The muon g-2: status from a theorist's point of view

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Selected Puzzles in Particle Physics Workshop
LNF - 22 December 2016

Outline

- μ: The muon g-2: recent theory progress
- e: Testing the Standard Model with the electron g-2
- τ: The tau g-2: opportunities or fantasies? Surprises?

Lepton magnetic moments: the basics

- Uhlenbeck and Goudsmit in 1925 proposed:

$$\vec{\mu} = g\frac{e}{2mc}\vec{s}$$
$$g = 2 \quad \text{(not 1!)}$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu)\gamma^\mu\psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation…

$$a\frac{e}{2m}\sigma_{\mu\nu}F^{\mu\nu}\psi \quad \to \quad g = 2(1+a)$$

…but there was no need for it! g=2 stood for ~20 yrs.

Theory of the g-2: the beginning

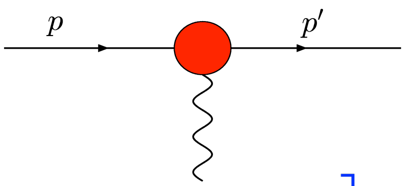
- **Kusch and Foley 1948:**

$$\mu_e^{\rm exp} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\rm th} = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**



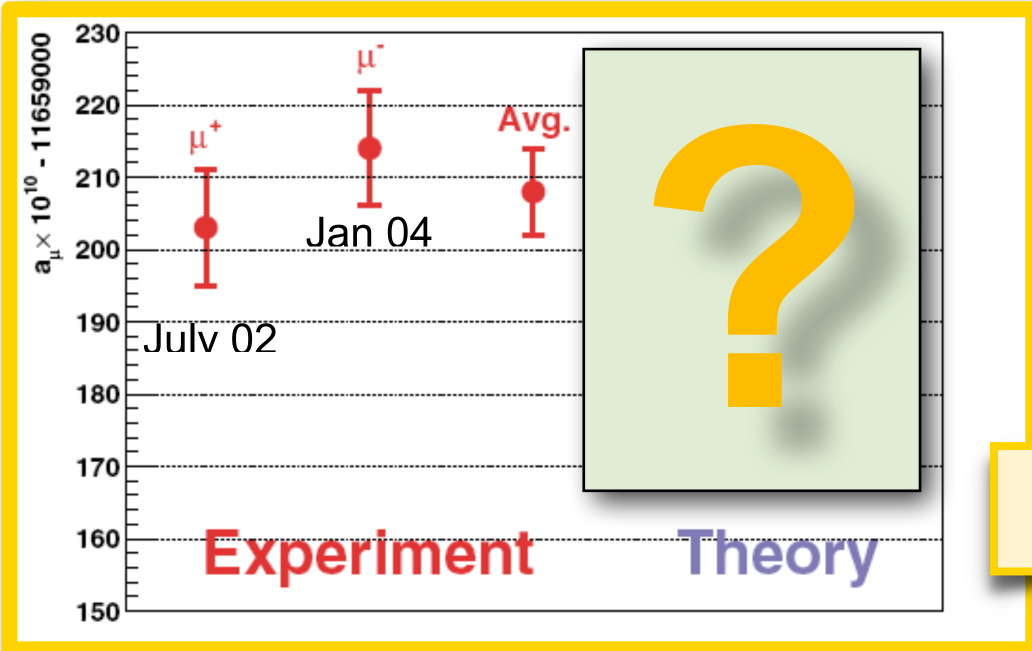
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

A pure "quantum correction" effect!

The muon g-2

The muon g-2: experimental status



See Alberto Lusiani's talk

- Today: $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- Future: new muon g-2 experiments at:
  - Fermilab E989: aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm. Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
  - J-PARC proposal: aiming at 2019 Phase 1 start with 0.4ppm.
- Are theorists ready for this (amazing) precision? Not yet

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+\ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+\ 24.05050988\ (28)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+\ 130.8773\ (61)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov[2], Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 (τ loops, analytic);
Steinhauser et al. 2015 & 2016 (all electron & τ loops, analytic).

$+\ 752.85\ (93)\ (\alpha/\pi)^5$ **COMPLETED!**

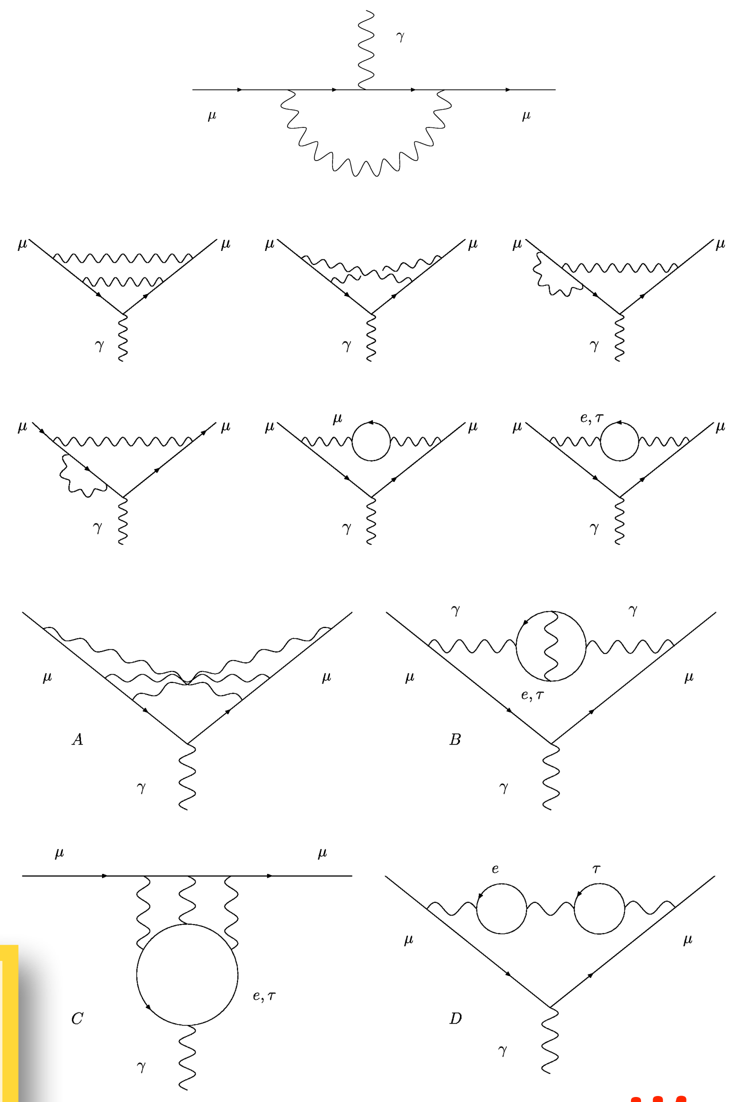
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015

Adding up, we get:

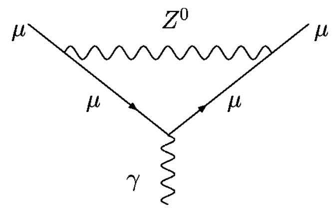
$a_\mu^{QED} = 116584718.941\ (21)(77) \times 10^{-11}$

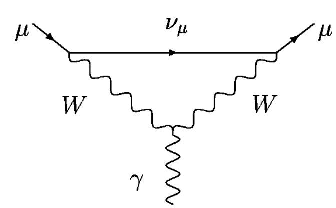
from coeffs, mainly from 4-loop unc ⮠ ⮡ from δα(Rb)

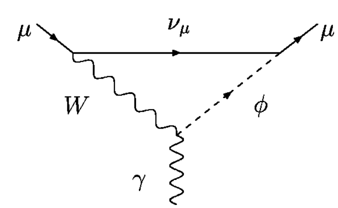
with $\alpha = 1/137.035999049(90)$ [0.66 ppb]



- **One-loop term:**







$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

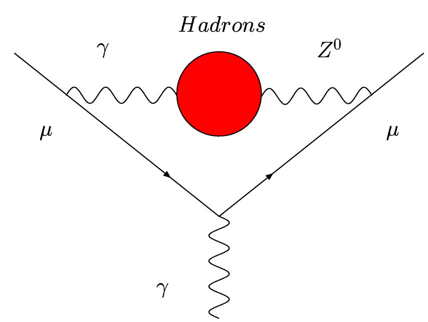
- **One-loop plus higher-order terms:**

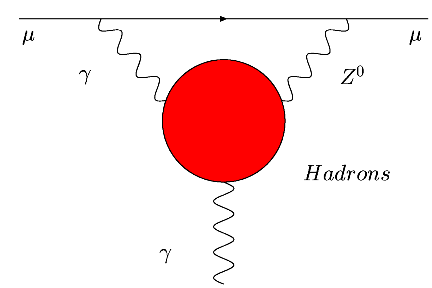
$a_\mu^{\text{EW}} = 153.6\ (1) \times 10^{-11}$

with M_{Higgs} = 125.6 (1.5) GeV

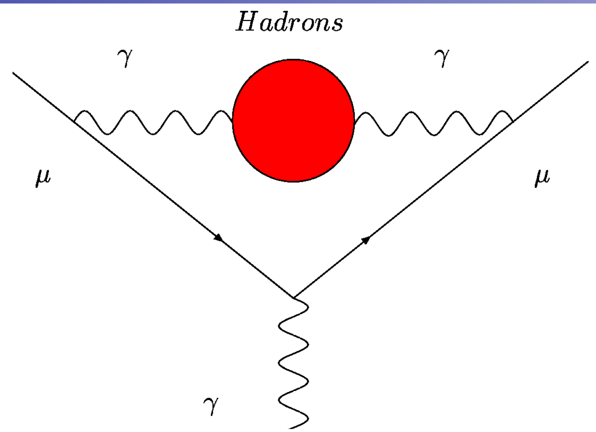
Hadronic loop uncertainties and 3-loop nonleading logs.

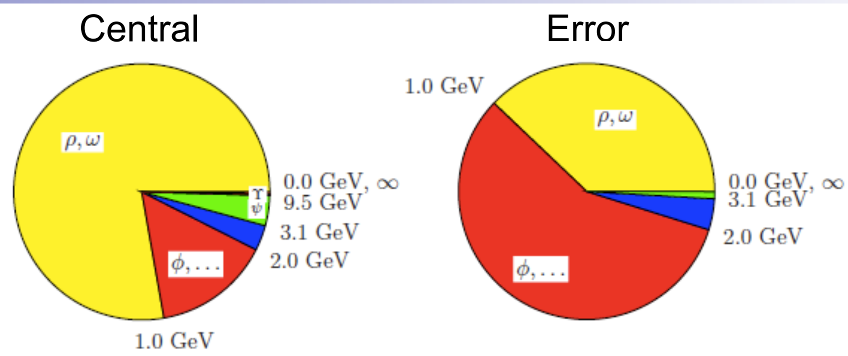
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.





The muon g-2: the hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds \, K(s) \, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$a_\mu^{\mathrm{HLO}} = 6870\,(42)_{\mathrm{tot}} \times 10^{-11}$ — F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

$= 6926\,(33)_{\mathrm{tot}} \times 10^{-11}$ — M. Davier, arXiv:1612.02743

$= 6949\,(37)_{\mathrm{exp}}\,(21)_{\mathrm{rad}} \times 10^{-11}$ — Hagiwara et al, JPG 38 (2011) 085003

- **Radiative Corrections are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Lots of progress in lattice calculations.** T. Blum et al, PRL116 (2016) 232002

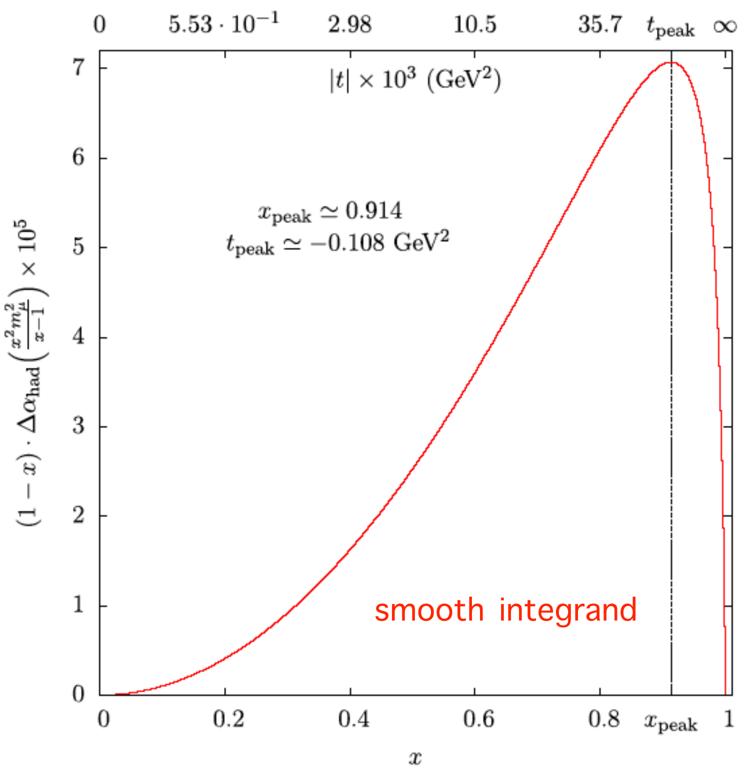
See Lusiani's talk

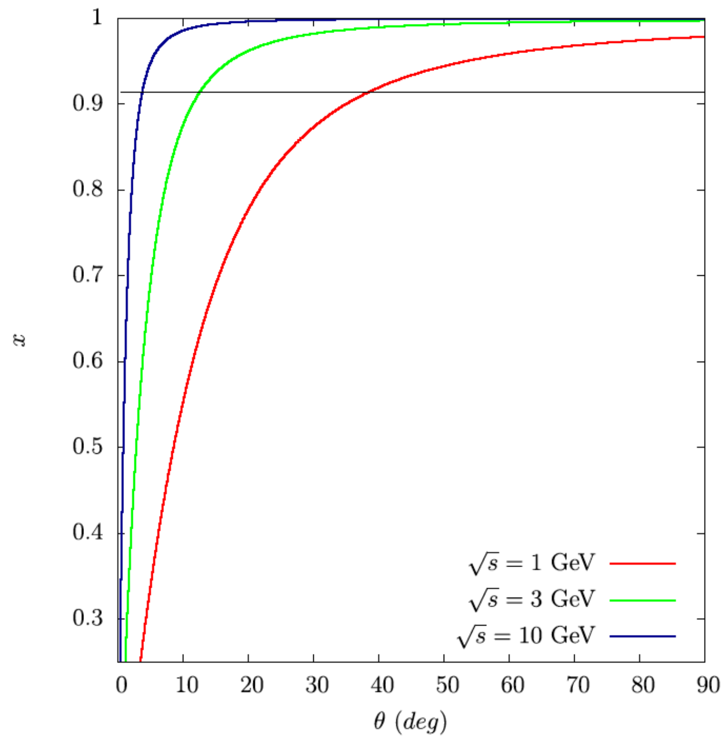
New space-like proposal for HLO

- **Alternatively, exchanging the x and s integrations in a_μ^{HLO}:**

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)] \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

which involves $\Delta\alpha_{had}(t)$, the hadr. contrib. to the running of α in the space-like region. It can be extracted from Bhabha scattering data!

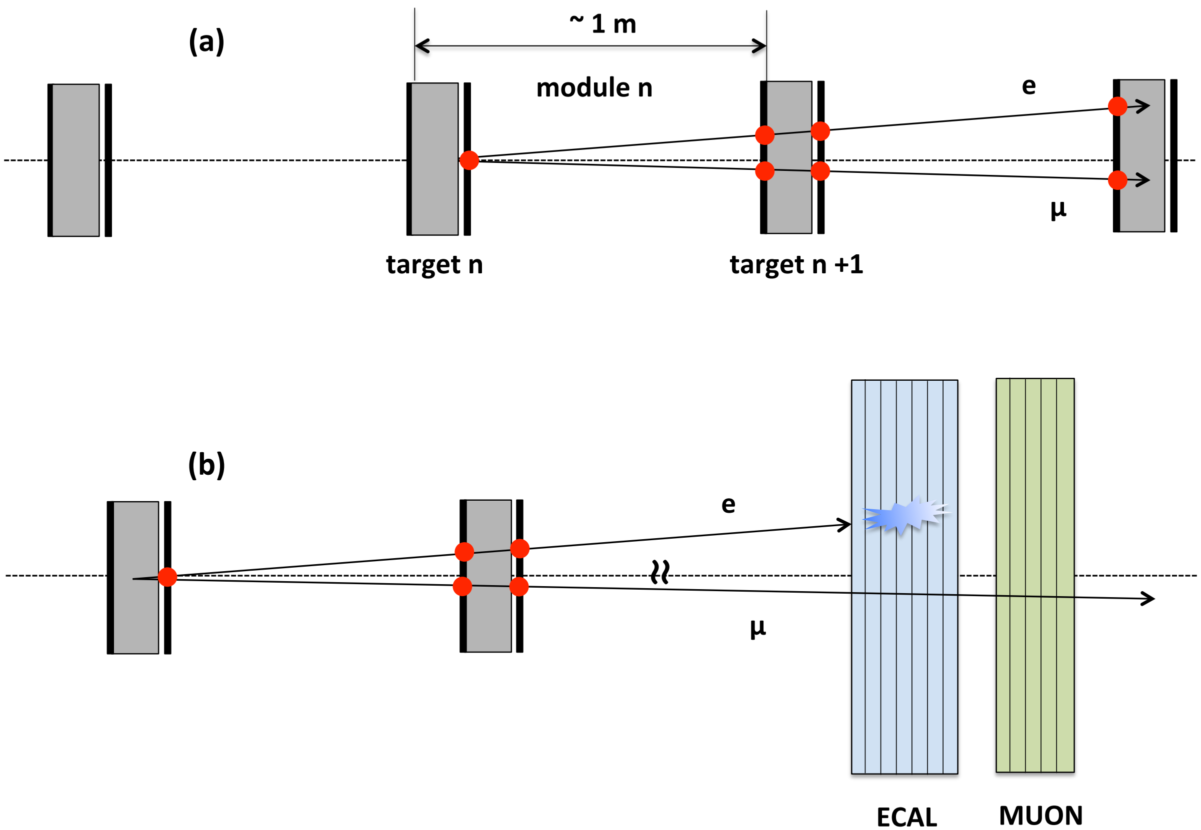




Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

New space-like proposal for HLO (2)

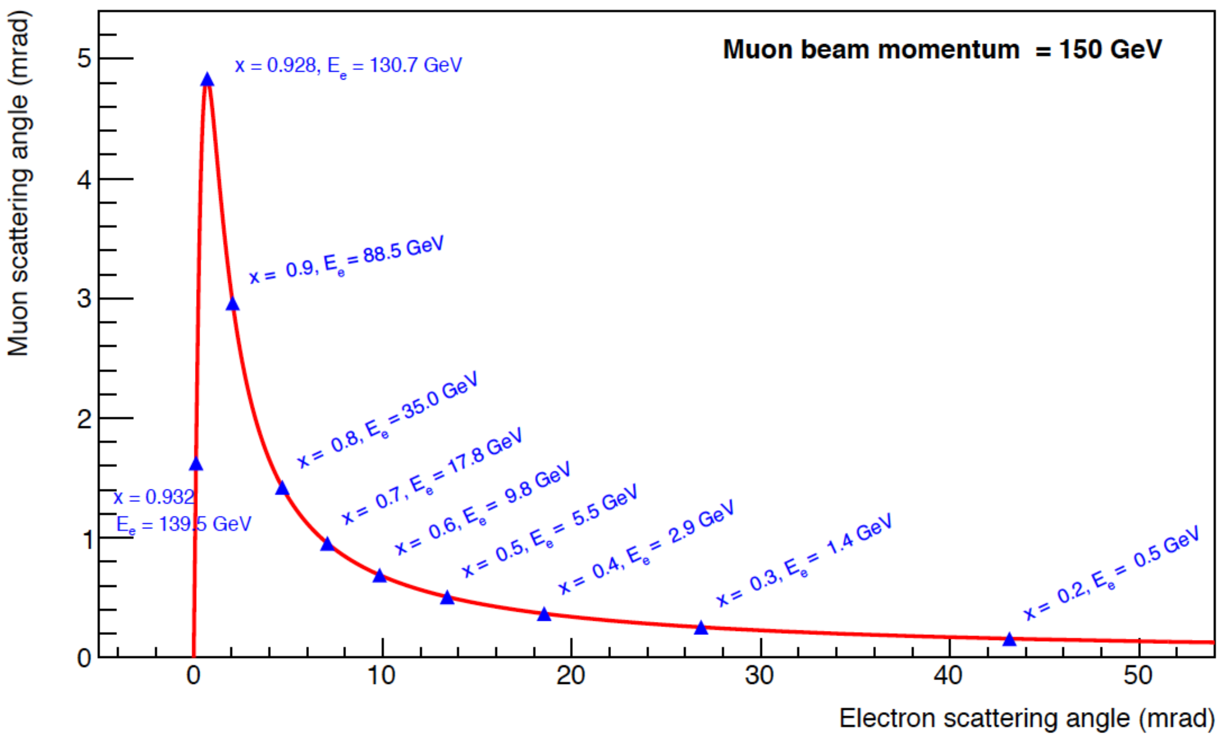
- $\Delta\alpha_{had}(t)$ can also be measured via the elastic scattering μ e → μ e.
- We propose to scatter a 150 GeV muon beam, presently available at CERN's North Area, on a fixed electron target:



G. Abbiendi et al, arXiv:1609.08987

New space-like proposal for HLO (3)

- With CERN's 150 GeV muon beam, which has an average of ~ 1.3×10^7 μ/s, incident on 20 Be layers, each 3 cm thick, and 2 years of data taking with a running time of 2×10^7 s/yr, one can reach an int. luminosity of $\mathcal{L}_{int} \sim 1.5 \times 10^7$ nb^{-1}.

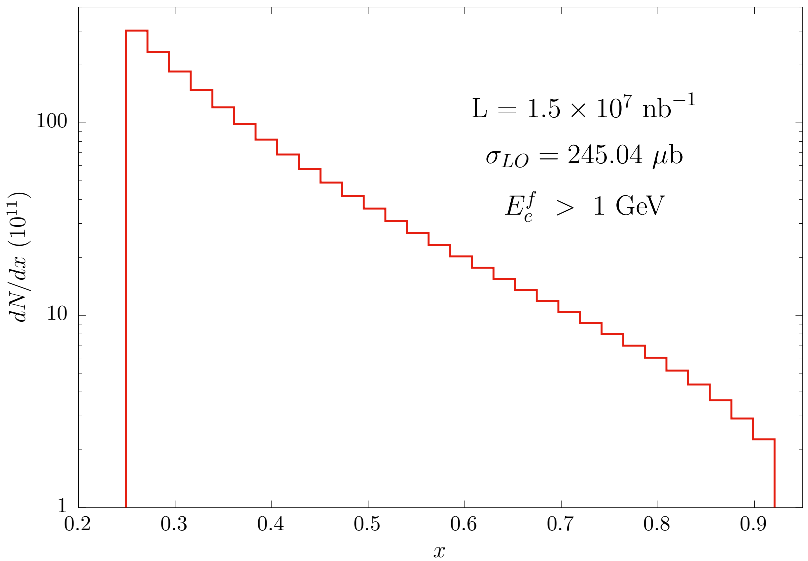


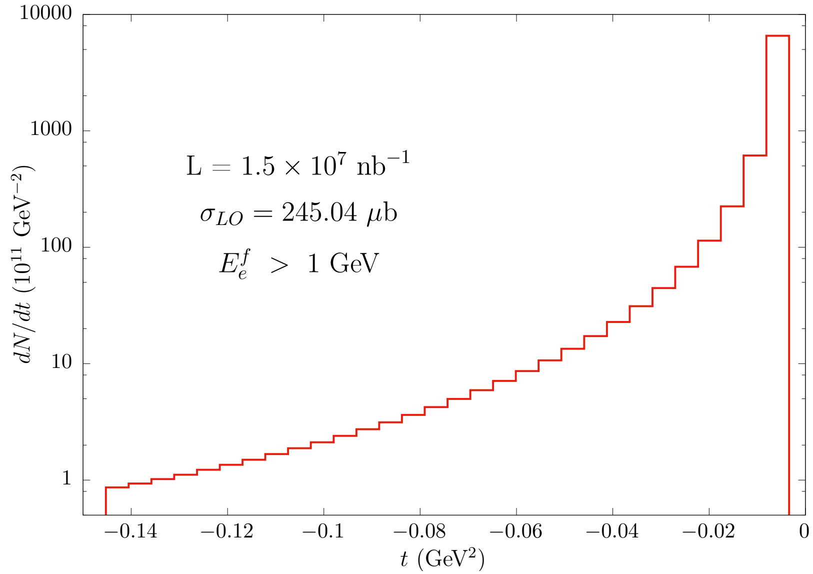
G. Abbiendi et al, arXiv:1609.08987

New space-like proposal for HLO (4)

- With $\mathcal{L}_{int} \sim 1.5 \times 10^7\ \mathrm{nb}^{-1}$ we estimate that we can reach a statistical sensitivity of ~ 0.3% on a_μ^{HLO}, ie 20×10^{-11}!

- The integrand in the small region $x \in [0.93,1]$ (the peak is at $x=0.91$), accounting for ~13% of the a_μ^{HLO} integral, cannot be reached by our experiment but can be determined using pQCD & time-like data, and/or lattice QCD results.

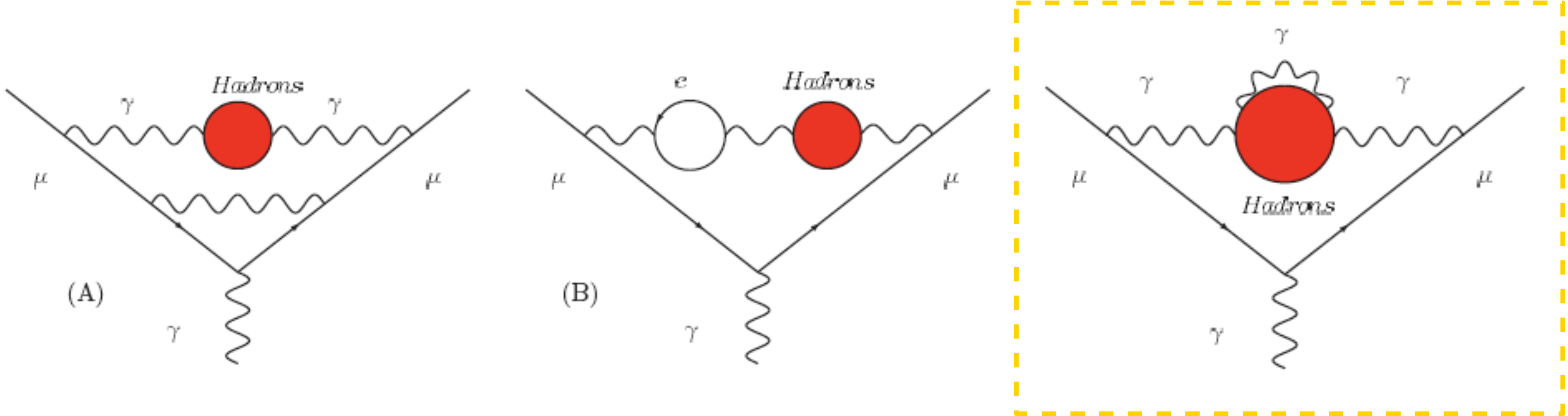




G. Abbiendi et al, arXiv:1609.08987

HNLO: Vacuum Polarization

Already included in a_μ^{HLO}

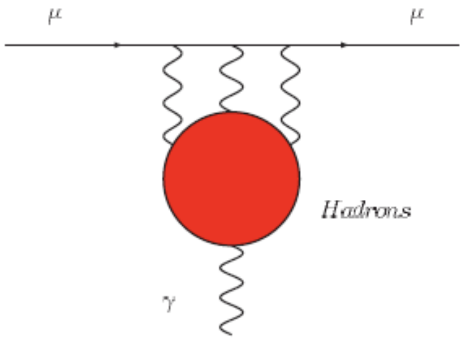


O(α³) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



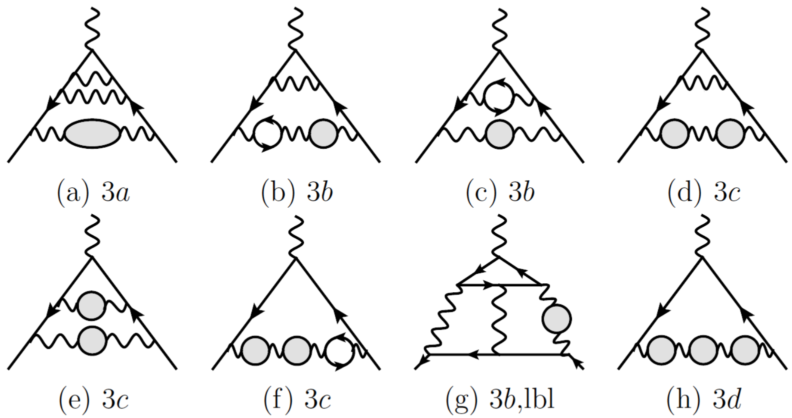
- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HNLO}(lbl) = +102\ (39) \times 10^{-11}$	Jegerlehner, arXiv:1511.04473

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- Improvements expected in the π^0 transition form factor A. Nyffeler 1602.03398
- Dispersive approach proposed Colangelo et al, 2014 & 2015, Pauk & Vanderhaeghen 2014.
- Progress on the lattice: $+53.5(13.5)\times10^{-11}$. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.
  Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, Lehner, arXiv:1610.04603

HNNLO: Vacuum Polarization



O(α^4) contributions of diagrams containing hadronic vacuum polarization insertions:

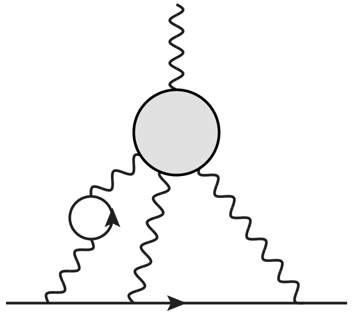
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

HNNLO: Light-by-light

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



Comparisons of the SM predictions with the measured g-2 value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda=\mu_\mu/\mu_p$ from CODATA'10

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$	σ
$116\,591\,761\ (57)$	$330\ (85) \times 10^{-11}$	3.9 [1]
$116\,591\,818\ (51)$	$273\ (81) \times 10^{-11}$	3.4 [2]
$116\,591\,841\ (58)$	$250\ (86) \times 10^{-11}$	2.9 [3]

with the recent "conservative" hadronic light-by-light $a_\mu^{HNLO}(lbl) = 102\ (39) \times 10^{-11}$ of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

[1] Jegerlehner, arXiv:1511.04473.
[2] Davier, arXiv:1612:02743.
[3] Hagiwara et al, JPG38 (2011) 085003.

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

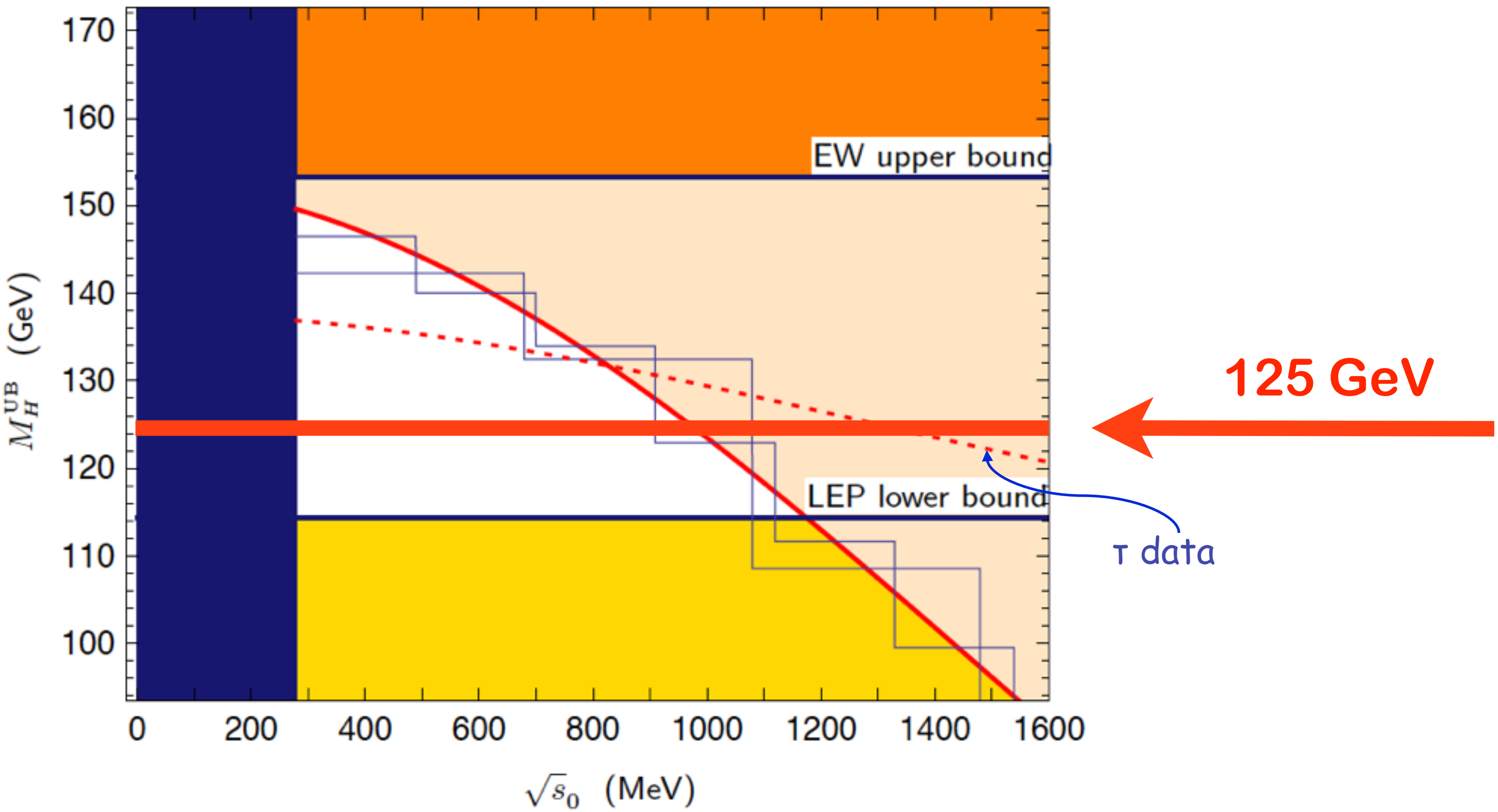
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (2)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

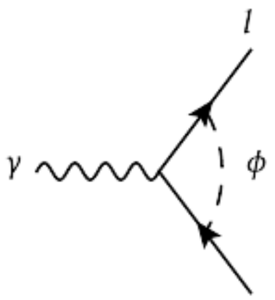
- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

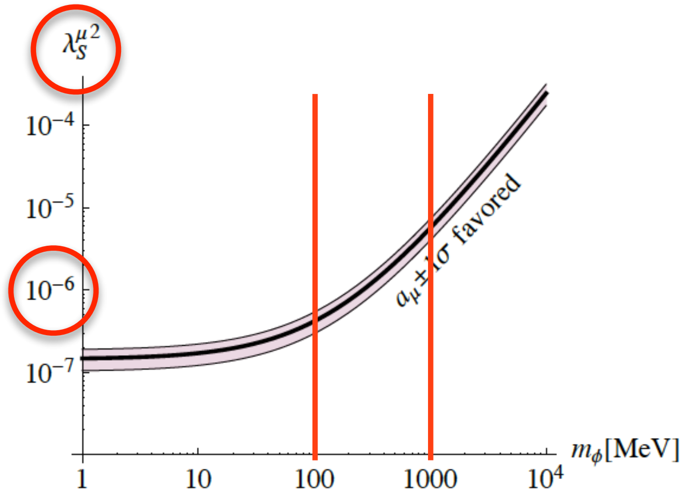
- Vice versa, assuming we now have a SM Higgs with M_H = 125 GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

- Light spin 0 scalars & pseudoscalars (axion-like-particles or ALPs), contribute to a_μ. We consider ALPs in the mass range ~ [0.1–1] GeV, where experimental constraints are rather loose.

- A possible resolution of Δa_μ by 1-loop contributions from scalar particles with relatively large Yukawa couplings to muons, of $O(10^{-3})$, was analyzed by Chen, Davoudiasl, Marciano & Zhang, PRD 93, 035006 (2016):





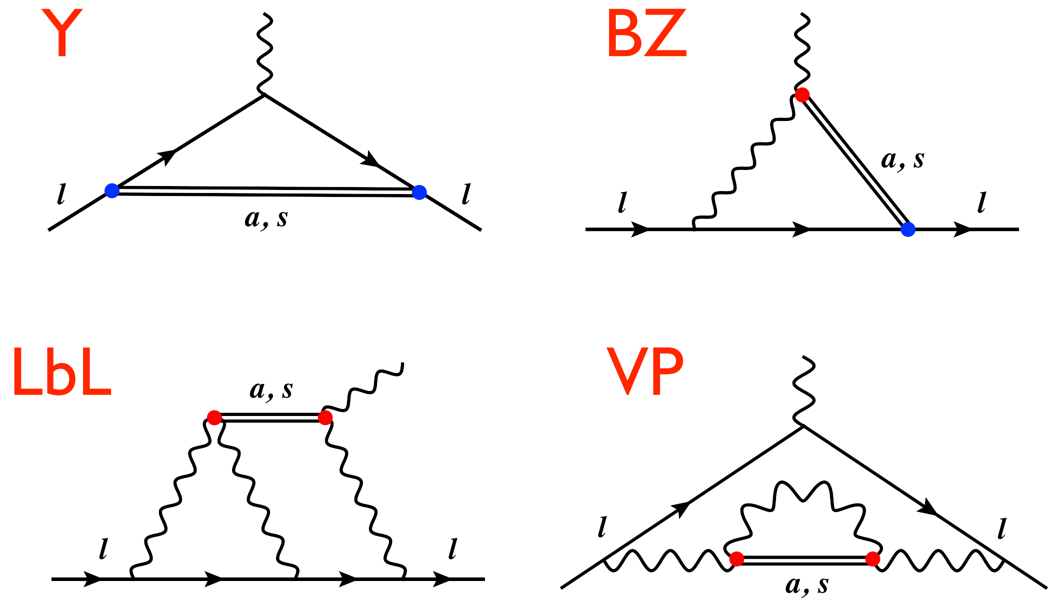
- For a pseudoscalar, the 1-loop contribution has the wrong sign (negative) to resolve the discrepancy on its own.

- Consider ALP-γγ couplings as well as Yukawa couplings:

$$\mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} + i\, y_{a\psi}\, a\, \bar{\psi}\gamma_5\psi\,,$$

$$\mathcal{L}_s = \frac{1}{4} g_{s\gamma\gamma}\, s\, F_{\mu\nu}F^{\mu\nu} + y_{s\psi}\, s\, \bar{\psi}\psi$$

- New, potentially important, ALP contributions to a_μ:



Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Pseudoscalar

leading log-enhanced terms

Y $$a_{\ell,a}^{\rm Y} < 0$$

BZ $$a_{\ell,a}^{\rm BZ} \simeq \left(\frac{m_\ell}{4\pi^2}\right) g_{a\gamma\gamma}\, y_{a\ell}\, \ln\frac{\Lambda}{m_a}$$

LbL $$a_{\ell,a}^{\rm LbL} \simeq 3\,\frac{\alpha}{\pi}\left(\frac{m_\ell\, g_{a\gamma\gamma}}{4\pi}\right)^2 \ln^2\frac{\Lambda}{m_a} > 0$$

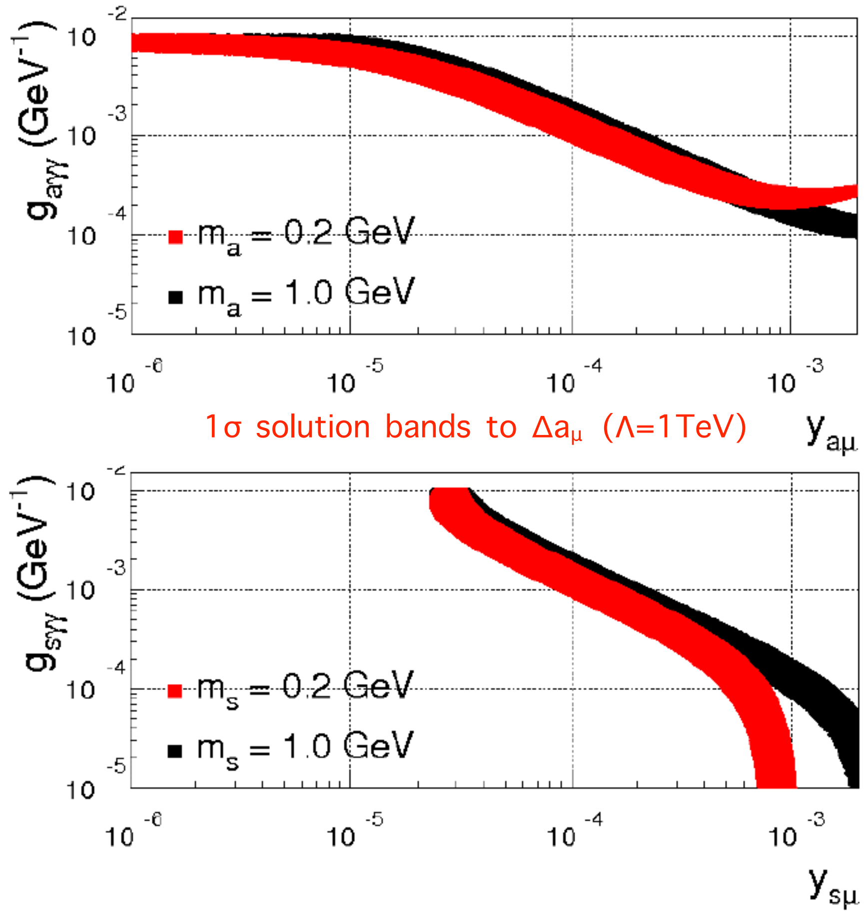
VP $$a_{\ell,a}^{\rm VP} \simeq \frac{\alpha}{\pi}\left(\frac{m_\ell\, g_{a\gamma\gamma}}{12\pi}\right)^2 \ln\frac{\Lambda}{m_a} > 0$$

- For a scalar ALP, change the signs of Y & LbL.
- The sign of BZ depends on the couplings. We assume it's > 0.
- VP is positive both for scalar & pseudoscalar, but negligible.

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Pseudoscalar

Scalar



- **Both pseudoscalar and scalar ALPs can solve Δa_μ for masses and couplings allowed by current exp. constraints.**
- **They can be tested at present low-energy e^+e^- experiments, via dedicated $e^+e^- \to e^+e^-$+ALP & $e^+e^- \to \gamma$+ALP searches.**

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Testing the SM with the electron g-2

The QED prediction of the electron g-2

$$a_e^{QED} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$$

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$$

$O(10^{-18})$ in a_e

$$+ 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$$

$O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$$

$$- 1.91206\,(84)\,(\alpha/\pi)^4$$

$0.2\ 10^{-13}$ in a_e

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012 & 2015;
Kurz, Liu, Marquard & Steinhauser 2014: analytic heavy virtual lepton part.

$$+ 7.79\,(34)\,(\alpha/\pi)^5$$

Complete Result! (12672 mass indep. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; PRD 91 (2015) 3, 033006

$0.2\ 10^{-13}$ in a_e

NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

No additional contribution of QED bound states beyond PT

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 17.10\,(17) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.66\,(11) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.234(14)_{vac} + 0.39(13)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- Which value of α should we use to compute a_e^{SM}?

The electron g-2 gives the best determination of alpha

e

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → best determination of alpha (2015):

  $\alpha^{-1} = 137.035\,999\,157\ (33)$ [0.24 ppb]

- Compare it with other determinations (independent of a_e):

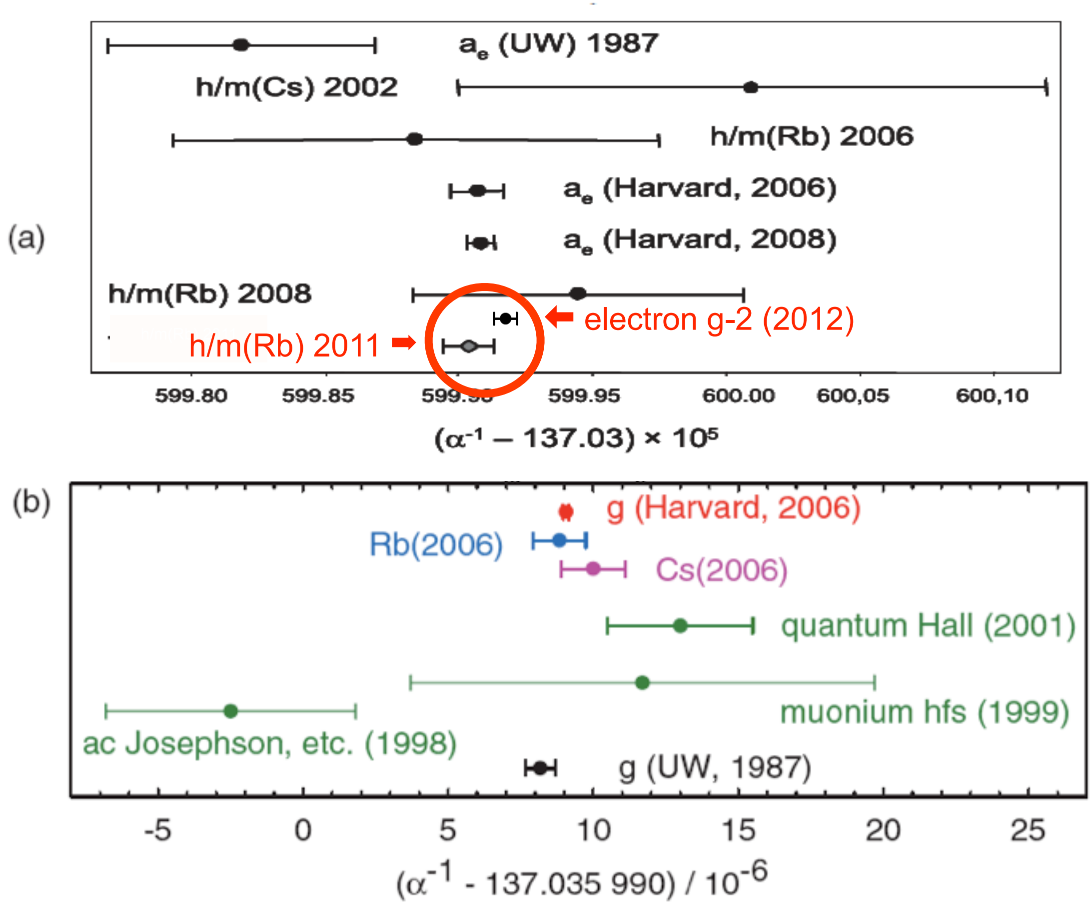
  $\alpha^{-1} = 137.036\,000\,0\ (11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

  $\alpha^{-1} = 137.035\,999\,049\ (90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

e

Old and recent determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
Bouchendira et al, PRL106 (2011) 080801

The electron g-2: SM vs Experiment

e

- Using $\alpha = 1/137.035\,999\,049\ (90)$ [^{87}Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,16.5\ (0.2)\ (0.2)\ (0.2)\ (7.6) \times 10^{-13}$$

δC_4^{qed} δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -9.2\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1σ).
NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}}_{(0.4)_{\text{TH}}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

← may drop to 0.2

- The $(g-2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to a_l scale as

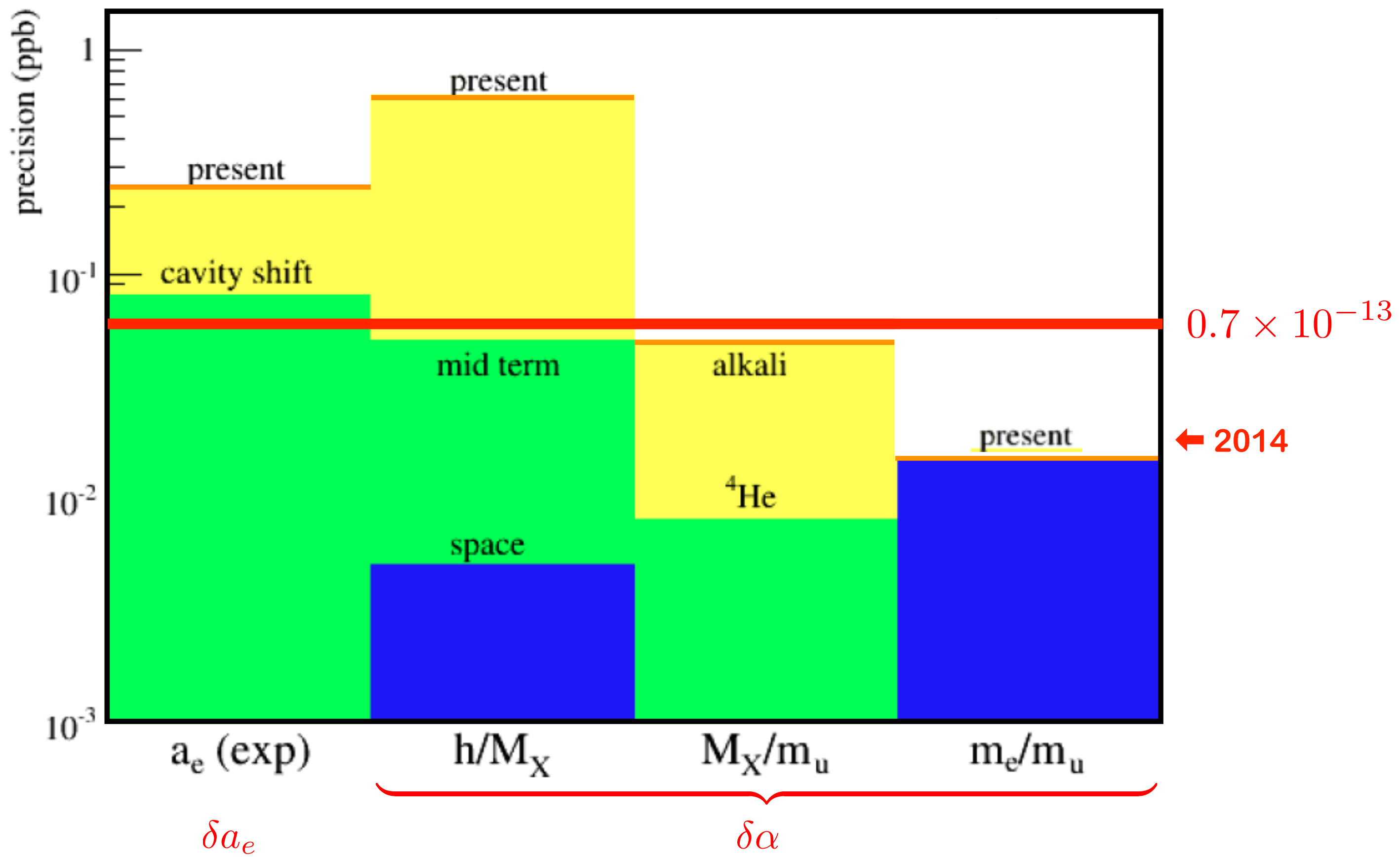
$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)

e



Summary of the exp. contributions to the relative uncertainty of $\delta\alpha$ and δa_e (in ppb).

F. Terranova & G.M. Tino, PRA89 (2014) 052118

The electron g-2 sensitivity and NP tests (3)

- The experimental sensitivity in Δa_e is not very far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi & MP, arXiv:1208.6583

The tau g-2: opportunities or fantasies?

The SM prediction of the tau g-2

τ

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM} =	117324 (2)	x 10^{-8}	QED
+	47.4 (0.5)	x 10^{-8}	EW
+	337.5 (3.7)	x 10^{-8}	HLO
+	7.6 (0.2)	x 10^{-8}	HHO (vac)
+	5 (3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

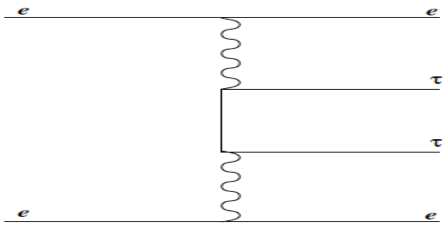
	Muon	Tau
aEW/aH	1/45	1/7
aEW/ δaH	3	10

… if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\,(17)$$

PDG 2014

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

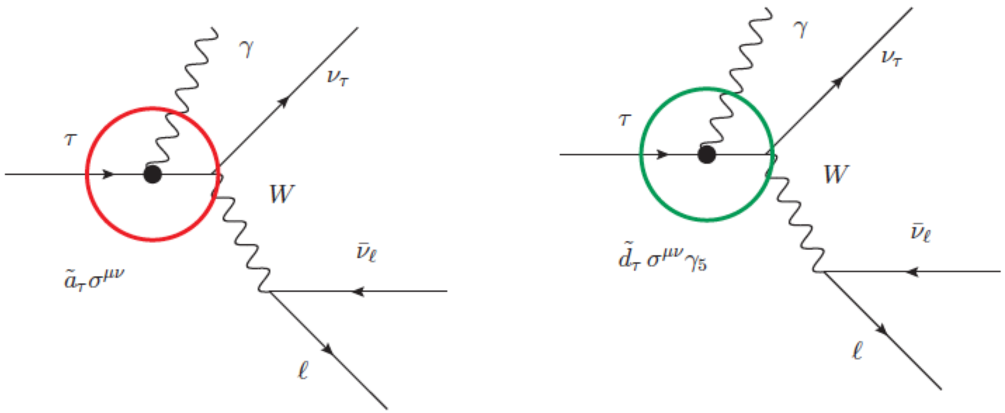
$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ \text{CL})$$

Gonzáles-Sprinberg et al 2000

- Bernabéu et al, proposed the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

- a_τ via the radiative leptonic decays $\tau \to e\bar{\nu}\nu\gamma,\ \tau \to \mu\bar{\nu}\nu\gamma$ comparing the theoretical prediction for the differential decay rates with precise data from high-luminosity B factories:

$$d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\mathrm{NLO}} + \tilde{a}_\tau\, d\Gamma_a + \tilde{d}_\tau\, d\Gamma_d$$



- **Detailed feasibility study performed in Belle-II conditions: we expect a (modest) improvement of the present PDG bound.**

Eidelman, Epifanov, Fael, Mercolli, MP, arXiv:1601.07987 (JHEP 2016)

Radiative leptonic tau decays: branching ratios

τ

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO corr.

$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

$(\rm th)$: combined $(n) \oplus (N)$

(τ): experimental error of τ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

† BABAR - PRD 91 (2015) 051103

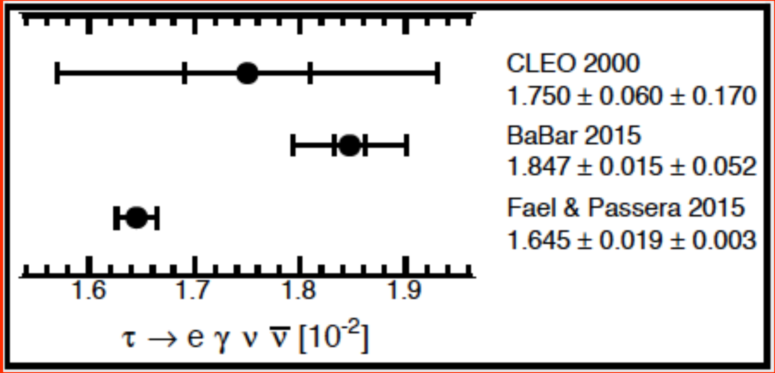
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\Delta^{\rm Exc}$	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

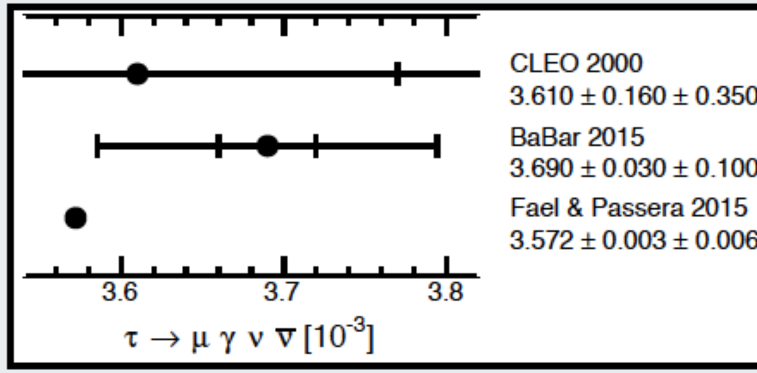
- **Agreement with MEG's recent μ→eννγ measurement [EPJ C76 (2016) 3, 108]**

Fael, Mercolli and MP, 1506.03416 (JHEP 2015)
Fael and MP, 1602.00457

Radiative leptonic tau decays: branching ratios (2)

Tau radiative leptonic decays ($E_\gamma > 10\,\text{MeV}$)





- (see also M.Passera presentation in this workshop)
- CLEO 2000: T. Bergfeld et al., PRL 84 (2000) 830
- *BABAR* 2015: PRD 91, 051103 (2015)
- Fael & Passera 2015: NLO calculation, JHEP 07 (2015) 153, arXiv:1602.00457 [hep-ph]
- 3.5σ discrepancy between *BABAR* 2015 and NLO calculation, to be investigated

Conclusions

- **Muon g-2: $\Delta a_\mu \sim 3.5\ \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadronic sector, but not yet ready!**

  New proposal to measure the leading hadronic contribution to the muon g-2 via μ-e elastic scattering at CERN.

  Δa_μ due to mistakes in the hadronic $\sigma(s)$? Very unlikely!

  Light spin 0 scalars & pseudoscalars can solve Δa_μ for masses and couplings allowed by current experimental bounds. Dedicated searches can test them at present low-energy e^+e^- colliders.

- **Electron g-2: Does the discrepancy in $(g\text{-}2)_\mu$ also manifests in $(g\text{-}2)_e$? NP sensitivity limited by exp. uncertainties, but a strong exp. program is under way to improve both α & a_e.**

- **Tau g-2: unknown. New proposal to measure it via radiative leptonic tau decays. Modest improvement expected. BaBar's precise measurement of $\mathcal{B}(\tau \to e\bar{\nu}\nu\gamma)$ differs from SM by 3.5 σ!**

The End