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Exact results for quenched disorder at criticality

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Based on:

G. Delfino, Phys. Rev. Lett. 118 (2017) 250601

G. Delfino and E. Tartaglia, arXiv:1709.00364, to appear in JSTAT

Introduction

quenched disorder: some degrees of freedom take too long to reach thermal equilibrium and can be considered as random variables

examples: impurities in ferromagnets, spin glasses, ...

disorder average is taken on the free energy $F(\{J\})$

$$\overline{F} = \sum_{\{J\}} P(\{J\}) F(\{J\})$$

with a probability distribution $P(\{J\})$

numerics/experiments: exists "random" criticality with new critical exponents

theory:

- perturbative results for exponents in very few cases for weak disorder
only numerics for strong disorder (short range interactions)

- surprising absence of exact results in 2D (pure systems solved in '80s)
moreover in 2D disorder softens 1st order transitions into 2nd order ones [Aizenman, Wehr '89] making more room for conformal invariance

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questions:

- does quenched disorder generally fall within the framework of renormalization group (RG) and field theory?

- do random critical points possess conformal invariance?

- if so, why the corresponding conformal field theories in 2D have never been found?

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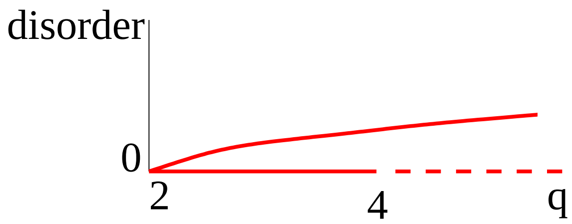
attempts of answers can hardly escape the Potts model

2D random bond Potts model:

$$H = -\sum_{\langle s_i, s_j \rangle} J_{ij} \delta_{s_i, s_j} \qquad s_i = 1, 2, \ldots, q$$



- permutational invariance S_q; exists continuation to q real

- transition of pure ferromagnet ($J_{ij} = J > 0$) 2nd order up to $q = 4$, 1st order after

- perturbative random critical point for $q \to 2$ [Ludwig '90, Dotsenko et al '95]

- numerical hints of q-independent exponents [Chen et al '92,'95; Domany, Wiseman '95; Kardar et al '95]. Superuniversality?

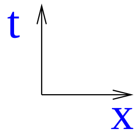
- q-dependent exponents (weakly for ν) [Cardy, Jacobsen '97 (numerical transfer matrix)]

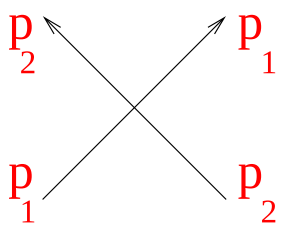
- no substantial progress afterwards

New insight: particles [GD '17]

Euclidean field theory in 2D $\longleftrightarrow$ relativistic quantum field theory in (1+1)D $\longrightarrow$ particles

at criticality ∞-dimensional conformal symmetry makes **scattering completely elastic**





center of mass energy only relativistic invariant, dimensionful $\Rightarrow$ **constant amplitudes** by scale invariance and unitarity

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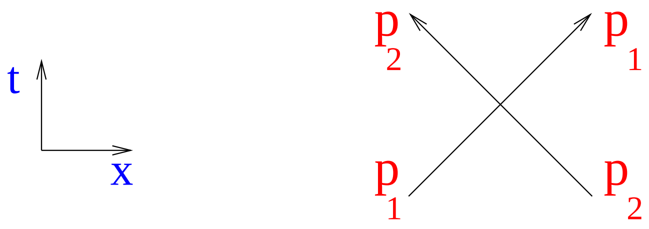

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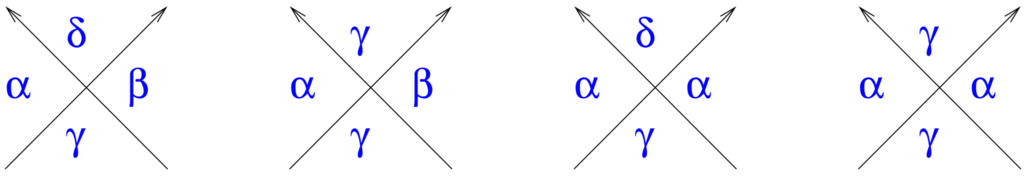
pure Potts: S_q symmetry $(\alpha = 1, 2, \dots, q)$



random Potts

$$\overline{F} = -\overline{\ln Z} = -\lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

n replicas coupled by average over disorder

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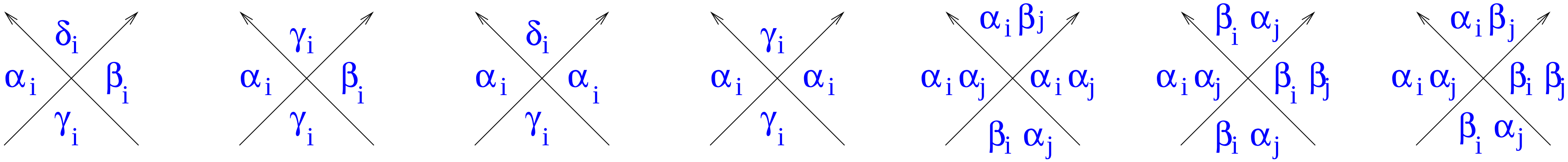

amplitudes $S_0, S_1, \ldots, S_6 \quad (i = 1, 2, \ldots, n)$

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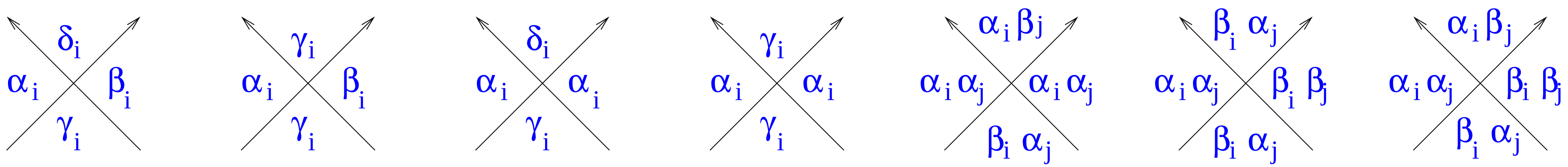

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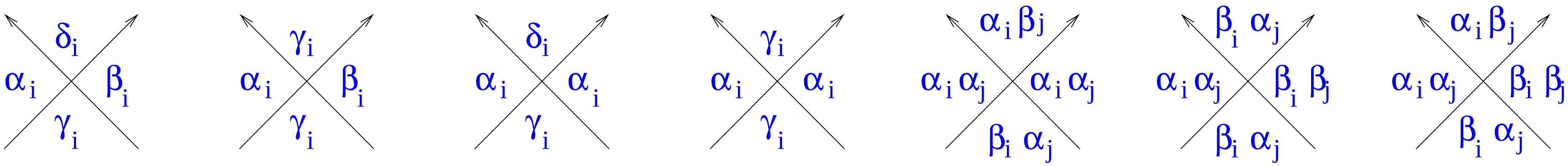
crossing:

$$S_0 = S_0^* \equiv \rho_0, \qquad S_1 = S_2^* \equiv \rho\, e^{i\varphi}, \qquad S_3 = S_3^* \equiv \rho_3, \qquad S_4 = S_5^* \equiv \rho_4\, e^{i\theta}, \qquad S_6 = S_6^* \equiv \rho_6$$

unitarity:

$$\rho_3^2 + (q-2)\rho^2 + (n-1)(q-1)\rho_4^2 = 1$$
$$2\rho\rho_3 \cos\varphi + (q-3)\rho^2 + (n-1)(q-1)\rho_4^2 = 0$$
$$2\rho_3\rho_4 \cos\theta + 2(q-2)\rho\rho_4 \cos(\varphi + \theta) + (n-2)(q-1)\rho_4^2 = 0$$
$$\rho^2 + (q-3)\rho_0^2 = 1$$
$$2\rho_0\rho \cos\varphi + (q-4)\rho_0^2 = 0$$
$$\rho_4^2 + \rho_6^2 = 1$$
$$\rho_4\rho_6 \cos\theta = 0$$



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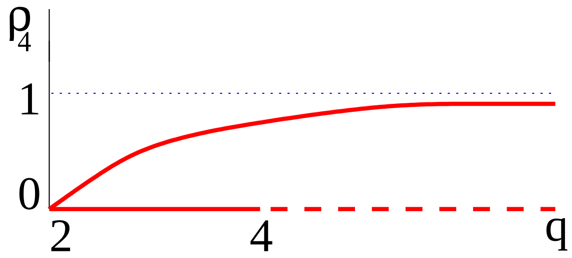
&\rho^2 + (q-3)\rho_0^2 = 1 \\
&2\rho_0\rho \cos\varphi + (q-4)\rho_0^2 = 0 \\
&\rho_4^2 + \rho_6^2 = 1 \\
&\rho_4\rho_6 \cos\theta = 0
\end{aligned}$$

- method yields exact scattering solutions
- solutions are RG fixed points with color and replica permutational symmetry
- q and n enter equations as continuous parameters
- replicas decouple for $\rho_4 = 0 \quad \Rightarrow \quad \rho_4 \sim$ disorder strength

random Potts $(n = 0)$

exists and is unique solution with disorder vanishing as $q \to 2$ and defined $\forall q \geq 2$: random ferromagnet

$$\cos\theta = \rho_0 = 0, \quad \rho = 1, \quad \rho_3 = 2\cos\varphi = -\frac{2}{q}, \quad \rho_4 = \sqrt{1 - \rho_6^2} = \frac{(q-2)}{q}\sqrt{\frac{q+1}{q-1}}$$



- softening of 1st order transition by disorder exhibited exactly random Potts $(n = 0)$

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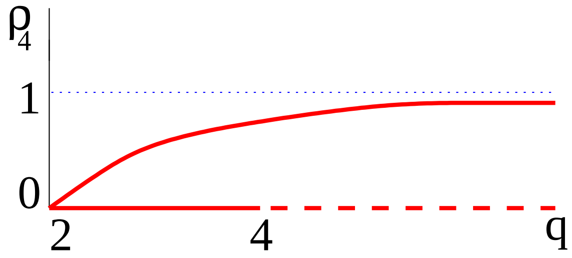
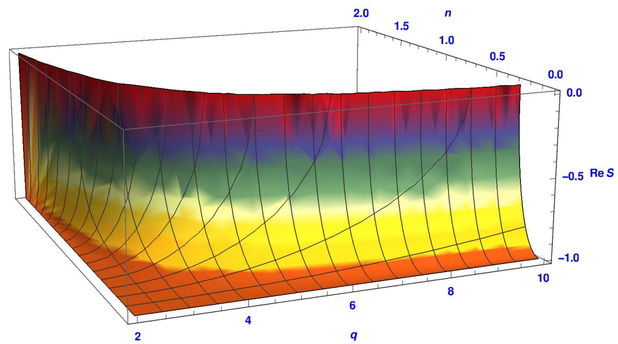


- softening of 1st order transition by disorder exhibited exactly

- color singlet sector becomes q-independent at $n = 0$:

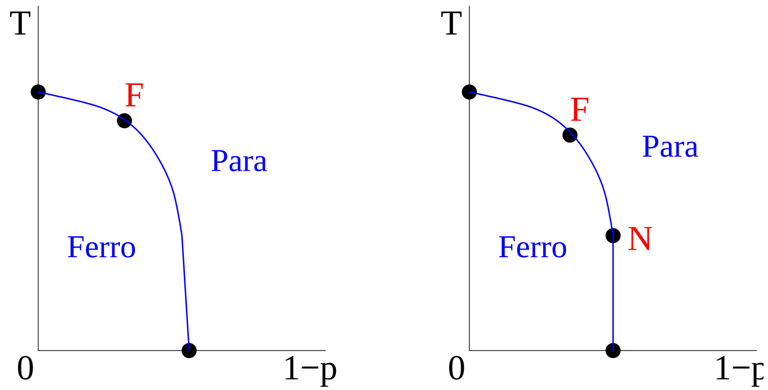
superuniversality of exponent ν
magnetic exponent η is q-dependent

consistent with numerics within their error bars



• all solutions have been determined [GD, Tartaglia '17]

• they should classify Potts critical points for any type of quenched disorder



dilute ferromagnet $\pm J$ model

$1-p =$ disorder strength

• one class has strong disorder for any q and includes:

– fully q-independent solution $\rightarrow$ zero-temperature percolation point of dilute ferromagnet

– candidate solution for the Nishimori point in $\pm J$ model

• all disordered solutions exhibit superuniversal sectors

Conclusion

- random criticality can be accessed exactly in 2D

- solutions show that quenched disorder (of any strength) falls in the realm of renormalization group and field theory

- solutions exhibit new phenomenon of symmetry-independent (superuniversal) sectors

- random critical points are described by conformal field theory of a new type allowing for superuniversality

- characterization of these conformal theories beyond scattering approach is one of the challenges ahead

- superuniversality also needs to be investigated numerically