QCD monopoles, abelian projections and gauge invariance

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Introduction.

- Free quarks never observed in Nature (Confinement of Colour): inhibited by a factor $\leq 10^{-15}$. Natural explanation: confinement is absolute, due to a symmetry, deconfinement a change of symmetry.

- Symmetry acting on degrees of freedom living on the boundary,$\rightarrow$ topology,$\rightarrow$ monopoles.

- Confinement produced by condensation of monopoles. ['tHooft, Mandelstam, 1975] . A dual statement to confinement of magnetic charge in superconductors.

- Condensation $\Longleftrightarrow$ $\langle\mu\rangle \equiv \langle 0|\mu|0\rangle \neq 0$
  μ a magnetically charged operator. $\langle\mu\rangle$ order parameter.

  More precise definition of the order parameter

  $$\langle\bar{\mu}\rangle = \frac{\langle 0|\mu 0\rangle}{\sqrt{(\langle 0|0\rangle)}\sqrt{(\langle 0\mu|\mu 0\rangle)}}$$

- True in compact $U(1)$ gauge theory [Frolich, Marchetti 1987, D. G., Paffuti 1997].
  Order parameter

  $$\mu(\vec{x},t) = \exp(i\int d^3y\vec{E}(\vec{y},t)\frac{m}{2g}\vec{A}_{tr}(\vec{x}-\vec{y}))$$

  $$\langle\mu\rangle = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)} = \exp(\int_0^\beta \rho(\beta')d\beta') \qquad \beta \equiv \frac{2N}{g^2}$$

  $$\rho \equiv \frac{\partial\log(\langle\mu\rangle)}{\partial\beta} = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{(S+\Delta S)}$$

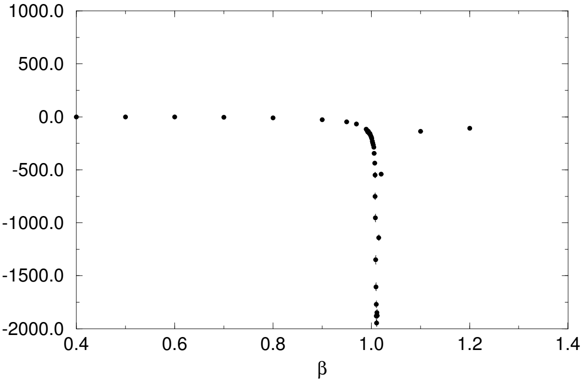

Figure : ρ versus β A.D.G. ,G.Paffuti P.R.D 56,6816 ,1997

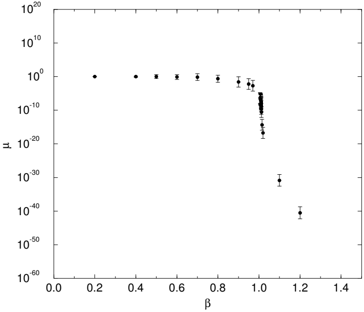

Figure : $\langle \mu \rangle$ for U(1) lattice gauge theory .

- In compact $SU(2)$ G.T. monopoles exist in presence of a Higgs field $\vec{\Phi}$ in the adjoint representation.
['tHooft, Polyakov 1974]
They are static classical solutions of the equations of motion.
Non trivial mapping of S_2 on $SU(2)/U(1)$: winding number $=$ m (magnetic charge) in units $\frac{1}{2g}$.
They look as abelian monopoles in the little group $U(1)$ of $\vec{\Phi}$.

- [G. 'tHooft Nucl. Phys. B 190 , 455 (1981)] : In absence of Higgs fields [QCD] any field in the adjoint representation can act as an effective Higgs, and monopoles are located in its zeroes. Each choice is called an ABELIAN PROJECTION .

  We shall analyse this statement in detail.

- In lattice $U(1)$ Gauge Theory monopoles are observed by detecting Dirac string as excess of magnetic flux trough a plaquette at the border of elementary volumes. [T.Degrand, D.Toussaint (1980)]
- In non abelian models the procedure is the same on the $U(1)$ subgroup selected by the abelian projection. Number and locations of monopoles strongly depend on the choice of it. If monopoles are physical the existence of a monopole should be a gauge invariant statement and the monopole a classical background configuration with the appropriate topology, as happens for instantons.
- — What projection identifies the monopoles which condense ?
  — What is the relation between monopoles from different abelian projections?

'tHooft Polyakov monopole

$$L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}D_\mu\vec{\Phi}D_\mu\vec{\Phi} - \frac{m^2}{2}\vec{\Phi}\vec{\Phi} - \frac{\lambda}{4}(\vec{\Phi}\vec{\Phi})^2 \qquad (1)$$

Hedgehog gauge

$$\Phi_a = \frac{x_a}{gx^2}H(\xi) \qquad \xi = g\mu x \quad \mu \equiv \sqrt{(}-\frac{m^2}{\lambda}) \qquad (2)$$

$$A_0^a = 0, \quad A_i^a = -\frac{1}{gx^2}\epsilon_{iab}x_b[1-K(\xi)] \qquad (3)$$

$H(\xi)_{\xi\to\infty} \approx \xi$, $\quad \lim_{\xi\to 0}\frac{H(\xi)}{\xi} = 0 \quad K(\infty) = 0, \quad K(0) = 1$

$$\xi^2\frac{d^2K}{d\xi^2} = KH^2 + K(K^2-1) \qquad (4)$$

$$\xi^2\frac{d^2H}{d\xi^2} = 2K^2H + \frac{\lambda}{g^2}H(H^2-\xi^2) \qquad (5)$$

At small ξ $\quad H(\xi) = \alpha\xi^2 + O(\xi^3) \quad K(\xi) = 1 + \gamma\xi^2 + O(\xi^3)$

Magnetic charge m

$$m = \frac{1}{g}\int_0^\infty d\xi \frac{d}{d\xi}\left[\frac{H(1-K^2)}{\xi}\right] = \frac{1}{g}\left[\frac{H(1-K^2)}{\xi}\right]_0^\infty \tag{6}$$

- Necessary conditions for the Higgs field Φ_a of a monopole in the hedgehog gauge

$$\Phi_a \quad \approx_{\vec{x}\to 0} \quad \alpha\mu^2 x_a \tag{7}$$

$$\Phi_a \quad \approx_{x\to\infty} \quad \mu\frac{x_a}{x} \qquad \mu \neq 0 \tag{8}$$

Every quantity $Q = Q_a\frac{\sigma_a}{2}$ transforming in the adjoint representation obeys the condition Eq(7) around one of its zeroes['tHooft 1981]. The argument is that, in the representation in which Q is diagonal in the vicinity of the zero $Q_a \approx \delta_{a3}x$, if there is rotation symmetry, and this is nothing but the required behaviour in the hedgehog gauge Eq(7). If Q is a polinomial in the fields $\vec{A}_a$ the solution is consistent since $\vec{A}_a = 0$ at $\vec{x} = 0$ Eq' s (2) (3).

- The condition at $x \to \infty$, Eq(8) is more difficult to satisfy. The gauge field $\vec{A}_i$ of the monopole vanishes at large x, and so does any polynomial of it transforming in the adjoint representation which then cannot satisfy the condition Eq.(8). The magnetic charge is zero and there is no mapping $S_2 \to SU(2)/U(1)$.
  The only possibility is to relax the condition $\vec{A}_0 = 0$ of the ansatz Eq(3), replace it by $\partial_0 \vec{A}_0 = 0$ (static solution) and assume it as gauge condition. In this case monopole solutions do exist [Julia, Zee (1975) , Bogomonlyi (1976) Prasad, Sommerfeld (1973)] in which $\vec{A}_4 = i\vec{A}_0$ plays the role of the Higgs field.
  In the gauge $\partial_0 \vec{A}_0 = 0$ the Polyakov line has a simple form

$$L(\vec{x}) = T\exp(i\int_0^{\frac{1}{T}} \vec{A}_0(\vec{x},t)dt) = \exp(i\frac{\vec{A}_0(\vec{x})}{T}) \qquad (9)$$

  The Polyakov line defines an Abelian Projection.

QCD Monopoles and Gauge Invariance.

- Consider a system in which there are two kinds of monopoles, with two Higgs fields $\hat{\Phi}$ and $\hat{\Phi}'$, fundamental or effective, each of length 1, on S_2. Since they both belong to the adjoint representation there exists a gauge transformation R , such that on S_2, $\hat{\Phi}' = R\hat{\Phi}$. The transformation between the two unitary gauges is global.

- In the unitary gauge the monopole field is, at large x, in the $U(1)$ which leaves Φ invariant, a Coulomb-like magnetic field plus a Dirac string or a Wu-Yang surface singularity. The Wu-Yang singularity can be replaced by a Dirac string, and the string rotated say along the positive z-axis: indeed, because of the quantisation of magnetic charge, a Dirac string which crosses physical space is invisible.

- The quantity M, computed on a small circle around the positive z-axis in the unitary gauge

$$M = \oint \hat{\Phi}(\vec{x})\vec{A}_i(\vec{x})dx_i = \int_\Sigma d\sigma_i \hat{\Phi}(\vec{\nabla} \wedge \vec{A}) \qquad (10)$$

  is the magnetic charge as defined by the abelian projection $\vec{\Phi}$. Indeed in the unitary gauge the 'tHooft tensor $F_{\mu\nu} \equiv (\hat{\Phi}\vec{G}_{\mu\nu} - \frac{1}{g}\hat{\Phi}(D_\mu\hat{\Phi} \wedge D_\nu\hat{\Phi}))$ reduces to $F_{\mu\nu} = \hat{\Phi}(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu))$.
- Transforming to the unitary gauge of $\hat{\Phi}'$ since the gauge transformation is global $M' = \oint \hat{\Phi}'(\vec{x})\vec{A}'_i(\vec{x})dx_i = \oint \hat{\Phi}(\vec{x})\vec{A}_i(\vec{x})dx_i = M$ The magnetic charge is abelian projection independent. The existence of a monopole is a projection independent fact. Adding a monopole in any abelian projection means adding it in all of them.

SU(3) gauge group. Generic compact Lie group.

Extension to higher groups straightforward. Monopoles can exist as mappings of S_2 on any $SU(2)$ subgroup of the gauge group G. An $SU(2)$ exists for each root $\vec{\alpha}$

- Generators $E_{\pm\vec{\alpha}}, H_i (i = 1...r)$ obeying the Lie algebra:

$$[H_i, H_j] = 0 \qquad [H_i, E_{\pm\vec{\alpha}}] = \pm\alpha_i E_{\pm\vec{\alpha}}$$

$$[E_{\vec{\alpha}}, E_{\vec{\beta}}] = N_{\vec{\alpha},\vec{\beta}} E_{\vec{\alpha}+\vec{\beta}} \qquad [E_{+\vec{\alpha}}, E_{-\vec{\alpha}}] = \vec{H}\vec{\alpha}$$

Simple roots $\vec{\alpha}_i (i = 1, .., r)$, fundamental weights $w_i = \vec{c}_i \vec{H}$,
$$\vec{c}_i \vec{\alpha}_j = \delta_{ij}$$

- $SU(2)$ attached to the root $\vec{\alpha}$

$$T^{\vec{\alpha}}_{\pm} = \sqrt{\frac{2}{(\vec{\alpha}\vec{\alpha})}} E_{\pm\vec{\alpha}} \qquad T^{\vec{\alpha}}_3 = \frac{\vec{\alpha}\vec{H}}{(\vec{\alpha}\vec{\alpha})}$$

$$[T^{\vec{\alpha}}_3, T^{\vec{\alpha}}_{\pm}] = \pm T^{\vec{\alpha}}_{\pm} \qquad [T^{\vec{\alpha}}_{+}, T^{\vec{\alpha}}_{-}] = 2T^{\vec{\alpha}}_3$$

- Monopole solutions

$$\Phi(\vec{x})^{(i)} = \chi^a(\vec{x}) T_a^{(i)} + \mu(w_i - T_3^{(i)}),\ \chi^a = \frac{x^a}{gx^2} H(\xi) \quad H(\xi) \approx_{\xi \to \infty} \xi$$

and

$$A_k^{(i)} = A_k^a(\vec{x}) T_a^{(i)} \qquad A_k^a(\vec{x}) = -\frac{1}{g}\epsilon_{akj}\frac{x^j}{x^2}[1 - K(\xi)]$$

Same form for monopoles attached to non simple roots if μ is i-independent. One single scale μ and same form for all roots.

- As in the $SU(2)$ case the 'tHooft tensor written in the unitary representation on S_2 has the simple form

$$F^i_{\mu\nu} = Tr[w^i(\partial_\mu A_\nu - \partial_\nu A_\mu)]$$

the monopole in the $U(1)$ projected by w^i is a point charge plus a Dirac string along the positive z-axis, and the line integral on a closed path encircling it

$$M = \oint Tr[w^i A_i(\vec{x}) dx^i] \qquad (11)$$

is the magnetic charge, projection invariant as in $SU(2)$.

Discussion.

A few comments:

- None of the arguments above is affected by the presence of matter fields, say quarks, in the theory.
- Creating a monopole along the 3 axis in color space for $SU(2)$ gauge theory or along the 3 axis of the appropriate $SU(2)$ subgroup of higher groups is equivalent to create it in any abelian projection. This justifies the approach used by the Pisa and Bari groups to define the operator μ which creates a monopole.
- The definition of order parameter used at that time

$$\mu(\vec{x}, x_0) = \exp(-i\frac{2\pi}{g}\int d^3y \vec{E}_3(\vec{y}, x_0) T^3 \vec{A}^{(mon)}(\vec{x}-\vec{y})) \quad (12)$$

$$\frac{\partial \ln(\langle\mu\rangle)}{\partial\beta} \equiv \rho = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{(S+\Delta S)} \quad (13)$$

with T_3 the third component of the appropriate $SU(2)$ subgroup, has infrared problems. [Cossu et al. 2007].

- $\rho_{V\to\infty} \propto -V^{\frac{1}{3}}$ or $\mu \to 0$ also in the confined phase.

Improvement needed. [Bonati et al 2012]

$$\langle\mu\rangle = \langle 0|\mu 0\rangle \qquad \longrightarrow \qquad \langle\bar{\mu}\rangle \equiv \frac{\langle 0|\mu 0\rangle}{\sqrt{(\langle 0|0\rangle)}\sqrt{(\langle \mu 0|\mu 0\rangle)}}$$

$$\rho \quad \rightarrow \qquad \bar{\rho} = \quad \frac{\partial \log(\langle\bar{\mu}\rangle)}{\partial_\beta} \quad = \quad \rho - \frac{1}{2}\rho_2$$

$$\rho_2 \equiv \frac{\partial \log(\langle \mu 0|\mu 0\rangle)}{\partial\beta} \quad = \quad \langle S\rangle_S - \langle S + \Delta_2 S\rangle_{S+\Delta_2 S}$$

Cluster Property. $SU(2)$.

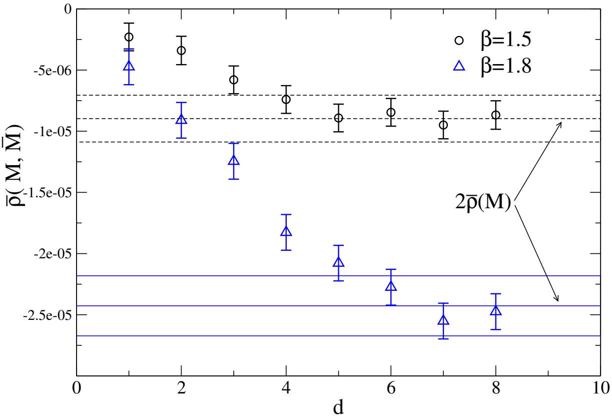

Figure : 3. Cluster property for $\bar{\rho}$. Group $SU(2)$; 4×20^3.

ρ at small β. SU(2).

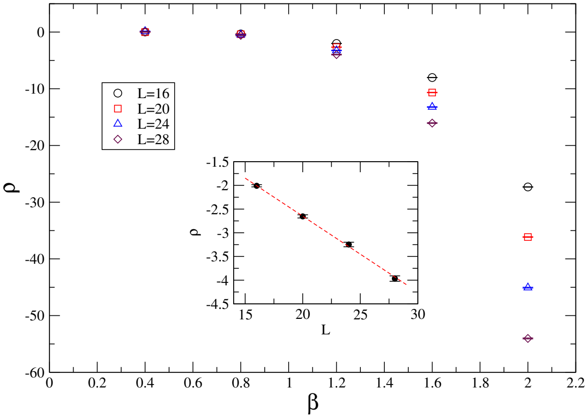

Figure : 4. ρ at small β vs L. $\beta = 1.2$, $4xL^3$, ($\beta_c \approx 2.3$.)

$\bar{\rho}$ at small β. SU(2).

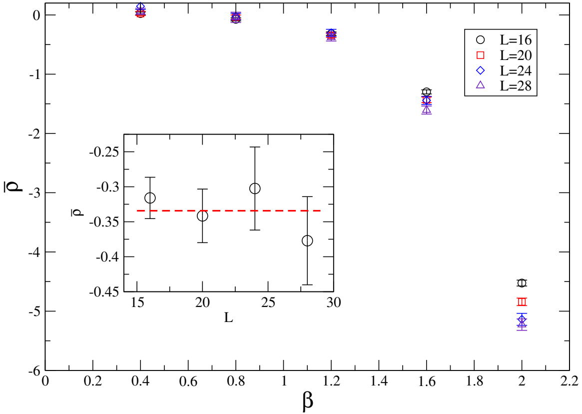

Figure : 5. $\bar{\rho}$ at small β vs L. $\beta = 1.2$, $4xL^3$, ($\beta_c \approx 2.3$).

$\bar{\rho}$ vs β at various L's.

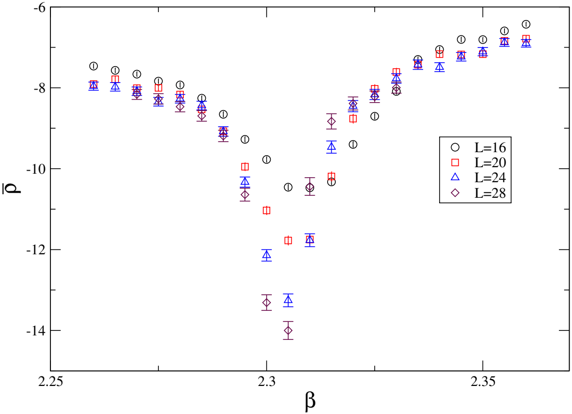

Figure : 6. $\bar{\rho}$ vs β at different L's. $4xL^3$

$\bar{\rho}$ Scaling.

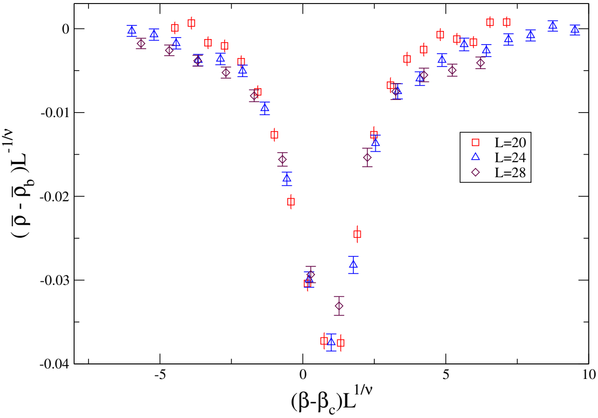

Figure : 7. $\bar{\rho}$ rescaled. $\beta_c = 2.2986(6), \nu = .6301(4)$ (3d Ising)
$\bar{\rho}_{peak} = \bar{\rho}_b + cL^{\frac{1}{\nu}}$, $4xL^3$

- Monopoles observed in lattice configurations for each choice of the Abelian Projection as in U(1) theory: any excess of abelian phase of a plaquette is interpreted as a Dirac string. Magnetic flux is conserved : Dirac lines form closed loops or end in elementary cubes containing a monopole or antimonopole. Number and position of the monopoles strongly depend on the abelian projection. $\longrightarrow$
  Many of them are unphysical.
  Strong dependence on local fluctuations.
- Maximal abelian gauge : monopole dominance [Kanazawa group 1980's]. No way to demonstrate condensation. [Polikarpov 1997].
- Cooling- smearing to detect monopoles as done for instantons. Start from the Polyakov abelian projection.
- $SU(3)$ works correctly. G_2 starting(in collaboration wirh C. Bonati)
- P, T in the Polyakov gauge.