Testing a non-perturbative mechanism for elementary fermion mass generation

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- Frezzotti and Rossi in [Phys. Rev. D92 (2015) 054505] conjectured a new non-perturbative mechanism for the elementary particle mass generation
- We are testing this conjecture in the "simplest" appropriate $d = 4$ "toy model"

$$\mathcal{L}_{\rm toy}(Q, A, \Phi) = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{\mu_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2$$
$$+ \overline{Q}_L \gamma_\mu \mathcal{D}_\mu Q_L + \overline{Q}_R \gamma_\mu \mathcal{D}_\mu Q_R + \frac{b^2}{2}\rho\left(\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.\right)$$
$$+ \eta\left(\overline{Q}_L \Phi Q_R + h.c.\right), \qquad \Phi \equiv \varphi_0 \mathbb{1} + i\tau_i \varphi_i$$

- "Wilson-like" $\propto \rho$ (naively irrelevant)
- UV cutoff $\sim b^{-1}$
- Fermionic chiral transformations $\tilde{\chi}$ are not a symmetry if $(\rho, \eta) \neq (0, 0)$

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}\mathrm{Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big] + \frac{\mu_0^2}{2}\mathrm{Tr}\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big(\mathrm{Tr}\big[\Phi^\dagger\Phi\big]\big)^2$$
$$+\overline{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \overline{Q}_R\gamma_\mu\mathcal{D}_\mu\, Q_R + \frac{b^2}{2}\rho\big(\overline{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + h.c.\big)$$
$$+\eta\,\big(\overline{Q}_L\Phi Q_R + h.c.\big), \qquad \Phi \equiv [\varphi| - i\tau^2\varphi^*]$$

- Symmetries & power counting $\begin{pmatrix}\text{in suitable}\\ \text{UV-regul.}\end{pmatrix}$ $\Longrightarrow$ Renormalizability
- Invariant under χ (global) $SU(2)_L\times SU(2)_R$ transformations

$$\bullet\,\chi_{L,R}: \quad \tilde{\chi}_L\otimes(\Phi\to\Omega_L\Phi) \quad \text{and/or} \quad \tilde{\chi}_R\otimes(\Phi\to\Phi\Omega_R^\dagger)$$
$$\tilde{\chi}_{L,R}: \begin{cases} Q_{L,R}\to\Omega_{L,R}Q_{L,R} & \Omega_{L,R}\in SU(2)_{L,R} \\ \overline{Q}_{L,R}\to\overline{Q}_{L,R}\Omega^\dagger_{L,R} & \end{cases}$$

- Not invariant under purely fermionic transformations $\tilde{\chi}$
- χ invariance forbids $\frac{1}{b}\overline{Q}Q$ terms and softens power like U.V. divergences

- Purely fermionic $\tilde{\chi}$ tranformations yield bare Schwinger Dyson Eq.s (SDEs)

$$\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{\mathcal{O}}(0)\rangle \delta(x) - \eta\, \langle O^{L,i}_{Yuk}(x)\, \hat{\mathcal{O}}(0)\rangle - b^2 \langle O^{L,i}_{Wil}(x)\, \hat{\mathcal{O}}(0)\rangle$$

$$\tilde{J}^{L,i}_\mu = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2} \Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$O^{L,i}_{Yuk} = \Big[\overline{Q}_L \frac{\tau^i}{2} \Phi Q_R - \text{h.c.}\Big] \qquad O^{L,i}_{Wil} = \frac{\rho}{2}\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2} \Phi \mathcal{D}_\mu Q_R - \text{h.c.}\Big]$$

- Mixing of $O^{L,i}_{Wil}$ under renormalization

$$b^2 O^{L,i}_{Wil} = (Z_{\tilde{\partial} J} - 1)\partial_\mu \tilde{J}^{L,i}_\mu - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O^{L,i}_{Yuk} + \underline{\ldots} + \mathrm{O}(b^2)$$

- Renormalized SDEs read

$$Z_{\partial \tilde{J}} \partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{\mathcal{O}}(0)\rangle \delta(x) - (\eta - \overline{\eta}(\eta))\, \langle O^{L,i}_{Yuk}(x)\, \hat{\mathcal{O}}(0)\rangle + \underline{\ldots} + \mathrm{O}(b^2)$$

where the ellipses ($\underline{\ldots}$) stand for possible NP mixing contributions

- At the critical $\eta_{cr}(g_s^2, \rho, \lambda_0)$ s.t. $\eta_{cr} = \overline{\eta}(\eta_{cr}; g_s^2, \rho, \lambda_0)$ the SDEs become WTIs

$$Z_{\partial\tilde{J}}\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \underline{\ldots} + \mathrm{O}(b^2)$$

- In Wigner phase $\big(\langle\Phi\rangle = 0\big)$ the Wilson-like term is uneffective for $\tilde{\chi}^{SSB}$

$$Z_{\partial\tilde{J}}\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \mathrm{O}(b^2)$$

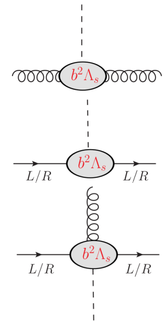
- In Nambu-Goldstone $\big(\langle\Phi\rangle = v1\!\!1_{2\times 2}\big)$ expect (conjecture)

$$Z_{\partial\tilde{J}}\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \langle C_1\Lambda_s[\overline{Q}_L \frac{\tau^i}{2}\mathcal{U} Q_R + \mathrm{hc}]\hat{\mathcal{O}}(0)\rangle + \mathrm{O}(b^2)$$

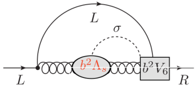
- The term $\propto C_1\Lambda_s$ can exist only in the NG phase where

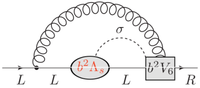
$$\mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \frac{(v + \sigma + i\overrightarrow{\tau}\overrightarrow{\varphi})}{\sqrt{(v+\sigma)^2 + \overrightarrow{\varphi}\overrightarrow{\varphi}}} = 1\!\!1 + i\frac{\overrightarrow{\tau}\overrightarrow{\varphi}}{v} + \ldots$$

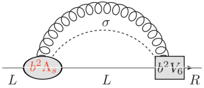
In Γ_{loc}^{NG} NP term $C_1\Lambda_s[\overline{Q}_L\mathcal{U}Q_R + \mathrm{hc}] \supset C_1\Lambda_s\overline{Q}Q_R \left\{ \begin{array}{l} \text{Natural mass} \\ \neq \text{Yukawa term} \\ C_1 = O(\alpha_s^2) \to \text{Hierarchy} \end{array} \right.$



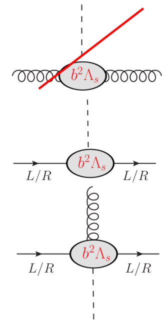
- Intuitive idea of the NP mass generation mechanism $O(b^2)$ NP corrections to ($\tilde{\chi}$-preserving) effective vertices combined in loop "diagrams" with $O(b^2)$ ($\tilde{\chi}$-breaking) vertices from the Wilson-like term







- b^{-4} loop divergency $\implies$ $O(b^0)C_1\Lambda_s$ mass term



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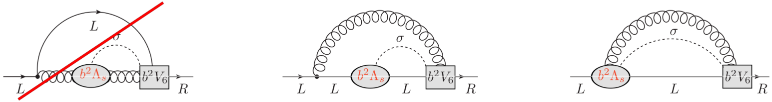


- b^{-4} loop divergency $\implies$ $O(b^0)C_1\Lambda_s$ mass term
- Phenomenon occurring even in the quenched fermion approximation

Choose a cheap lattice regularization of $\int d^4x \mathcal{L}_{toy}$

- First NP study of a theory with gauge, $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$ & $(\varphi_0, \vec{\varphi})$
- "Naive" fermions (good for quenched approximation only)

$\mathcal{S}_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \sum_g \overline{\Psi}_g D_{lat}[U,\Phi]\Psi_g \right\}$

$\mathcal{L}_{kin}^{YM}[U]$: SU(3) plaquette action

$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\,\mathrm{tr}\,[\Phi^\dagger(-\partial^*_\mu \partial_\mu)\Phi] + \frac{\mu_0^2}{2}\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$

where $\Phi = \varphi_0 \mathbb{1} + i\varphi_j \tau^j$ and $F(x) \equiv [\varphi_0 \mathbb{1} + i\gamma_5 \tau^j \varphi_j](x)$

$$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu \widetilde{\nabla}_\mu \Psi(x) + \eta F(x)\Psi(x) - b^2\rho\tfrac{1}{2}F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) +$$
$$- b^2\rho\tfrac{1}{4}\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial^*_\mu F)(x)U^\dagger_\mu(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\right]$$

- Yukawa ($d=4$) term $\propto \eta$, Wilson-like ($d=6$) term $\propto \rho$
- Unquenched studies will require overlap, DW or staggered fermions

- Wilson-like term does not remove the doublers because it involves the scalar field Φ and it has dimension 6
- In S_{latt} there are only symmetric derivatives $\tilde{\nabla}_\mu \implies$ at $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all doublers up to cutoff effects
- Quenched $(U, \Phi) \implies$ exceptional configurations: at large $|\eta|$ and $|\rho|$ enhanced by Φ fluctuations
- Add twisted mass term: $S_{lat}^{toy+tm} = S_{lat} + i\mu b^4 \sum_x \overline{\Psi}\gamma_5\tau_3\Psi$ control over exceptional confs. at the price of harmless breaking of $\chi_{L,R}$ (and $\widetilde{\chi}_{L,R}$ when restored)

- Quenched lattice study: independent M.C. update of U and Φ

$$Z = \int \mathcal{D}\Phi \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\overline{\Psi} e^{S[U]+S[\phi]} \cancel{\det\left(D_{latt}[U,\phi]\right)} = Z[\Phi]Z[U]$$

- Action parameter renormalization can be carried out separately for gauge and scalars:
  - In the fermionic sector effective Yukawa coupling vanishes at $\eta = \eta_{cr}$
  - Gauge coupling renormalized keeping $r_0 \sim 0.5 fm$ fixed
    M. Guagnelli, R. Sommer and H. Wittig (1998) S. Necco, R. Sommer (2001)
  - Scalars parameters μ_0, λ_0 fixed by the renormalization condition

$$m_\sigma^2 r_0^2 = 1.285 \qquad \lambda_R = \frac{m_\sigma^2}{2v_R^2} = 0.4408$$

- Wilson-like coupling ρ: free parameter as long as we are only interested to see if the mechanism exists, relevant for the magnitude of the NP mass (if any)

- Determine η_{cr} in the Wigner phase from the renormalized SDE

$$Z_{\partial \tilde{J}}\partial_0\langle \tilde{J}_0^{V,3}(x)\tilde{D}^{S,3}(0)\rangle = (\eta-\bar{\eta})\langle \tilde{D}^{S,3}(x)\tilde{D}^{S,3}(0)\rangle + O(b^2)$$

$$\tilde{J}_0^{V,\,3}(x) = \tilde{J}_0^{L,\,3}(x) + \tilde{J}_0^{R,\,3}(x),$$

$$\tilde{J}_0^{L/R,\,3}(x) = \frac{1}{2}\left[\bar{Q}_{L/R}(x-\hat{0})\gamma_0\frac{\tau_3}{2}U_0(x-\hat{0})Q_{L/R}(x) + \bar{Q}_{L/R}(x)\gamma_0\frac{\tau_3}{2}U_0^\dagger(x-\hat{0})Q_{L/R}(x-\hat{0})\right]$$

$$\tilde{D}^{S,\,3}(x) = \bar{Q}_L(x)\left[\Phi,\frac{\tau^3}{2}\right]Q_R(x) - \bar{Q}_R(x)\left[\frac{\tau^3}{2},\Phi^\dagger\right]Q_L(x)$$

- We look for the value of η where $r_{WTI} = 0$

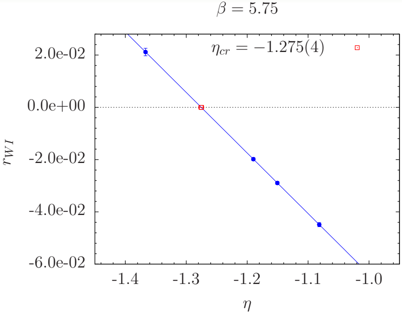
$$r_{WI} = \frac{\partial_0\langle \tilde{J}_0^{V,3}(x)\tilde{D}^{S,3}(0)\rangle}{\langle \tilde{D}^{S,3}(x)\tilde{D}^{S,3}(0)\rangle} = Z_{\partial\tilde{J}}^{-1}(\eta-\bar{\eta}) + O(b^2)$$

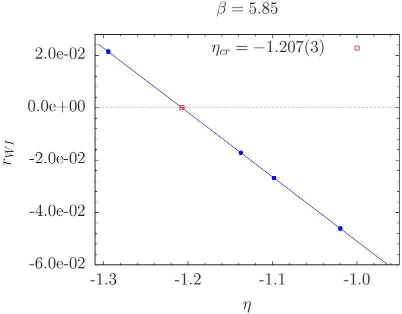
- $30 \sim 40 gauges \times 8 scalars$ for each set of the parameters

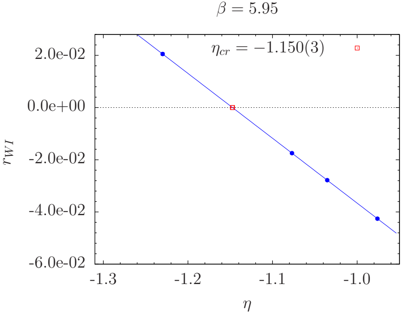
b (fm)	β	λ_0	$(m_0^2-m_{cr}^2)b^2$	$b^2 m_0^2$
0.152	5.75	0.5807	0.1119(12)	-0.4150
0.123	5.85	0.5917	0.0742(11)	-0.461538
0.102	5.95	0.6022	0.0504(10)	-0.4956

- For each set 3 values of the twisted mass μ to extrapolate r_{WI} for $\mu \to 0$

- result for the 3 lattice spacing for η_{cr} from $r_{WI}(\eta_{cr}) = 0$

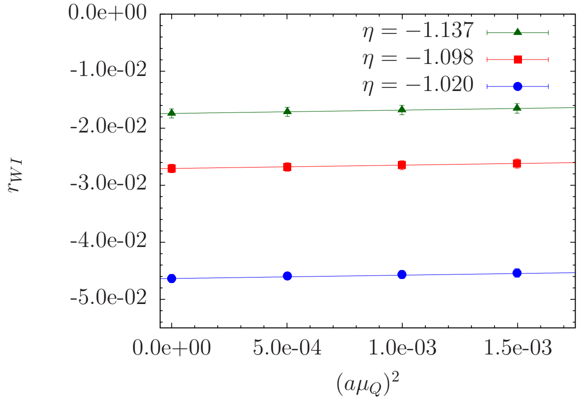






b (fm)	β	η_{cr}
0.152	5.75	-1.275(4)
0.123	5.85	-1.207(3)
0.102	5.95	-1.150(3)

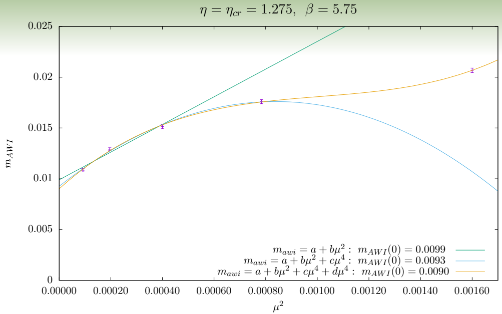
- Twisted mass term $S_{lat}^{toy+tm} = S_{lat} + i\mu b^4 \sum_x \overline{\Psi}\gamma_5\tau_3\Psi$
- Extrapolation to $\mu^2 \to 0$ in the Wigner phase
- e.g. at $\beta = 5.85$

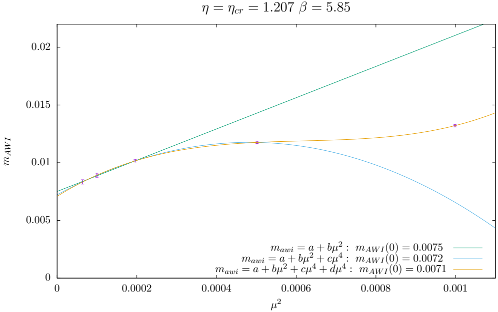


- In the Nambu-Goldstone phase of the theory $\langle\Phi\rangle = v$ at $\eta = \eta_{cr}$ to check if a non zero quark mass is generated
- At $\eta = \eta_{cr}$ a renormalized measure of the NP $\widetilde{\chi}$ breaking is given by

$$m_{AWI}^{ren} \equiv \frac{Z_A}{Z_P}\frac{\partial_0\langle 0|\widetilde{J}_0^{A^1}(x)P^1(0)|0\rangle}{2\langle 0|P^1(x)P^1(0)|0\rangle} \propto C_{1,ren}\Lambda_s$$

$$\widetilde{J}_\mu^{A^1} = \overline{Q}\gamma_\mu\gamma_5\frac{\tau^\pm}{2}Q(x)\Big|_{\text{point split}} \qquad P^1 = \overline{Q}_L\frac{\tau^\pm}{2}Q_R - h.c.$$





- Parity implies $m_{AWI}(\mu^2)$
- Different fit antsaz: linear, parabolic, cubic in $x = \mu^2$
- $\beta = 5.75 : \ r_0 m_{AWI} = 0.032(3)$
- $\beta = 5.85 : \ r_0 m_{AWI} = 0.030(3)$
    - Systematic error fit antsaz: Δ_{Sys}
    - Statistical error: Δ_{Stat}
    - Error on η_{cr}: $\frac{dm_{AWI}}{d\eta}\Delta\eta$
- Total error $\sqrt{\Delta_{Sys}^2 + \Delta_{Stat}^2 + (\frac{dm_{AWI}}{d\eta}\Delta\eta)^2}$
- Finer lattice spacing $\beta = 5.95$ in progress

- Renormalized quantity

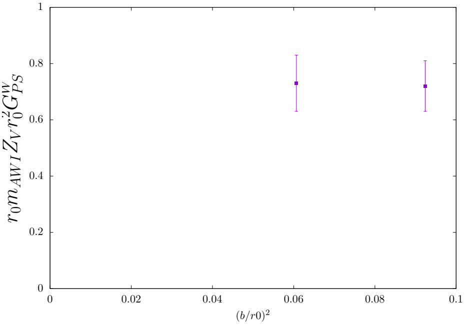
$$m_{AWI}^{ren} \equiv \frac{Z_A}{Z_P} \frac{\partial_0 \langle 0|\widetilde{J}_0^{A^1}(x) P^1(0)|0\rangle}{2\langle 0|P^1(x) P^1(0)|0\rangle} \propto C_{1,ren}\Lambda_s$$

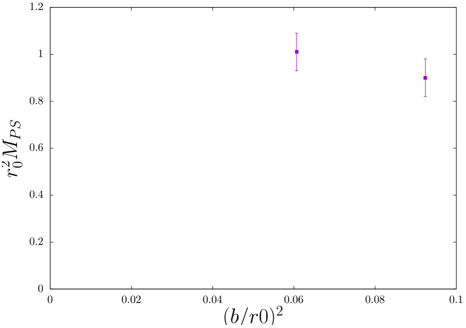
- The $\chi_L \otimes \chi_R$ symmetry implies that $Z_A = Z_V$

$$Z_V \langle \widetilde{J}_\mu^{V,i}(x) P^1(0)\rangle = 2\mu \langle P^1(x) P^1(0)\rangle$$

- Take value at $\mu = 0$ with a linear fit
  - $\beta = 5.75$, $Z_V = 0.957(1)$
  - $\beta = 5.85$, $Z_V = 0.959(1)$
- Alias of $Z_{P,alias}^{-1} = r_0^2 G_{PS}^W = r_0^2 \langle 0|P^1|PS_{meson}\rangle\big|_{Wigner}^{\mu\to 0}$
  - $\beta = 5.75$, $r_0^2 G_{PS}^W = 23.6(6)$
  - $\beta = 5.85$, $r_0^2 G_{PS}^W = 25.6(8)$

- Renormalized quantity looking flat towards the continuum limit
- $\beta = 5.75$: $r_0 m_{AWI} Z_V (r_0^2 G_{PS}) = 0.72(9)$
- $\beta = 5.85$: $r_0 m_{AWI} Z_V (r_0^2 G_{PS}) = 0.73(10)$

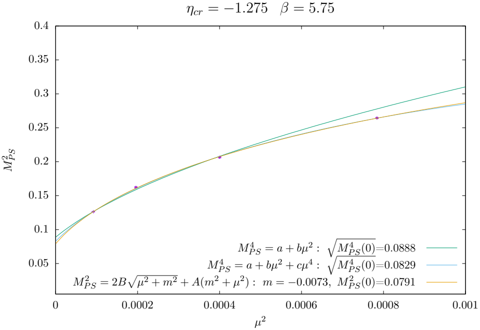


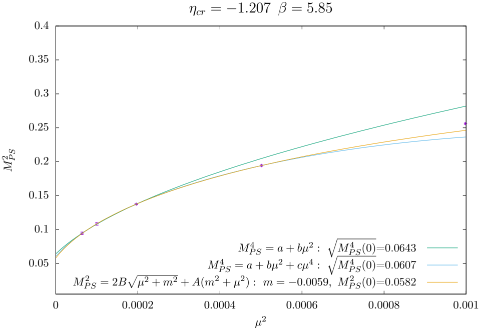


- finer lattice spacing in progress

- An other evidence of the NP mass generation is a non zero pseudoscalar meson M_{PS}
- M_{PS} dependency on μ [here $m = Z_V m_{AWI}(\mu = 0)$]

$$M_{PS}^2(\mu) = 2B\sqrt{\mu^2 + m^2} + A(\mu^2 + m^2) + O\left((\mu^2 + m^2)^{3/2}\right)$$





- Finer lattice spacing $\beta = 5.95$ in progress
  - Systematic error fit antsaz: Δ_{Sys}
  - Statistical error: Δ_{Stat}
  - Error on η_{cr}: $\frac{dM_{PS}}{d\eta}\Delta\eta$
- Total error $\sqrt{\Delta^2_{Sys} + \Delta^2_{Stat} + (\frac{dM_{PS}}{d\eta}\Delta\eta)^2}$

b (fm)	β	$r_0^2 M_{PS}^2$
0.152	5.75	0.90(8)
0.123	5.85	1.01(8)
0.102	5.95	in progress

in QCD $1/r_0 \sim 400$ MeV

- M_{PS} is not going to zero in the continuum limit
- Finer lattice spacing to confirm the results

Conclusion

- We check that for in the Wigner phase there is a value of the Yukawa coupling η_{cr} where the fermionic $\tilde{\chi}_L \otimes \tilde{\chi}_L$ are restored and are not spontaneously broken
- At η_{cr} in the Nambu-Goldston phase $m_{AWI}^{ren} \neq 0$ and $M_{PS} \neq 0$ in the continuum limit

Outlook

- Finer lattice spacing in progress
- Reduce the errors in m_{AWI} and M_{PS}

Thank you for your attention

Backup

- Using staggered formalism to analyze naive valence fermions
- $\Psi(x)$ contain 4 replicas $B = 1,..,4$
  $\Psi(x) = \mathcal{A}_x \chi(x)$, $\mathcal{A}_x = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_2^{x_2}\gamma_2^{x_2}$, Spin diagonalization of S_{latt} in χ^B basis
- $\chi(x)$ contain 4 tastes a=1,...,4
  $q^B_{\alpha,a}(y) = \frac{1}{8}\sum_\xi \overline{U}(2y, 2y+\xi)[\Gamma_\xi]_{\alpha,a}(1 - b\sum_\mu \xi_\mu \widetilde{\nabla}_\mu)\chi^B(2y+\xi)$,
  $q^B_{\alpha,a}(y)$ taste basis, $x_\mu = 2y_\mu + \xi_\mu$, $\xi_\mu = 0, 1$
- Flavour content: $\underbrace{(4 \text{ replicas:} B) \times (4 \text{ tastes:} a)\times}_{16 \text{ doublers}}$ (2 isospin)$\times$ generations
- Following Kluberg-Stern et al. ('83),..., Sharpe et al. ('93), Luo ('96) and adding scalars we get the small b expansion of S^{fer}_{lat} on smooth U,Φ configuration

$$S^{fer}_{lat} = \sum_{y,B} \bar{q}^B(y)\left\{\sum_\mu (\gamma_\mu \otimes \mathbb{1})D_\mu + (\eta - \bar{\eta})\mathcal{F}(y)\right\} q^B(y) + O(b^2)$$

$\mathcal{F}(y) = \varphi_0(2y)(\mathbb{1}\otimes\mathbb{1}) + S^B i\tau^i\varphi_i(2y)(\gamma_5 \otimes t_5)$, $S^A = \pm 1$, taste matrices $t_\mu = \gamma^*_\mu$

- Quark bilinear in Ψ basis that have the classical continuum limit in q^B basis
- Point split vector current

$$\widetilde{J}^{V^i}_\mu(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu \frac{\tau^i}{2} U^\dagger_\mu(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}^{V^i}_\mu(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu \otimes 1\!\!1)\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Point split axial current

$$\widetilde{J}^{A^i}_\mu(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu\gamma_5 \frac{\tau^i}{2} U^\dagger_\mu(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}^{A^i}_\mu(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu\gamma_5 \otimes t_5)\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Correlators with generation off-diagonal operator $\Rightarrow$ no disconnected diagrams

e.g. $\langle \overline{\Psi}_\ell(x)\Gamma\tau\Psi_h(x)\overline{\Psi}_h(y)\Gamma\tau\Psi_\ell(y)\rangle \qquad \ell = (u,d) \quad h = (c,s)$

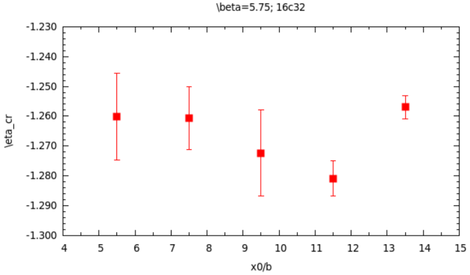
- Loop effects do not generate $d \leq 4$ operator besides $F_{\mu\nu}F_{\mu\nu}$, $\partial_\mu \Phi^\dagger \partial_\mu \Phi$
  $q^B(\gamma_\mu \otimes 1\!\!1)\tilde{\nabla} q^B$, $\Phi^\dagger\Phi$, $(\Phi^\dagger\Phi)^2$, $\eta \bar{q}^B(y)\mathcal{F}^B(y)q^B(y)$ (all in S_{lat})

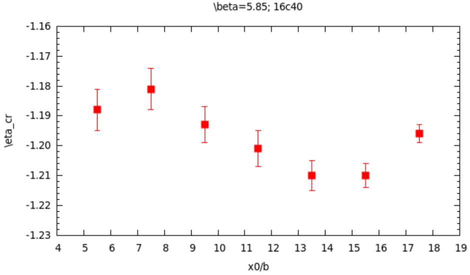
- Argument: In S_{latt} there are only $\widetilde{\nabla}_\mu$ acting on fermions $\Longrightarrow$
  Spectrum Doubling Symmetry :
  $\Psi \to e^{-ix\cdot\pi_H} M_H \Psi \quad \overline{\Psi} \to \overline{\Psi} M_H^\dagger e^{+ix\cdot\pi_H}$, $H = \{\mu_1, ..., \mu_h\}$ ordered,
  16 vectors $\pi_H(\pi_{H,\mu} = \pi$ if $\mu \in H)$ with $M_H = (i\gamma_5\gamma_1)...(i\gamma_5\gamma_{\mu_h})$

- It is a symmetry of S_{latt}, thus also of the $\Gamma_{lat}[U, \Phi, \Psi]$. So the latter can only have terms with symmetric covariant derivatives $\widetilde{\nabla}_\mu$ acting on Ψ.
  Close to the continuum limit among the local terms of Γ_{lat} only those with no or one $\widetilde{\nabla}_\mu$ fermionic derivative are relevant

- At $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all tastes up to cutoff effects

- $\eta_{cr}(x_0)$


\beta=5.75; 16c32
\eta_cr
x0/b
-1.230
-1.240
-1.250
-1.260
-1.270
-1.280
-1.290
-1.300
4
5
6
7
8
9
10
11
12
13
14
15


\beta=5.85; 16c40
\eta_cr
x0/b
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-1.17
-1.18
-1.19
-1.20
-1.21
-1.22
-1.23
4
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- Plateu at $\beta = 5.85$ in NG phase