

# Non Equilibrium dynamics Models and Excited state properties of low-dimensional SYStems (NEMESYS)



UNIVERSITÀ DELLA CALABRIA  
Dipartimento di Fisica

Via P. Bucci, Cubo 30C, I-87036 Rende (CS), Italy



Dr. Antonello SINDONA\*

(National Coordinator)

[antonello.sindona@fis.unical.it](mailto:antonello.sindona@fis.unical.it)

Prof. Giovanni Falcone

Prof. Francesco Plastina

Prof. Arturo Tagliacozzo (UniNa)

Dr. Pierfrancesco Riccardi



Lab. Nazionali  
di Frascati

Dr. Stefano Bellucci  
(Local Coordinator)

Dr. Danilo BABUSCI

Dr. Maurizio BENFATTO

Prof. Maurizio BOZZI (UNIPV)

Prof. Antonio MAFFUCCI (UNICAS)

Prof. Antonio PANTANO (UNIPA)

Prof. Luca PIERANTONI (UNVPM)

Dr. Davide MENCARELLI (UNVPM)



Dr. Gianluca Stefanucci

(Local Coordinator)

Prof. Olivia Pulci

Dr. Maurizia Palummo



**TIFPA**

Trento Institute for  
Fundamental Physics and  
Applications

Dr. Simone Taioli

(Local Coordinator)

Dr. Maurizio Dapor

Dr. Giovanni Garberoglio



UNIVERSITÀ  
DEGLI STUDI DI BARI  
ALDO MORO

XVII workshop on Statistical Mechanics  
and non Perturbative Field Theory  
Bari (15.12.2017)

**technological  
Interests**  
(from nano-  
electronics  
To health-care)

**Computational  
methods: Density  
Functional Theory  
(DFT), Time Dependent  
(TD) DFT, Many Body  
Perturbation Theory  
(MBPT)**

**ensembles of ultra-cold  
atomic gases [atomic  
gases in mono(bi)-  
chromatic traps];  
magnetic and spin  
systems**

**fundamental  
properties**  
(High energy  
physics in solid-  
state setting!)

**topological quantum  
field theory on space-  
time (2+1) and (1+1)  
manifolds; quantum  
Montecarlo; semi-  
classical multiscale  
approaches**

**NEMESYS**  
**out-of-equilibrium, non-  
adiabatic and excited-  
state features of  
interacting many fermion  
and boson systems  
confined to low-  
dimensions**

**electrons in  
honeycomb-like lattice  
potentials: graphene,  
graphene related and  
beyond graphene  
nanostructures**

**Huge  
Computatio  
nal Costs**  
( $10^6$  Coreh  
per simulation)

**spectral features,  
dielectric screening,  
conductivity and electro-  
mechanical properties of  
charge-carriers**

**irreversible properties  
and quantum  
thermodynamics of  
ultra cold Fermi and  
Bose gases, following a  
change of their trapping  
potentials**

**Huge  
Investments**  
(H2020 flagships  
for graphene  
and quantum  
information)

## Parameter-free Density Functional Theory (DFT)

### Time Dependent DFT (TDDFT)

## Many Body Perturbation Theory (MBPT)

### Green's Function (GF) techniques

HPC modules (abinit, cpmd, crystal14, nwchem, qe, vasp, yambo) & self-developed programs

#### MOST RELEVANT PUBLICATIONS in 2017

APS (2 PRL, 2 PRA, 8 PRB, 1 PR App.);

AIP (1 Appl. Phys. Lett., 1 AIP Advances);

ACS (1 Nanoletters, 1 Nanophotonics);

Nature (1 Nature Communications, 1 Nature Nanotechnology, 2 Scientific Reports, 1 NPJ Quantum Information);

AAAS (1 Science)

#### Today's Topic

PHYSICAL REVIEW B 96, 201408(R) (2017)

RAPID COMMUNICATIONS

DOI: [10.1103/PhysRevB.96.201408](https://doi.org/10.1103/PhysRevB.96.201408)

### Calibration of the fine-structure constant of graphene by time-dependent density-functional theory

A. Sindona,<sup>1,2,\*</sup> M. Pisarra,<sup>3</sup> C. Vacacela Gomez,<sup>1,4</sup> P. Riccardi,<sup>1,2</sup> G. Falcone,<sup>1,2</sup> and S. Bellucci<sup>5</sup>

<sup>1</sup>Dipartimento di Fisica, Università della Calabria, Via P. Bucci, Cubo 30C, I-87036 Rende (CS), Italy

<sup>2</sup>INFN, sezione LNF, Gruppo collegato di Cosenza, Cubo 31C, I-87036 Rende (CS), Italy

<sup>3</sup>Departamento de Química, Universidad Autónoma de Madrid, C/Fco. Tomás y Valiente 7, E-28049 Madrid, Spain

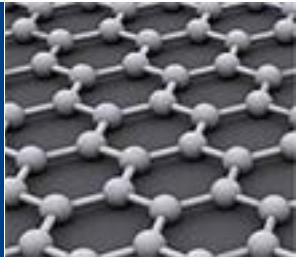
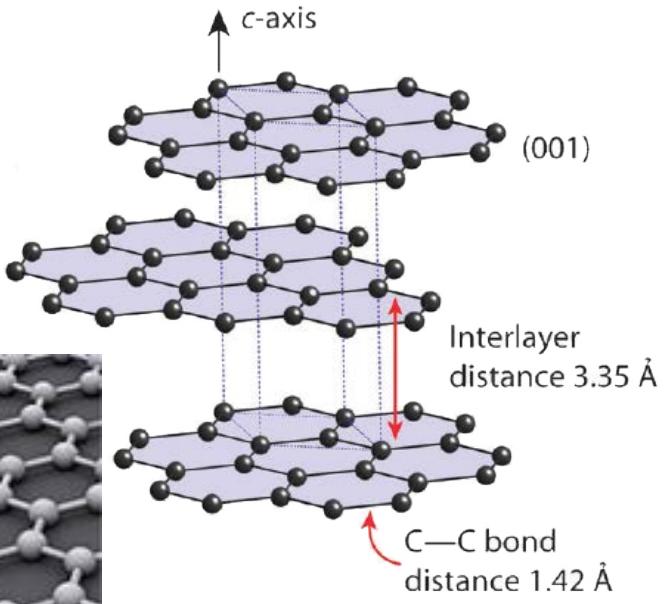
<sup>4</sup>Escuela Superior Politécnica de Chimborazo, Panamericana Sur Km 1 1/2, Riobamba EC-060155, Ecuador

<sup>5</sup>INFN-Laboratori Nazionali di Frascati (LNF), Via E. Fermi 40, I-00044 Frascati, Italy

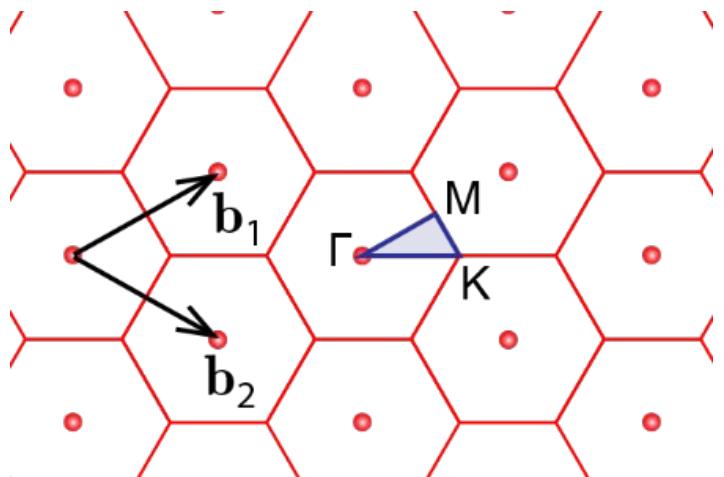
(Received 28 September 2017; published 17 November 2017)



## GRAPHITE vs GRAPHENE

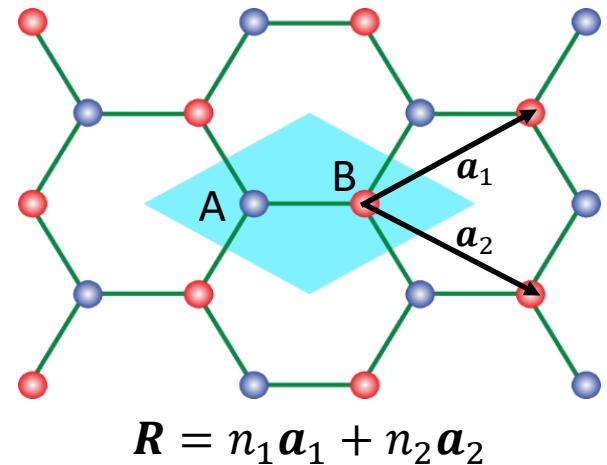


## geometry (reciprocal space)

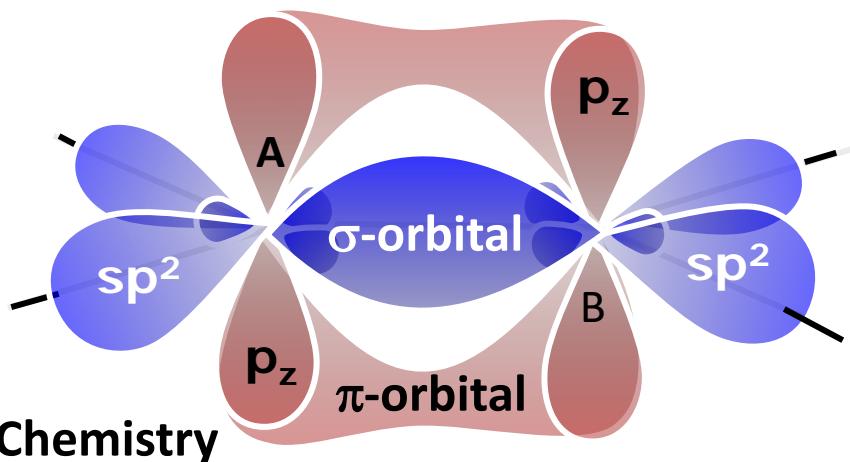


$$G = m_1 b_1 + m_2 b_2$$

## geometry (real space)



Wallace, Phys. Rev. 71, 622 (1947)  
a simple exercise in quantum mechanics



## $\pi$ states (Block's Theorem, Tight-binding approach)

$$|\pi_k\rangle = a_k \sum_R e^{ik \cdot R} |p_z(R)\rangle + b_k \sum_R e^{ik \cdot (R+\delta)} |p_z(R+\delta)\rangle$$

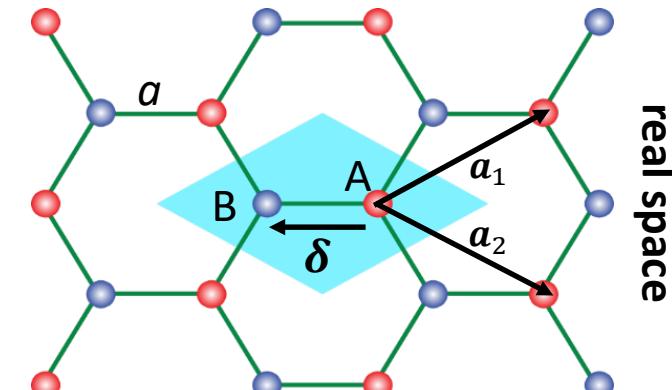
minimizing  $E[\pi_k] = \frac{\langle \pi_k | \hat{H} | \pi_k \rangle}{\langle \pi_k | \pi_k \rangle}$

with respect to  $a_k$  and  $b_k$  leads to

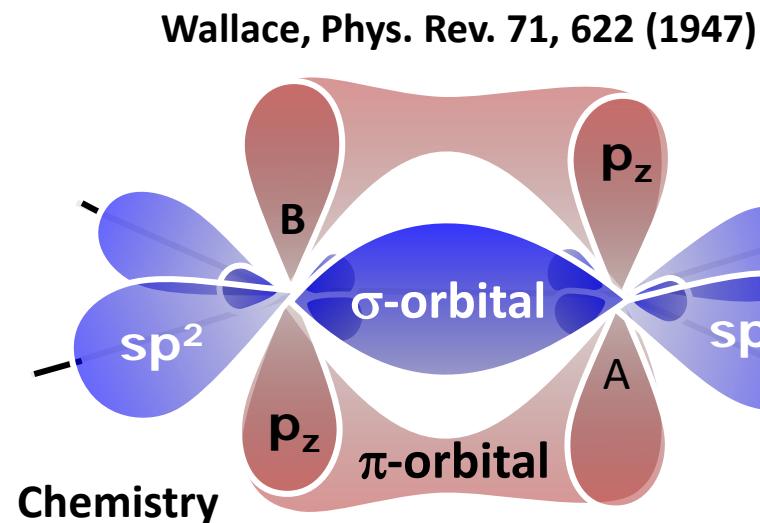
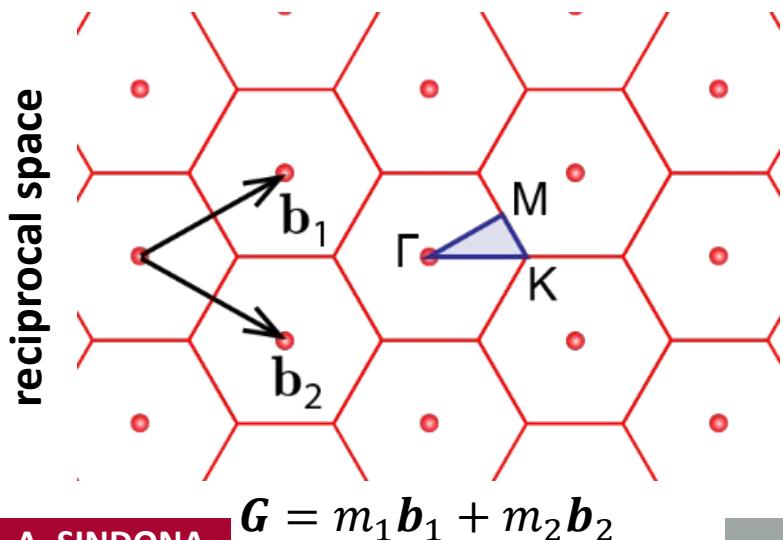
$$\left| \begin{pmatrix} E_{2p} & V_0 f_k \\ V_0 f_k^* & E_{2p} \end{pmatrix} - E \begin{pmatrix} 1 & S_0 f_k \\ S_0 f_k^* & 1 \end{pmatrix} \right| = 0$$

with  $f_k = e^{-iak_x} + 2 e^{\frac{iak_x}{2}} \cos \frac{\sqrt{3}ak_y}{2}$

one-electron Hamiltonian  $H=T+V$ , with  $V$  translationally invariant under translations of  $R$



$$R = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$$



## $\pi$ states (Block's Theorem, Tight-binding approach)

$$|\pi_k\rangle = a_k \sum_R e^{ik \cdot R} |p_z(R)\rangle + b_k \sum_R e^{ik \cdot (R+\delta)} |p_z(R+\delta)\rangle$$

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with  $f_k = e^{-iak_x} + 2 e^{\frac{iak_x}{2}} \cos \frac{\sqrt{3}ak_y}{2}$

$$S_0 = \langle p_z(R) | p_z(R+\delta) \rangle \sim 0.1$$

and

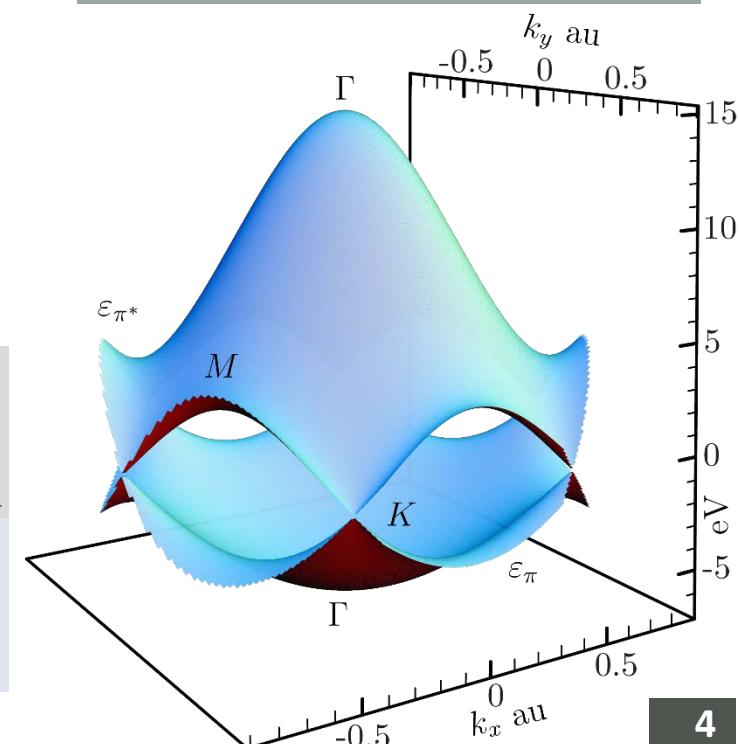
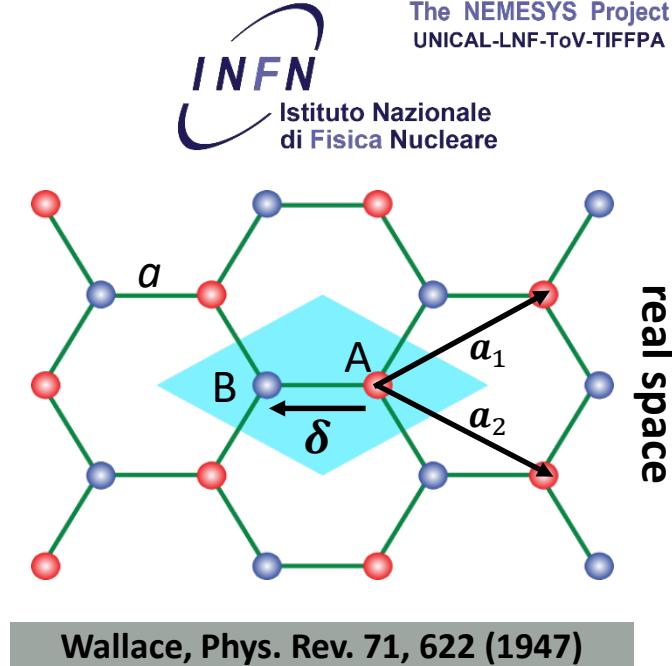
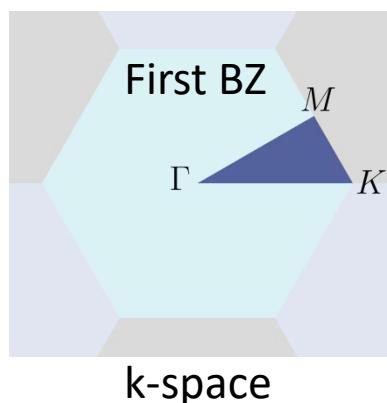
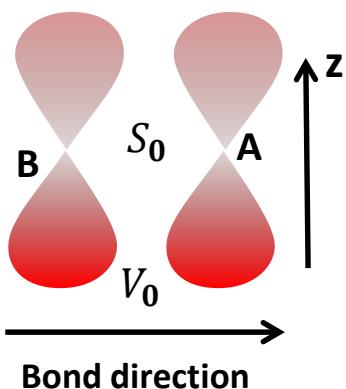
$$V_0 = \langle p_z(R) | \hat{V} | p_z(R+\delta) \rangle \sim 3.2 \text{ eV}$$

Then, assuming  $E_{2p}=0$ , we get:

$$\varepsilon_\pi(k) = \frac{V_0 |f_k|^2}{1 + S_0 |f_k|^2}$$

$$\varepsilon_{\pi^*}(k) = \frac{-V_0 |f_k|^2}{1 - S_0 |f_k|^2}$$

one-electron Hamiltonian  $H=T+V$ , with  $V$  translationally invariant under translations of  $R$



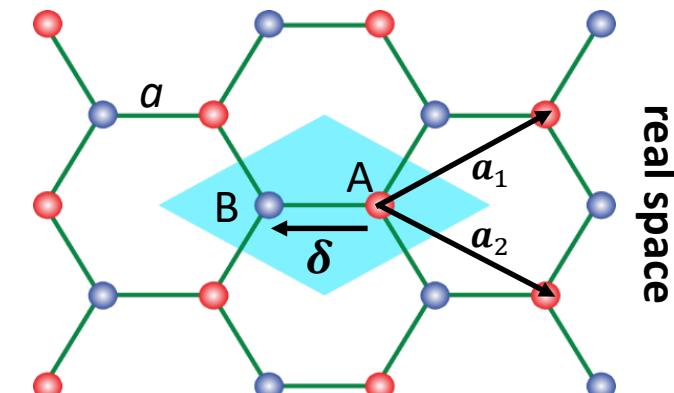
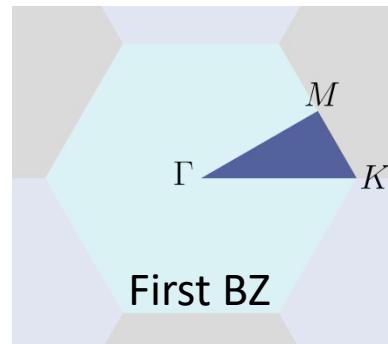
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minimizing  $E[\pi_k] = \frac{\langle \pi_k | \hat{H} | \pi_k \rangle}{\langle \pi_k | \pi_k \rangle}$

with respect to  $a_k$  and  $b_k$  leads to

$$\left| \begin{pmatrix} E_{2p} & V_0 f_k \\ V_0 f_k^* & E_{2p} \end{pmatrix} - E \begin{pmatrix} 1 & S_0 f_k \\ S_0 f_k^* & 1 \end{pmatrix} \right| = 0$$



with  $f_k = e^{-iak_x} + 2 e^{\frac{iak_x}{2}} \cos \frac{\sqrt{3}ak_y}{2}$

$$S_0 = \langle p_z(R) | p_z(R+\delta) \rangle \sim 0.1$$

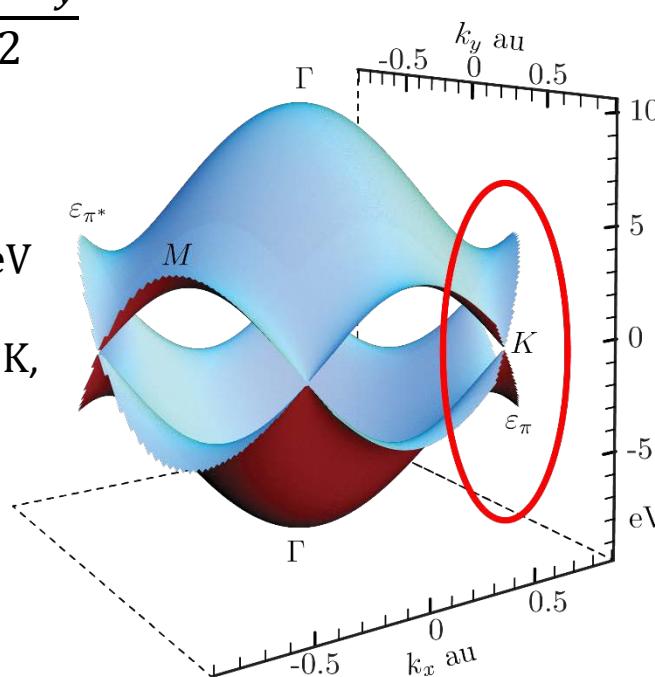
and

$$V_0 = \langle p_z(R) | \hat{V} | p_z(R+\delta) \rangle \sim 3.2 \text{ eV}$$

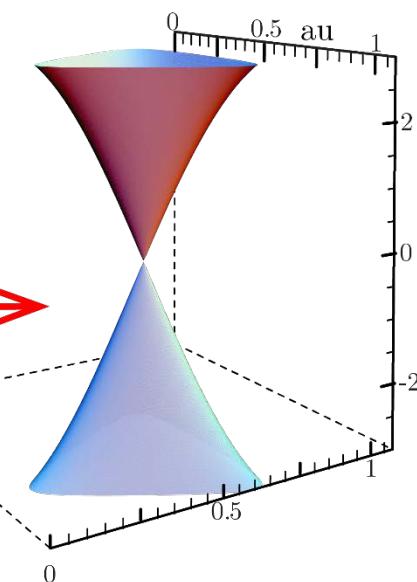
Then, assuming  $E_{2p}=0$  and letting  $k \rightarrow K$ , we get:

$$\varepsilon_\pi(k) \approx -\hbar v_F |k - \mathbf{K}|$$

$$\varepsilon_{\pi^*}(k) \approx \hbar v_F |k - \mathbf{K}|$$

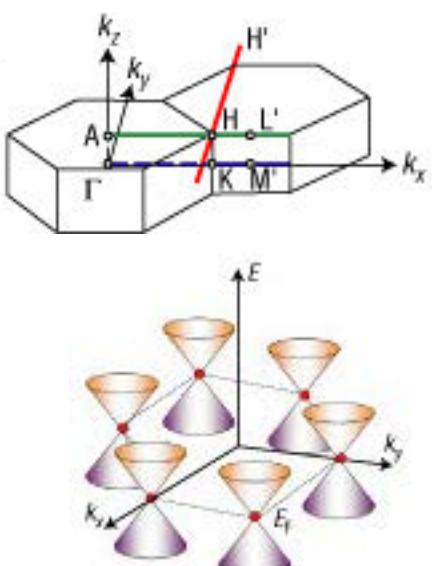
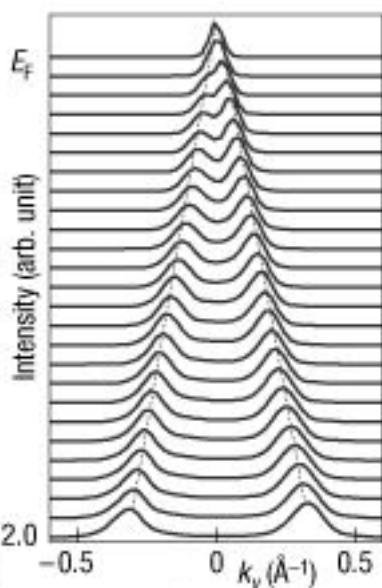
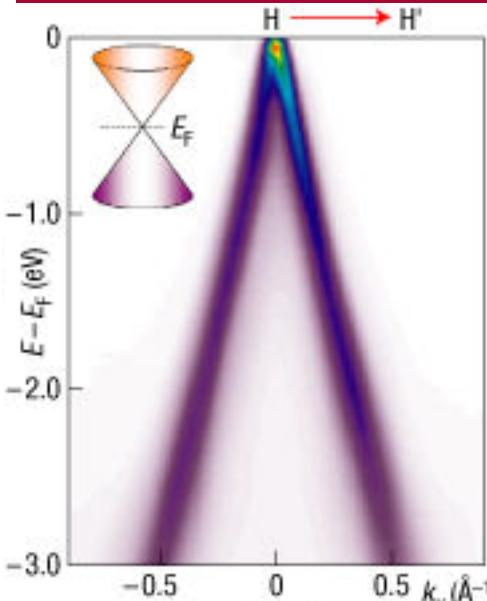


Wallace, Phys. Rev. 71, 622 (1947)

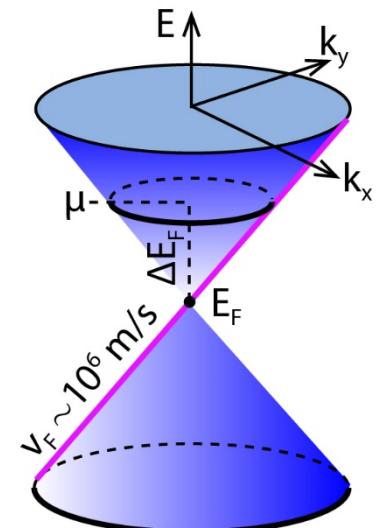


## Massless Dirac Fermions

S. Y. Zhou et al., "First direct observation of Dirac fermions in graphite", *Nature Physics* 2, 595–599 (2006)



## Dirac Cone (Positive Doping)



$$\omega(k) = v_F |k| \quad v_F \sim c/300$$

## Interaction with light

**QED**

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \xrightarrow{e=\hbar=\frac{1}{4\pi\epsilon_0}=1} \frac{1}{c} \sim \frac{1}{137}$$

## Graphene

$$\alpha_g = \frac{e^2}{4\pi\epsilon_0\hbar v_F} \xrightarrow{\text{au}} \frac{1}{v_F} \sim 2.2!$$

## Dressed interaction

The bare fine-structure constant of graphene is too large for any perturbation treatments and too small for close coupling approaches!!

$$\alpha_g^*(\mathbf{q}, \omega) = \frac{e^2}{4\pi\hbar\epsilon_0 v_F} \langle \epsilon_r^{-1} \rangle(\mathbf{q}, \omega) \\ \xrightarrow{\text{au}} \frac{1}{v_F} \langle \epsilon_r^{-1} \rangle(\mathbf{q}, \omega)$$

## Static limiting value

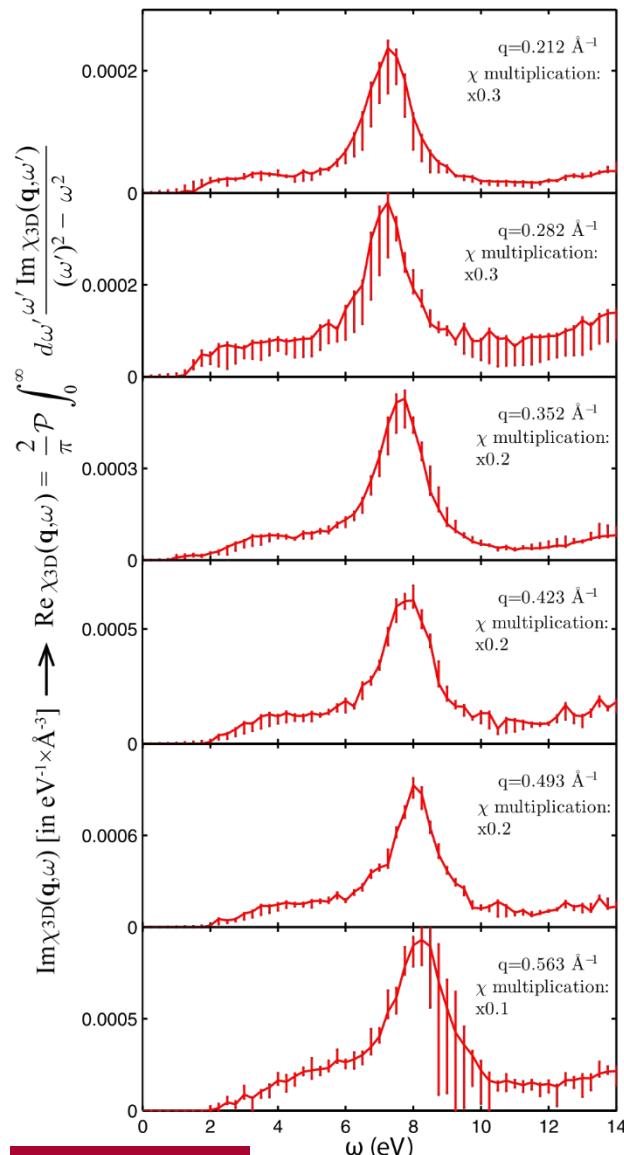
$$\tilde{\alpha}_g = \alpha_g^*(\mathbf{0}, 0)$$

A more complete 'view' of the interaction strength amounts to replacing the vacuum permittivity with the dynamic permittivity, obtained by transferring an energy  $\omega$  and a momentum  $\mathbf{q}$  to the system

# How to measure $\alpha$ in graphene?

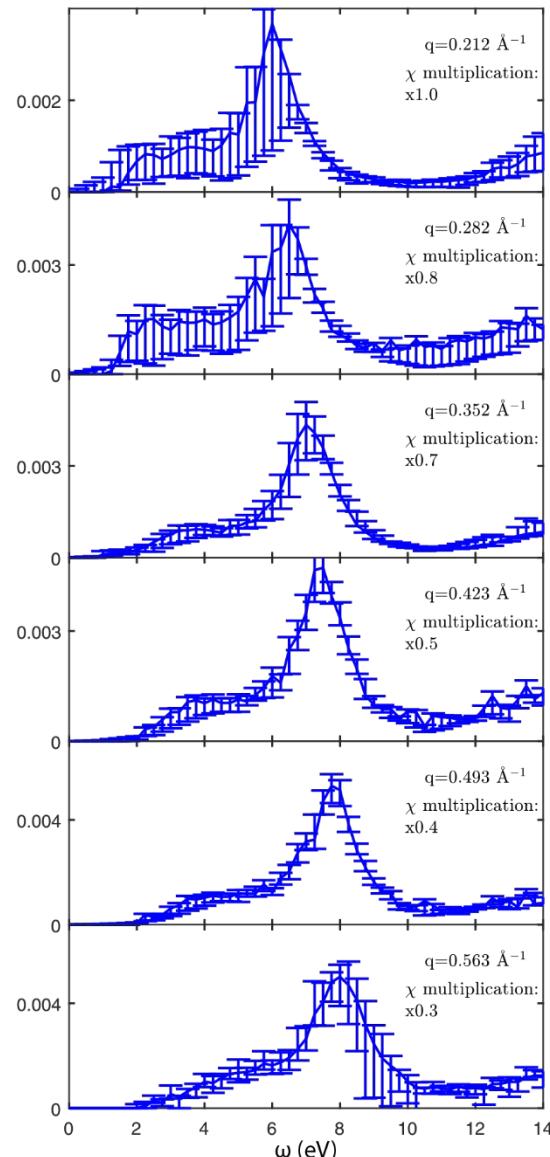
inelastic x-ray scattering (IXS) experiments on graphite:

$$\text{measured intensity} \propto -\frac{1}{\pi} \frac{1}{1 - e^{-\beta\hbar\omega}} \text{Im } \chi_{3D}(\mathbf{q}, \omega)$$

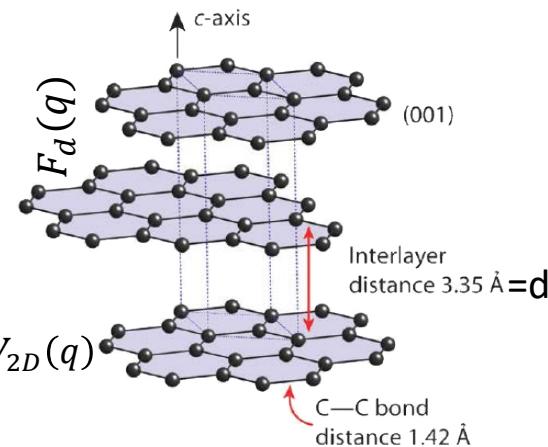


reconstruction of the imaginary susceptibility of graphene:

$$\chi_{2D}(\mathbf{q}, \omega) = \frac{\chi_{3D}(\mathbf{q}, \omega)}{1 - [V_{2D}(\mathbf{q})d - V_{3D}(\mathbf{q})]\chi_{3D}(\mathbf{q}, \omega)} d$$



$$V_{2D}(q) = \frac{2\pi}{q} \quad V_{3D}(q) = V_{2D}(q)F_d(q)$$

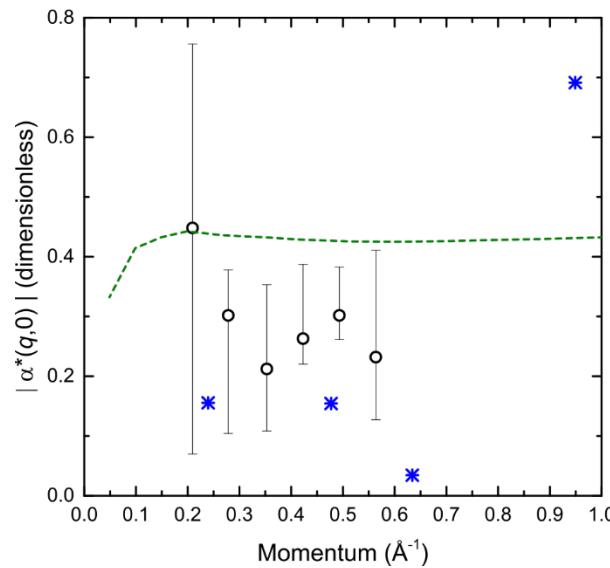


Reconstruction of the dressed fine-structure constant

$$\alpha_g^*(\mathbf{q}, \omega) = \frac{\alpha}{\epsilon_{2D}} = \alpha_g [1 + V_{2D}(\mathbf{q})\chi_{2D}(\mathbf{q}, \omega)]$$

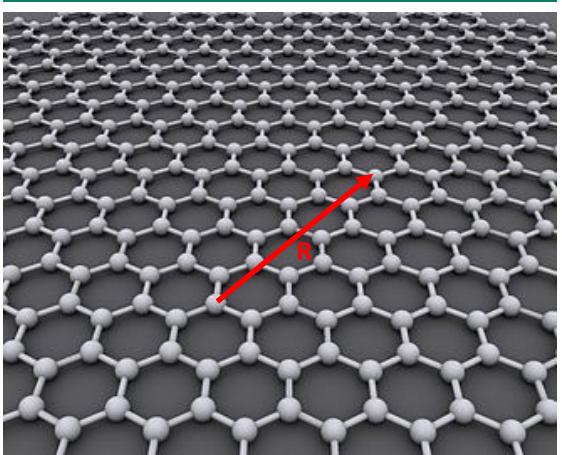
Experiments

- PHYSICAL REVIEW B 93, 195150 (2016)
- Science 330, 805 (2010)



## Ab initio calculation of $\alpha$ in graphene

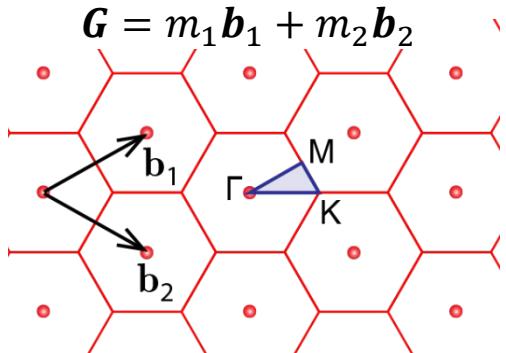
Stationary Schrödinger  
Equation ( $e = m_e = \hbar = 1$ )



The ground state properties are uniquely determined from the electron density

PW-DFT  
 $i = (\nu, \mathbf{k})$

$$\psi_{\nu k}(\mathbf{r}) = \sum_{\mathbf{G}} \frac{c_{\nu k+\mathbf{G}}}{\sqrt{\Omega}} e^{i(\mathbf{k}+\mathbf{G}) \cdot \mathbf{r}}$$



$$\frac{|\mathbf{k} + \mathbf{G}|^2}{2} < 50 \text{ eV}$$

$\mathbf{k} \in \Gamma KM + 12 \text{ symm.}$   
1-10<sup>5</sup> core h depending on the resolution  
10<sup>4</sup>-10<sup>5</sup> coefficients  $c_{\nu k+\mathbf{G}}$  per wave function

## Ground-state properties (Density Functional Theory)

Available on Marconi: abinit, qe, vasp

$$H \Psi = \left[ \sum_i^N \frac{-\nabla_i^2}{2} + \sum_i^N \sum_R \frac{-Z}{|\mathbf{r}_i - \mathbf{R}|} + \frac{1}{2} \sum_{i \neq j}^N \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} \right] \Psi = E \Psi$$

Kohn-Sham DFT for electrons

Variational Principle

$$\min_{\Psi} \langle \Psi | \hat{H} | \Psi \rangle = E_0[n]$$

$$n(\mathbf{r}) = \sum_{i \text{ occ}} |\psi_i(\mathbf{r})|^2$$

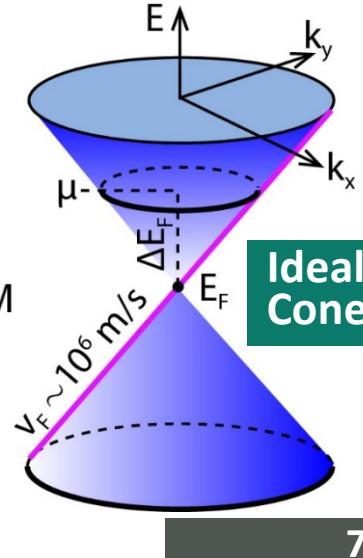
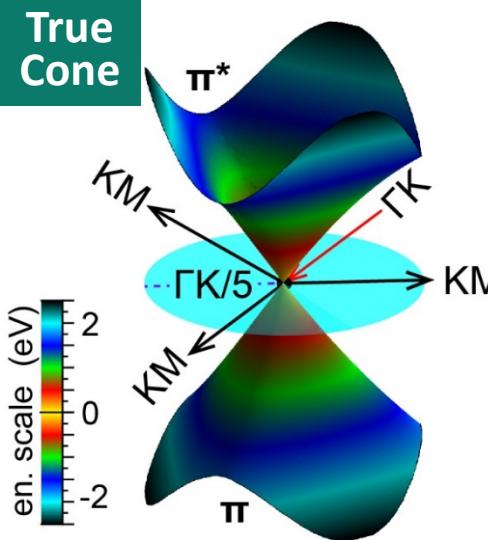
Self-Consistent KS Equations

$$\left[ \frac{-\hbar^2 \nabla^2}{2m} + V_{KS}[n] \right] \psi_i = \varepsilon_i \psi_i$$

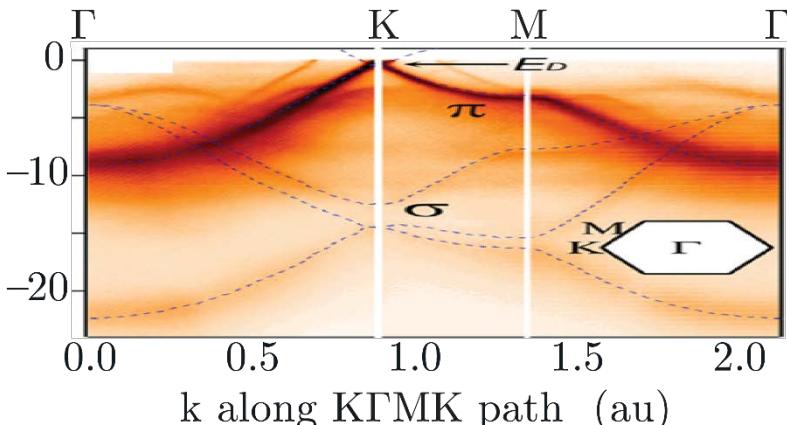
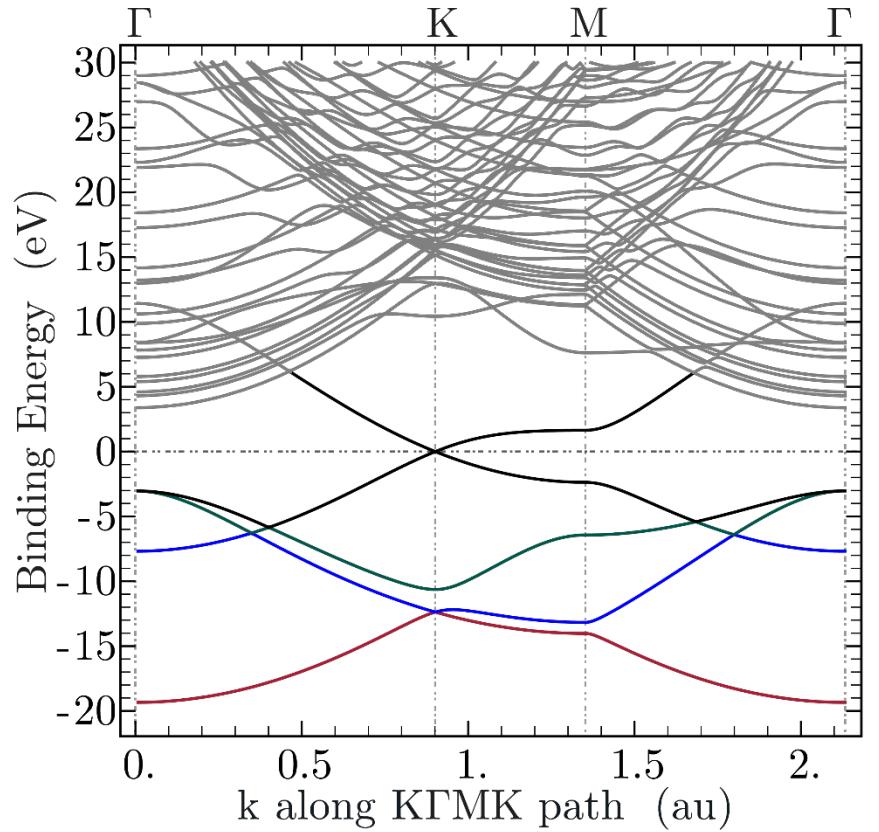
LDA

$$V_{KS}[n] = \sum_R \frac{-Z}{|\mathbf{r} - \mathbf{R}|} + \int d^3 r' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + \frac{\delta E_{xc}}{\delta n(\mathbf{r})}$$

True Cone

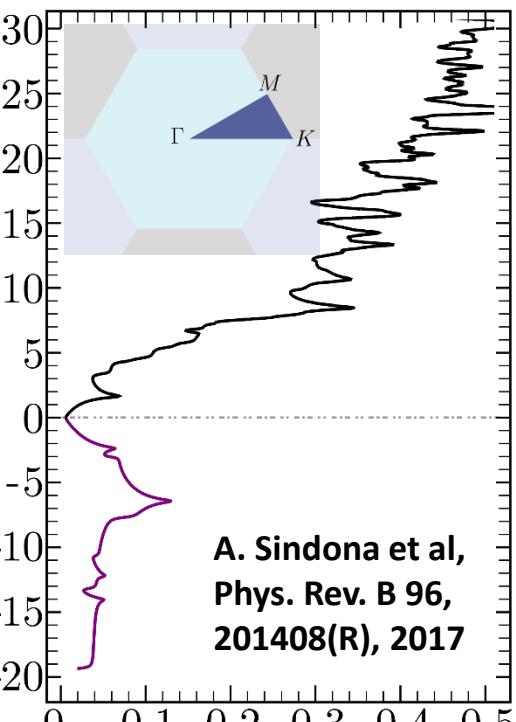


## Ab initio calculation of $\alpha$ in graphene

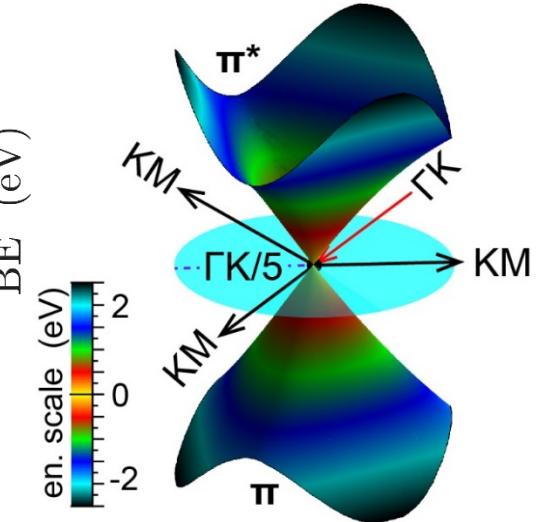
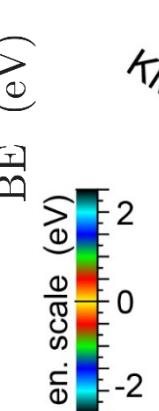


Bostwick et al, Nature Physics 3, 36 - 40 (2007)

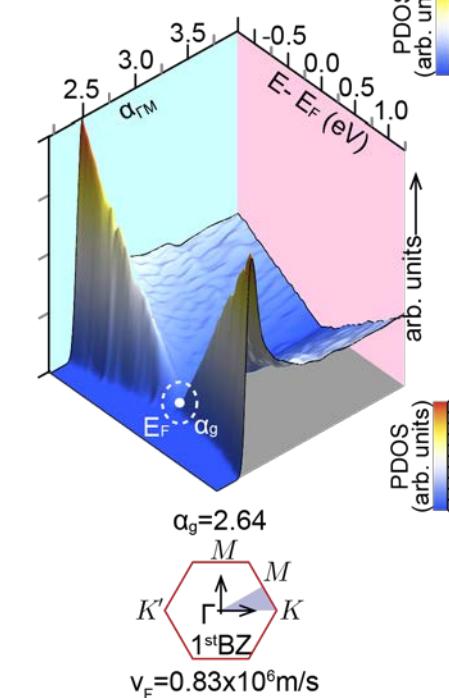
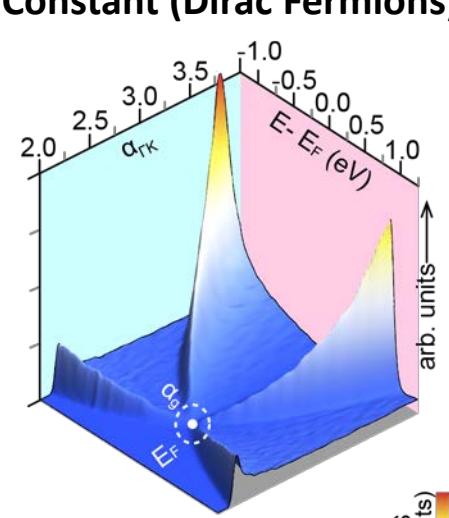
## Full Band Structure (PW-DFT)



A. Sindona et al,  
Phys. Rev. B 96,  
201408(R), 2017



## Static fine-structure Constant (Dirac Fermions)



## PW-DFT

Eigenvectors

$$\psi_{\nu k}(\mathbf{r}) = \sum_{\mathbf{G}} \frac{c_{\nu \mathbf{k} + \mathbf{G}}}{\sqrt{\Omega}} e^{i(\mathbf{k} + \mathbf{G}) \cdot \mathbf{r}}$$

v band index  
k wave vector in IrrBZ

Eigenvalues  $\varepsilon_{\nu k}$ 

Self-Consistent KS Potential

$$V_{KS}(\mathbf{r}) = V_{ext}(\mathbf{r}) + V_H[n](\mathbf{r}) + V_{XC}[n](\mathbf{r})$$

Electron Density

$$n(\mathbf{r}) = \sum_{\nu k \in \text{IrrBZ}}^{\text{occ}} w_k |\psi_{\nu k}(\mathbf{r})|^2$$

## Linear Response Theory

Small perturbation

$$\delta V_{\text{ext}} \rightarrow \delta V_{KS} \rightarrow \delta n = \chi_0 \delta V_{KS}$$

Extra electron  
or incident  
photon ( $\mathbf{q}, \omega$ )

Unperturbed density-density response

$$\chi_0^\pm(\mathbf{r}, \mathbf{r}'; t) = \mp \frac{i}{\hbar} \lim_{\eta \rightarrow 0} \langle [n_I(r, t), n(r')] \rangle e^{\pm \frac{\eta t}{\hbar}},$$

$$\chi_{GG'}^{\pm 0}(\omega, \mathbf{q}) = \frac{2}{\Omega} \sum_{\mathbf{k}} \sum_{\nu, \nu'} \frac{\left( f_{\varepsilon_{\nu k}} - f_{\varepsilon_{\nu' k+q}} \right) \rho_{\nu \nu'}^{\mathbf{kq}}(\mathbf{G}) \rho_{\nu \nu'}^{\mathbf{kq}}(\mathbf{G}')^*}{\omega + \varepsilon_{\nu k} - \varepsilon_{\nu' k+q} \pm i\eta}$$

$$\rho_{\nu \nu'}^{\mathbf{kq}}(\mathbf{G}) = \sum_{\mathbf{G}'} c_{\nu \mathbf{k} + \mathbf{G}'}^* c_{\nu' \mathbf{k} + \mathbf{q} + \mathbf{G} + \mathbf{G}'}$$

$$\text{Full Polarizability } \chi_{GG'} = \chi_{GG'}^0 + (\chi^0 \nu \chi)_{GG'} = [\chi^0 (1 - \nu \chi_0)^{-1}]_{GG'}$$

$$\begin{aligned} \text{Screening} \quad (\epsilon^{-1})_{GG'} &= (1 + \nu \chi)_{GG'} \\ \text{(Plasmon Properties)} \quad &\approx [(1 - \nu \chi^0)^{-1}]_{GG'} \end{aligned}$$

Dynamically  
screened  
interaction

$$W(\mathbf{r}, \mathbf{r}') = \int \frac{d^3 r''}{4\pi} \frac{\epsilon^{-1}(\mathbf{r}, \mathbf{r}'')}{|\mathbf{r}'' - \mathbf{r}'|}$$

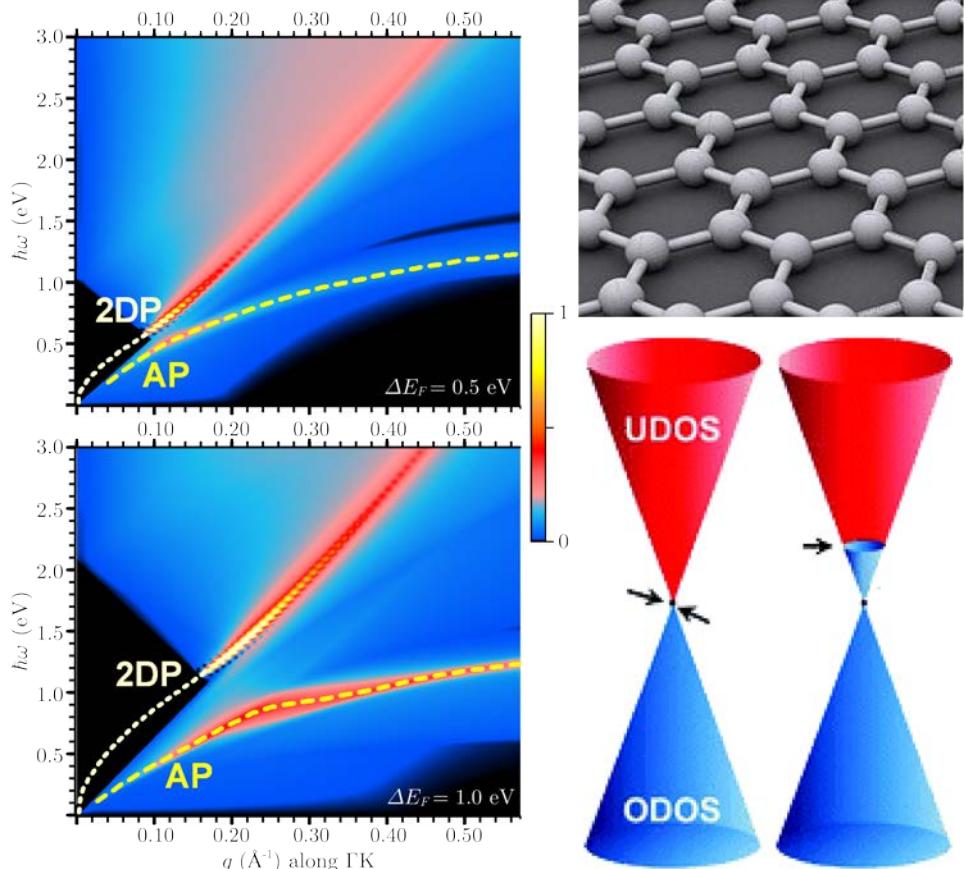
## Macroscopic Average

$$\text{Re}[\epsilon_{\alpha\alpha}(q, \nu)] = 0$$

$$\text{Im}[\epsilon_{\alpha\alpha}(q, \nu)]$$

$$-\text{Im}[\epsilon_{\alpha\alpha}(q, \nu)^{-1}]$$

## Graphene



**~1000 Core h per q point!!!**

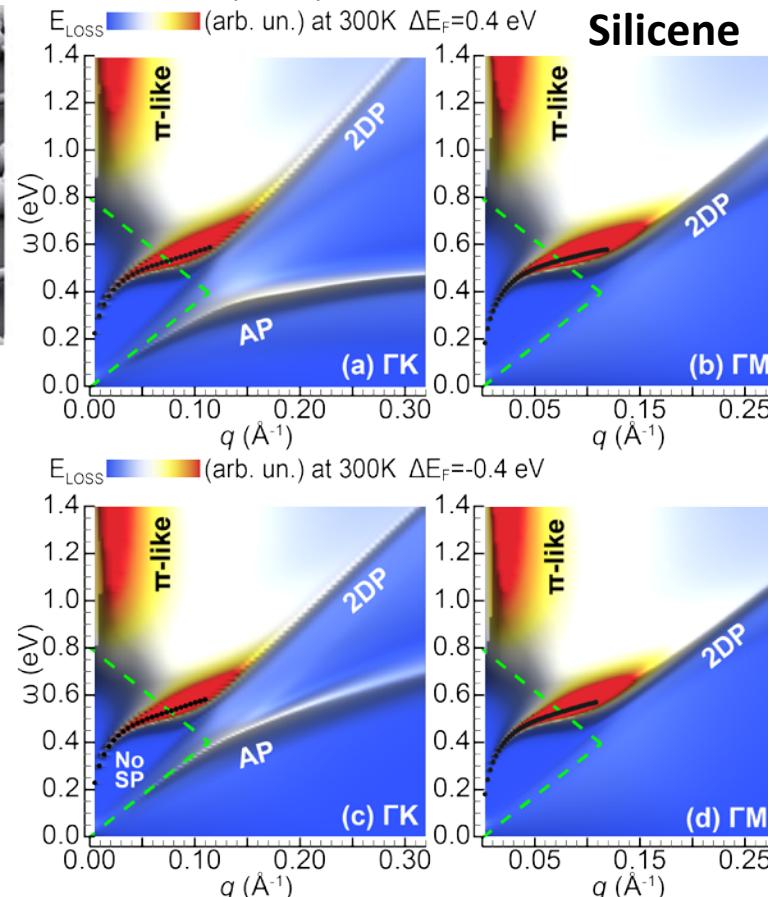
$$\epsilon_{\alpha\alpha}(q, \omega)^{-1} = [\epsilon_{\mathbf{G}\mathbf{G}'}(q\mathbf{u}_\alpha, \omega)^{-1}]_{\mathbf{G}=\mathbf{G}'=0}$$

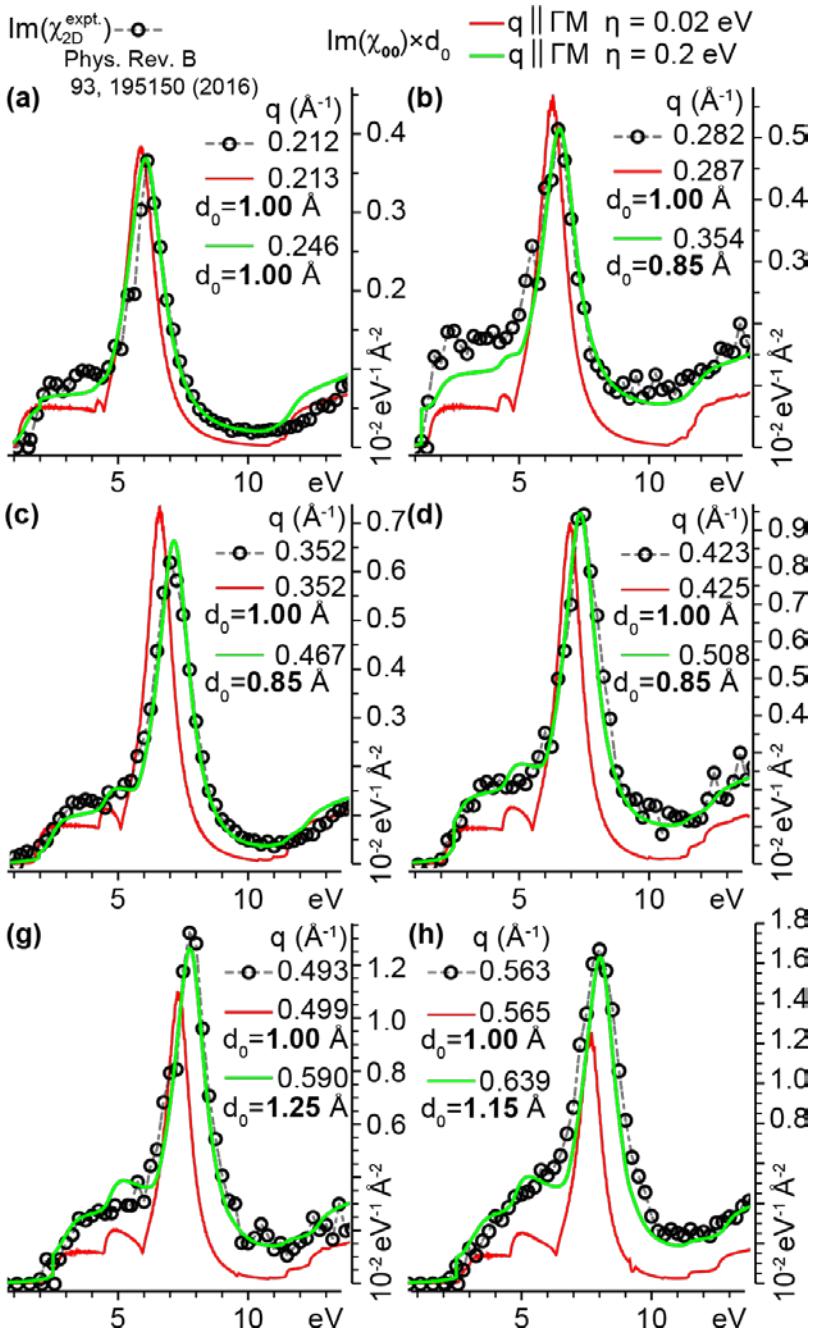
## Plasmon Resonances

## Absorption Spectrum

## Loss Function (Plasmon Spectrum)

M. Pisarra, A. Sindona, P. Riccardi, V. M. Silkin, J. M., Pitarke, **New Journal of Physics** 16, 083003 (2014); M. Pisarra, A. Sindona, M. Gravina, V. M. Silkin, J. M., Pitarke, **Physical Review B** 93, 035440 (2016); C. Vacacela Gomez, M. Pisarra, M. Gravina, J. M., Pitarke and A. Sindona, **Physical Review Letters** 117, 116801; (2016); C. Vacacela Gomez, M. Pisarra, M. Gravina and A. Sindona, **Physical Review B** 00, 005400 (2017)



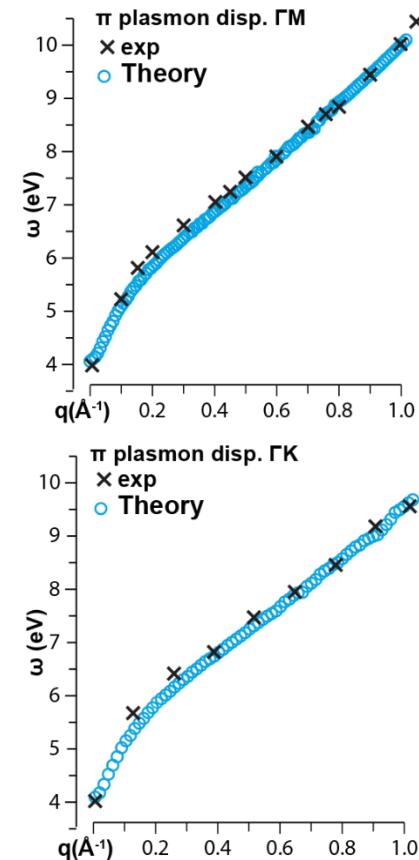
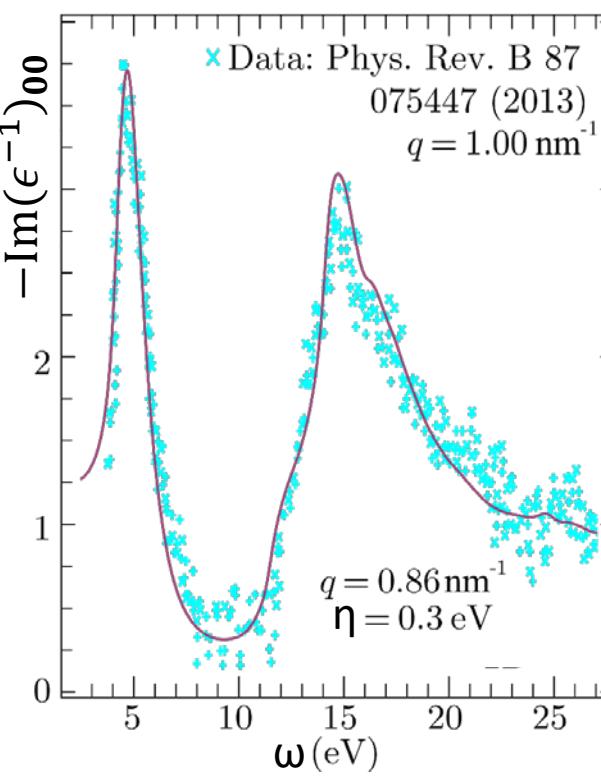


$$(\epsilon^{-1})_{GG'} = (1 + v\chi)_{GG'}$$

$$\begin{aligned}\chi_{GG'} &= (\chi^0 + \chi^0 v\chi)_{GG'} \\ &= [\chi^0 (1 - v\chi^0)^{-1}]_{GG'}\end{aligned}$$

Effective electron-electron interaction

$$v_{GG'} = \frac{2\pi}{|q + g|} \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' e^{iG_z z - iG'_z z' - |q+g||z-z'|}$$



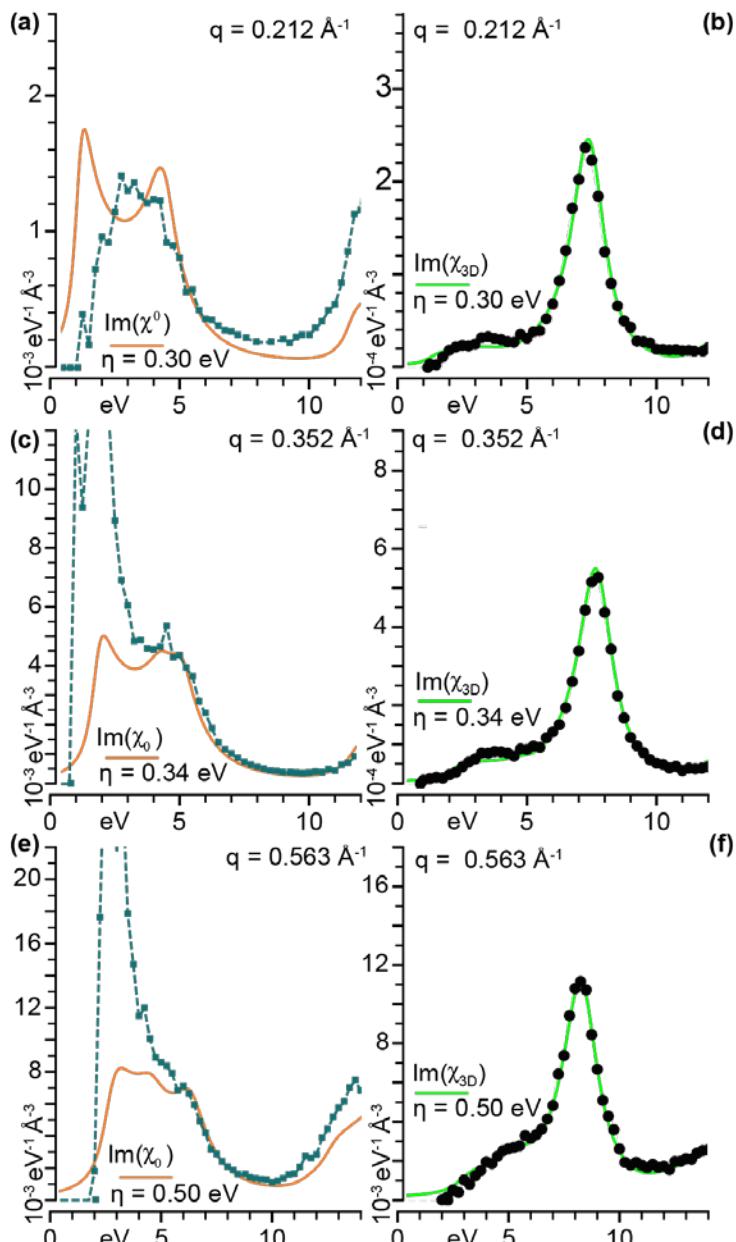
$\text{Im}(\chi_0^{\text{expt.}})$

$\text{Im}(\chi_{3D}^{\text{expt.}})$

Yu Gan et al, Phys. Rev. B 93,  
195150 (2016)

$$\chi_{00}(\mathbf{q}, \omega) = \frac{\chi_{3D}(\mathbf{q}, \omega) d^*}{1 - v_{00} d^* [1 - F^*(q)] \chi_{3D}(\mathbf{q}, \omega)} \quad \text{our model}$$

$\mathbf{q} = 0.212 \text{ \AA}^{-1}$        $\mathbf{q} = 0.212 \text{ \AA}^{-1}$



$$\chi_{GG'}^{+0} = \frac{2}{\Omega} \sum_{\mathbf{k}} \sum_{\nu, \nu'} \frac{\left( f_{\varepsilon_{\nu \mathbf{k}}} - f_{\varepsilon_{\nu' \mathbf{k}+\mathbf{q}}} \right) \rho_{\nu \nu'}^{\mathbf{k} \mathbf{q}}(\mathbf{G}) \rho_{\nu \nu'}^{\mathbf{k} \mathbf{q}}(\mathbf{G}')^*}{\omega + \varepsilon_{\nu \mathbf{k}} - \varepsilon_{\nu' \mathbf{k}+\mathbf{q}} + i\eta}$$

$$\chi_{GG'} = [\chi^0 (1 - v \chi^0)^{-1}]_{GG'}$$

### Effective electron-electron interaction

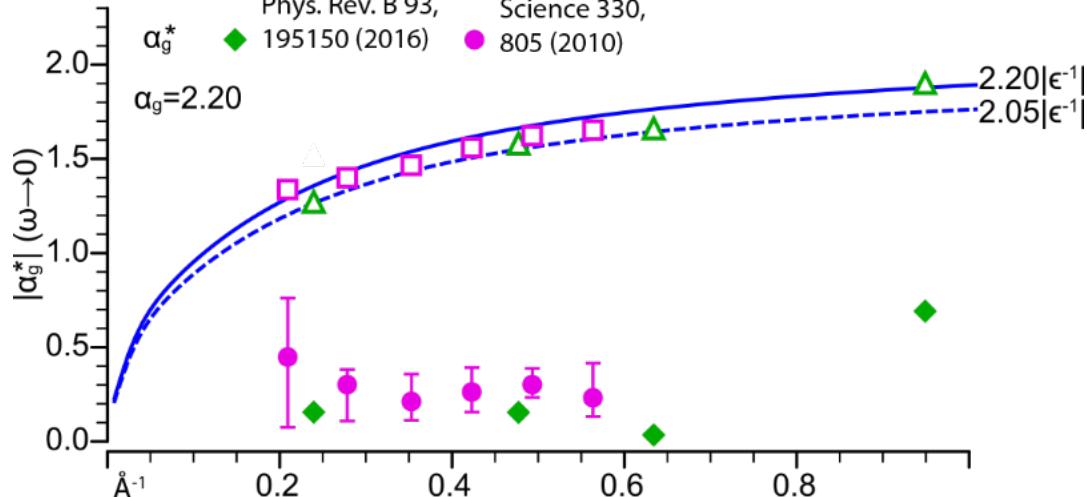
$$v_{GG'} = \frac{2\pi}{|\mathbf{q} + \mathbf{g}|} \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' e^{iG_z z - iG'_z z' - |\mathbf{q} + \mathbf{g}| |z - z'|}$$

### Reconstruction of the dressed fine-structure constant

$$\alpha_g^*(\mathbf{q}, \omega) = \alpha_g [1 + V_{2D}(\mathbf{q}) \chi_{2D}(\mathbf{q}, \omega)]$$

$$(\alpha_g^*)^C(\mathbf{q}, \omega) = \alpha_g [1 + \langle v_{00}(\mathbf{q}) \rangle \chi_{2D}(\mathbf{q}, \omega)]$$

$(\alpha_g^*)^C$  ▲ Yu Gan et al,  
Phys. Rev. B 93,  
195150 (2016)  
 $\alpha_g^*$  ◆ J. P. Reed et al,  
Science 330,  
805 (2010)



# HPC Resources

## Requirements:

**Large cells** (>50 atoms) or **supercells** (4x4 or more!)

**Large k-space grids** (up to 2400x2400!) to properly sample the Dirac cones features

**Multiple-Step** simulations: DFT-SCR-GW-BSE & post-processing (DOS,PDOS ect)

## Average Core h per simulations on a case study:

-5000 Core h for a typical relaxation run + scf + nscf

(x # systems) = 100.000 Core h for all DFT runs

-6000 Core h for a typical GW run

(x # systems x convergence tests ) = 300.000 Core h for GW runs

-4000 Core h for a typical BSE run

(x # systems x convergence tests ) = 200.000 Core h for BSE runs

## Suggested Reading

1. A. Sindona, M. Pisarra, C. Vacacela Gomez, P. Riccardi, G. Falcone, and S. Bellucci, "Calibration of the fine-structure constant of graphene by time-dependent density-functional theory", **Physical Review B** 96, 201408(R) – Published 17 November 2017
2. A. Sindona, A. Cupolillo, et al., "Interband  $\pi$ -like plasmon in silicene grown on silver", arXiv:1708.03858 [cond-mat.mtrl-sci], to appear as a rapid communication in **Physical Review B** (21 dec 2017)
3. C. Vacacela Gomez, M. Pisarra, M. Gravina, J. M. Pitarke and A. Sindona, "Plasmon Modes of Graphene Nanoribbons with Periodic Planar Arrangements", **Physical Review Letters** 117, 116801 (2016)
4. A. Sindona et al, "Orthogonality Catastrophe and Decoherence in a Trapped-Fermion Environment", **Physical Review Letters** 111, 165303 (2013)