A proposal for the classical limit in Bohm's theory

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The classical limit problem

- Quantum mechanics (QM) $\longrightarrow$ Classical mechanics (CM)
- Phenomenological problem: quantum probability distributions must converge to classical quantities
(example: Ehrenfest's theorem for position distribution)
- Conceptual problem: two different ontologies for QM and CM
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Ehrenfest's theorem

- Example: Ehrenfest's theorem
- If $\Psi(x)$ is a narrow wave packet in position space
- $m\frac{d^2}{dt^2}\langle X\rangle = -\nabla V(\langle X\rangle)$ (for linear and quadratic V)
- the mean values of the position and momentum operators follow a "Newtonian flow"
- Note: formal derivation $\neq$ real physical derivation
- $\rightarrow$ This is not a classical trajectory:
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de Broglie-Bohm theory

- Quantum theories describing stuff moving in space
  1. de Broglie-Bohm theory (wave function + particles)
  2. GRW (wave function + density matter or flashes)
  3. Everett-Many Worlds Interpretation (wave function)
- CM ontology: particles following Newtonian trajectories in 3D
- dBB ontology: particles following Bohmian trajectories in 3D + wave function in 3N-D
- The classical limit in the Bohmian framework reduces to:
  1. Making the wave function disappear (from quantum "holistic" non-locality to classical non-locality)
  2. Bohmian trajectories → Newtonian trajectories

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Holistic regime

- N-particle system: $(\mathcal{Q}, \Psi)$
  Wave function: $\Psi = \Psi(x_1, \ldots, x_N, t)$
  Configuration: $\mathcal{Q} = \mathcal{Q}(q_1, \ldots, q_N)$
- Guiding equation: $\dot{q}_k = \frac{\hbar}{m_k} \Im \frac{\nabla_k \Psi(q_1,\ldots,q_N,t)}{\Psi(q_1,\ldots,q_N,t)}$
- The velocity of one particle depends on the position of all the other particles of the configuration
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The quantum potential Q

- $Q = \frac{\hbar^2}{2m}\frac{\nabla^2 R(x_1,...,x_N)}{R(x_1,...,x_N)}$
- Q does not generally decrease with distance
- Q does not decrease when the amplitude R decreases
- Example. Gaussian state: $\psi = e^{-\beta(x-x_0)^2}$
- $F_{Q,Gaussian} = \frac{4\hbar^2\beta^2}{m}(x - x_0)$ [Bowman 2011]
- Q is selective: $Q(\psi)$ affects only the particles belonging to ψ

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Factorization of states

- $\Psi_{tot}(x_1, \ldots, x_n, t) = \psi_1(x_1, t) \ldots \psi_N(x_N, t)$
- $\frac{dq_k}{dt} = \frac{\hbar}{m_k} \Im \frac{\nabla_k \Psi_k(q_k, t)}{\Psi_k(q_k, t)}$ (local regime: classical non-locality)
- $Q_k = \frac{\hbar^2}{2m_k} \frac{\nabla_k^2 R_k(q_k, t)}{R_k(q_k, t)}$ (non-classical trajectory)
- *factorability implies physical independence* (Holland (1993))
- Problem: factorization is not realistic in the macroscopic world
- In fact, I will take the opposite view:
  interaction between systems leads to *effective factorization*

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Effective factorization by decoherence

- Bohm (1987): effective factorization is produced by spontaneous interactions with the environment (decoherence)
- $\psi(x)\phi(y) = (\psi_1(x)+\psi_2(x))\phi(y) \xrightarrow{H_{int}} \psi_1(x)\phi_1(y)+\psi_2(x)\phi_2(y)$
- if $supp(\psi_1(x)\phi_1(y)) \cap supp(\psi_2(x)\phi_2(y)) \approxeq \emptyset$
- The WF is effectively factorized in two separate channels
- $X \in supp(\psi_1(x))$ and $Y \in supp(\phi_1(y))$
- $\psi_1(x)$ becomes the EWF of the system
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- $\psi_1(x)\phi_1(y) + \psi_2(x)\phi_2(y) \approx \psi_1(x)\phi_1(y)+\psi_2(x)\phi_2(y)$

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Classical non-locality and effective wave functions

- Effective factorization by decoherence $\rightarrow$ production of EWFs
- EWFs encodes a local dynamics $\rightarrow$ classical non-locality
- Zurek, Habib & Paz (1993): pointer states are Gaussian states
  EWFs $\rightarrow$ Gaussian states
- Reduced density matrix in dBB theory in decoherence regime:
  proper mixtures of EWFs
  improper mixtures of Gaussian states

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Holland (1993)

- $\Psi(x) = R(x)e^{\frac{i}{\hbar}S(x)}$ in Schrödinger equation
- quantum Hamilton-Jacobi: $\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$
- 2^{nd} order formulation: $\frac{d^2X}{dt^2} = -\nabla V - \nabla Q$
- Newtonian motion: $Q \to 0$ (negligible $|Q|$ and ∇Q)
- 2 major problems:
  1. Q is state dependent
  2. Approach for closed systems (particle in a box problem)
- $\to$ need of considering open quantum systems (decoherence)

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Proposal: open quantum potential approach

- Idea: applying Holland's strategy to post-decohered states. In dBB theory, they are EWFs described by Gaussian states
- Gaussian state: $\psi(x) = Ae^{-\frac{x^2}{4a^2}}$
- Quantum potential: $Q = \frac{\hbar^2}{4M\beta^2}(1 - \frac{x^2}{\beta^2})$ $(\beta^2 = a^2[1 + (\frac{\hbar t}{2Ma^2})^2]$
- $Q \to 0$ for $M \to \infty$ or $\hbar \ll A_{cl}$ (macroscopic regime)
- The Bohmian trajectory of a macroscopic system becomes approximately Newtonian

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Emergence of the classical world from dBB theory

- de Broglie-Bohm theory $\rightarrow$ Newtonian mechanics under the following conditions (conditions for the classical limit):
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