A review of the conceptual problems in tests of the Pauli Exclusion Principle

Edoardo Milotti
Dipartimento di Fisica, Univ. di Trieste
and
INFN – Sezione di Trieste

A review of ~~the~~ some conceptual problems in tests of the Pauli Exclusion Principle

Edoardo Milotti
Dipartimento di Fisica, Univ. di Trieste
and
INFN – Sezione di Trieste

As physicists we are accustomed to putting to stringent tests all the basic principles of our science.

The Pauli Exclusion Principle is an exception: it is believed to be valid on the basis of extensive and vaguely defined multiparticle phenomena rather than on a controlled few-particles basis.

But strictly controlled few-particle tests are just what we need to detect minute violations of the principle.

This is easier said than done.

Experimental studies of the Pauli Exclusion Principle (PEP for short) bring us closer than ever to the conceptual boundaries of physics, and to the very essence of science.

First of all, an experimental study of PEP means carrying out some form of test to detect possible (small? large?) violations.

In this context we have first to ask some important questions

- What is it that we study? (is it a property of individual particles, or is it actually something else?)
- Is there a mathematical framework that predicts violations? And if there is none, does it make sense to search for violations?

(see Matteo Morganti's talk for some partial answers ...)

It may seem strange that physicists ask this kind of questions, but tests of PEP have always been controversial.

Now let’s consider an unconventional view of PEP violation, one that appears to be totally disconnected from QFT.

VOLUME 78, NUMBER 22 PHYSICAL REVIEW LETTERS 2 JUNE 1997

Properties of Particles Obeying Ambiguous Statistics

M. V. Medvedev*,†

Physics Department, University of California at San Diego, La Jolla, California 92093

(Received 9 December 1996)

A new class of identical particles which may exhibit both Bose and Fermi statistics with respective probabilities p_b and p_f is introduced. Such an uncertainty may be either an intrinsic property of a particle or can be viewed as an “experimental uncertainty.” Statistical equivalence of such particles and particles obeying parastatistics of infinite order is shown. Generalized statistical distributions are derived, and statistical and thermodynamical properties of an ideal gas of the particles are investigated. The physical nature of such particles and the implications of this investigation for the statistics of extremal black holes are discussed. [S0031-9007(97)03281-X]

PACS numbers: 05.30.–d

we admit only "primary" Bose-Einstein and Fermi-Dirac statistics as existing. Assume now that a particle is neither a pure boson or pure fermion. Let another particle, which interacts with the first one, play the role of an external observer. During the interaction, it performs a measurement at the first particle and identifies it as either a boson or a fermion with respective probabilities p_b and p_f. According to the result of this measurement, it interacts with the first particle as if the last is a boson or fermion, respectively. The first particle, of course, is the observer for the second particle, thus the process is symmetric. Note that $p_b + p_f$ is not necessarily equal to one, and, if not, it means that the second particle (observer) does not detect the first particle.

Discrete violation

The violation has a given probability of being observed, basically the underlying stochastic model is binomial

This model can be interpreted in another manner. Assume a particle can oscillate between two types of statistics, then the model we propose represents a system of such particles averaged over time scales much larger than the oscillation period. The probabilities p_b and p_f, thus, are those portions of time during which a particle resides in a Fermi- or Bose-type state.

Dynamical reinterpretation of the model

All this means that the average commutation relation between two particles is the average of

$$\begin{aligned} a_i a_j^+ - a_j^+ a_i &= \delta_{i,j} \quad \text{with probability } p_b^2 \\ a_i a_j^+ + a_j^+ a_i &= \delta_{i,j} \quad \text{with probability } p_f^2 \\ a_i a_j^+ &= \delta_{i,j} \quad \text{with probability } 2 p_b p_f \end{aligned}$$

i.e.,

$$(p_b^2 + p_f^2 + 2p_b p_f) a_i a_j^+ - (p_b^2 - p_f^2) a_j^+ a_i = (p_b^2 + p_f^2 + 2p_b p_f)\delta_{i,j}$$

which corresponds to the deformed commutator

$$\boxed{a_i a_j^+ - q a_j^+ a_i = \delta_{i,j} \quad \text{with } q = \frac{p_b^2 - p_f^2}{(p_b + p_f)^2} = \frac{p_b - p_f}{p_b + p_f}}$$

In the context of a simple binomial model (i.e., with no alternative to observing a boson or a fermion), this means that

$$\boxed{q = p_b - p_f = 1 - 2p_f}$$

Then for a pure boson (p_b = 1) we expect q = 1, and for a pure fermion q = -1.

This kind of violation leads to thermodynamical consequences in multiparticle states.

The downside is that there is no QFT that describes this simple scheme.

Already in 1980, Amado and Primakoff published a short and very critical paper on the meaning of experimental tests of PEP.

PHYSICAL REVIEW C VOLUME 22, NUMBER 3 SEPTEMBER 1980

Comments

The Comments section is for short papers which comment on papers previously published in The Physical Review. *Manuscripts intended for this section must be accompanied by a brief abstract for information retrieval purposes and a keyword abstract.*

Comments on testing the Pauli principle

R. D. Amado and H. Primakoff
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 1 February 1980)

Particle identity in quantum mechanics makes it impossible to test the Pauli exclusion principle by looking for "forbidden" x rays or γ rays. Such experiments do, however, test particle stability. Evidence for the indistinguishability of electrons and nucleons is discussed.

[NUCLEAR STRUCTURE Particle identity and the Pauli exclusion principle, particle stability.]

Amado&Primakoff: the symmetrization principle is extremely robust

- Hamiltonians that are symmetric with respect to particle exchange in closed systems cannot change the symmetry of any given state, and therefore they cannot connect a symmetric or antisymmetric state to a state of mixed symmetry, even if it exists, and so there cannot be either large or small violations (always holds in nonrelativistic QM)

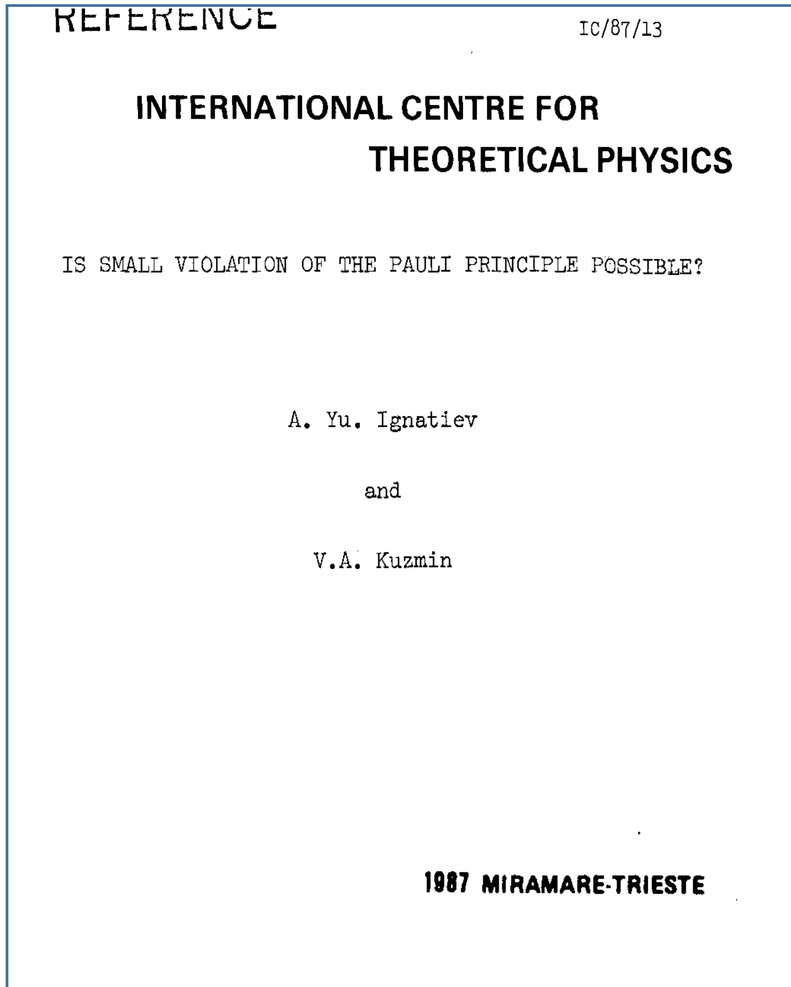
- If there are nonidentical electrons, then they should show up as additional particle-antiparticle pairs in production experiments (doubling or more of cross section, not observed)

- the appearance of additional particle pairs would change virtual diagrams like those that contribute to g-2, etc, and heavily influence the theoretical predictions (again, no small violation)

- small PEP violations could possibly leave a trace as electric charge nonconservation; this is also unobserved

Amado and Primakoff – unlike Medvedev – frame possible violations in a conventional scheme.

To better understand how to set up a conventional scheme of **small violations**, let us start from QM and a simple QM model *of small violations* proposed long ago (1987) by Ignatiev and Kuzmin



REFERENCE

IC/87/13

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

IS SMALL VIOLATION OF THE PAULI PRINCIPLE POSSIBLE?

A. Yu. Ignatiev

and

V.A. Kuzmin

1987 MIRAMARE-TRIESTE

A Fermi oscillator usually has only two base states: $|0\rangle, |1\rangle$

with annihilation and creation operators:

$$a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad a^\dagger = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

<u>In the IK model, three base states</u>: $|0\rangle, |1\rangle, |2\rangle$

$$\begin{aligned} a^\dagger|0\rangle &= |1\rangle & a|0\rangle &= 0 \\ a^\dagger|1\rangle &= \beta|2\rangle & a|1\rangle &= |0\rangle \\ a^\dagger|2\rangle &= 0 & a|2\rangle &= \beta|1\rangle \end{aligned}$$

$$\begin{aligned} a^\dagger|0\rangle &= |1\rangle & a|0\rangle &= 0 \\ a^\dagger|1\rangle &= \beta|2\rangle & a|1\rangle &= |0\rangle \\ a^\dagger|2\rangle &= 0 & a|2\rangle &= \beta|1\rangle \end{aligned}$$

Matrix representation

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad |2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \beta & 0 \end{pmatrix}; \quad a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix}$$

The matrix representation of the creation and annihilation operators has a simple orthogonal basis

$$M_{11} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ;\ M_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ;\ \ldots ;\ M_{33} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Using this basis we can write all the possible operator products, up to triple.

Example:

$$a^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \beta M_{13}$$

linear and bilinear relations

$$a = (a^\dagger)^\dagger = M_{12} + \beta M_{23}; \qquad a^\dagger = M_{21} + \beta M_{32}$$

$$a^2 = \left[(a^\dagger)^2\right]^\dagger = \beta M_{13}; \qquad (a^\dagger)^2 = \beta M_{31}$$

$$a^\dagger a = M_{22} + \beta^2 M_{33}; \qquad a a^\dagger = M_{11} + \beta^2 M_{22};$$

$$a^3 = \left[(a^\dagger)^3\right]^\dagger = 0; \qquad a^2 a^\dagger = \left[a(a^\dagger)^2\right]^\dagger = \beta^2 M_{12};$$

$$a a^\dagger a = \left[a^\dagger a a^\dagger\right]^\dagger = M_{12} + \beta^3 M_{23}; \qquad a^\dagger a^2 = \left[(a^\dagger)^2 a\right]^\dagger = \beta M_{23}$$

trilinear relations

And now, what about the commutation relations: are there any bilinear commutation relations as in the standard Fermi oscillator?

If there are bilinear relations, the commutator must be a linear combination with the following general form

$$\left[C_1 a^\dagger a + C_2 a a^\dagger + C_3 I\right] + \left[C_4 a + C_5 a^\dagger + C_6 a^2 + C_7 \left(a^\dagger\right)^2\right] = 0$$

The terms in square brackets at right are all linearly independent among them and they do not depend on the terms in the brackets at left, therefore

$$C_4 = C_5 = C_6 = C_7 = 0$$

$$\begin{aligned}
&a = (a^\dagger)^\dagger = M_{12} + \beta M_{23}; && a^\dagger = M_{21} + \beta M_{32} \\
&a^2 = \left[(a^\dagger)^2\right]^\dagger = \beta M_{13}; && (a^\dagger)^2 = \beta M_{31} \\
&a^\dagger a = M_{22} + \beta^2 M_{33}; && a a^\dagger = M_{11} + \beta^2 M_{22}; \\
&a^3 = \left[(a^\dagger)^3\right]^\dagger = 0; && a^2 a^\dagger = \left[a (a^\dagger)^2\right]^\dagger = \beta^2 M_{12}; \\
&a a^\dagger a = \left[a^\dagger a a^\dagger\right]^\dagger = M_{12} + \beta^3 M_{23}; && a^\dagger a^2 = \left[(a^\dagger)^2 a\right]^\dagger = \beta M_{23}
\end{aligned}$$

Then

$$0 = C_1 a^\dagger a + C_2 a a^\dagger + C_3 I$$
$$= C_1(M_{22} + \beta^2 M_{33}) + C_2(M_{11} + \beta^2 M_{22}) + C_3(M_{11} + M_{22} + M_{33})$$

$$\Rightarrow \begin{cases} C_2 + C_3 = 0 \\ C_1 + \beta^2 C_2 + C_3 = 0 \\ \beta^2 C_1 + C_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} 0 & 1 & 1 \\ 1 & \beta^2 & 1 \\ \beta^2 & 0 & 1 \end{vmatrix} = -1 + \beta^2 - \beta^4 = 0$$

The algebraic equation has no real solution, only the trivial solution exists, and therefore there are no bilinear commutators.

$$a = (a^\dagger)^\dagger = M_{12} + \beta M_{23}; \qquad a^\dagger = M_{21} + \beta M_{32}$$

$$a^2 = \left[(a^\dagger)^2\right]^\dagger = \beta M_{13}; \qquad (a^\dagger)^2 = \beta M_{31}$$

$$a^\dagger a = M_{22} + \beta^2 M_{33}; \qquad a a^\dagger = M_{11} + \beta^2 M_{22};$$

$$a^3 = \left[(a^\dagger)^3\right]^\dagger = 0; \qquad a^2 a^\dagger = \left[a(a^\dagger)^2\right]^\dagger = \beta^2 M_{12};$$

$$a a^\dagger a = \left[a^\dagger a a^\dagger\right]^\dagger = M_{12} + \beta^3 M_{23}; \qquad a^\dagger a^2 = \left[(a^\dagger)^2 a\right]^\dagger = \beta M_{23}$$

However, it is easy to see from the M representation that the following trilinear relations hold:

$$a^2 a^\dagger + \beta^2 a^\dagger a^2 = \beta^2 a$$

$$a^2 a^\dagger + \beta^4 a^\dagger a^2 = \beta^2 a a^\dagger a$$

$$a^3 = (a^\dagger)^3 = 0$$

Moreover, the number operator is

$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = M_{22} + 2M_{33}$$

and it satisfies the usual commutation relations

$$[N, a] = -a; \quad [N, a^\dagger] = a^\dagger$$

We can also find a more explicit representation of the number operator using equations

$$
\begin{aligned}
a^\dagger a &= M_{22} + \beta^2 M_{33}; \\
a a^\dagger &= M_{11} + \beta^2 M_{22}; \\
I &= M_{11} + M_{22} + M_{33}
\end{aligned}
$$

These equations can be solved to express the M matrices in terms of *a*'s and I:

$$
\begin{aligned}
M_{22} &= \frac{a^\dagger a - \beta^2 I + \beta^2 a a^\dagger}{1 - \beta^2 + \beta^4} \\
M_{33} &= \frac{I - a^\dagger a - a a^\dagger + \beta^2 a^\dagger a}{1 - \beta^2 + \beta^4}
\end{aligned}
$$

and finally we find the number operator as a linear combination of bilinears:

$$
\boxed{
\begin{aligned}
N &= M_{22} + 2M_{33} \\
&= \frac{1}{1 - \beta^2 + \beta^4}\left[(2 - \beta^2)I + (-1 + 2\beta^2)a^\dagger a + (-2 + \beta^2) a a^\dagger\right]
\end{aligned}
}
$$

Now we consider the following toy model (also introduced by IK) to understand the role of the violation parameter

$$H = H_0 + H_{\mathrm{int}} = EN + \epsilon V$$
$$V = a^2 a^\dagger + a^\dagger a^2 + a a^\dagger a + h.c.$$

with $\epsilon \ll E$.

Using the matrix representation of all operators we find

$$H = \begin{pmatrix} 0 & \epsilon(1+\beta^2) & 0 \\ \epsilon(1+\beta^2) & E & \epsilon\beta(1+\beta^2) \\ 0 & \epsilon\beta(1+\beta^2) & 2E \end{pmatrix}$$

$$\approx \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & E & \epsilon\beta \\ 0 & \epsilon\beta & 2E \end{pmatrix} = EN + \epsilon\beta(a + a^\dagger)$$

Using the approximate Hamiltonian one finds

1. the energy eigenvalues (up to order $(\epsilon/E)^2$)

$$E_0 = \frac{\epsilon^2}{E}; \quad E_1 = E + \frac{\epsilon^2(1-\beta^2)}{E}; \quad E_2 = 2E + \frac{\epsilon^2\beta^2}{E}$$

2. the transition rates (from standard perturbation theory)

$$W_{01} = 2\frac{\epsilon^2}{E^2}(1-\cos Et)$$

$$W_{02} = 0$$

$$W_{12} = 2\beta^2\frac{\epsilon^2}{E^2}(1-\cos Et)$$

Pauli-violating transition rate proportional to β^2

When we try to relate the IK theory to some deformed commutator, we find

$$\begin{aligned} aa^+ - qa^+a &= M_{11} + \beta^2 M_{22} - q(M_{22} + \beta^2 M_{33}) \\ &= M_{11} + (\beta^2 - q)M_{22} - q\beta^2 M_{33} \end{aligned}$$

Therefore, if we neglect the $|2\rangle$ state, in the $|0\rangle,\ |1\rangle$ subspace we find that this commutation relation falls back into the usual scheme if we let

$$\beta^2 = 1 + q$$

Mohapatra and Greenberg suggested a QFT for the IK scheme, and also two different experimental ideas

VOLUME 59, NUMBER 22 PHYSICAL REVIEW LETTERS 30 NOVEMBER 1987

Local Quantum Field Theory of Possible Violation of the Pauli Principle

O. W. Greenberg and R. N. Mohapatra
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 5 August 1987)

We generalize to a local relativistic quantum field theory a proposal of Ignatiev and Kuzmin for a single oscillator which has small violation of the Pauli principle and thus provide a theoretical framework which, for the first time, allows quantitative tests of the Pauli principle. Our theory provides a continuous interpolation between fully hindered parafermi statistics of order 2 ($\beta=0$), which is equivalent to Fermi statistics, and ordinary parafermi statistics of order 2 ($\beta=1$). We suggest two types of experiments which can place bounds on β.

We suggest two types of experiment to put bounds on β. The probability of finding an atom in which an electron violates the Pauli principle is of order β^2. In stable matter, such electrons would long ago have made transitions to the lowest allowed state; thus we do not expect to observe x rays. Rather such atoms could be detected by exciting them and observing their spectra. It will be difficult to bound β^2 by less than 10^{-8} with spectroscopy. Our second suggestion is to bring slow electrons in contact with an atom and look for x rays coming, with probability β^2, from a transition of an electron in a high Pauli-principle-violating state to a low-lying such state. An efficient way to do this would be to run a high current through a metal and to look for x rays while the current is running. This should give strong bounds on β^2. The old experiments of Ref. 4 are not tests of the Pauli principle; indeed, no high-precision tests of the Pauli principle have been made. Analogous experiments can be made for nucleons in nuclei. We will give further phenomenological analysis elsewhere.

We thank Palash Pal for discussions in the initial stages of this work, Shmuel Nussinov for raising stimulating questions, and George Snow for suggesting the experiment using a current running through a metal. This work was supported in part by the National Science Foundation.

Search for non-Paulian atoms

Search for anomalous X-rays from non-Paulian transitions in non-equilibrium systems

The second suggestion is actually due to George Snow

However, it turns out that the model of IK is a nice little theory … but it cannot be extended to a true QFT !!!

Some of the problems of the IK model are common to all schemes that incorporate "small" violation of the spin-statistics connection.

It is important to note that already in 1950, Green proved that there could be alternative discrete statistics which he called "parastatistics".

Green proved first that common Fermions satisfy the trilinear relations

$$\left[a_r^\dagger, [a_s, a_t]\right] = \delta_{rs} a_t - \delta_{rt} a_s$$

$$\left[a_r^\dagger, [a_s^\dagger, a_t^\dagger]\right] = 0$$

These trilinear relations are satisfied by the common operators that anticommute

$$\{a_r, a_s\} = 0; \quad \{a_r^\dagger, a_s\} = \delta_{rs}$$

BUT they are also satisfied by those tha satisfy the set of trilinear relations

$$a_r a_s a_t + a_t a_s a_r = 0$$
$$a_r^\dagger a_s a_t + a_t a_s a_r^\dagger = \delta_{rs} a_t$$
$$a_r a_s^\dagger a_t + a_t a_s^\dagger a_r = \delta_{rs} a_t + \delta_{ts} a_r$$

which are incompatible with common anticommutators.

There are infinite other multilinear relations. Each set of (incompatible) relations represents a given situation with a maximum occupation number *n*.

As a simple example, consider the situation where *r=s=t*, then

$$
\begin{aligned}
a^3 &= 0 \\
a^\dagger a^2 + a^2 a^\dagger &= a \\
a a^\dagger a + a a^\dagger a &= a
\end{aligned}
$$

Now, note the action of the second trilinear on the vector $|1\rangle$

$$(a^\dagger a^2 + a^2 a^\dagger)|1\rangle = a^2 \left(a^\dagger |1\rangle\right) = a|1\rangle = |0\rangle$$

Clearly, $a^\dagger |1\rangle \neq |0\rangle$ and $a^\dagger |1\rangle \neq |1\rangle$ because a^2 would annihilate it, then another vector must exist such that

$$|2\rangle = a^\dagger |1\rangle; \quad a^2 |2\rangle = |0\rangle$$

In addition to these multistate parafermions there are also the parabosons. However **Green's parastatistics is ruled out by experiment, we do not observe these multistate oscillators.**

We seem to be in a dead end, discrete violations are not observed, while small violations are theoretically inconsistent.

The proofs of inconsistency are due to A. B. Govorkov who explored these problems in depth in a series of papers.

Govorkov noted that the IK trilinears can be written in the general form

$$[[a_m^\dagger, a_l]_\epsilon, a_k] = -\alpha \delta_{km} a_l$$

$$[a_m^\dagger, a_l]_\epsilon = a_m^\dagger a_l + \epsilon a_l a_m^\dagger$$

where the parameters can be related to IK's β

$$\epsilon = \frac{2-\beta^2}{1-2\beta^2}; \quad \alpha = -\frac{1-\beta^2+\beta^4}{1-2\beta^2}$$

Govorkov used these commutation relations and the assumptions that:

1. ϵ is finite and nonzero
2. there is a unique vacuum state $|0\rangle$ such that $a_m|0\rangle = 0$ for all m
3. the norm of vectors is positive definite
4. the number of particles in a symmetric (antisymmetric) state does not exceed a given integer $M \geq 2$

and he was able to show that in such a case one must have ether $\epsilon = -1$ or $\epsilon = +1$ and $\alpha > 0$.

This strongly restricts the allowed statistics, and provides a sort of generalized proof of PEP.

This result also rules out "small" violations of PEP in any parafermionic system with a finite number of states.

In 1990 Greenberg proposed a new QFT with a small violation of PEP, based on the deformed commutators

$$a_k a_l^+ - q a_l^+ a_k = \delta_{kl} \quad (-1 < q < 1)$$

The new theory led to fields called ***quons***, an example of *infinite statistics* – which is the statistics of indistinguishable particles with infinite degrees of freedom – and escaped Govorkov's criticism.

The case *q* = 0, which corresponds to the "commutator"

$$a_k a_l^+ = \delta_{kl}$$

a sort of average between a standard commutator and an anticommutator – turns out to be specially important, because its algebra can be used to generate the algebra of the *q* ≠ 0 cases.

Moreover the statistical mechanics of the $q = 0$ case matches the classical Maxwell-Boltzmann statistics, possibly because – as first noted by Govorkov – the infinite degrees of freedom endow the particles of the theory with an effective distinguishability.

Happily, it also turned out that the q parameter of the theory could be related to the β parameter of the earlier Ignatiev-Kuzmin model,

$$\frac{\beta^2}{2} = \frac{1+q}{2}$$

Greenberg noted that the theory is nonlocal, and initially it was not clear whether this nonlocality could also be relativistically invariant – and therefore whether the theory could be a true relativistic QFT or not.

This was decided in 1993, when Govorkov showed that the existence of antiparticles rules out a "small" deviation from PEP even with infinite statistics.

Physica A 203 (1994) 655–670
North-Holland

SSDI 0378-4371(93)E0392-R

PHYSICA A

The existence of antiparticles seems to forbid violations of statistics

A.B. Govorkov[†]

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Head Post Office P.O. Box 79, 101000 Moscow, Russian Federation

Received 22 March 1993
Revised manuscript received 9 July 1993

I try to prove the impossibility of small violations of Fermi- and Bose-statistics even within the nonlocal quantum field theory corresponding to the infinite statistics. The existence of antiparticles plays the crucial role in this proof.

Since the existence of antiparticles is bound to the relativistic nature of a QFT, quon theory cannot be a relativistic theory.

Given these considerations, is it conceivable to ever find a model of (small) violations of PEP that fits the constraints of QFT?

Maybe ... and to this end it is useful to review the axiomatic basis of a standard proof of the spin-statistics connection.

Lüders and Zumino lay out a very clean set of assumptions in their 1958 proof:

I. The theory is invariant with respect to the proper inhomogeneous Lorentz group (includes translations, does not include reflections)
II. Two operators of the same field at points separated by a spacelike interval either commute or anticommute (Locality)
III. The vacuum is the state of lowest energy
IV. The metric of the Hilbert space is positive definite
V. The vacuum is not identically annihilated by a field

(G. Lüders and B. Zumino, Phys. Rev. **110** (1958) 1450)

In principle any theory that breaks one or more of these axioms is a candidate for PEP violation.

However, the results of Govorkov seem to indicate that most important is the breaking of Lorentz invariance.

In the absence of a true theoretical framework, experiments also present difficult challenges in the interpretation of their results

The kinds of tests devised up till now are as follows:

- anomalous X-ray emission from atoms (anomalous electronic transitions)
- anomalous X-ray emission from nuclei (anomalous nuclear transitions)
- searches for non-Paulian isotopes
- anomalous X-rays from materials where "new" or "fresh" electrons are injected in some way

The first two kinds of searches have been inspired by the work of Reines, but unfortunately the claim that they test PEP is flawed.

So, what is the problem with anomalous X-rays from small closed systems?

When we rule out relativistic effects, we are bound to consider the standard QM situation described above, and the symmetry of the global wavefunction has a symmetry described by a Young tableau such as the one in the paper by Rahal and Campa (PRA, 38 (1988) 3728).

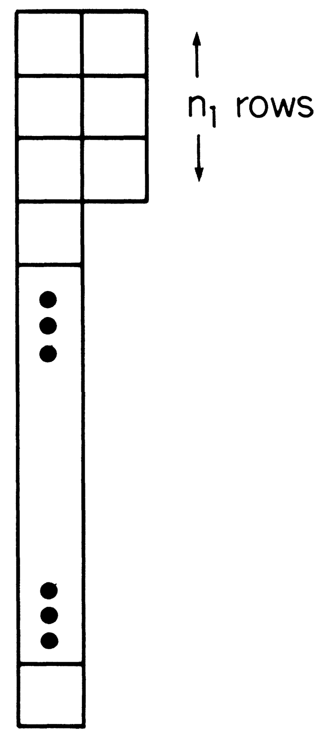
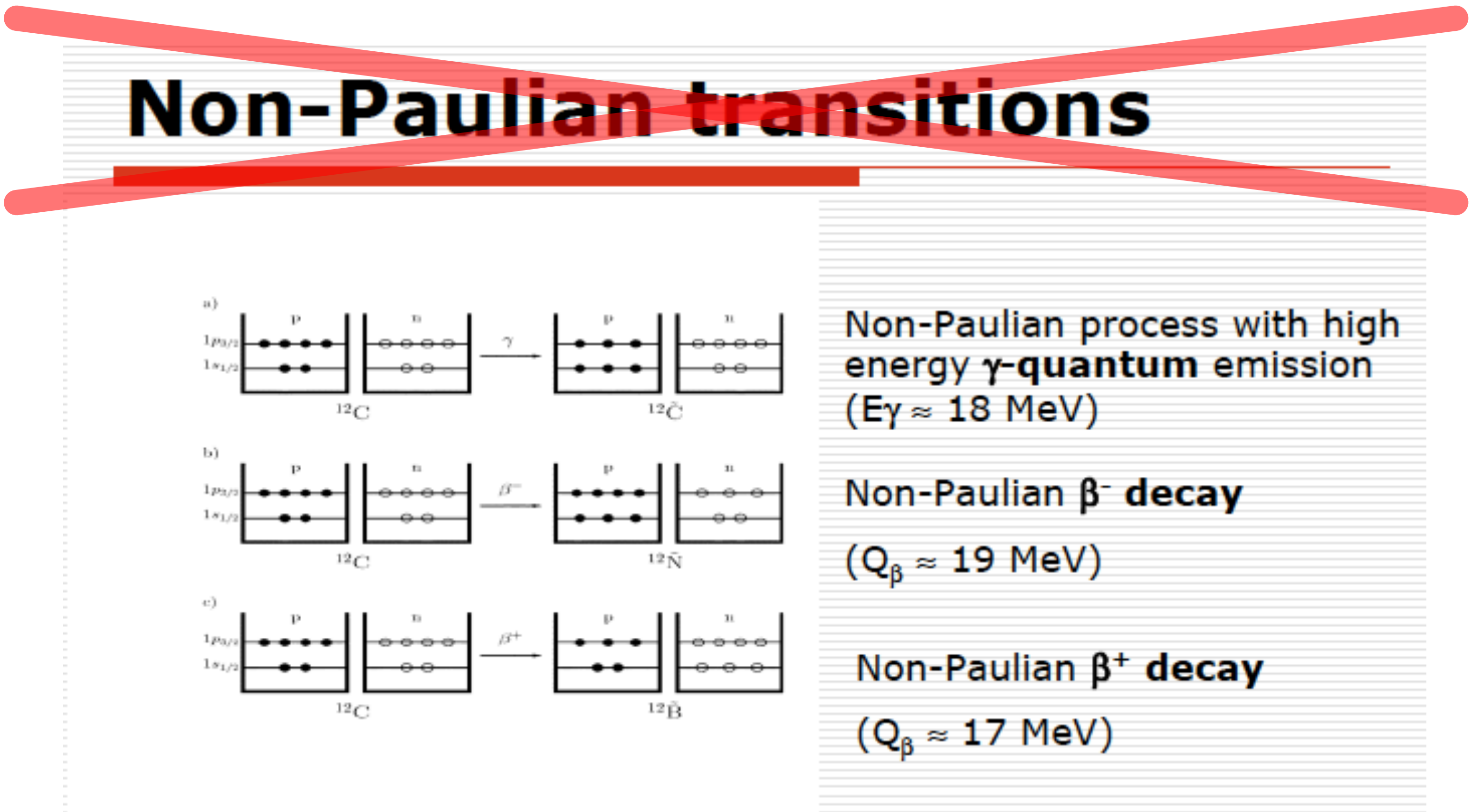


FIG. 2. Young tableau for an N-particle system violating the PEP. In the figure $n_1 \ll N$, N being the total number of boxes.

In a small system with anomalous wavefunction (because some of the electrons or nucleons in it are those associated with the n_1 rows) there can be transitions with anomalous X-rays, ***but since we observe "old" systems, they should have emitted those X-rays long ago, given the high transition rates.***

This means that recently announced bounds on the validity of PEP by experiments such as DAMA, Borexino, NEMO-2, etc. are actually bounds on the stability of electrons or nucleons.



Such tests could only be valid in the framework of an unconventional theory of PEP violation like that of Medvedev (PRL 78 (1997) 4147).

What about the other tests?

Searches for non-Paulian isotopes have been performed, with negative results, however they are limited by the accuracy of the chemical analyses necessary to carry out the extraction of minute amounts of “wrong” atoms amid a score of others.

Finally we are left with those tests that inject “new” electrons in a system, and search for anomalous X-rays. The prototype experiment is that of Goldhaber and Scharff-Goldhaber, which started out as a totally different experiment, and was interpreted much later as a test of the Pauli principle for electrons (Phys. Rev. 73 (1948) 1472 and PRL 32 (1974) 954).

Identification of Beta-Rays with Atomic Electrons

M. Goldhaber and Gertrude Scharff-Goldhaber
Department of Physics, University of Illinois, Urbana, Illinois
May 8, 1948

The experiment is based on the following consideration: when beta-rays are stopped in matter, their final fate will depend on whether or not they are identical with atomic electrons. If they were not identical with atomic electrons, they would not obey Pauli's exclusion principle and could therefore be captured into bound orbits “filled” with atomic electrons. Their transition to the lowest orbit would take place within an extremely short time and would be accompanied by K x-rays, slightly longer in wavelength than the K x-rays characteristic of the capturing atom, because of the additional screening. A test for the absence or presence of these x-rays can thus decide whether or not beta-rays are identical with electrons.

Following discussions with Greenberg and Mohapatra, Ramberg and Snow started a new experimental line

Volume 238, number 2,3,4 PHYSICS LETTERS B 5 April 1990

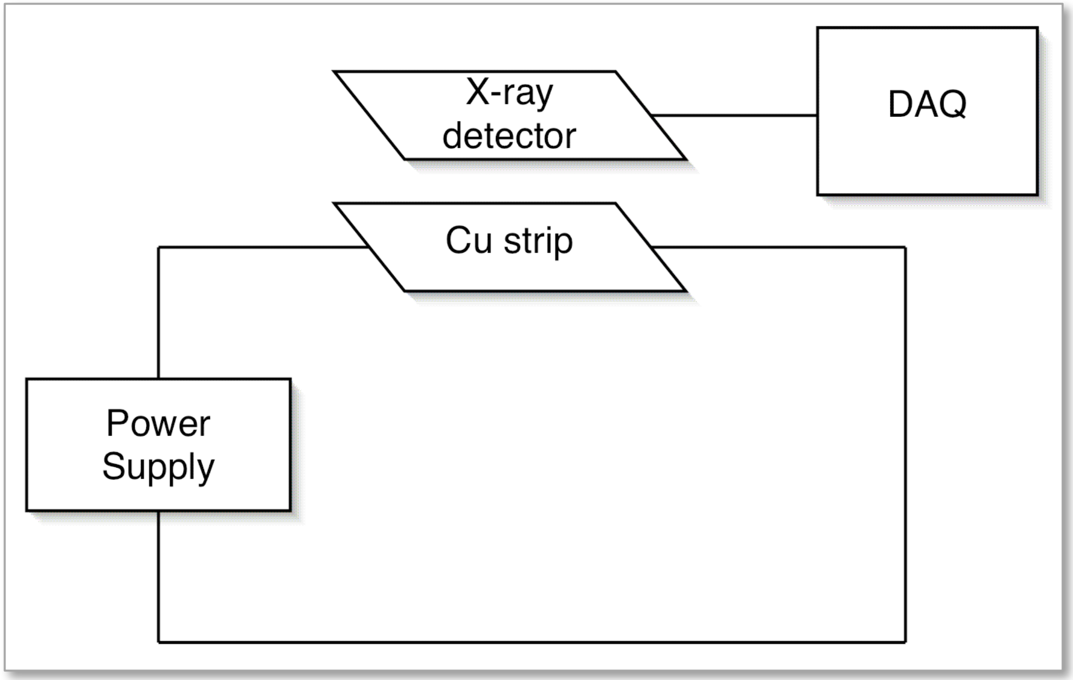
EXPERIMENTAL LIMIT ON A SMALL VIOLATION OF THE PAULI PRINCIPLE

Erik RAMBERG and George A. SNOW
Department of Physics and Astronomy, University of Maryland, College Park, MD 20742, USA

Received 3 November 1989

We have made a search for anomalous X-rays arising from a small violation of the Pauli exclusion principle in current carrying copper. No such signal was found. From a minimal set of assumptions we conclude that the probability that a new electron introduced into copper would form a mixed symmetry state with respect to the electrons already present in the copper sample, thus violating the Pauli principle, is less than 1.7×10^{-26}.

In the RS experiment, “new” electrons are not injected by a radioactive source – as in the experiment by Goldhaber and Scharff-Goldhaber, but by an electric current source.



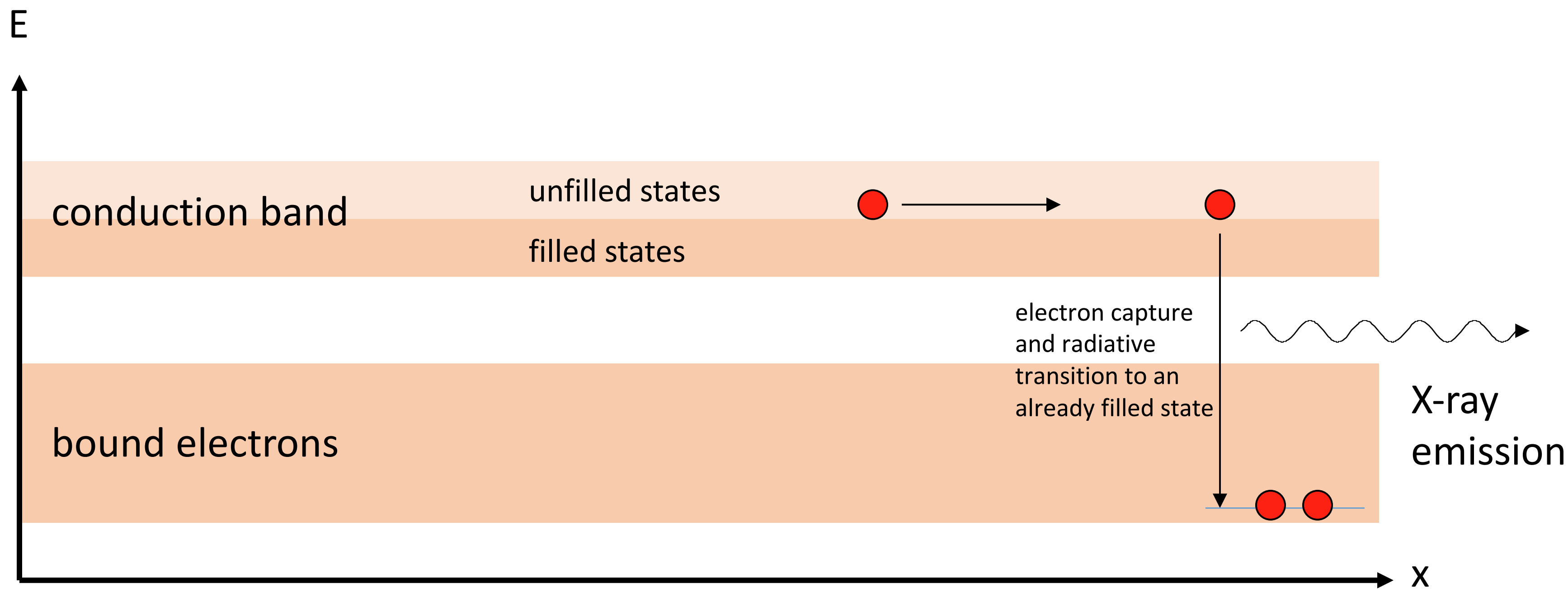
Conceptually very simple experimental scheme, replicated – however, with much better detectors and shielding – in VIP:

- electrons injected by a power supply (current source); ideally this should be connected to a large metal block that acts as a source of “new” electrons
- large area conductor strip where electrons circulate
- large area X-ray detector, with good energy resolution, to detect and pinpoint any anomalous X-ray

Unfortunately, “the devil hides in the details”, and here there are quite a few difficult, and sometimes very conceptual, details …

1. what is β ?
2. what is an anomalous X-ray?
3. how many scatterings are there?
4. what is a ”new electron”?
5. how many anomalous X-rays are there?
6. ... and finally, **what does all this mean?**

Here I consider in greater depth only item 2: what is an anomalous X-ray?



Estimate of X-ray energy (Cu $K_{\alpha 1}$ =8.05 KeV; $K_{\beta 1}$ =8.90 KeV)

This estimate is necessary to define the region of interest in the X-ray spectrum

Available methods:

1. *Naive estimates*
2. *Hartree-Fock methods*
3. *Thomas-Fermi and modified Thomas-Fermi methods*

A **simple estimate** is based on the remark that if we assume that the effective charge "seen" by the captured anomalous electron is approximately (Z-1) (because of the partial screening of the K-shell electrons), then - in the case of Copper - the emitted photon has an energy which is approximately that of the Nickel K X-rays, i.e.

$K_{\alpha 1}$ =7.48 KeV;

$K_{\beta 1}$ =8.26 KeV

A somewhat better estimate starts from an approximate calculation of the screening effect of the other electrons. Here we calculate the charge inside the orbit of the 1S electron in a hydrogenoid atom:

$$
\begin{aligned}
\int_0^{r_{1S}} |\psi(r)|^2 \cdot 4\pi r^2 dr &= \frac{1}{2}\int_0^{a_0/Z} \exp\left(-\frac{2Zr}{a_0}\right) \left(\frac{2Zr}{a_0}\right)^2 d\left(\frac{2Zr}{a_0}\right) \\
&= \frac{1}{2}\int_0^{2} x^2 e^{-x} = 1 - 5e^{-2} \approx 0.32
\end{aligned}
$$

Thus, we obtain a naive estimate of the energy for both H-like ions and He-like ions:

$$
E_H \approx (Z - 0.32)^2 R_\infty; \quad E_{He} \approx (Z - 0.64)^2 R_\infty;
$$

$$E_H \approx (Z - 0.32)^2 R_\infty; \quad E_{He} \approx (Z - 0.64)^2 R_\infty;$$

ionization energies (eV)

Element	*Observed E_H*	*E_H from naive estimate*	*Observed E_{He}*	*E_{He} from naive estimate*
C	490.0	489.8	392.1	439
Cu	11568	11443	11063	11192

ionization energies from: C. E. Moore, *Ionization Potentials and Ionization Limits Derived from the Analysis of Optical Spectra*, NBS Pub. NSRDS-NBS 34 (1970)

If we use this “naive estimate” to correct the energy of K_α X-rays for non-Paulian copper, and we find:

$$E \approx (8.05 \text{ KeV}) \cdot \left(\frac{Z - 0.64}{Z} \right)^2 \approx 7.70 \text{ KeV}$$

Hartree-Fock methods are usually quite precise, but in this case there is some awkwardness of implementation.

We must exclude the “anomalous” electron from the Hartree-Fock antisymmetrization determinant, we have to deal with a global electron wavefunction of the form

$$\Psi = \Psi_{HF,Z-1} \cdot \Psi_{NP}$$

This implies that the added electron has no specific symmetry with respect to all other electrons in the atom and this is an additional assumption that may not be true.

Moreover, this approach requires an *ad hoc* HF procedure, it is not possible to use existing programs without modifications

To compute the transition energies VIP utilizes the relativistic multiconfiguration Dirac-Fock package (MCDF) by J.P. Desclaux et al.

Transition	$\hbar\omega_{\text{Paulian}}$ (eV)	$\hbar\omega_{\text{non-Paulian}}$ (eV)	$\lambda_{\text{non-Paulian}}$ (Hz)	$(\hbar\omega_{\text{non-Paulian}} - \hbar\omega_{\text{Paulian}})$ (eV)
$2p_{1/2} \to 1s_{1/2}$ $(K_{\alpha2})$	8047.78	7728.92	2.64×10^{14}	318.86
$2p_{3/2} \to 1s_{1/2}$ $(K_{\alpha1})$	8027.83	7746.73	2.57×10^{14}	279.84
$3p_{1/2} \to 1s_{1/2}$ $(K_{\beta2})$	8905.41	8529.54	2.77×10^{13}	375.87
$3p_{3/2} \to 1s_{1/2}$ $(K_{\beta1})$	8905.41	8531.69	2.67×10^{13}	373.72
$3d_{3/2} \to 2p_{3/2}$ $(L_{\alpha2})$	929.70	822.84	5.99×10^{7}	106.86
$3d_{5/2} \to 2p_{3/2}$ $(L_{\alpha1})$	929.70	822.83	3.49×10^{8}	106.87
$3s_{1/2} \to 2p_{1/2}$	832.10	762.04	3.70×10^{11}	70.06
$3s_{1/2} \to 2p_{3/2}$	811.70	742.97	7.84×10^{11}	68.73
$3d_{5/2} \to 1s$	8977.14	8570.82	1.21×10^{6}	406.32

… the accurate estimate is not so far off the naive estimate after all ...

For more details see Sergio di Matteo’s talk.

Final considerations ...

We have seen that tests of PEP confront us with problems that go deep into the heart of science.

This is unusual in physics: in most cases questions are laid out clearly.

In this regard it is interesting to compare the situation to the statistical inference in physics, starting from Bayes' Theorem

Likelihood

Prior distribution

Posterior distribution

$$p(\theta|D,I) = \frac{p(D|\theta,I)}{\int_\Theta p(D|\theta,I)p(\theta,I)d\theta}\, p(\theta,I)$$

The usual (frequentist) approach that maximizes the likelihood is a stripped-down version of Bayes' Theorem

Posterior distribution of the parameter: according to frequentists this does not make sense because the parameter is normally a constant.

Prior distribution: main target of frequentist criticisms, as a source of subjective information.

$$p(\theta|D,I) = \frac{p(D|\theta,I)}{\int_\Theta p(D|\theta,I)p(\theta,I)d\theta}\, p(\theta,I)$$

$$L(\theta;D) = p(D|\theta,I)$$

Likelihood: frequentists (many physicists) are happy with this, however the likelihood also embeds a great deal of prior information as it is the physical model of the distribution of data.

John Tukey put the statistical argument beautifully in a 1980 paper

We Need Both Exploratory and Confirmatory

JOHN W. TUKEY*

We often forget how science and engineering function. Ideas come from previous exploration more often than from lightning strokes. Important questions can demand the most careful planning for confirmatory analysis. Broad general inquiries are also important. Finding the question is often more important than finding the answer. Exploratory data analysis is an attitude, a flexibility, and a reliance on display, NOT a bundle of techniques, and should be so taught. Confirmatory data analysis, by contrast, is easier to teach and easier to computerize. We need to teach both; to think about science and engineering more broadly; to be prepared to randomize and avoid multiplicity.

KEY WORDS: Exploratory data analysis; Confirmatory data analysis; Paradigms of science and engineering; Sources of ideas; Randomization; Multiplicity.

© *The American Statistician, February 1980, Vol. 34, No. 1* 23

Find the right question!

1. *An incomplete paradigm*. We are, I assert, all too familiar with the following straight-line paradigm—asserted far too frequently as how science and engineering function:

(*) question → design → collection → analysis → answer

Any attempt to claim that this straight-line, confirmatory pattern is more than a substantial part of the story neglects crucial questions (and their answers):

1. How are questions generated? (Mainly by quasi-theoretical insights and the exploration of past data.)
2. How are designs guided? (Usually, by the best qualitative and semiquantitative information available, obtained by exploration of past data.)
3. How is data collection monitored? (By exploring the data, often as they come in, for unexpected behavior.)
4. How is analysis overseen; how do we avoid analysis that the data before us indicate should be avoided? (By exploring the data—before, during, and after analysis—for hints, ideas, and, sometimes, a few conclusions-at-5%/k.)

I assert, and I count upon most of you to agree after reflection, that to implement the very confirmatory paradigm (*) properly we need to do a lot of exploratory work.

Neither exploratory nor confirmatory is sufficient alone. To try to replace either by the other is madness. We need them both.

2. *The origin of ideas*. Reorganizing the early stage of the last paradigm can help us understand better what is going on. What often happens is better diagrammed thus:

(∗) idea → question / design → collection → analysis → answer

Maybe, following Tukey's advice we shall be able to grasp something more of the essence of PEP!

For more on the experiments, see Hans Marton's talk.