Proton radius puzzle

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Measurements of atomic spectra

Measurement of transition frequencies can be very accurate [Garching, 2013]

- $\nu(1S - 2S)_{\mathrm{H}} = 2466\,061\,413\,187\,035(10)\ \mathrm{Hz}$
- sensitive to the nuclear size and the nuclear polarizability
- from $\nu(1S - 2S)_{H-D}$: $r_d^2 - r_P^2 = 3.820\,07(65)\ \mathrm{fm}^2$
- determination of fundamental constants

Another example: electron mass from the g-factor measurement in hydrogen-like C, [Sturm, et al, Nature 2014]

- $m_e = 0.000\,548\,579\,909\,067(14)(9)(2)$ au

Proton charge radius puzzle

- global fit to H and D spectrum: $r_p = 0.8758(77)$ fm (CODATA 2010)
- $e - p$ scattering: $r_p = 0.8791(79)$ (Bernauer, 2010)
- from muonic hydrogen: $r_p = 0.84089(39)$ fm (PSI, 2010, 2012)

If all these measurements and Lamb shift calculations are correct, this discrepancy does not find explanation within the known description of electroweak and strong interactions

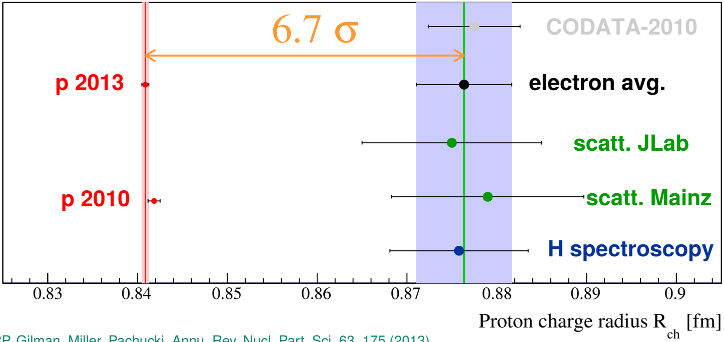
Potential to discover new physics . . .

The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm



RP, Gilman, Miller, Pachucki, Annu. Rev. Nucl. Part. Sci. 63, 175 (2013).

Proton charge radius puzzle

- $\delta_{\rm fs} E = \frac{2\,\pi\,\alpha}{3}\,\phi^2(0)\,\langle r_{\rm ch}^2\rangle$
  - this formula is universal for all light atoms
  - the energy shift is proportional to the mean square charge radius $r_{\rm ch}^2$
- two-photon exchange $O(Z\,\alpha\,r_{\rm ch}/\lambda\!\!\!^{-})$, pretty small
- nuclear polarizability effects are in general quite small, significant only for muonic atoms
- how come r_p from (electronic) H differs by 4% from that of μH ?

Proton charge radius puzzle

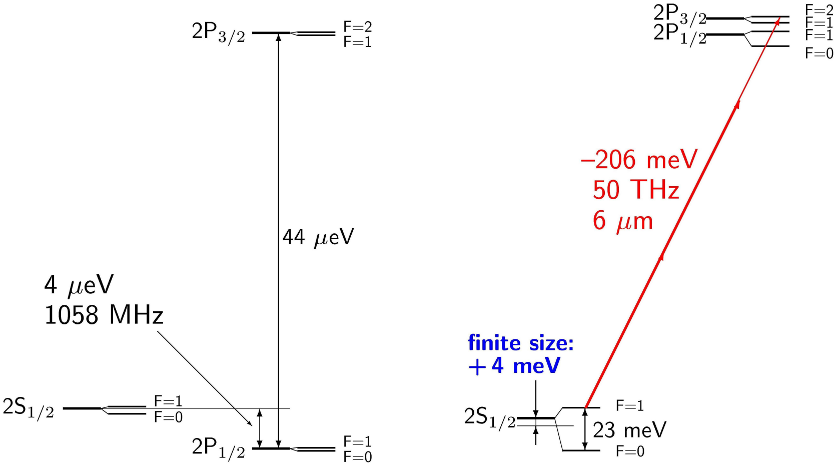
The only solution which does not violate SM is the assumption that the hydrogen spectroscopy and e-p scattering measurements, although in agreements, are both incorrect

How it can be verified ?

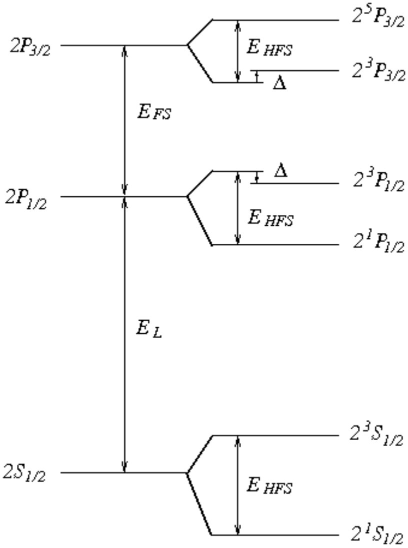
- muon-proton scattering: MUSE project (PSI)
- low Q^2 e-proton scattering: PRad (Jefferson lab)
- μHe: CREMA collaboration ($r^2_{\rm He}$ from e-α scattering)
- $r^2_{\rm He}$ from spectroscopy of He: $2^3S - 2^3P$ (Warsaw)
- H-spectroscopy: 2S-4P (Garching), 2S-3S (Paris)
- $He^+(1S - 2S)$ (Amsterdam, Garching)

Let us say few words about μH theory, why is it so reliable.

energy levels of μH in comparison to H



energy levels of μH



$E_L = 202.1\ meV$

$E_{FS} = 8.4\ meV$

$E_{HFS}(2S_{1/2}) = 22.7\ meV$

$E_{HFS}(2P_{1/2}) = 8.0\ meV$

$E_{HFS}(2P_{3/2}) = 3.4\ meV$

$\Delta = 0.1\ meV$

μH energy levels

- μH is essentially a nonrelativistic atomic system
- muon and proton are treated on the same footing
- $m_\mu / m_e = 206.768 \Rightarrow \beta = m_e/(\mu\,\alpha) = 0.737$
  the ratio of the Bohr radius to the electron Compton wavelength
- the electron vacuum polarization dominates the Lamb shift in muonic hydrogen

Theory of μH energy levels

- nonrelativistic Hamiltonian $H_0 = \frac{p^2}{2\,m_\mu} + \frac{p^2}{2\,m_p} - \frac{\alpha}{r}$
- and the nonrelativistic energy $E_0 = -\frac{m_r\,\alpha^2}{2\,n^2}$
- the evp dominates the Lamb shift

$$\begin{aligned} E_L = \langle 2P|V_{vp}(r)|2P\rangle - \langle 2S|V_{vp}(r)|2S\rangle &= 205.007\,3\,\mathrm{meV} \\ \text{complete result but without finite size} &= 206.033\,6(5)\,\mathrm{meV} \end{aligned}$$

- important corrections: second order, two-loop vacuum polarization, and the muon self-energy
- other corrections are much smaller than the discrepancy of 0.3 meV, while finite nuclear size is -3.9 meV.

Leading relativistic correction

Breit-Pauli Hamiltonian

$$
\begin{aligned}
H_{BP} \quad = \quad & -\frac{p^4}{8\,m_\mu^3} - \frac{p^4}{8\,m_p^3} - \frac{\alpha}{2\,m_\mu\,m_p}\,p^i\left(\frac{\delta^{ij}}{r} + \frac{r^i\,r^j}{r^3}\right)p^j \\
& + \frac{2\,\pi\,\alpha}{3}\left(\langle r_p^2\rangle + \frac{3}{4\,m_\mu^2} + \frac{3}{4\,m_p^2}\right)\delta^3(r) \\
& + \frac{2\,\pi\,\alpha}{3\,m_\mu\,m_p}\,g_\mu\,g_p\,\vec{s}_\mu\cdot\vec{s}_p\,\delta^3(r) - \frac{\alpha}{4\,m_\mu\,m_p}\,g_\mu\,g_p\,\frac{s_\mu^i\,s_p^j}{r^3}\left(\delta^{ij} - 3\,\frac{r^i\,r^j}{r^2}\right) \\
& + \frac{\alpha}{2\,r^3}\,\vec{r}\times\vec{p}\left[\vec{s}_\mu\left(\frac{g_\mu}{m_\mu\,m_p} + \frac{(g_\mu-1)}{m_\mu^2}\right) + \vec{s}_p\left(\frac{g_p}{m_\mu\,m_p} + \frac{(g_p-1)}{m_p^2}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\delta_{\rm rel}E_L \quad = \quad & \langle 2P_{1/2}|H_{BP}|2P_{1/2}\rangle - \langle 2S_{1/2}|H_{BP}|2S_{1/2}\rangle \\
= \quad & \frac{\alpha^4\,m_r^3}{48\,m_p^2} = 0.05747\ \mathrm{meV}
\end{aligned}
$$

- valid for an arbitrary mass ratio
- quite small and higher order relativistic corrections are negligible

Important corrections

- second order $V_{\rm vp}$: $\delta E_L = 0.1509$ meV
- two-loop vp: $\delta E_L = 1.5081$ meV
- three-loop vp: $\delta E_L = 0.0053$ meV
- hadronic vp: $\delta E_L = 0.0112(4)$ meV
- muon self-energy and muon vp: $\delta E_L = -0.6677$ meV

Small corrections

- relativistic correction to vp

$$\delta_{\mathrm{vp,rel}} E_L = \langle \delta_{\mathrm{vp}} H_{BP} \rangle + 2 \left\langle V_{\mathrm{vp}} \frac{1}{(E-H)'} H_{\mathrm{BP}} \right\rangle = 0.01876\,\mathrm{meV}.$$

  If one used the Dirac equation in the infinite nuclear mass limit, the obtained result would be 0.021 meV

- muon self-energy combined with evp: $\delta E_L = -0.0025$ meV
- light by light diagrams $\delta E_L = -0.0009$ meV
- proton (electromagnetic) self-energy

Proton self-energy

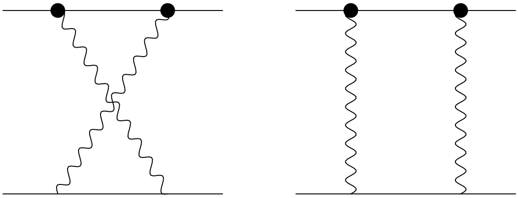
- The proton self energy leads to the modification of elastic form factors in such a way that they depend on a fictitious photon mass
- one takes the simplest possible point of view and use the formula for the low energy part of the proton self-energy

$$\delta E = \frac{4\, m_r^3\, (Z^2\, \alpha)\, (Z\, \alpha)^4}{3\, \pi\, n^3\, m_p^2} \left(\delta_{l0}\, \ln\left(\frac{m_p}{m_r\, (Z\, \alpha)^2} \right) - \ln k_0(n, l) \right) .$$
$$= -0.0099\, \mathrm{meV} .$$

  the high energy part of the Lamb shift is by definition included in the charge radius and the magnetic moment anomaly
- how this definition corresponds to r_p from the electron scattering ?

Nuclear structure effects

- if nuclear excitation energy is much larger than the atomic energy, the two-photon exchange scattering amplitude gives the dominating correction
- the total proton structure contribution $\delta E_L = 0.035\,1(20)$ meV is much too small to explain the discrepancy, but its calculation is uncertain [Carlson, Vanderhaeghen, 2011; Pascalutsa *et al*, 2013]



Summary of theoretical predictions

$$
\begin{aligned}
\Delta E_{\mathrm{LS}} &= 206.0336(15) - 5.2275(10)\, r_p^2 + \Delta E_{\mathrm{TPE}} \\
\Delta E_{\mathrm{FS}} &= 8.3521\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2S_{1/2}} &= 22.8089(51)\,\mathrm{meV}, \quad \text{(exp. value)} \\
\Delta E_{\mathrm{HFS}}^{2P_{1/2}} &= 7.9644\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2P_{3/2}} &= 3.3926\,\mathrm{meV} \\
\Delta &= 0.1446\,\mathrm{meV} \\
\Delta E_{\mathrm{TPE}} &= 0.0351(20)\,\mathrm{meV}
\end{aligned}
$$

Does $e - p$ scatt. and μH measure the same r_p ?

- $G_E(-\vec{Q}^2) = 1 - \frac{\langle r^2 \rangle}{6}\,\vec{Q}^2 + O(Q^4)$
- Low energy Hamiltonian with EM field

$$\delta H = e\,A^0 - e\left(\frac{\langle R^2 \rangle}{6} + \frac{\delta_I}{M^2}\right)\vec{\nabla}\cdot\vec{E} - \frac{e}{2}\,Q\,(I^i\,I^j)^{(2)}\,\nabla^j E^i - \vec{\mu}\cdot\vec{B}$$

- for a scalar particle $\delta_0 = 0$
- for a half-spin particle $\delta_{1/2} = 1/8$
- difference appears at the level of proton self-energy corrections

Possible sources of r_p puzzle: theory

- mistake in $e - H$ calculations: all corrections calculated independently by at least two groups, uncertainty in the two-loop correction enters at 1 kHz level for 1S state, but this discrepancy corresponds to 100 kHz
- mistake in $\mu - H$: QED theory is quite simple, dominated by nonrelativistic vacuum polarization, everything checked and verified
- large Zemach moment $(r_p^{(2)})^3$ ruled out by the low energy electron-proton scattering [Friar, Sick, 2005], [Cloët, Miller, 2010], [Distler, Bernauer, Walcher, 2010]

Possible sources of r_p puzzle: theory

- underestimation of proton structure correction ? many doubts in the literature, but all different calculations lead to similar value, Estimated value is 10 times smaller than the dicrepancy
- possible new light particles ? ruled out by muon $g-2$ and other low energy Standard Model tests: Barger *et al.*, Phys. Rev. Lett. **106**, 153001 (2011), **108**, 081802 (2012)
- violation of the universality in the lepton-proton interaction of different origin

New interactions

If discrepancy in r_p is to be explained by a new type of interaction between the proton (neutron) and leptons, than we have two options

- long range $\sim \lambda\!\!\!^{-}_e$,
- short range $\sim$ 1fm (or shorter), can be seen in μp scatt.

Comparison of nuclear charge radii for H,D,^{3}He and ^{4}He will give hints on the range of new interactions

If it is local, than discrepancy for all these elements can be parametrized by

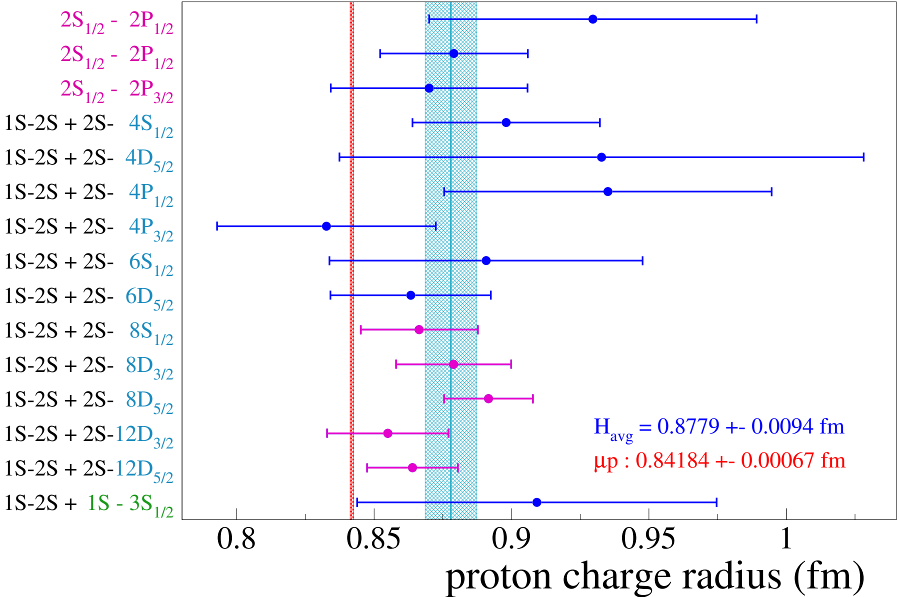
$$\delta E = (Z\,\delta r_p^2 + (A - Z)\,\delta r_n^2)\,\frac{2\,\delta_{l0}}{3\,n^3}\,Z^3\,\alpha^4\,\mu^3$$

Determination of r_N from muonic atoms spectra requires an accurate calculation of the nuclear polarizability correction, not necessarily easy task

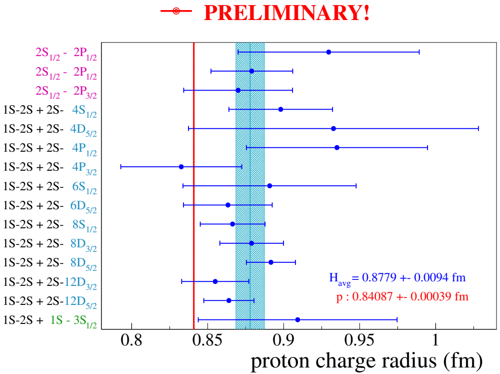
Possible sources of the proton radius discrepancy: experiment

- the determination of r_p from $e - p$ scattering data requires extrapolation to $q^2 = 0$, subject of systematic uncertainties and model dependence, there is an intensive discussion in the literature with contradicting results
  - Horbatsch, *at al.*, arXiv: 1610.09760
  - Bernauer *et al*, arXiv: 1606.02159
  - Arrington, arXiv: 1506.00873
  - Arrington, Sick, arXiv: 1505.02680
  - Kraus *et al.*, arXiv: 1405.4735
  - Griffioen, *et al*, Phys. Rev. C **93**, 065207 (2016)
  - Lorenz, *et al*, arXiv: 1205.6628
- $2S - nS, D$ measurements (mostly from one laboratory, LKB Paris), not confirmed by independent and equally accurate measurements. Highly excited states of H are affected by various systematics. As a result the Rydberg constant might be not as accurate as claimed

Experimental results for hydrogen



New hydrogen 2S→4P at MPQ!



$2S \rightarrow 4P_{1/2}$ and $4P_{3/2}$

cold H(2S) beam
optically excited $(1S \rightarrow 2S)$

$\Delta\nu \sim 2\,\text{kHz} \quad \equiv \Gamma/10'000$!!!

Beyer, Maisenbacher, Matveev, RP, Khabarova, Grinin, Lamour, Yost, Hänsch, Kolachevsky, Udem, submitted (2016)

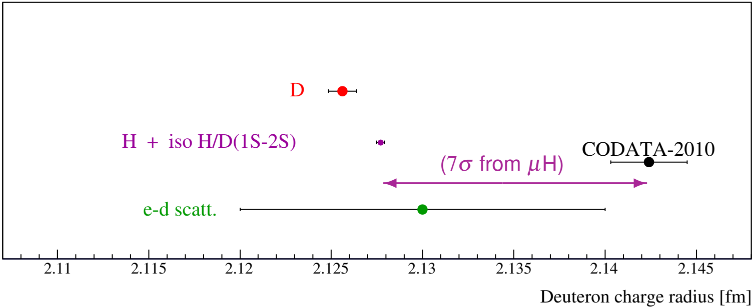
Ongoing experimental tests

- determine *Ry* by another accurate measurement in
  - 1S-2S in He^+
  - 1S-3S (Paris, . . .)
  - transitions between Rydberg states of heavy H-like ions (NIST, N.D. Guise talk)
- determine r_p
  - low Q^2 e-p scattering (PRad)
  - $2S - 2P$ in H (Hessels)
  - $\mu - p$ elastic scattering (MUSE collaboration)
- compare charge radii from electronic and muonic spectra of other atomic systems
  - μD data just published, r_D from very accurate H-D isotope shift (Garching)
  - μHe, $r_{\rm He}$ charge radius from scattering or $2^3S - 2^3P$ transition in He,

Deuteron charge radius

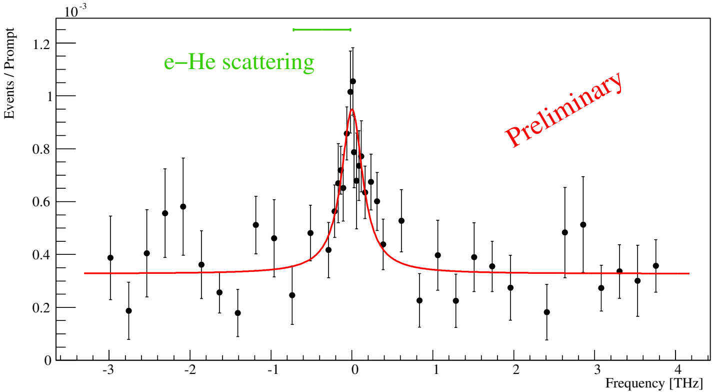
H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65)\ \mathrm{fm}^2$ C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)

CODATA 2010	$r_d = 2.14240(210)$ fm
$r_p$from μH gives	$r_d = 2.12771(\ 22)$ fm $\leftarrow 7\sigma$ from r_p
Muonic DEUTERIUM	$r_d = 2.12562(\ 13)_{exp}\,(77)_{theo}$ fm RP *et al.*, Science **353**, 417 (2016)



1st resonance in muonic He-4

$\mu^4\mathrm{He}(2S_{1/2} \rightarrow 2P_{3/2})$ at ~ 813 nm wavelength



Sick, PRD 77, 040302(R) (2008) Borie, Ann. Phys. 327, 733 (2012)

α charge radius from He $2^3S - 2^3P$

- $E(2^3S - 2^3P, {}^4\mathrm{He})_{\mathrm{centroid}} = 276\,736\,495\,649.5(2.1)$ kHz, Florence, 2004
- finite size effect: $E_{\mathrm{fs}} = 3\,427$ kHz
- since E_{fs} is proportional to r^2

$$\frac{\Delta r}{r} = \frac{1}{2}\,\frac{\delta E_{\mathrm{fs}}}{E_{\mathrm{fs}}} \approx \frac{1}{2}\,\frac{10}{3\,427} = 1.5 \cdot 10^{-3}$$

- electron scattering gives $r_{\mathrm{He}} = 1.681(4)$ fm, what corresponds to about $2.5 \cdot 10^{-3}$ relative accuracy
- $\sim$ 10 kHz accuracy requires calculation of $m\,\alpha^7$ correction

$2^3S - 2^3P$ transition in ^{4}He in MHz

	$(m/M)^0$	$(m/M)^1$	$(m/M)^2$	Sum
α^2	−276 775 637.536	102 903.459	−4.781	−276 672 738.857
α^4	−69 066.189	−6.769	−0.003	−69 072.961
α^5	5 234.163	−0.186	—	5 233.978
α^6	87.067	−0.029	—	87.039
α^7	−8.0 (1.0)	—	—	−8.0(1.0)
FNS	3.427	—	—	3.427
NPOL	−0.002	—	—	−0.002
Theory				−276 736 495.41 (1.00)
Exp.				−276 736 495.649 (2)