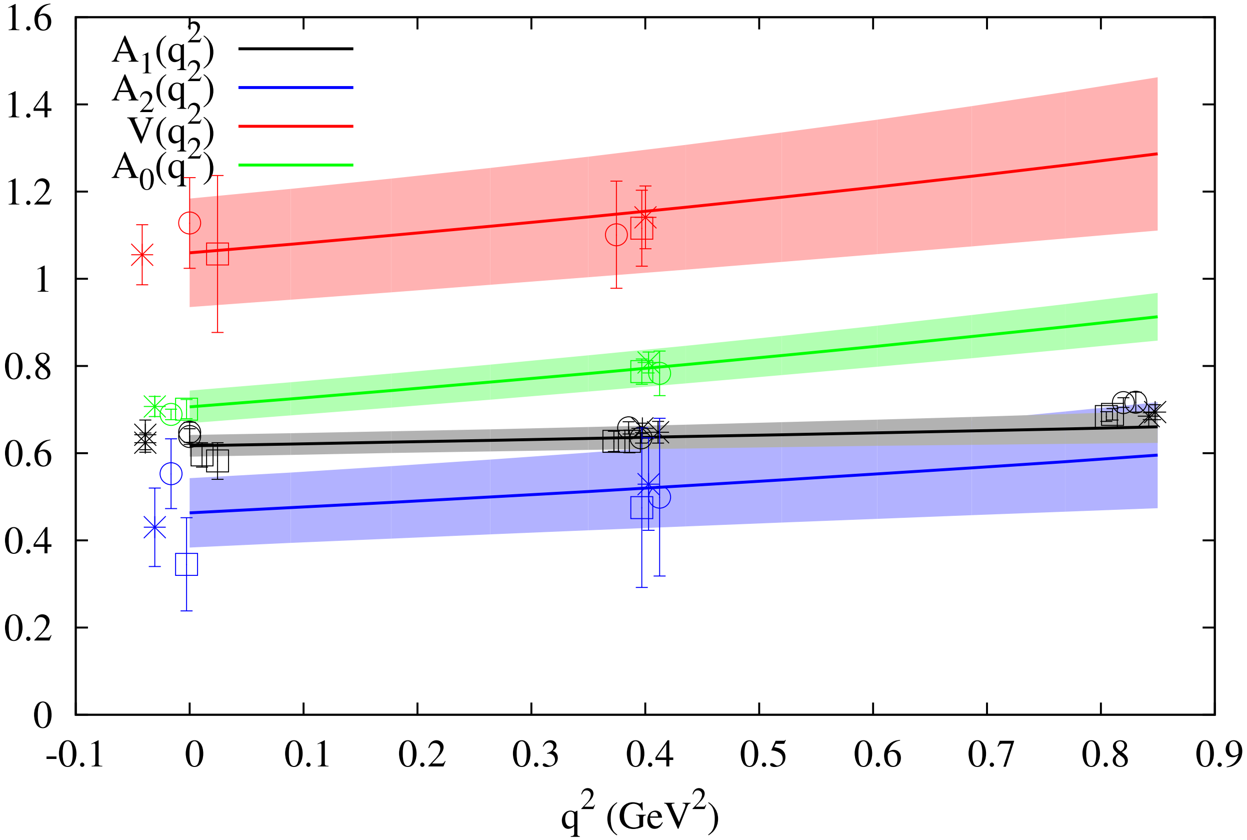


B_c^*, B_c

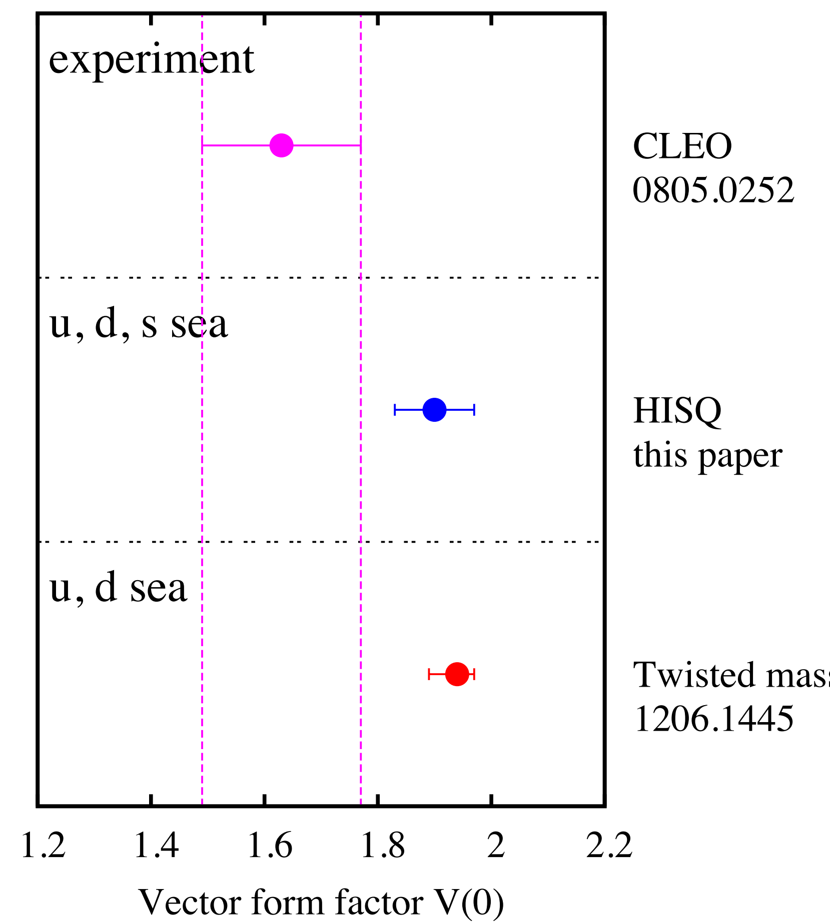
$$\frac{R_c}{R_s} \equiv \left(\frac{f_{B_c^*}\sqrt{M_{B_c^*}}}{f_{B_c}\sqrt{M_{B_c}}}\right)\left(\frac{f_{B_s}\sqrt{M_{B_s}}}{f_{B_s^*}\sqrt{M_{B_s^*}}}\right)$$

Other decays involving vector mesons

weak decay $D_s \to \phi \ell \nu$



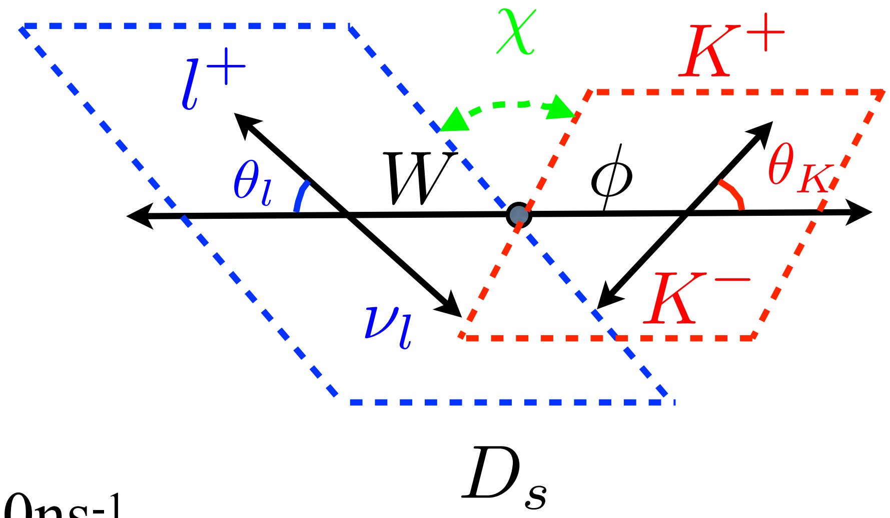
charmonium radiative decay $J/\psi \to \eta_c \gamma$

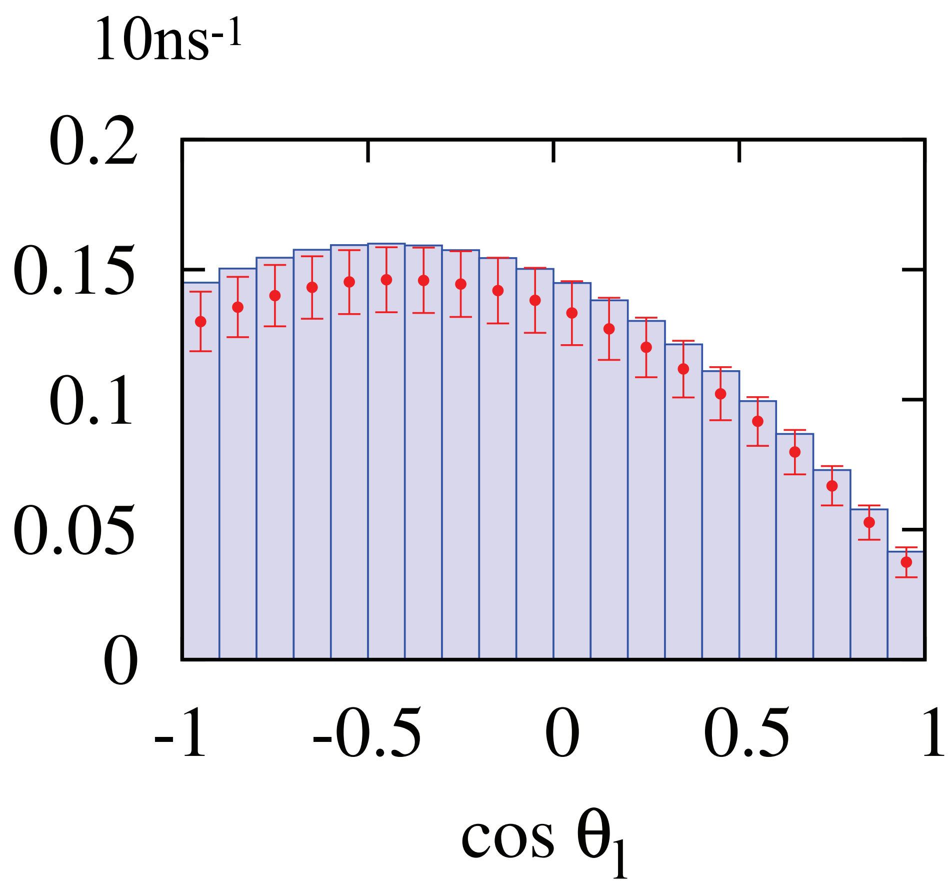


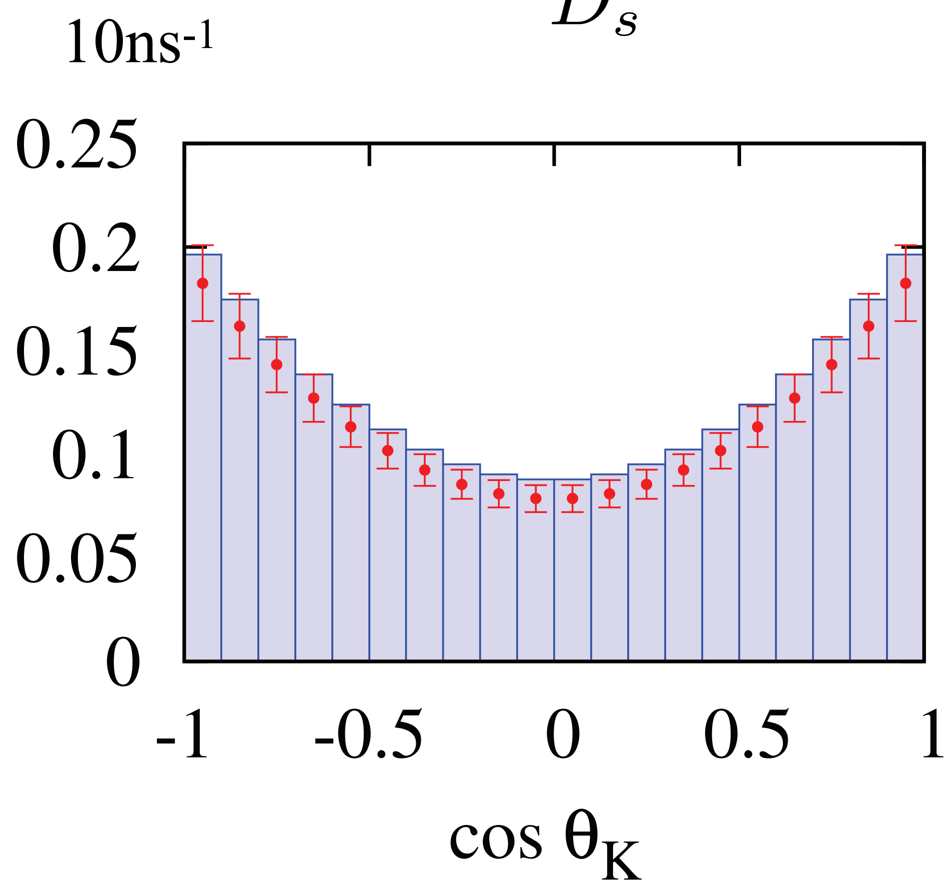
Figs. by G. Donald, HPQCD, arXiv:1311.6669 and 1208.2855

$D_s \to \phi \ell \nu$ angular distributions

G. Donald, HPQCD, Lattice 2013







Experimental data from BaBar, PRD 78, 051101(R) (2008)

$D_s \to \phi \ell v$ differential decay rate

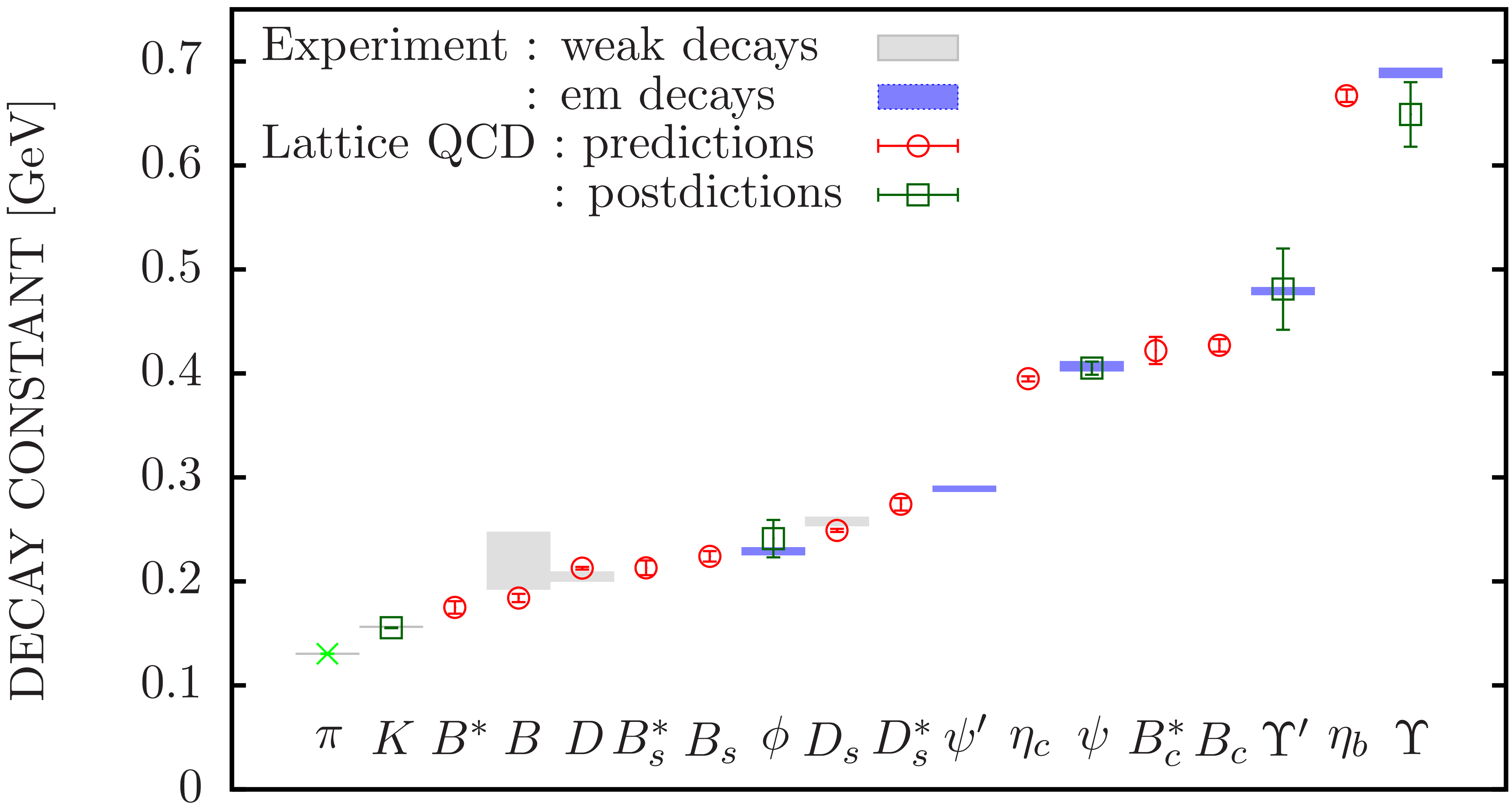
$$\frac{\mathrm{d}\Gamma(P\to V\ell\nu, V\to P_1P_2)}{\mathrm{d}q^2\mathrm{dcos}\,\theta_V\mathrm{dcos}\,\theta_\ell\mathrm{d}\chi} = \frac{3}{8(4\pi)^4}G_F^2|V_{q'Q}|^2\frac{p_V q^2}{M^2}\mathcal{B}(V\to P_1P_2)$$
$$\times\ \{(1-\eta\cos\theta_\ell)^2\sin^2\theta_V|H_+(q^2)|^2$$
$$+\ (1+\eta\cos\theta_\ell)^2\sin^2\theta_V|H_-(q^2)|^2$$
$$+\ 4\sin^2\theta_\ell\cos^2\theta_V|H_0(q^2)|^2$$
$$-\ 4\eta\sin\theta_\ell(1-\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2)$$
$$+\ 4\eta\sin\theta_\ell(1+\eta\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\theta_\chi H_+(q^2)H_0(q^2)$$
$$-\ 2\sin^2\theta_\ell\sin^2\theta_V\cos2\chi H_+(q^2)H_-(q^2)\},$$

where the helicity amplitudes are

$$H_0(q^2) = \frac{1}{2m_\phi\sqrt{q^2}}\left[(M^2-m_\phi^2-q^2)(M+m_\phi)A_1(q^2) - 4\frac{M^2p_\phi^2}{M+m_\phi}A_2(q^2)\right]$$

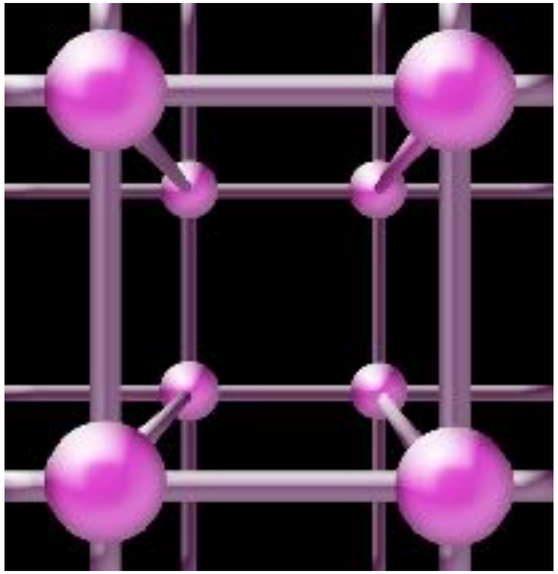
$$H_\pm(q^2) = (M+m_\phi)A_1(q^2) \mp \frac{2Mp_\phi}{M+m_\phi}V(q^2)$$

Meson decay constants: summary



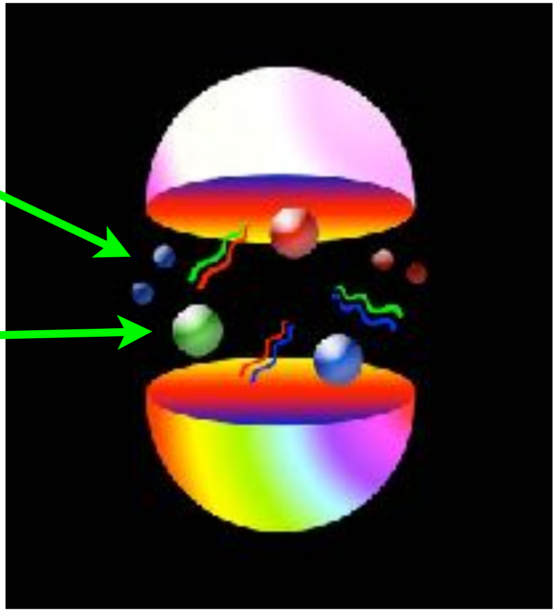
Lattice QCD

= fully nonperturbative QCD calculation

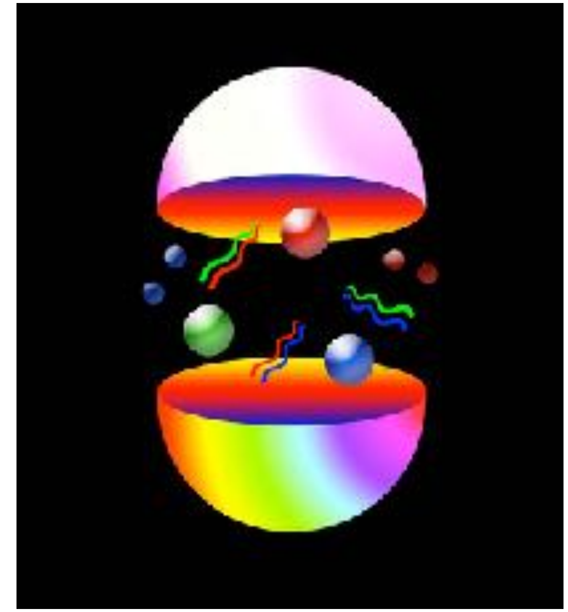


RECIPE

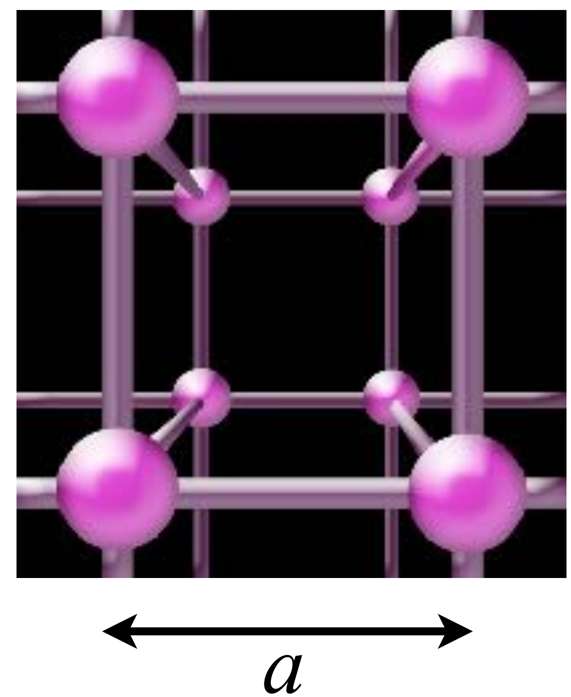
- Generate sets of gluon fields for Monte Carlo integration of path integral (including effects of u, d, s and c sea quarks)
- Calculate averaged "hadron correlators" from valence quark propagators
- Fit as a function of time to obtain masses and simple matrix elements



Lattice QCD RECIPE
continued



- Determine lattice spacing a and fix m_q using experimental information (often meson masses) to get results in physical units
- extrapolate to a=0, physical u/d quark mass for real world
  - lattices with physical $m_{u,d}$ now available: chiral extrapolation becoming a small correction



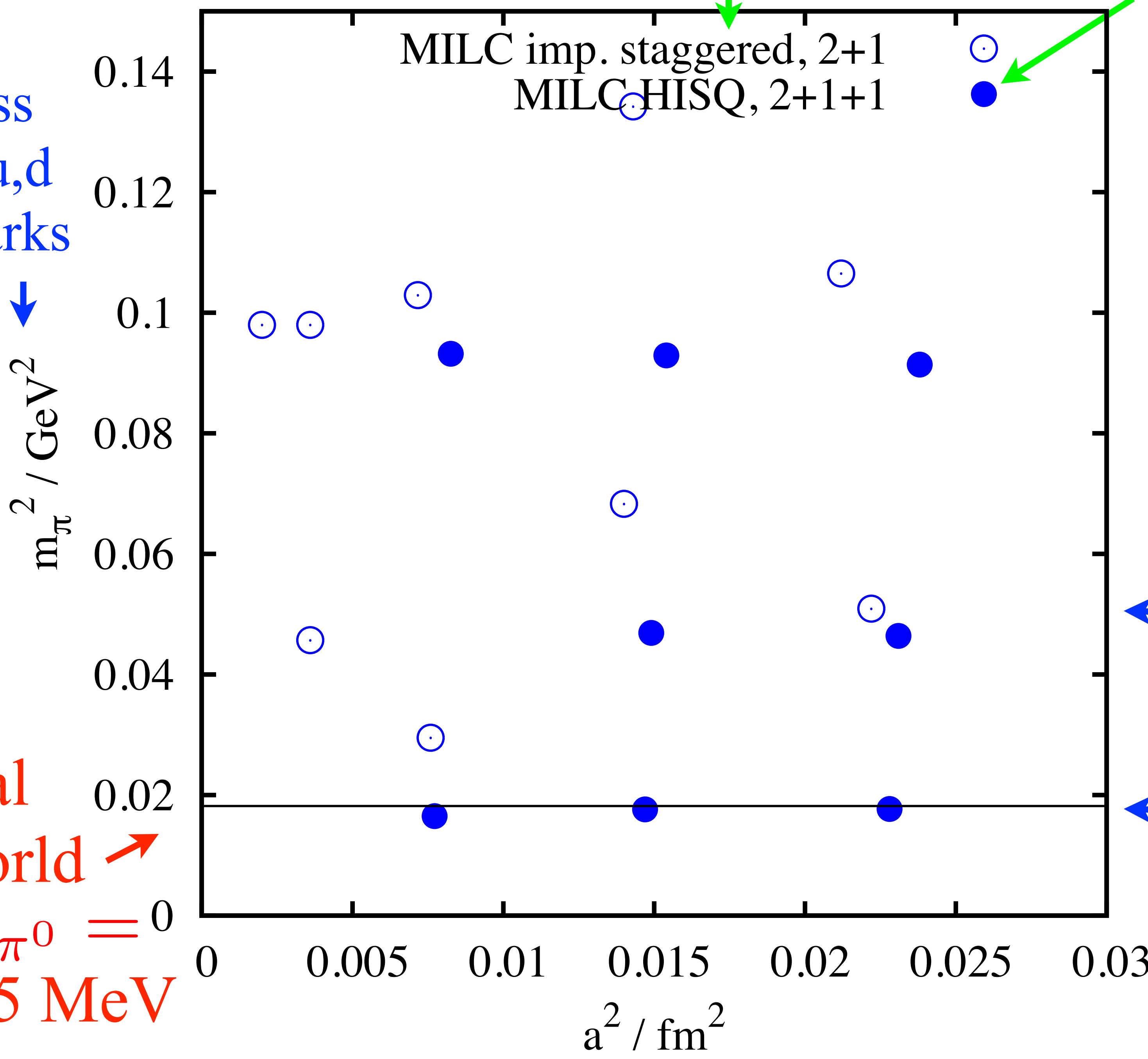
Example parameters for calculations now being done with ‘staggered’ quarks.

C.Davies et al, HPQCD/FNAL/MILC, hep-lat/0304004.

“2nd generation” lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana et al, HPQCD, hep-lat/0610092.



mass of u,d quarks

real world

$m_{\pi^0} =$ 135 MeV

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

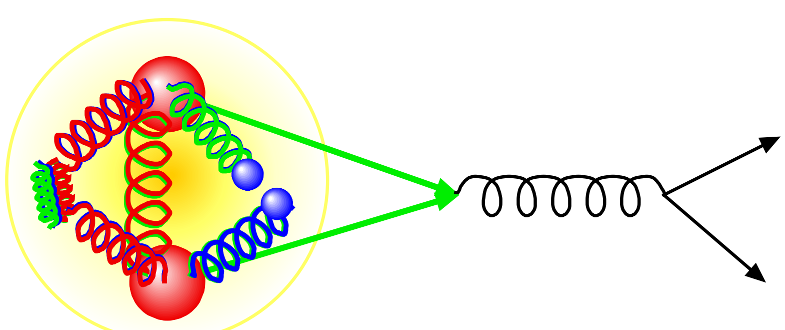
Hadron correlation functions ('2-point functions') give masses and decay constants.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \xrightarrow{T\ \text{large}} A_0 e^{-m_0 T}$$

masses of all hadrons with quantum numbers of H

$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD. Relate to experimental decay rate.



1% accurate experimental info. for f and m for many mesons! Need accurate determination from lattice QCD to match

Darwin@Cambridge,
part of UK's HPC facility
for theoretical particle physics
and astronomy

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Allows us to calculate quark propagators rapidly and store them for flexible re-use.

State-of-the-art commodity cluster: 9600 Intel Sandybridge cores, infiniband interconnect, fast switch and 2 Pbytes storage