

# Physics and applications of Thomson/Compton back scattering

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Introduction to Thomson and Compton back scattering.

Properties of the photon distribution

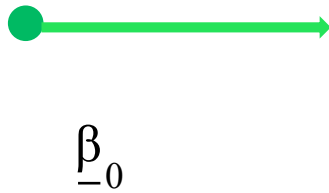
Electron distribution, deformation and information.

Examples of X and Gamma sources.

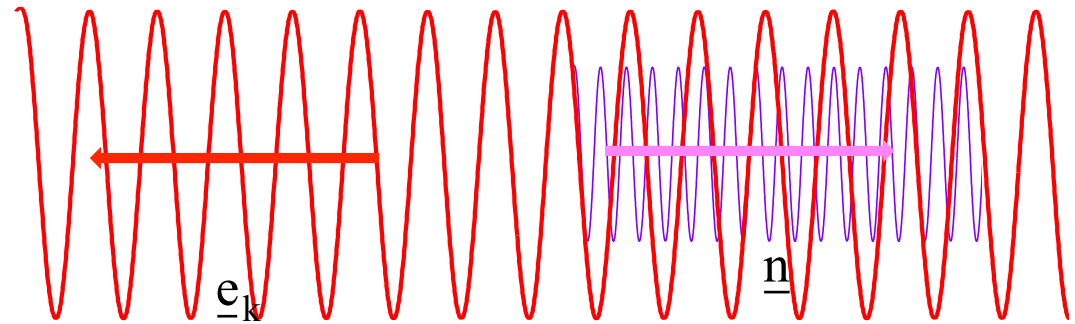
Comments and conclusions.

# Classical model

Relativistic electron



Laser pulse



X/gamma radiation

From the double Doppler effect :

$$\nu = \nu_L \frac{1 - \underline{e}_k \cdot \underline{\beta}_0}{1 - \underline{n} \cdot \underline{\beta}_0} \approx 4\gamma_0^2 \nu_L$$

$$\lambda = \lambda_L \frac{1 - \underline{n} \cdot \underline{\beta}_0}{1 - \underline{e}_k \cdot \underline{\beta}_0}$$

Head to head scattering  
Radiation on axis

$$\lambda = \lambda_L \frac{1}{4\gamma_0^2}$$

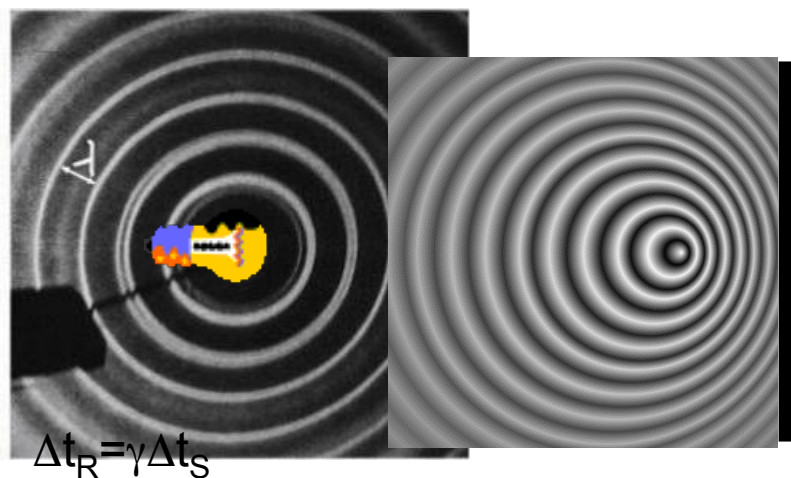


## effetto Doppler relativistico

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Sorgente ferma:  
 frequenza  $\nu_0 = 1/T_0$   
 lunghezza d'onda  $\lambda_0 = c/\nu_0$   
 N creste  
 per un tempo  $\Delta t_S$   
 $\lambda_0 = c\Delta t_S/N$

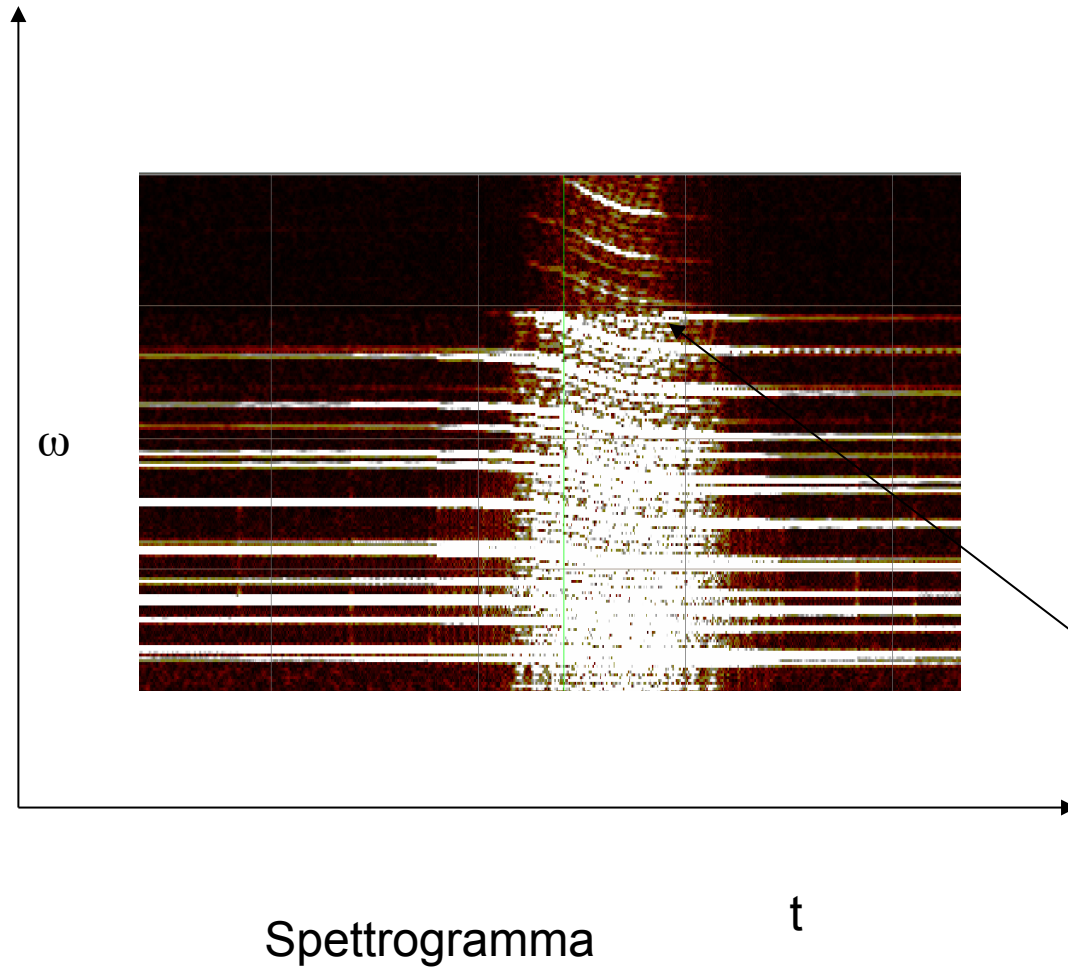


Sorgente in moto di avvicinamento  
 con velocità V vista dal ricevitore:  
 lunghezza d'onda  $\lambda = (c\Delta t_R - V\Delta t_R)/N$   
 N creste per un tempo  $\Delta t_R$

**Il numero N di creste si conserva**

$$\frac{c\Delta t_S}{\lambda_0} = \frac{(c - V)\Delta t_R}{\lambda} = \frac{\gamma(c - V)\Delta t_S}{\lambda} = \gamma v \frac{(c - V)}{c} \Delta t_S \quad \nu_0 = \sqrt{\frac{1 - \frac{V}{c}}{1 + \frac{V}{c}}} \nu$$

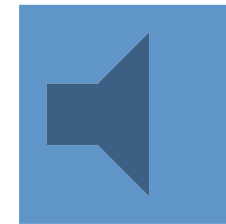
# Effetto Doppler non relativistico: esempio delle onde acustiche



(è l'analogo di uno spartito)

$$v_0 = \sqrt{\frac{(1 + \frac{V}{c})}{(1 - \frac{V}{c})}} v$$

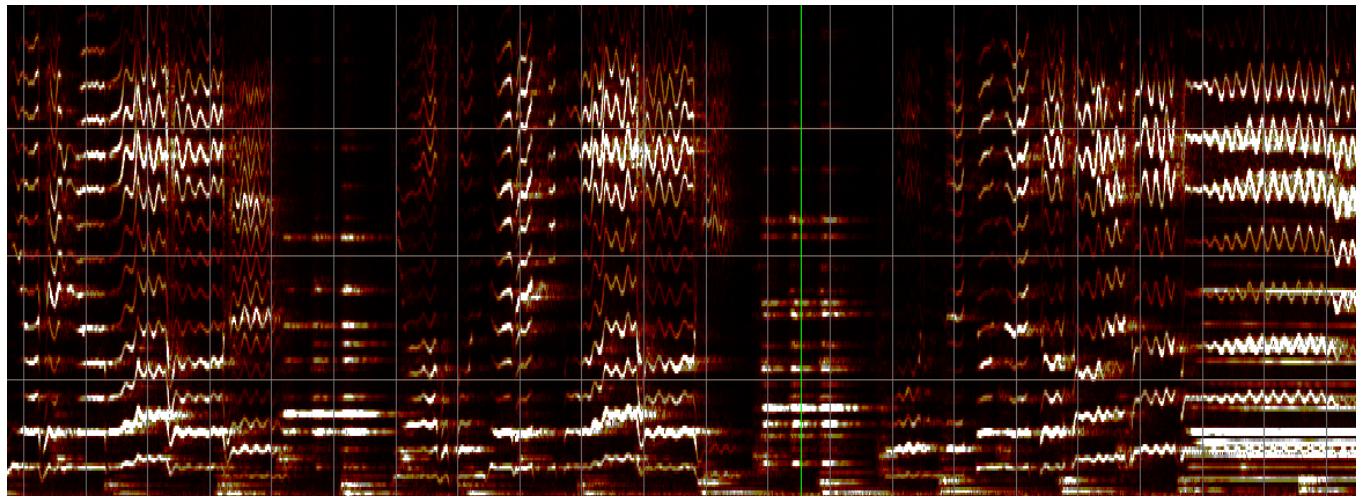
$$v = \sqrt{\frac{(1 - \frac{V}{c})}{(1 + \frac{V}{c})}} v_0$$



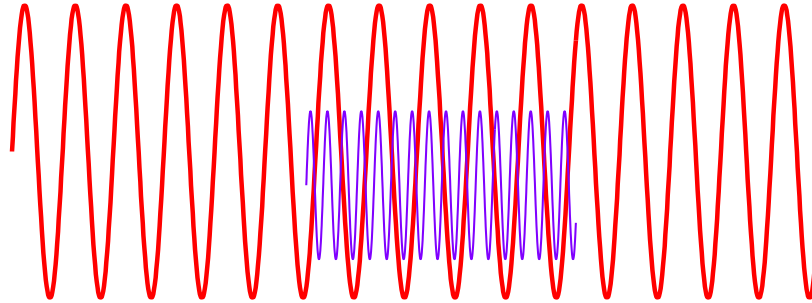
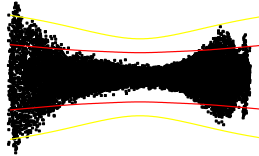
red shift della frequenza  
quando la sorgente si allontana



Spartito e Spettrogramma: Di Stefano,  
Callas, Gobbi, Lazzarini  
Bella figlia dell'amore  
Verdi: Rigoletto



# Scattering Thomson come applicazione dell'effetto Doppler



$$\nu_{L\_SRE} = \sqrt{\frac{(1 + \frac{V_e}{c})}{(1 - \frac{V_e}{c})}} \nu_{L\_Lab}$$

scattering o emissione  
nel sistema di riferimento  
dell'elettrone

$$\nu_{ST\_lab} = \sqrt{\frac{(1 + \frac{V_e}{c})}{(1 - \frac{V_e}{c})}} \nu_{ST\_sre}$$

$$\nu_{ST\_SRE} = \nu_{L\_SRE}$$

$$\nu_{ST\_lab} = \sqrt{\frac{(1 + \frac{V_e}{c})}{(1 - \frac{V_e}{c})}} \nu_{L\_SRE}$$

$$= \sqrt{\frac{(1 + \frac{V_e}{c})^2}{(1 - \frac{V_e}{c})^2}} \nu_{L\_Lab}$$

$$\frac{(1 + \frac{V_e}{c})}{(1 - \frac{V_e}{c})} \nu_{L\_Lab} = \frac{(1 + (\frac{V_e}{c}))^2}{(1 - (\frac{V_e}{c})^2)} \nu_{L\_Lab}$$

$$\approx 4\gamma^2 \nu_{L\_Lab}$$

## A Frascati: SL\_Thomson

Laser Ti::Sa

$\lambda_0=800$  nm, P=1 TW

Elettroni

E=30 MeV

$\gamma=60$

Radiazione emessa:

$$\lambda=\lambda_0/4\gamma^2$$

$$=8 \cdot 10^{-7}/(4 \cdot 60^2)\text{m}$$

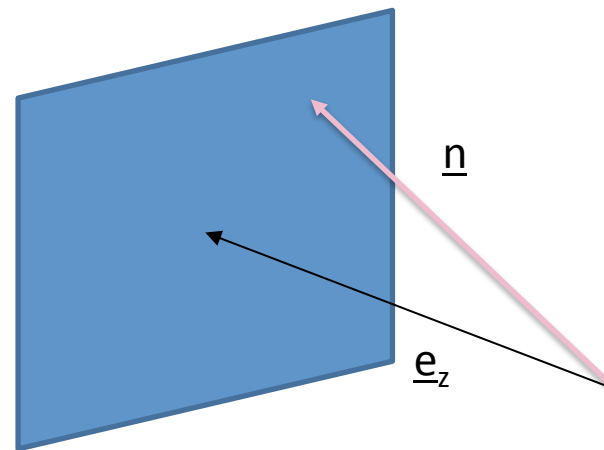
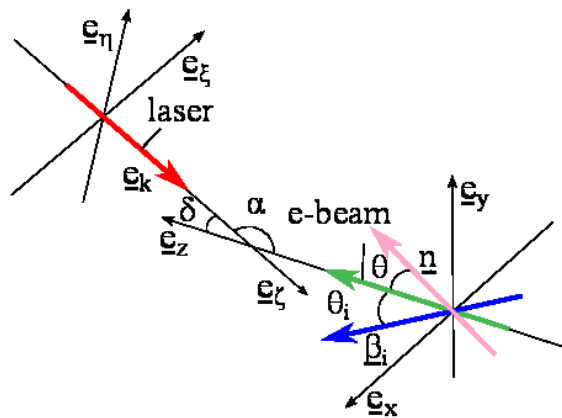
$$=5.55 \cdot 10^{-10}\text{m}$$

**Raggi X!!**



From the electron orbits and the Liénard-Wiechert potentials **in the far zone** one can write the expression of the electric field [Jackson..]:

$$\mathbf{E} = \frac{e}{c} \left[ \frac{\mathbf{n} \times \left[ (\mathbf{n} - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t') \right]}{R(1 - \mathbf{n} \cdot \boldsymbol{\beta}(t'))^3} \right]_{ret}$$



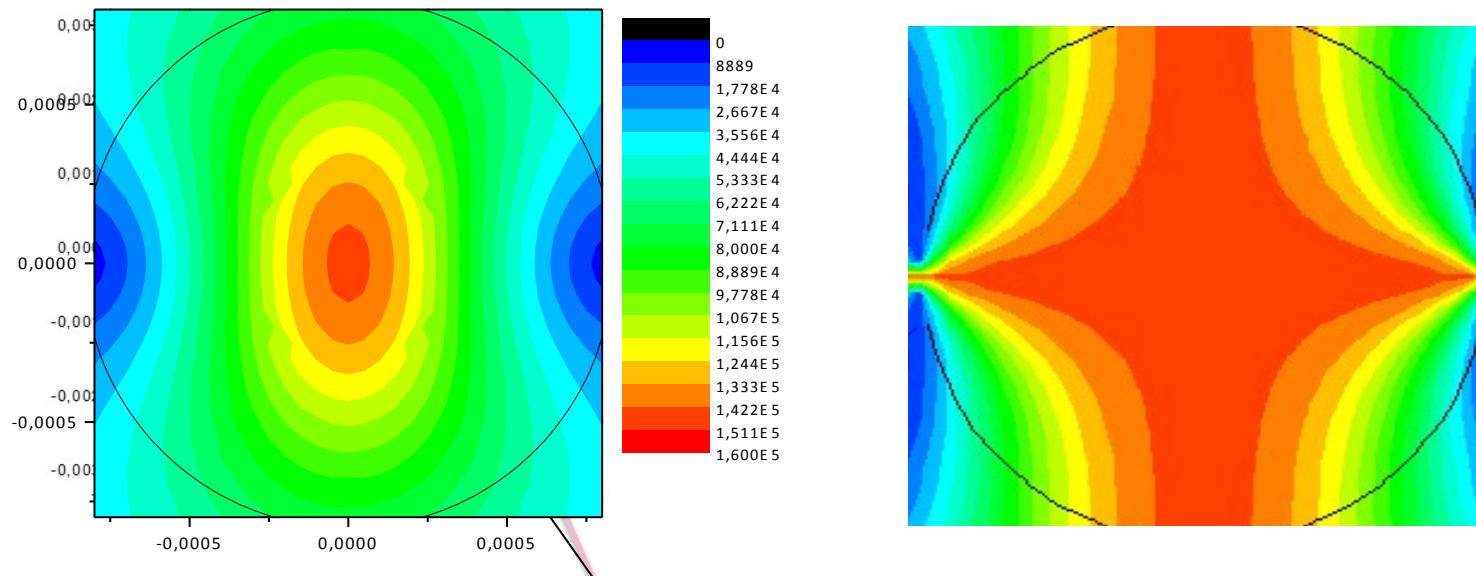
From the motion equation of the electrons

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E}_L + \boldsymbol{\beta} \times \mathbf{B}_L)$$

If  $\mathbf{E}$  and  $\mathbf{B}=\mathbf{k}\times\mathbf{E}$  are electric and magnetic field of the incoming laser,

$$\dot{\boldsymbol{\beta}} = \frac{d\boldsymbol{\beta}}{dt} = -\frac{e}{mc\gamma} (\mathbf{E}_L (1 - \boldsymbol{\beta} \cdot \mathbf{e}_k) + \boldsymbol{\beta} \cdot \mathbf{E}_L (\mathbf{k} - \boldsymbol{\beta}))$$

Total intensity and Stokes parameter  $|E_x|^2 - |E_y|^2$  on the screen at 1 m,  $\gamma=1200$



# Classical double differential spectrum

The double differential spectrum for **one electron** is:

$$\frac{d^2 W_i}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} dt e^{i\omega t} \frac{\mathbf{n} \times \left[ (\mathbf{n} - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t') \right]}{(1 - \mathbf{n} \cdot \boldsymbol{\beta}(t'))^3} \right|^2 = \hbar\omega \frac{d^2 N_i}{d\omega d\Omega}$$

**And for all the beam:**

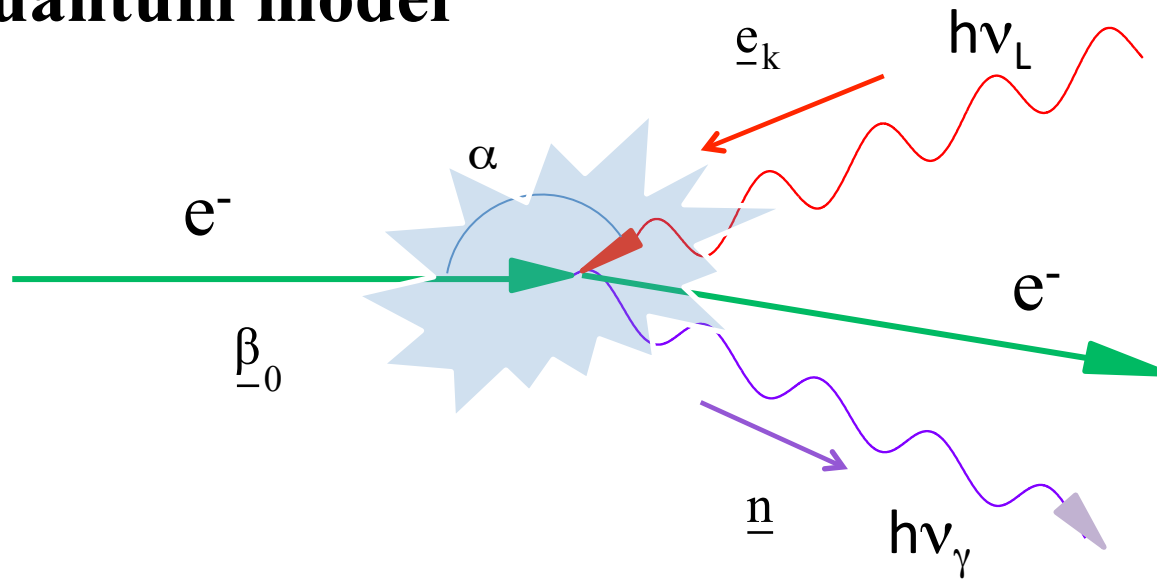
$$\hbar\omega \frac{d^2 N}{d\omega d\Omega} = \hbar\omega \sum_i \frac{d^2 N_i}{d\omega d\Omega}$$

$$\Psi \equiv \gamma \vartheta_M$$

$$N(\Psi) \equiv \pi \alpha \mathfrak{S} N_e \left( \frac{cT}{\lambda} \right) a_0^2 \Psi^2 \frac{(1 + \Psi^2 + \frac{2}{3} \Psi^4)}{(1 + \Psi^2)^3}$$

Full treatment of linear and nonlinear TS for a plane-wave laser pulse with analytical expression of the distributions as well as several approximate expressions in *P. Tomassini et al.*, Appl. Phys. B **80**, 419 (2005).

# Quantum model



Before the scattering

Electron energy  $mc^2\gamma_0$   
momentum  $mc\underline{\beta}\gamma_0$

Photon energy  $h\nu_L$   
momentum  $h\underline{k}_L/2\pi$

$$mc^2\gamma_0 + h\nu_L = h\nu + mc^2\gamma$$

$$mc\underline{\beta}_0\gamma_0 + h\underline{k}_L/2\pi = mc\underline{\beta}\gamma + h\underline{k}/2\pi$$

After the scattering

Electron energy  $mc^2\gamma$   
momentum  $mc\underline{\beta}\gamma$

Photon energy  $h\nu$   
momentum  $h\underline{k}/2\pi$

Energy and momentum conservation laws  $\gamma_0$ : initial Lorentz factor

$$\left\{ \begin{array}{l} mc^2(\gamma - \gamma_0) = -h(\nu - \nu_L) \\ mc(\underline{\beta}\gamma - \underline{\beta}_0\gamma_0) = -h(\underline{k} - \underline{k}_L)/2\pi \end{array} \right.$$

$$\left\{ \begin{array}{l} mc^2(\gamma - \gamma_0) = -h(\nu - \nu_L) \\ mc(\underline{\beta}\gamma - \underline{\beta}_0\gamma_0) = -h(\underline{k} - \underline{k}_L)/2\pi \end{array} \right.$$

Energy and momentum conservation laws

$\gamma_0$ : initial Lorentz factor

$$\left\{ \begin{array}{l} (\gamma - \gamma_0) = -h(\nu - \nu_L)/mc^2 \\ (\underline{\beta}\gamma - \underline{\beta}_0\gamma_0) = -h(\underline{k} - \underline{k}_L)/(2\pi mc) \end{array} \right.$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta^2 = 1 - \frac{1}{\gamma^2}$$

$$\beta^2\gamma^2 = \beta_0^2\gamma_0^2 + h^2(k^2 + k_L^2 - 2\underline{k} \cdot \underline{k}_L)/(2\pi mc)^2 - 2h\gamma_0(\underline{\beta}_0 \cdot \underline{k} - \underline{\beta}_0 \cdot \underline{k}_L)/(2\pi mc)$$

$$\cancel{\gamma^2 - 1} \quad \cancel{\gamma_0^2 - 1}$$

$$\beta^2 = 1 - 1/\gamma^2 \quad \Rightarrow \quad \gamma^2\beta^2 = \gamma^2 - 1 \quad \beta_0^2 = 1 - 1/\gamma_0^2 \quad \Rightarrow \quad \gamma_0^2\beta_0^2 = \gamma_0^2 - 1$$

$$\gamma^2 = \gamma_0^2 + h^2(k^2 + k_L^2 - 2\underline{k} \cdot \underline{k}_L)/(2\pi mc)^2 - 2h\gamma_0(\underline{\beta}_0 \cdot \underline{k} - \underline{\beta}_0 \cdot \underline{k}_L)/(2\pi mc)$$

$$\gamma^2 = \gamma_0^2 + \frac{h^2(k^2 + k_L^2 - 2\underline{k} \cdot \underline{k}_L)}{(2\pi mc)^2} - 2h\gamma_0 (\underline{\beta}_0 \cdot \underline{k} - \underline{\beta}_0 \cdot \underline{k}_L) / (2\pi mc)$$

$$(\gamma - \gamma_0) = -h(v - v_L) / mc^2$$

$$\gamma = \gamma_0 - h(v - v_L) / mc^2$$

$$\gamma^2 = \gamma_0^2 - 2\gamma_0 h(v - v_L) / mc^2 + h^2(v - v_L)^2 / (mc^2)^2$$

~~$$\gamma_0^2 - 2\gamma_0 h(v - v_L) / mc^2 + h^2(v - v_L)^2 / (mc^2)^2$$~~

~~$$\gamma_0^2 = \frac{h^2(k^2 + k_L^2 - 2\underline{k} \cdot \underline{k}_L)}{(2\pi mc)^2} - 2h\gamma_0 (\underline{\beta}_0 \cdot \underline{k} - \underline{\beta}_0 \cdot \underline{k}_L) / (2\pi mc)$$~~

$$-2\gamma_0 h(v - v_L) / mc^2 + h^2(v - v_L)^2 / (mc^2)^2$$

$$= \frac{h^2(k^2 + k_L^2 - 2\underline{k} \cdot \underline{k}_L)}{(2\pi mc)^2} - 2h\gamma_0 (\underline{\beta}_0 \cdot \underline{k} - \underline{\beta}_0 \cdot \underline{k}_L) / (2\pi mc)$$

$$\begin{aligned}
 & -2\gamma_0 (\nu - \nu_L)/mc^2 + h(\nu - \nu_L)^2/(mc^2)^2 \\
 & = h(k^2 + k_L^2 - 2\mathbf{k} \cdot \mathbf{k}_L)/(2\pi mc)^2 - 2\gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L)/(2\pi mc)
 \end{aligned}$$

$$\begin{aligned}
 & -2\gamma_0 (\nu - \nu_L)/c^2 + h(\nu - \nu_L)^2/(mc^4) \\
 & = h(k^2 + k_L^2 - 2\mathbf{k} \cdot \mathbf{k}_L)/m(2\pi c)^2 - 2\gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L)/(2\pi c)
 \end{aligned}$$

$$\nu = c/\lambda = ck/2\pi \qquad \nu_L = c/\lambda_L = ck_L/2\pi$$

$$\begin{aligned}
 & -2\gamma_0 (\nu - \nu_L)/c^2 + h(\nu^2 + \nu_L^2 - 2\nu\nu_L)/(mc^4) \\
 & = h(k^2 + k_L^2 - 2\mathbf{k} \cdot \mathbf{k}_L)/m(2\pi c)^2 - 2\gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L)/(2\pi c)
 \end{aligned}$$

$$\begin{aligned}
 & -2\gamma_0 (\nu - \nu_L)/c^2 - h 2\nu\nu_L/(mc^4) \\
 & \qquad = -h 2\mathbf{k} \cdot \mathbf{k}_L/m(2\pi c)^2 - 2\gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L)/(2\pi c)
 \end{aligned}$$



$$\begin{aligned}
 & -2\gamma_0 (v-v_L)/c^2 - h 2v v_L / (mc^4) \\
 & = -h 2\mathbf{k} \cdot \mathbf{k}_L / m(2\pi c)^2 - 2\gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L) / (2\pi c)
 \end{aligned}$$

$$\gamma_0 (v-v_L)/c^2 + h v v_L / (mc^4) = h \mathbf{k} \cdot \mathbf{k}_L / m(2\pi c)^2 + \gamma_0 (\beta_0 \cdot \mathbf{k} - \beta_0 \cdot \mathbf{k}_L) / (2\pi c)$$

$$\begin{aligned}
 v = c/\lambda = ck/2\pi & \quad v_L = c/\lambda_L = ck_L/2\pi & \quad \mathbf{n} = \mathbf{k}/k & \quad \mathbf{e}_{k_L} = \mathbf{k}_L/k_L \\
 \mathbf{k} = 2\pi v \mathbf{n} / c & \quad \mathbf{k}_L = 2\pi v_L \mathbf{e}_{k_L} / c
 \end{aligned}$$

$$\gamma_0 (v-v_L)/c^2 + h v v_L / (mc^4) = h v v_L \mathbf{n} \cdot \mathbf{e}_{k_L} / (mc^4) + \gamma_0 (v\beta_0 \cdot \mathbf{n} - v_L \beta_0 \cdot \mathbf{e}_{k_L}) / c^2$$

$$\gamma_0 (v-v_L) + h v v_L / (mc^2) = h v v_L \mathbf{n} \cdot \mathbf{e}_{k_L} / (mc^2) + \gamma_0 (v\beta_0 \cdot \mathbf{n} - v_L \beta_0 \cdot \mathbf{e}_{k_L})$$

$$\gamma_0 (v-v_L) - \gamma_0 (v\beta_0 \cdot \mathbf{n} - v_L \beta_0 \cdot \mathbf{e}_{k_L}) = -h v v_L / (mc^2) + h v v_L \mathbf{n} \cdot \mathbf{e}_{k_L} / (mc^2)$$

$$\gamma_0 (v-v_L) - \gamma_0 (v\beta_0 \cdot \mathbf{n} - v_L \beta_0 \cdot \mathbf{e}_{k_L}) = -h v v_L (1 - \mathbf{n} \cdot \mathbf{e}_{k_L}) / (mc^2)$$

$$\gamma_0 c(1/\lambda - 1/\lambda_L) - c\gamma_0 (\beta_0 \cdot \mathbf{n} / \lambda - \beta_0 \cdot \mathbf{e}_{k_L} / \lambda_L) = -h (1 - \mathbf{n} \cdot \mathbf{e}_{k_L}) / (\lambda \lambda_L m)$$

$$\gamma_0 c(1/\lambda - 1/\lambda_L) - c\gamma_0 (\underline{\beta}_0 \cdot \underline{n}/\lambda - \underline{\beta}_0 \cdot \underline{e}_{kL}/\lambda_L) = -h(1 - \underline{n} \cdot \underline{e}_{kL})/(\lambda\lambda_L m)$$

$$\gamma_0 c \frac{\lambda_L - \lambda}{\lambda_L \lambda} - \gamma_0 c \frac{\underline{\beta}_0 \cdot \underline{n} \lambda_L - \underline{\beta}_0 \cdot \underline{e}_{kL} \lambda}{\lambda_L \lambda} = \frac{-h(1 - \underline{n} \cdot \underline{e}_{kL})}{m \lambda_L \lambda}$$

$$(\lambda_L - \lambda) - (\underline{\beta}_0 \cdot \underline{n} \lambda_L - \underline{\beta}_0 \cdot \underline{e}_{kL} \lambda) = \frac{-h(1 - \underline{n} \cdot \underline{e}_{kL})}{m c \gamma_0}$$

$$(1 - \underline{\beta}_0 \cdot \underline{n}) \lambda_L - (1 - \underline{\beta}_0 \cdot \underline{e}_{kL}) \lambda = \frac{-h(1 - \underline{n} \cdot \underline{e}_{kL})}{m c \gamma_0}$$

$$\lambda = \lambda_L \frac{1 - \underline{n} \cdot \underline{\beta}_0}{1 - \underline{e}_{kL} \cdot \underline{\beta}_0} + \frac{h}{m c \gamma_0} \frac{1 - \underline{e}_{kL} \cdot \underline{n}}{1 - \underline{e}_{kL} \cdot \underline{\beta}_0}$$

$$\gamma_0 (v - v_L) - \gamma_0 (v \beta_0 \cdot \underline{n} - v_L \beta_0 \cdot \underline{e}_{kL}) = -h \nu v_L (1 - \underline{n} \cdot \underline{e}_{kL}) / (mc^2)$$

$$\gamma_0 (v) - \gamma_0 v \beta_0 \cdot \underline{n} + h \nu v_L (1 - \underline{n} \cdot \underline{e}_{kL}) / (mc^2) = -\gamma_0 v_L \beta_0 \cdot \underline{e}_{kL} + \gamma_0 v_L$$

$$v (\gamma_0 - \gamma_0 \beta_0 \cdot \underline{n} + h \nu v_L (1 - \underline{n} \cdot \underline{e}_{kL}) / (mc^2)) = -\gamma_0 v_L (\beta_0 \cdot \underline{e}_{kL} - 1)$$

$$v = v_L \frac{1 - \underline{e}_k \cdot \beta_0}{1 - \underline{n} \cdot \beta_0 + \frac{h \nu v_L}{mc^2 \gamma_0} (1 - \underline{e}_k \cdot \underline{n})}$$

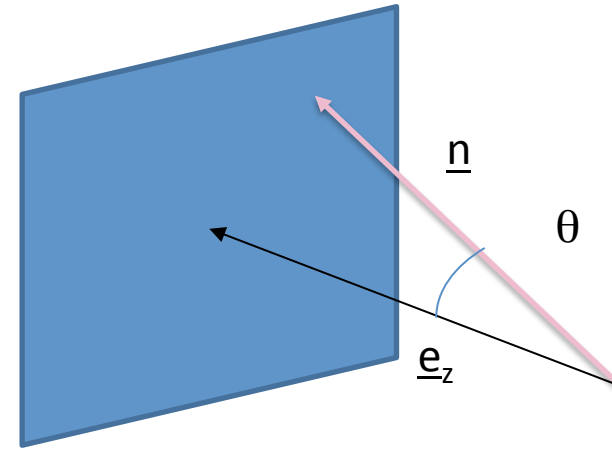
$$\lambda = \lambda_L \frac{1 - \underline{n} \cdot \underline{\beta}_0}{1 - \underline{e}_k \cdot \underline{\beta}_0} + \frac{h}{mc\gamma_0} \frac{1 - \underline{e}_k \cdot \underline{n}}{1 - \underline{e}_k \cdot \underline{\beta}_0}$$

If  $\underline{e}_k = -\underline{e}_z$  and  $\underline{\beta}_0 = \beta_0 \underline{e}_z$

$$\lambda = \lambda_L \frac{1 - \beta_0 \cos \theta}{1 + \beta_0} + \frac{h}{mc\gamma_0} \frac{1 + \cos \theta}{1 + \beta_0}$$

For  $\beta=0, \gamma=1$

$$\lambda = \lambda_L + \frac{h}{mc} (1 + \cos \theta)$$

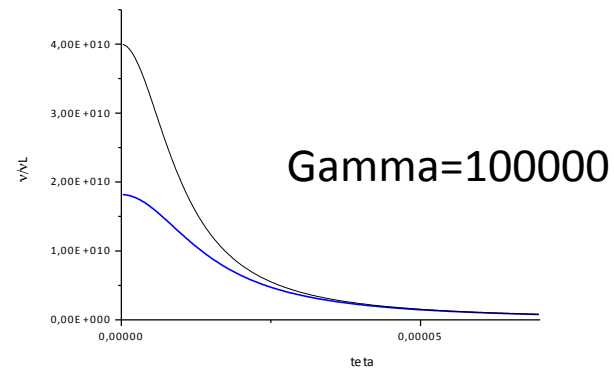
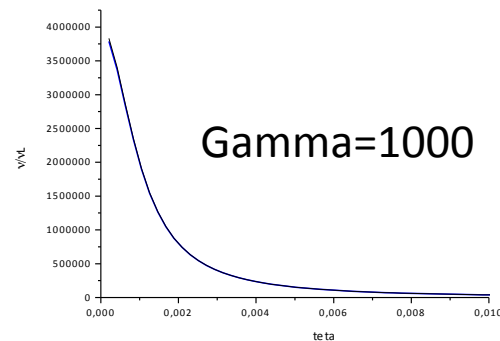


For example  
 $\lambda_L = 0.8 \cdot 10^{-6}$

$$\frac{h\nu_L}{mc^2} = \frac{1.5}{0.511 \cdot 10^6} = 3 \cdot 10^{-6}$$

$$\nu = \nu_L \frac{1 - \underline{e}_k \cdot \underline{\beta}_0}{1 - \underline{n} \cdot \underline{\beta}_0 + \frac{h\nu_L}{mc^2\gamma_0} (1 - \underline{e}_k \cdot \underline{n})}$$

$$\nu = \nu_L \frac{1 + \beta_0}{1 - \beta_0 \cos \theta + \frac{h\nu_L}{mc^2\gamma_0} (1 + \cos \theta)}$$



On the axis:  $\theta=0$

$$\lambda = \lambda_L \frac{1 - \beta_0 \cos \theta}{1 + \beta_0} + \frac{h}{mc\gamma_0} \frac{1 + \cos \theta}{1 + \beta_0}$$

$$\lambda = \lambda_L \frac{1 - \beta_0}{1 + \beta_0} + \frac{h}{mc\gamma_0} \frac{2}{1 + \beta_0}$$

And with  $\gamma \gg 1$

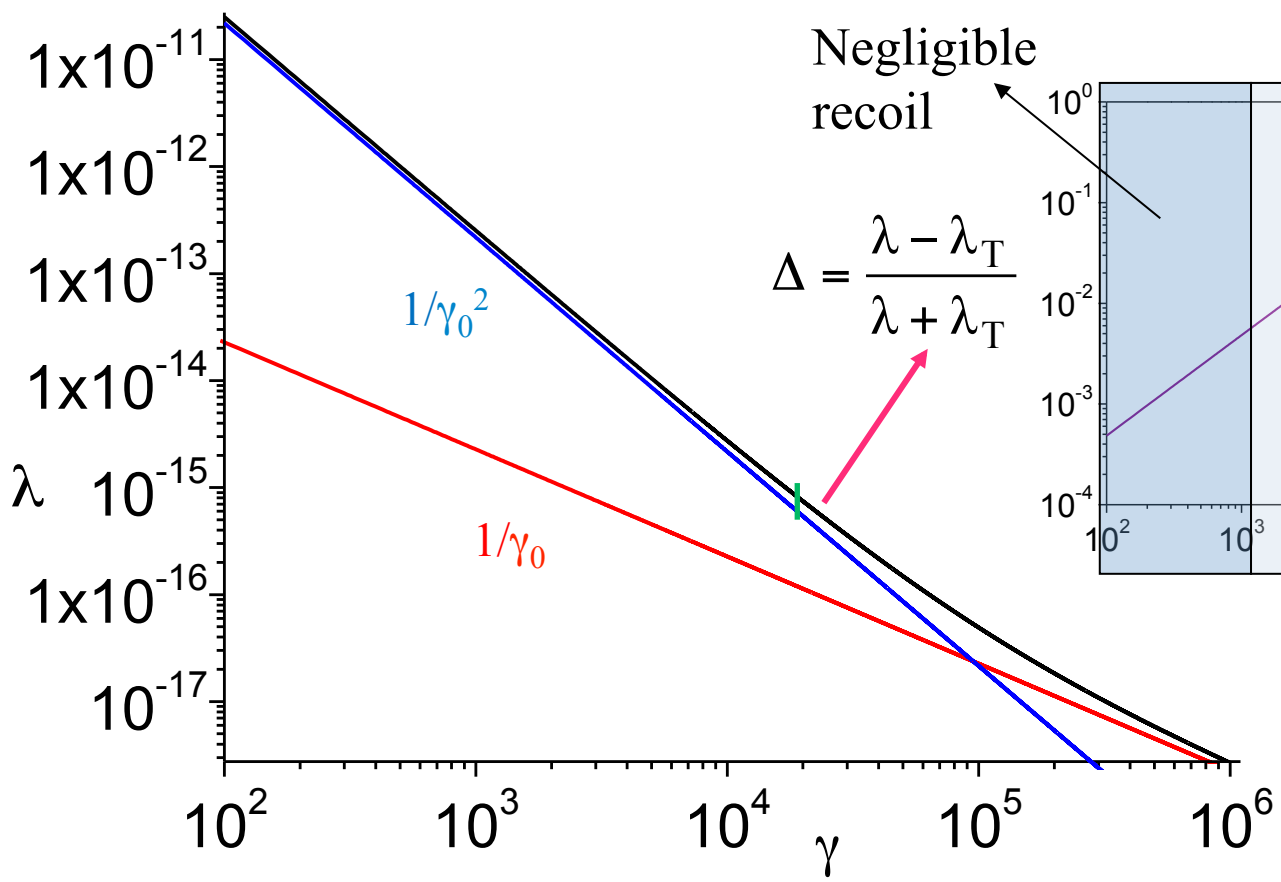
$$\lambda = \frac{\lambda_L}{4\gamma_0^2} + \frac{h}{mc\gamma_0}$$

Back-scattering  
Radiation on-axis

$$\lambda = \lambda_L \frac{1}{4\gamma_0^2} + \frac{h}{mc\gamma_0}$$

Thomson  
factor  $\lambda_T$

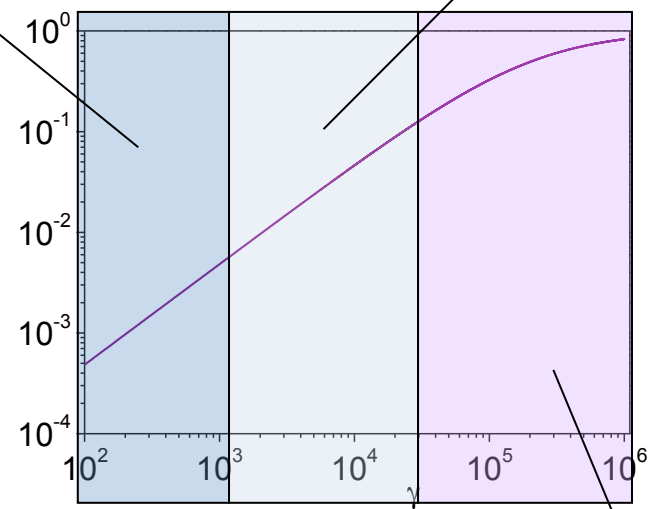
Compton  
red shift



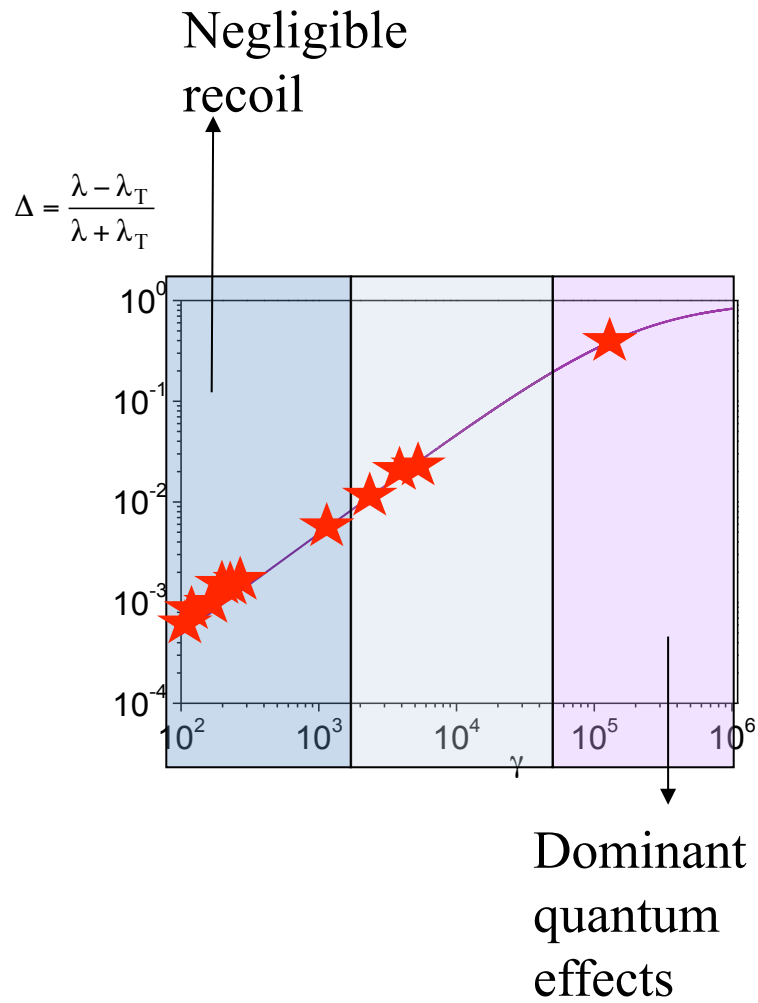
Negligible  
recoil

$$\Delta = \frac{\lambda - \lambda_T}{\lambda + \lambda_T}$$

Intermediate region



Dominant  
quantum  
effects



CLS	25 MeV	12 keV
PhoeniX	25-30 MeV	12-20 keV
Pleiades	20-100 MeV	20-500 keV
PlasmonX	30-150 MeV	20-500 keV
LSS (ATF)	60 MeV	80 keV
STAR		

# Quantum cross-section for electron-photon interaction (Klein and Nishina)

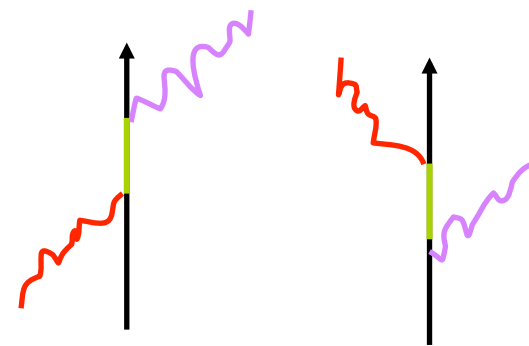
Dirac Equation: 
$$i \frac{\partial \psi}{\partial t} = (\hat{H}_0 + \hat{H}_{\text{int}}) \psi$$

Radiation potential:

$$\hat{A} = hc(\underline{e}_L \sqrt{\frac{1}{\omega_L}} (e^{i(\underline{k}_L \cdot \underline{r} - \omega_L t)} \hat{a}_L + e^{-i(\underline{k}_L \cdot \underline{r} - \omega_L t)} \hat{a}_L^\dagger) + \underline{e} \sqrt{\frac{1}{\omega}} (e^{i(\underline{k} \cdot \underline{r} - \omega t)} \hat{a} + e^{-i(\underline{k} \cdot \underline{r} - \omega t)} \hat{a}^\dagger))$$

Transition probability (perturbation theory or Feynman-Dyson graphs):

$$W_{n,m} = \frac{2\pi}{\hbar} \rho \left| \sum_{n'} \frac{H_{m,n'} H_{n',n}}{E_m - E_{n'}} + \sum_{n''} \frac{H_{m,n''} H_{n'',n}}{E_m - E_{n''}} \right|^2$$





Cross section **in the electron frame:**

$$\left( \frac{d\sigma}{d\Omega} \right)' = \frac{r_0^2}{4} \left( \frac{v'}{v'_L} \right)^2 \left[ 4(\underline{e}' \cdot \underline{e}'_L)^2 - 2 + \frac{v'}{v'_L} + \frac{v'_L}{v'} \right]$$

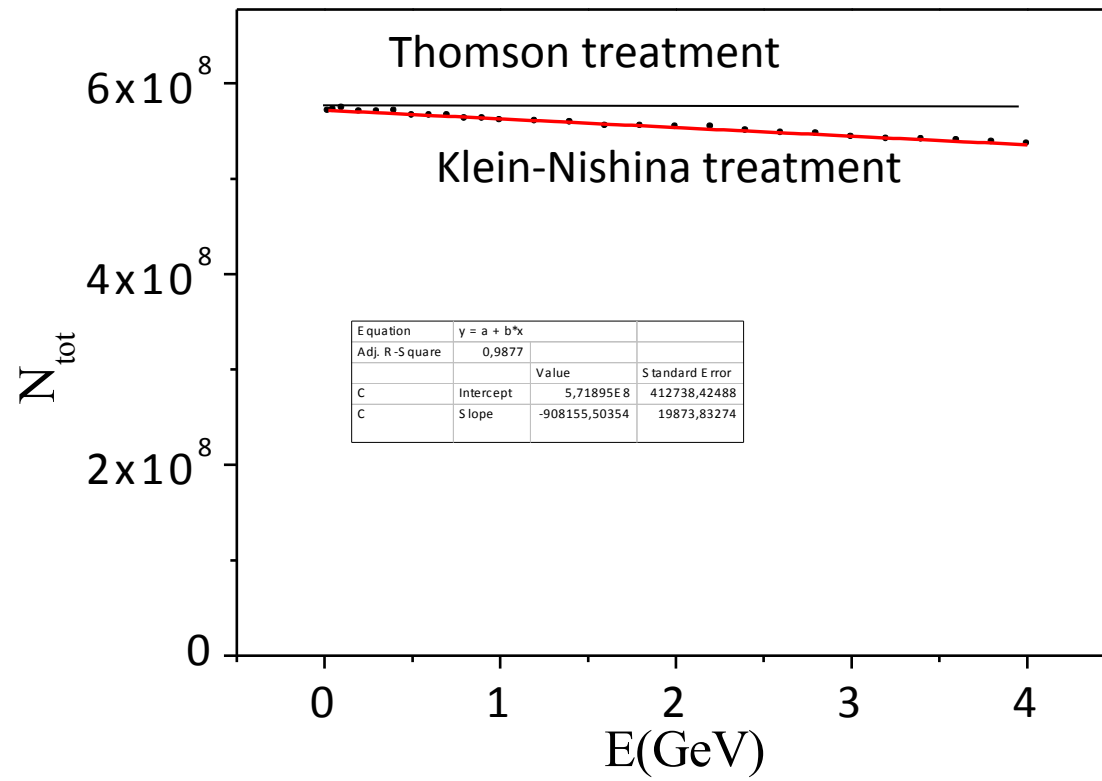
By Lorentz transforming frequencies, polarizations and differentials:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2 X}{4\gamma_0^2 (1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L)^2} \left( \frac{v}{v_L} \right)^2$$

$$X = \frac{v_L}{v} \frac{(1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L)}{1 - \underline{n} \cdot \underline{\beta}_0} + \frac{v}{v_L} \frac{1 - \underline{n} \cdot \underline{\beta}_0}{(1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L)} - 1 + \left( 1 + \frac{mc^2}{h\nu\gamma_0} \frac{1}{1 - \underline{n} \cdot \underline{\beta}_0} - \frac{mc^2}{h\nu_L\gamma_0} \frac{1}{1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L} \right)^2$$

Double differential cross section **in the laboratory frame:**

$$\frac{d^2\sigma}{d\nu d\Omega} = \frac{r_0^2 X}{2\pi 2\gamma_0^2 (1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L)^2} \left( \frac{v}{v_L} \right)^2 \delta(v - v_L \frac{1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L}{1 - \underline{n} \cdot \underline{\beta}_0 + \frac{h\nu_L}{mc^2\gamma_0} (1 - \underline{n} \cdot \hat{\underline{k}}_L)})$$



$$\sigma_{\text{K-N}} \approx \sigma_{\text{T}} \left( 1 - \frac{2\hbar\omega_{\text{L}}}{mc^2} \gamma_{\text{e}} \right)$$

## Extension to electron-laser beams

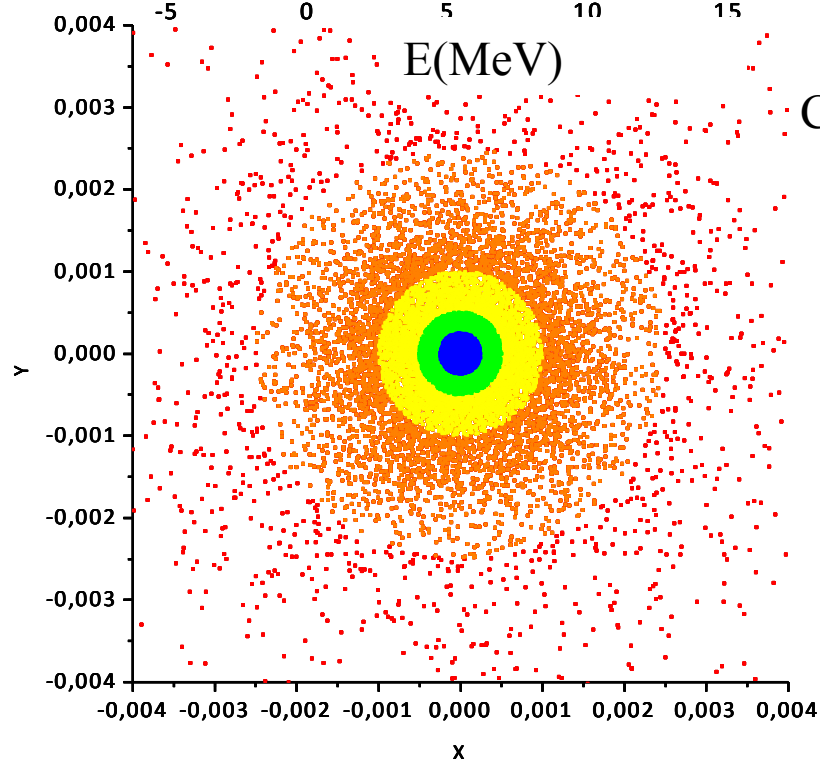
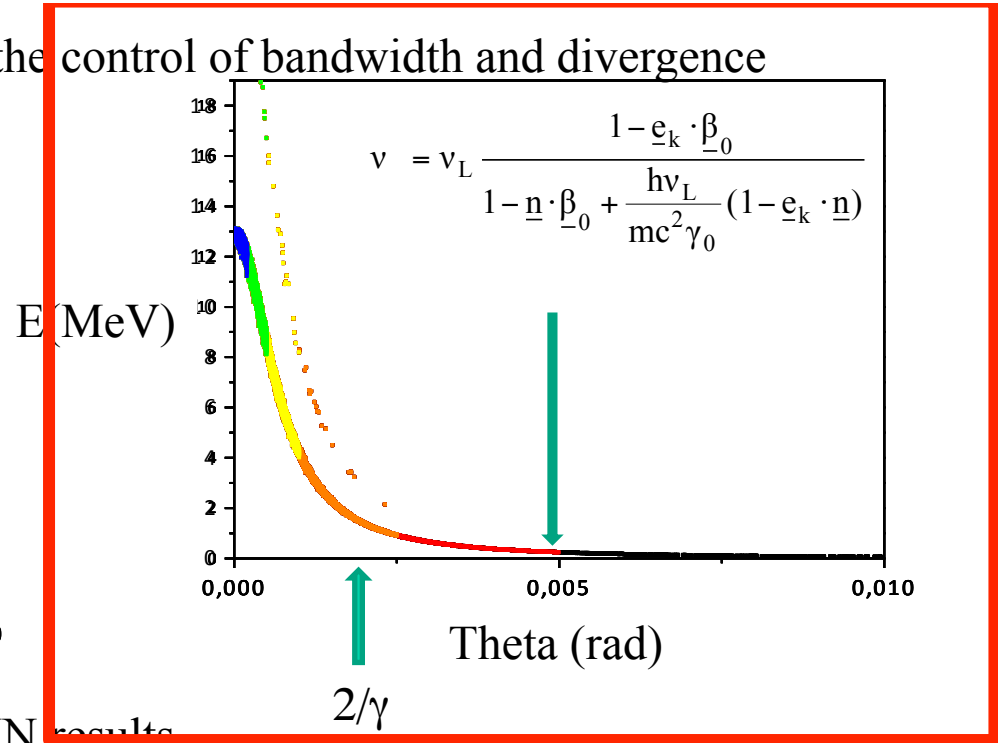
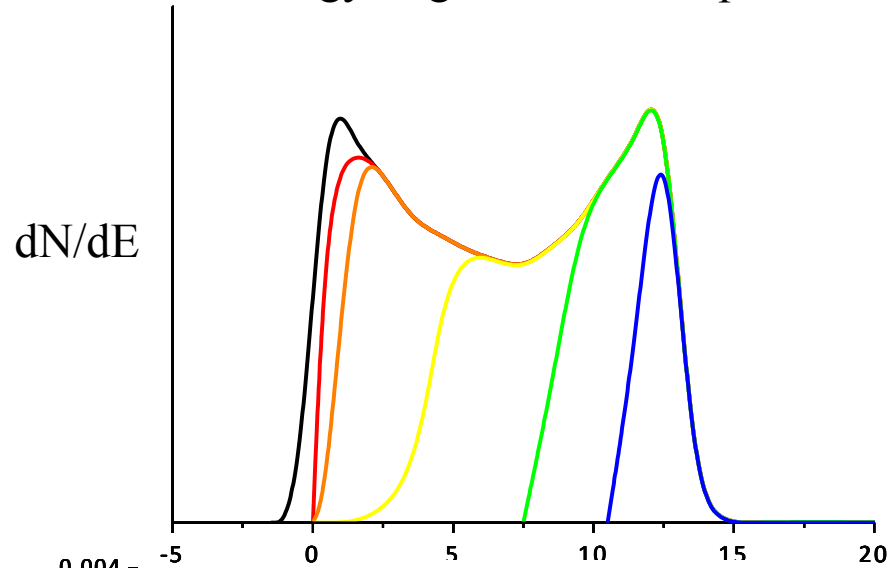
$$\frac{d^2N}{dv d\Omega} = \hbar c \int \frac{d^2\sigma}{dv d\Omega} \frac{d^2N_e}{d\underline{x} d\underline{p}} \frac{d^2N_L}{d\underline{x}_L d\underline{k}_L} (1 - \underline{\beta}_0 \cdot \hat{\underline{k}}_L) dt d\underline{x} d\underline{p} d\underline{k}_L$$

$$\frac{d^2N_e}{d\underline{x} d\underline{p}} = \delta(\underline{x} - \underline{x}_j(t)) \delta(\underline{p} - \underline{p}_j(t))$$

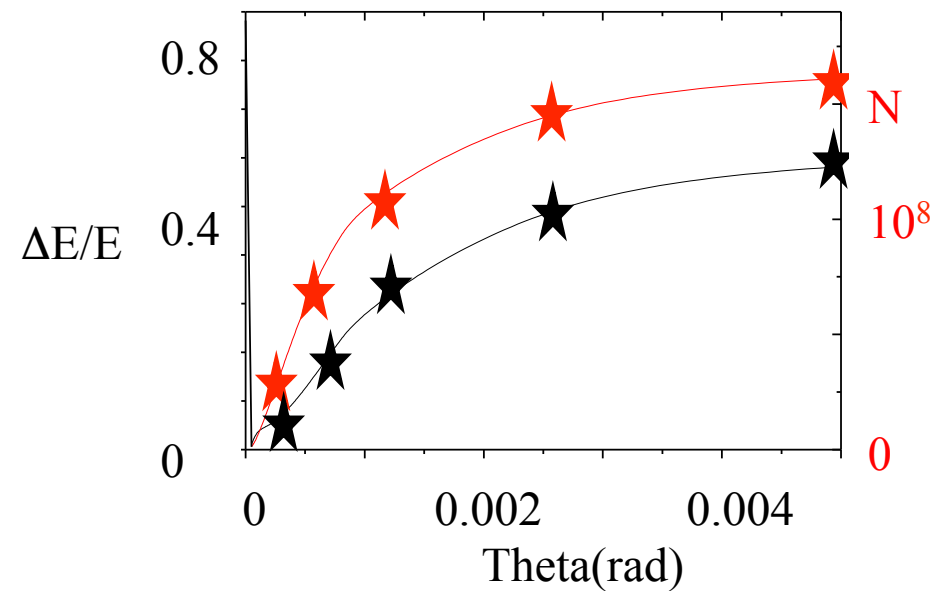
$$\frac{d^2N_L}{d\underline{x}_L d\underline{k}_L}$$

Longitudinal and transverse Gaussian profiles

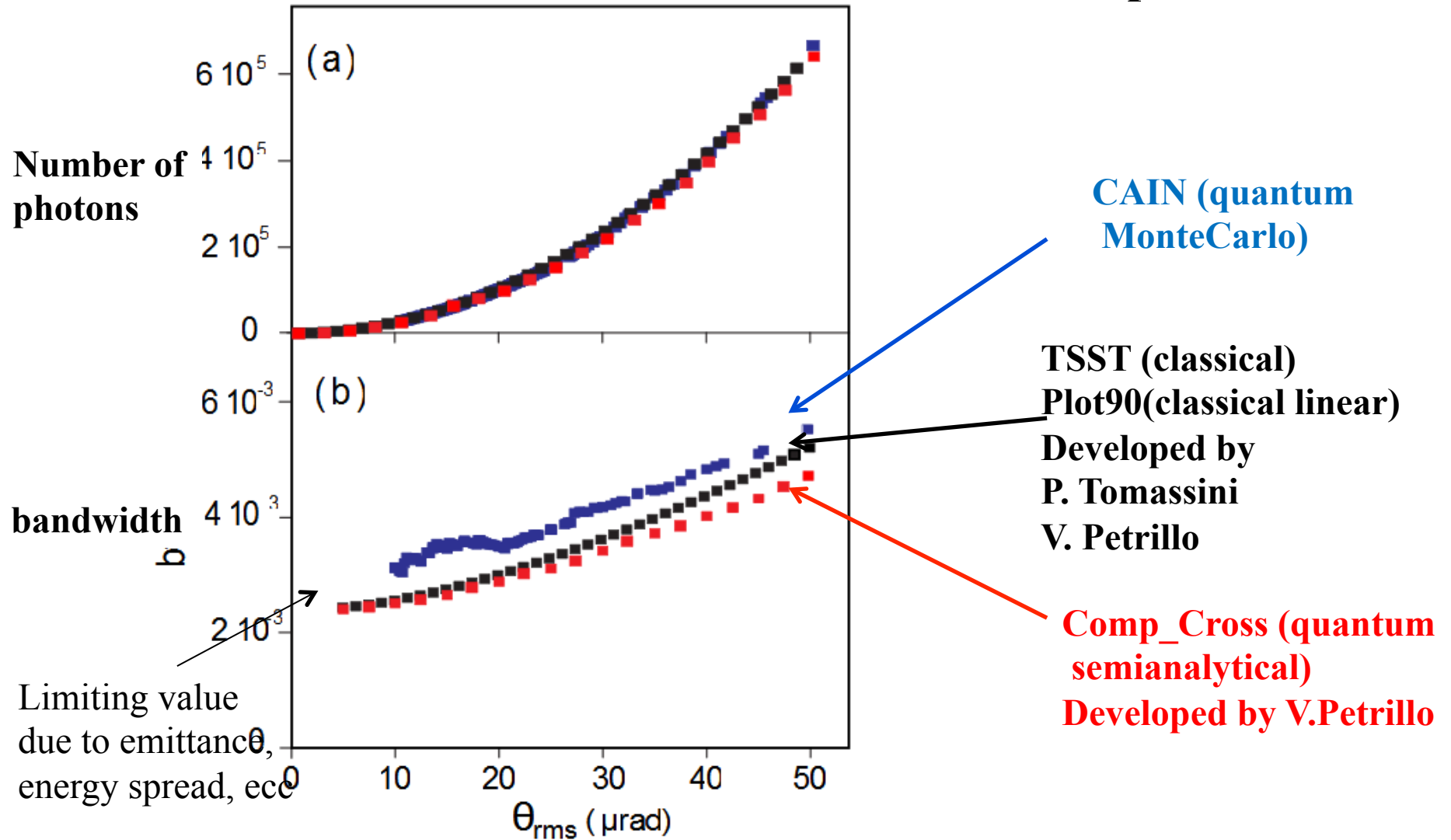
The energy-angle correlation permits the control of bandwidth and divergence

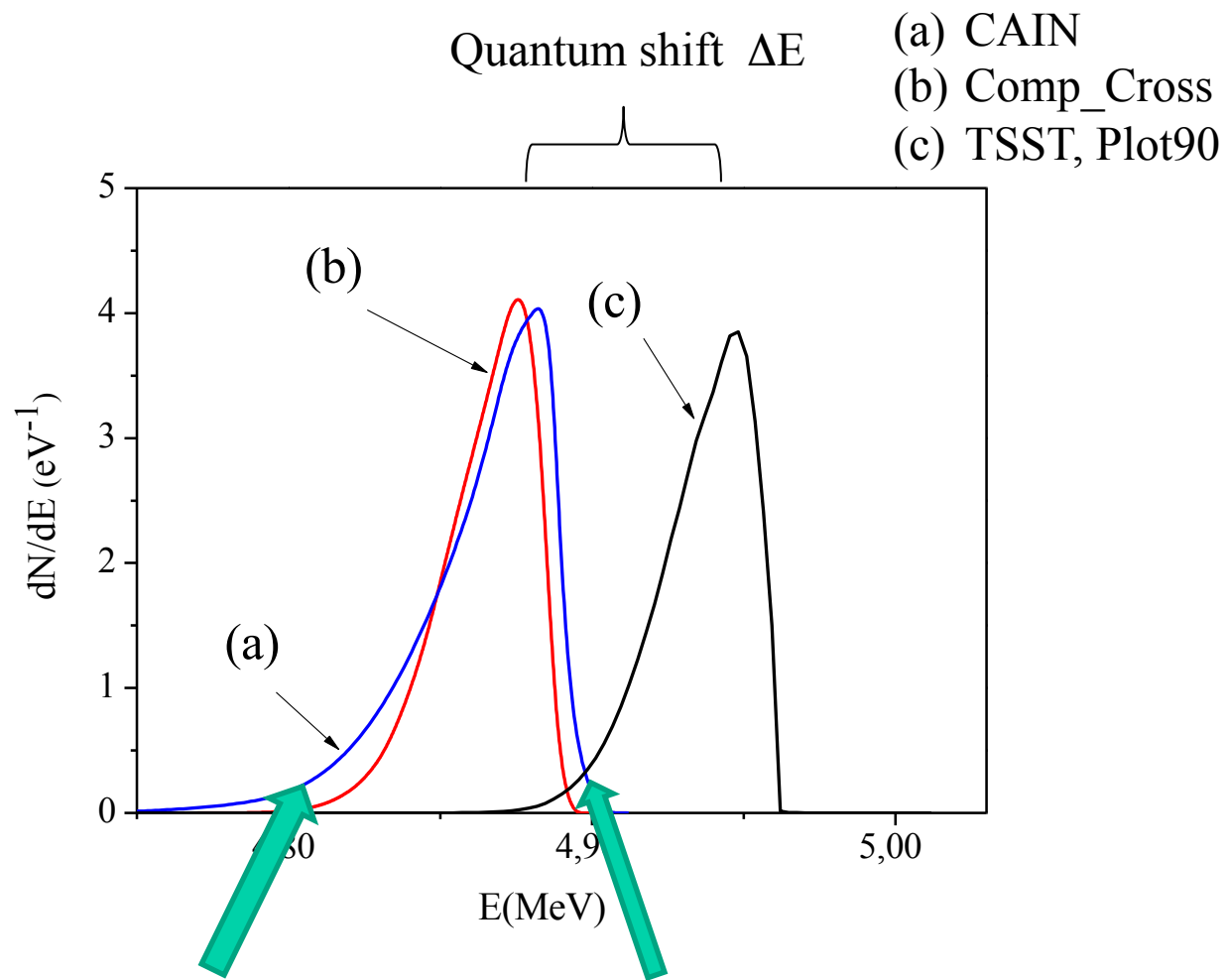


CAIN results



# COMPARISON between classical (TSST), quantum semianalytical (Comp\_cross) and quantum MonteCarlo (CAIN) ELI-np data





A part from the quantum shift, the spectra are very similar

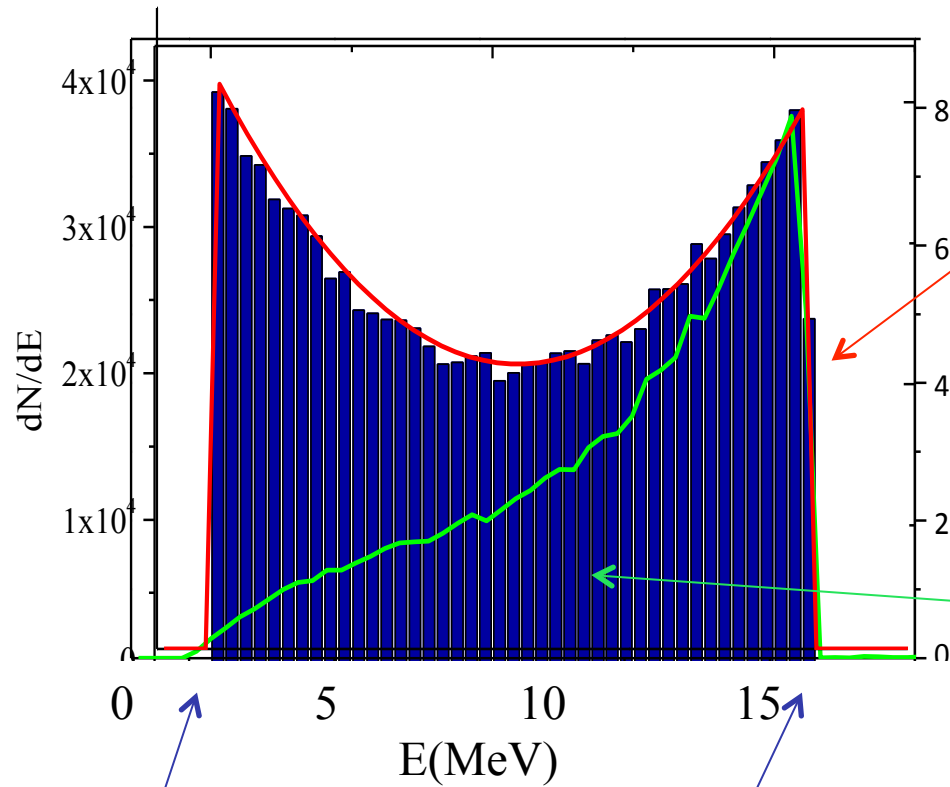
# **Electron distribution during and after the scattering**

Encodes important informations about the scattering

Deterioration in the longitudinal phase space

Eventual transverse cooling

# Photon distribution , linear case, Eli-np data



Total photon distribution

$$\frac{dN_{ph}}{dE} \propto \frac{3}{\Delta E} \left[ \left( \frac{E - E_{min}}{\Delta E} \right)^2 - \frac{E - E_{min}}{\Delta E} + \frac{1}{2} \right] \times H(E_{max} - E)H(E - E_{min})$$

Photon energy distribution

$$\frac{dE_{ph}}{dE} \propto \frac{E}{\Delta E} \left[ \left( \frac{E - E_{min}}{\Delta E} \right)^2 - \frac{E - E_{min}}{\Delta E} + \frac{1}{2} \right] \times H(E_{max} - E)H(E - E_{min})$$

$$E_{min} = \hbar\omega_L$$

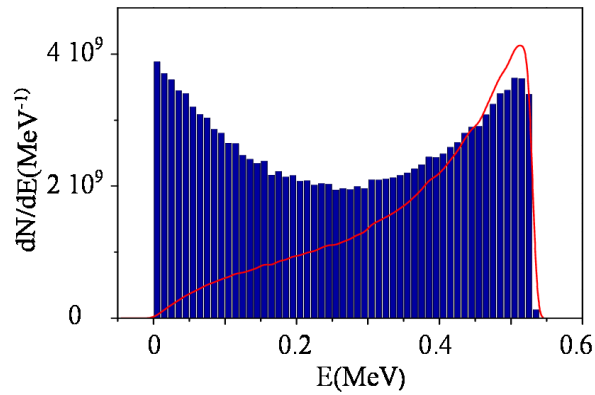
$$E_{max} = \frac{(1 + \beta_e)\hbar\omega_L}{1 - \beta_e + 2\hbar\omega_L/\gamma_e m_0 c^2}$$

$$\Delta N_{ph}/\langle N_{ph} \rangle \approx 61\%$$

$$\Delta E_{ph}/\langle E_{ph} \rangle \approx 25\%$$



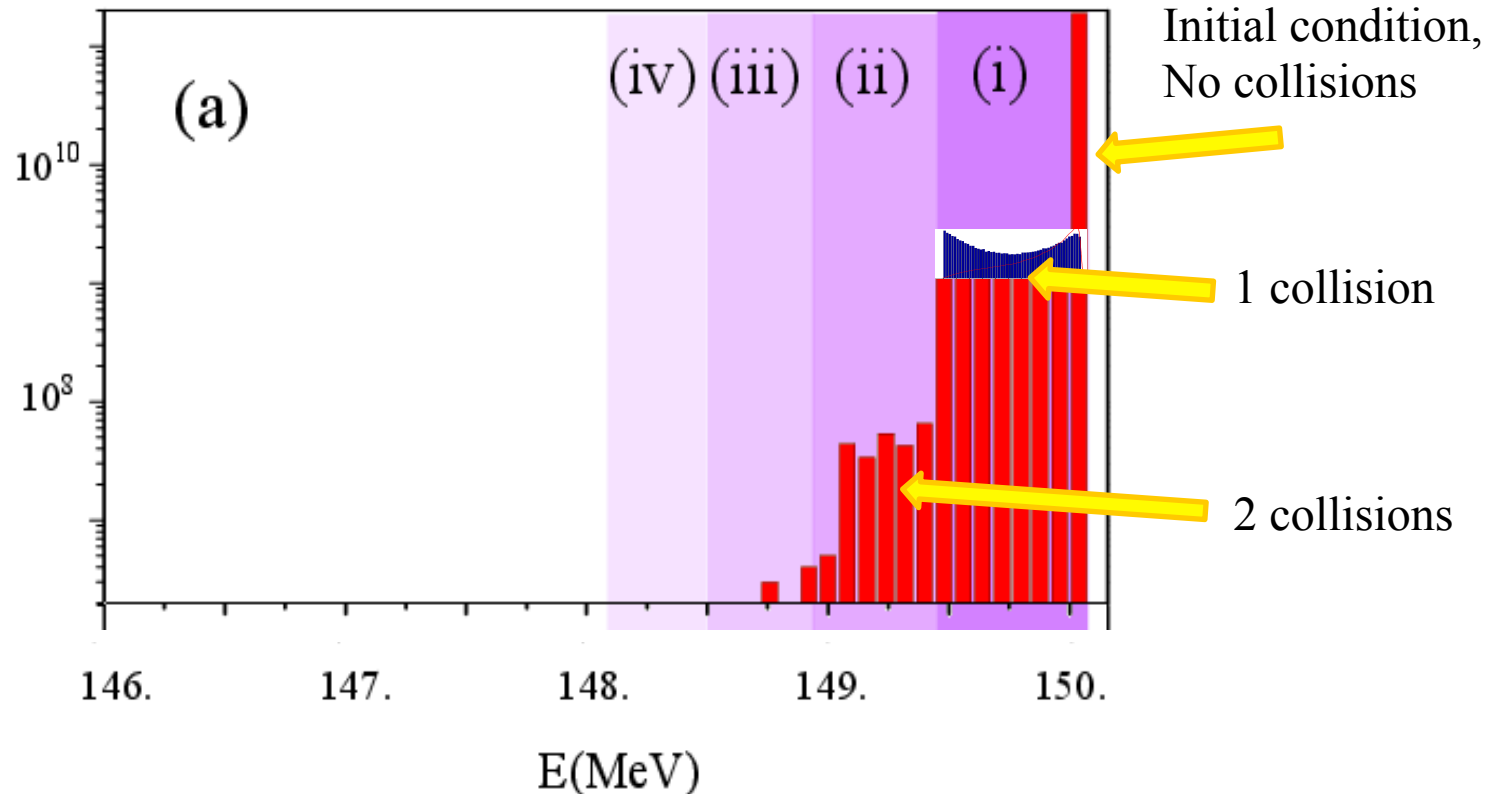
# Electron energy distribution at varying laser time duration. Simulation made with CAIN.



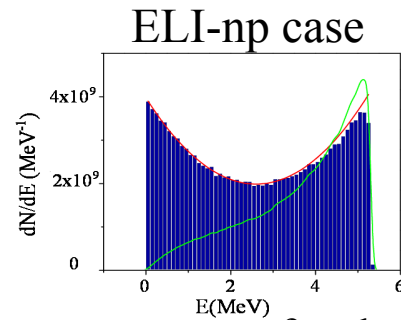
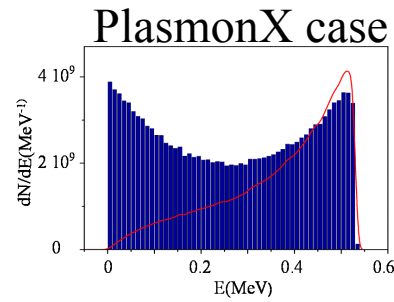
Thomson SPARC-LAB at 150 MeV

Photon distribution vs electron energy for  $a_0 = 0.083$  and

$\Delta t = 0.083$  ps

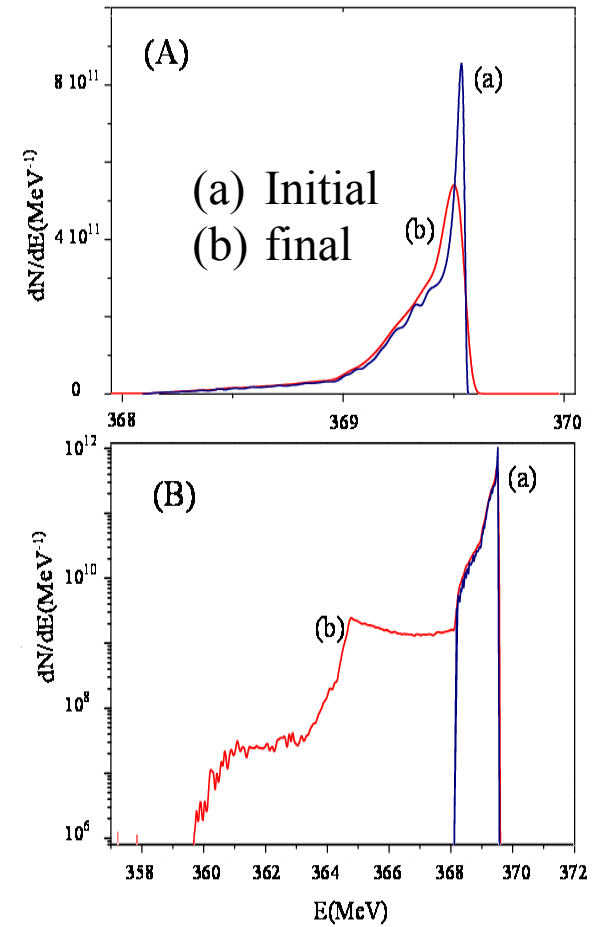
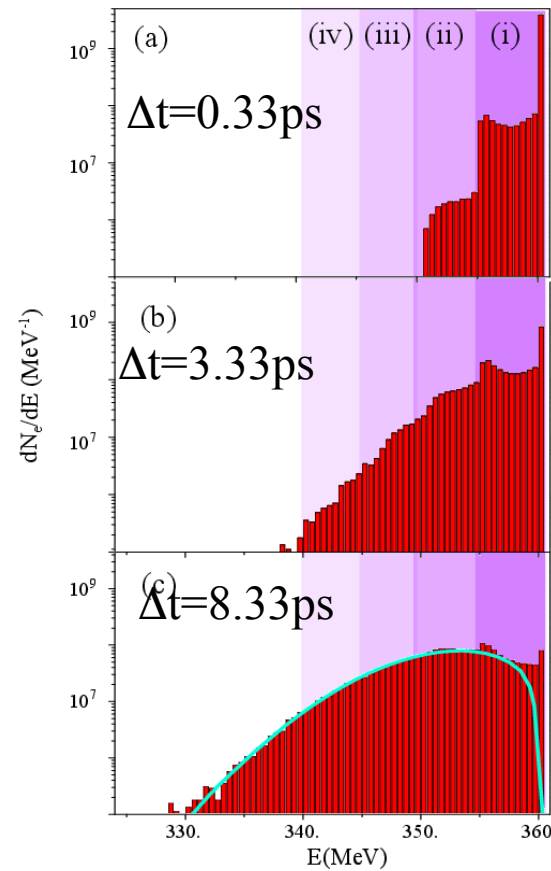
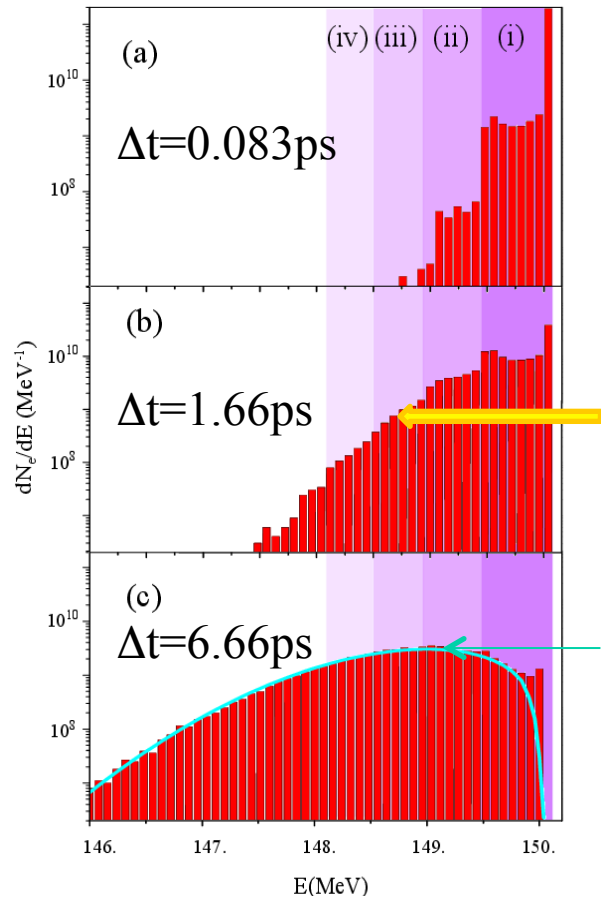


# Electron energy distribution at varying laser time duration. Simulation with CAIN

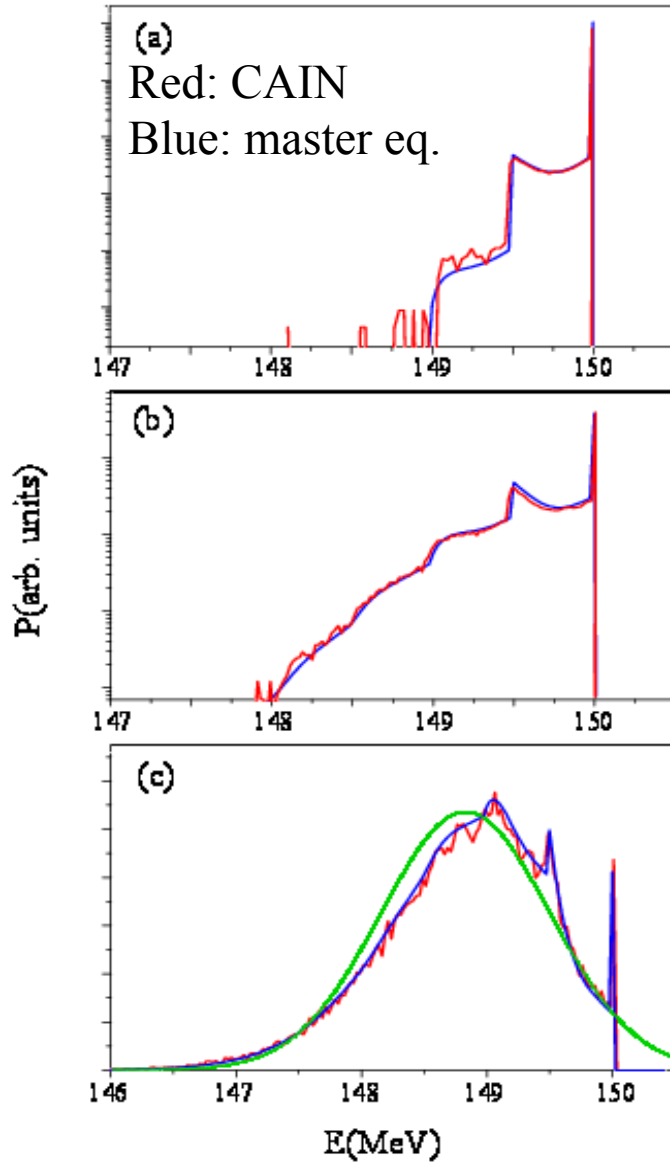


ELI simulated beam

Electron distribution vs electron energy for the  $a_0 = 0.083$



## Thomson SPARC-LAB data, 150 MeV



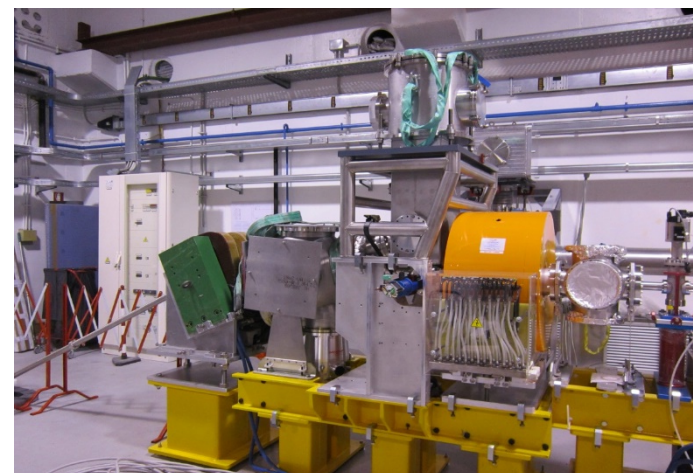
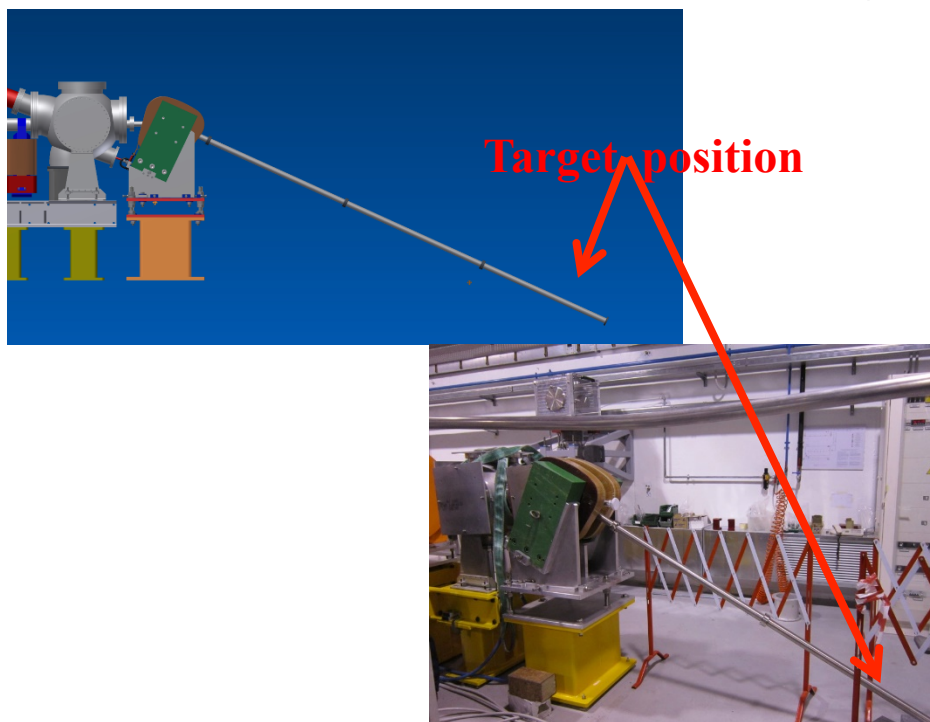
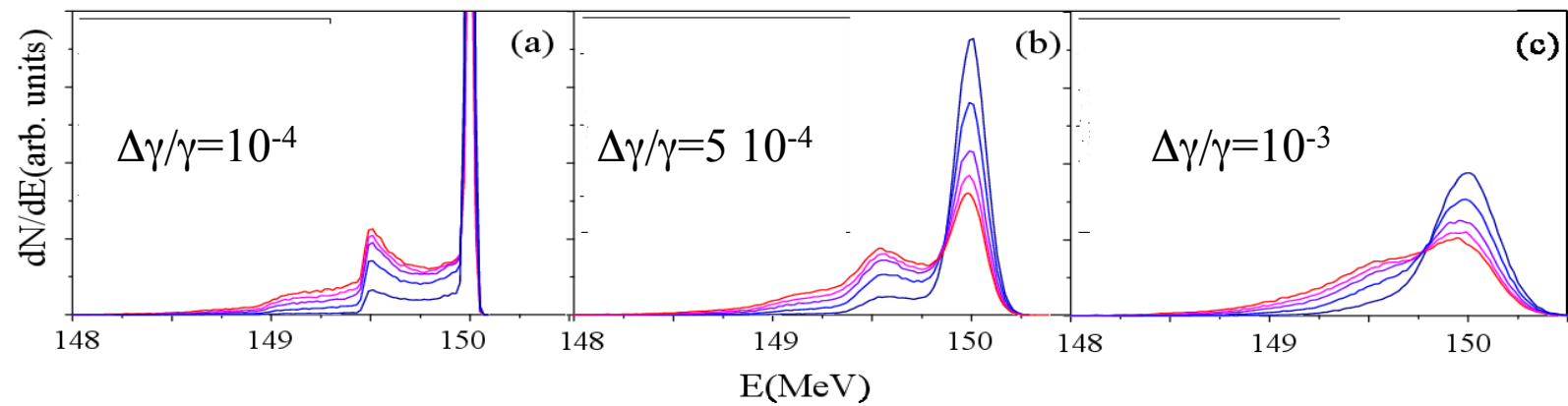
$$\frac{\partial P(E, t)}{\partial t} = \alpha \int dE^* W(E, E^*) P(E^*, t) - \alpha P(E, t)$$

$$W = \frac{dN_{\text{ph}}}{dE}$$

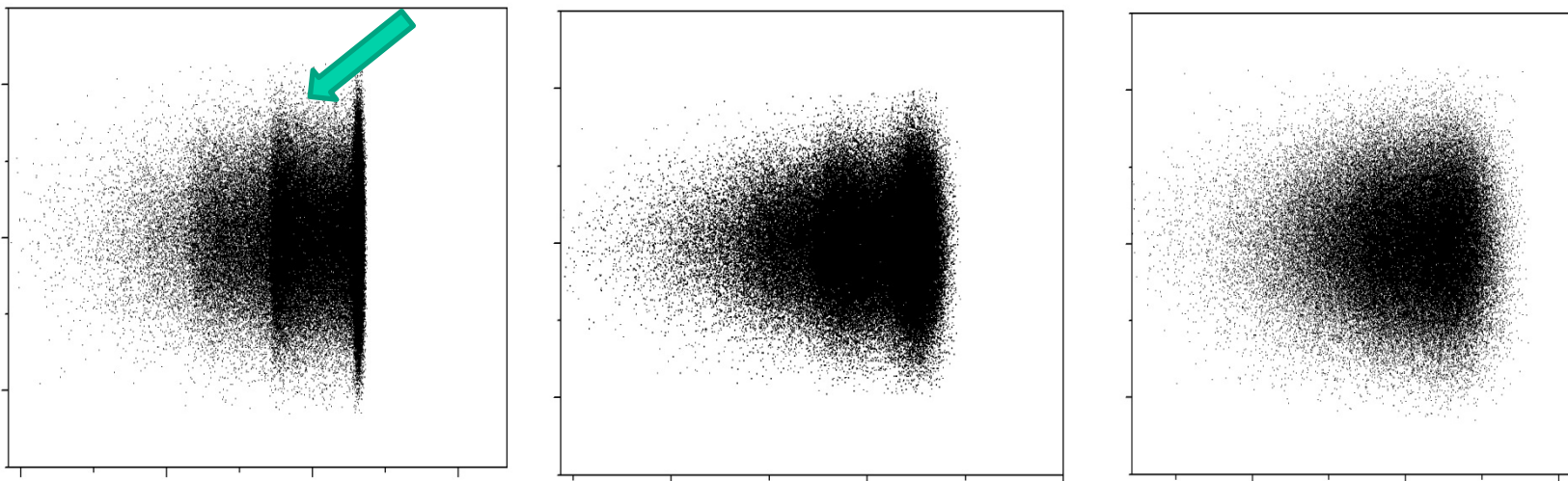
P is the electron distribution

Chapman-Kolmogorov master-equation for Markov process

$$\frac{dN_{\text{ph}}}{dE} \propto \frac{3}{\Delta E} \left[ \left( \frac{E - E_{\text{min}}}{\Delta E} \right)^2 - \frac{E - E_{\text{min}}}{\Delta E} + \frac{1}{2} \right] \times H(E_{\text{max}} - E) H(E - E_{\text{min}})$$



Quantum structures exist only at low energy spread



# CONCLUSIONS

## Regarding the photon distribution

The **quantum model** is important to determine the radiation frequency for gamma factors larger than 1000.

The **other characteristics of the radiation**, i.e., for instance, the shape of the spectrum, the total number of photons, the bandwidth **are not substantially affected by quantum effects**.

If the laser is intense **non linear effects** play a significant role, but in the range analysed the linear model is convenient

**Ambiguous mathematical procedures** in the Klein-Nishina cross section derivation (such as, i.e., the use of improper eigenfunctions and squared Dirac delta functions), **should be eliminated by a rigorous revision**.

## Regarding the electron distribution

The electron distribution evolves in time during the collision, **presenting a sequence of stripes**, connected to the quantum nature of the scattering

At **longer times the process becomes diffusive**, following the Fokker-Planck equation

A **master equation** derived by the Kolmogorov equation for Markov phenomena is able to describe the process

By deflecting the electron on a screen, one can detect the details of the energy distribution, **confirming the quantum nature of the collisions**

# Scaling laws and their validation

Total number of photons  
In the bandwidth

$$N = \frac{4.1 \times 1.5 \cdot 10^8 E_L (\text{J}) Q (\text{pC}) \psi^2}{h\nu_L (\text{eV}) \left( \frac{w_0^2}{4} + \sigma_x^2 \right) \sqrt{1 + \frac{\delta^2 (\sigma_x^2 + \sigma_{x,e}^2)}{(w_0^2 + 2\sigma_x^2)}}$$

Bandwidth

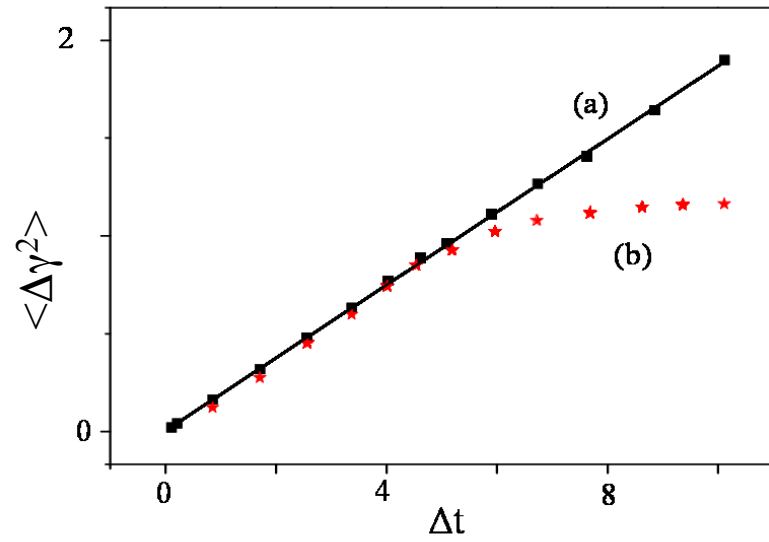
$$\frac{\Delta\nu_p}{\nu_p} = \sqrt{\psi^4 + \left( \frac{\varepsilon_n}{\sigma_x} \right)^4 + 4 \left( \frac{\Delta\gamma}{\gamma} \right)^2 + \left( \frac{\Delta\nu_p}{\nu_p} \right)_L^2}$$

acceptance
emittance
energy spread
broadening due to laser

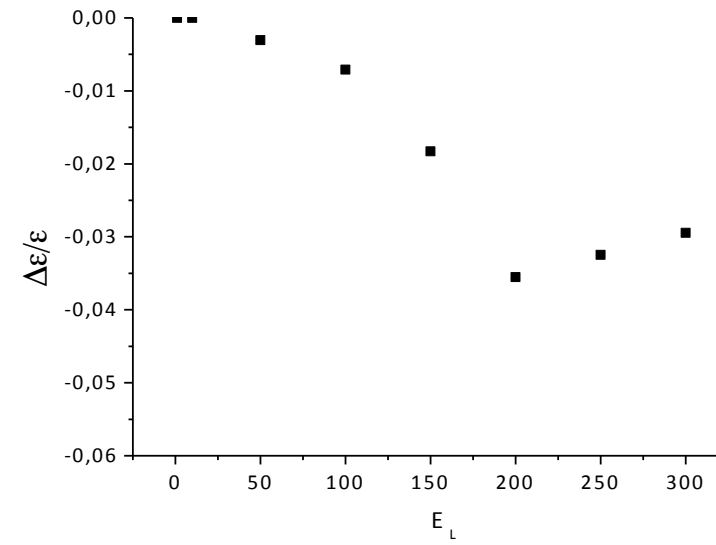
Spectral density

$$S = \left[ \frac{dN}{d\nu} \right]_{\text{peak}}$$

Increase in the energy spread



decrease in emittance



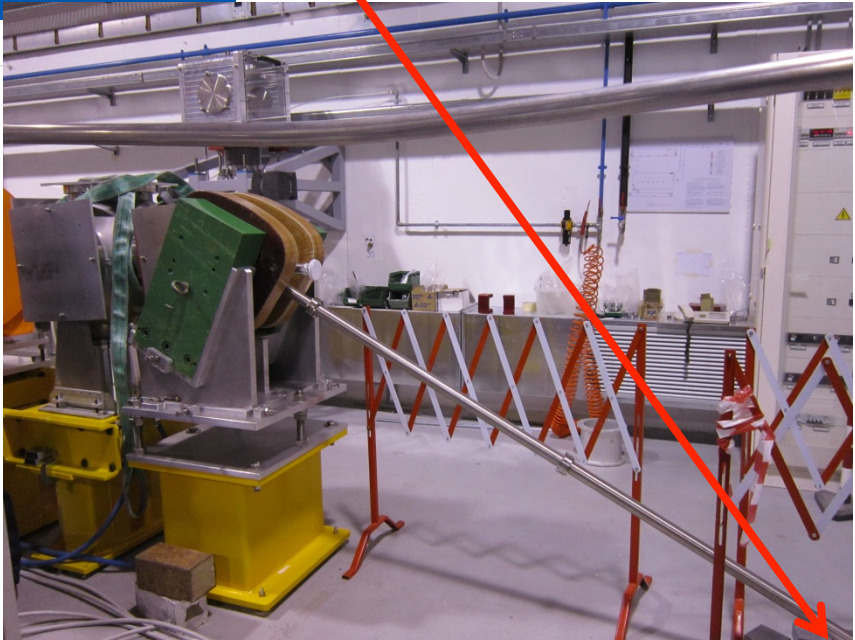
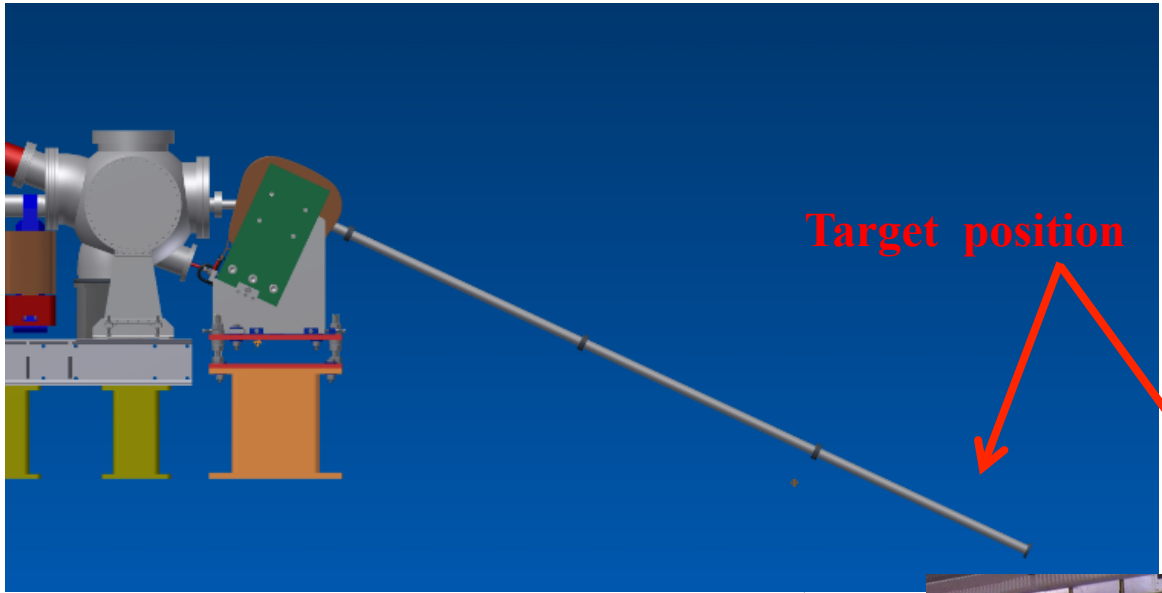


Contributors to the work:

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A. Variola, I. Chaikowska, P. Tomassini

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G. Geloni, R. Bonifacio, G. Robb for stimulating discussions.

Thank You for the attention



# Application to a gamma source: ELI-np

With an optimization procedure we obtain: **Working point at 460 MeV**

Charge Q (C) 0.2510<sup>-9</sup>  
 Energy E (MeV) 460  
 $\Delta\gamma/\gamma$  5. 10<sup>-4</sup>  
 Emittance(mm-mrad) 0. 4/0.39  
 $\sigma_x=18 \mu\text{m}$   
 $\sigma_y=15 \mu\text{m}$

Laser photon energy (eV) 3.1  
 Laser energy E<sub>L</sub>(J) 0.2  
 Laser rms time duration (ps) 1.5  
 Laser waist ( $\mu\text{m}$ ) 25  
**Interaction angle 7.5°**

Photon energy(MeV) 10.

**Bandwidth rms=5 10<sup>-3</sup>**

$N_{\text{band}}=1.6 \cdot 10^5$

**S=1.56 photons/eV**

Acceptance angle=52 microrad

Laser  
recirculation

$$S_{\text{tot}}=1.56*32*100=0.5 \cdot 10^4 \text{ photons/eV}$$

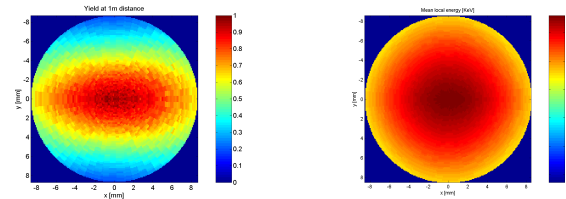
Multibunch  
techniques

$$\langle B_{\text{tot}} \rangle = 3.8 \cdot 10^{14}$$

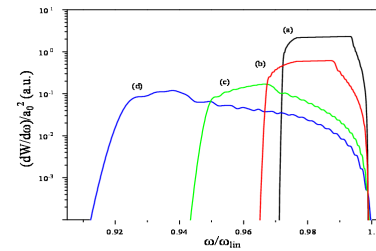
$$B_{\text{peak}} = 1.3 \cdot 10^{23}$$

# Experiments on Thomson@SPARC-LAB

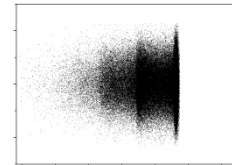
Characterization of the source



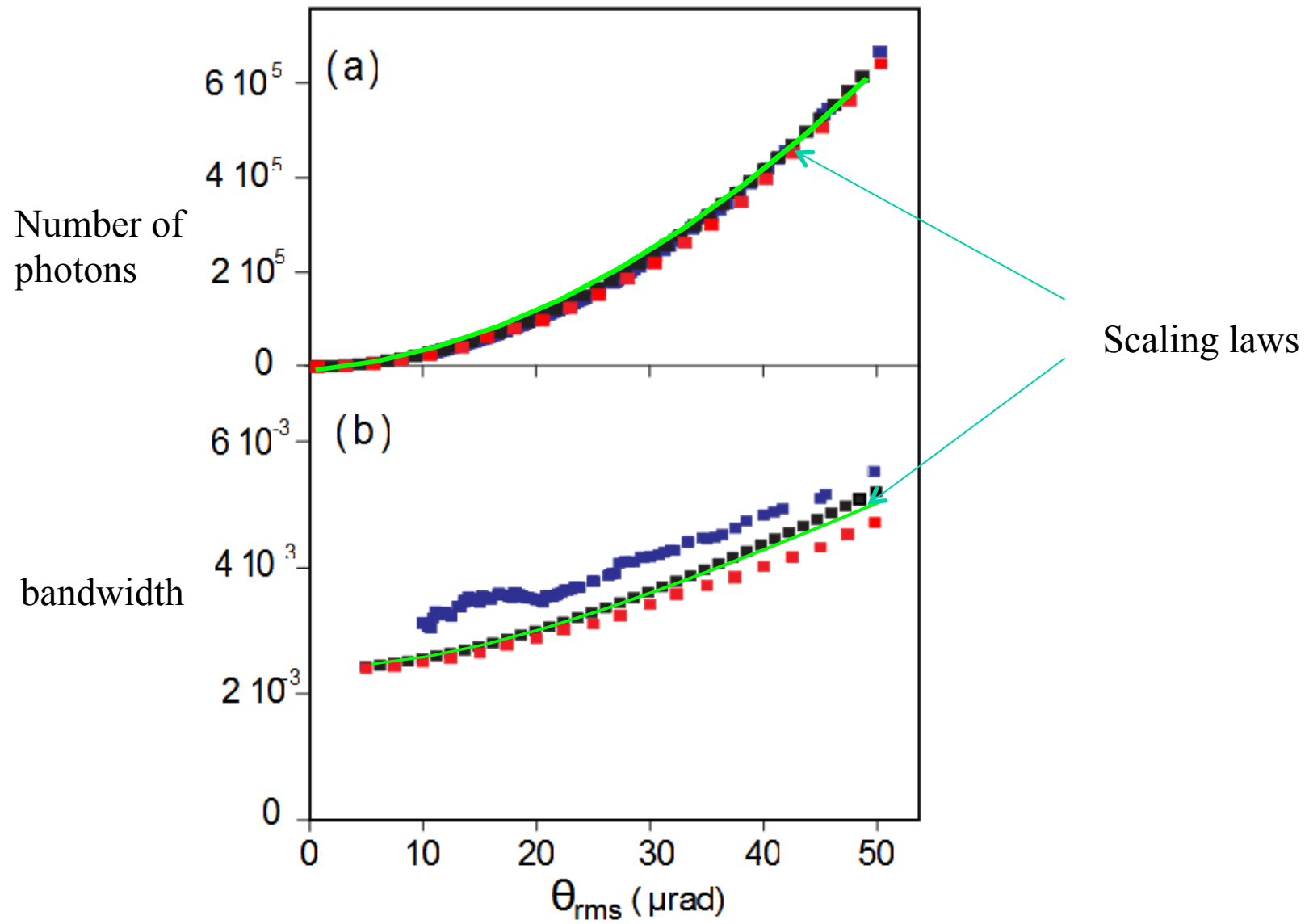
Insurgence of non linear effects



Quantum electron grouping



X two colors radiation



# Non linear effects

The derivation of the Klein-Nishina cross section is based on hypothesis of weak non linearity (development in series), quantum theory is not able to treat realistic laser fields.

For the evaluation of the non linearities due to high laser energy we use the classical model inserting the numerical trajectories obtained under a realistic laser pulse (taking into account gaussian profiles, curvature of the wave front, diffraction) into the radiation integral.

shift of the spectrum towards lower energies

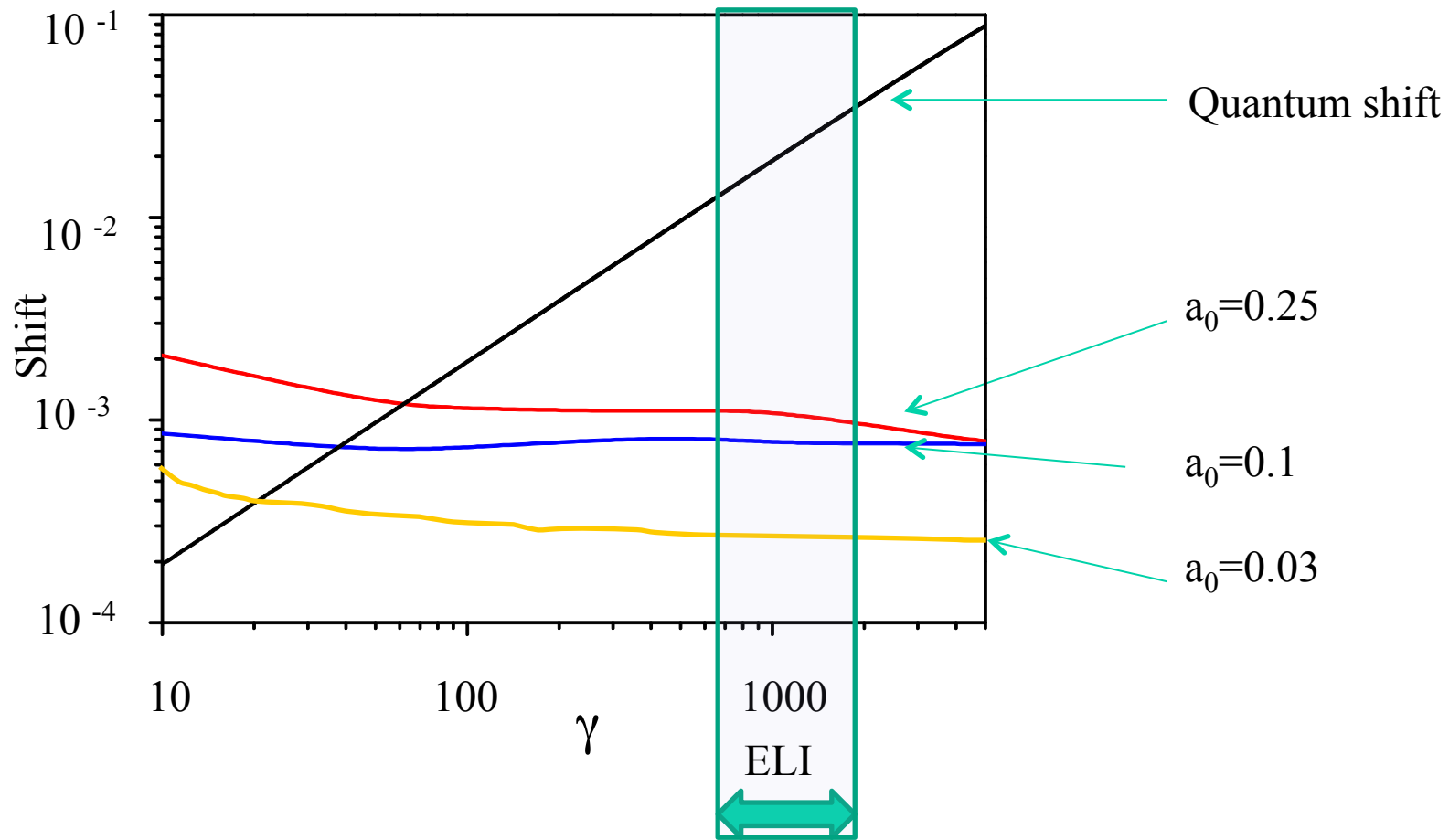
broadening of the bandwidth

rising of side bands,

the growth of harmonics

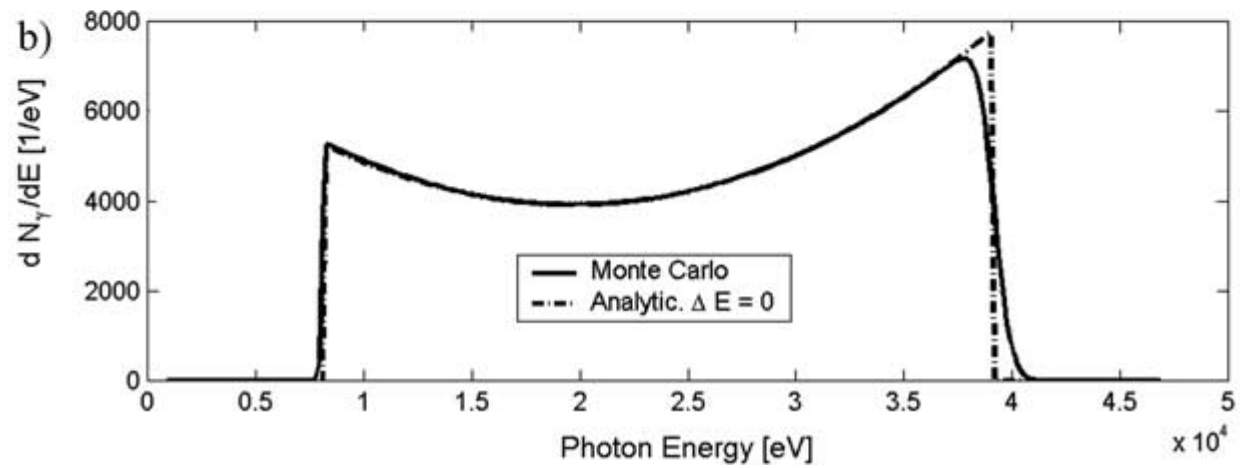
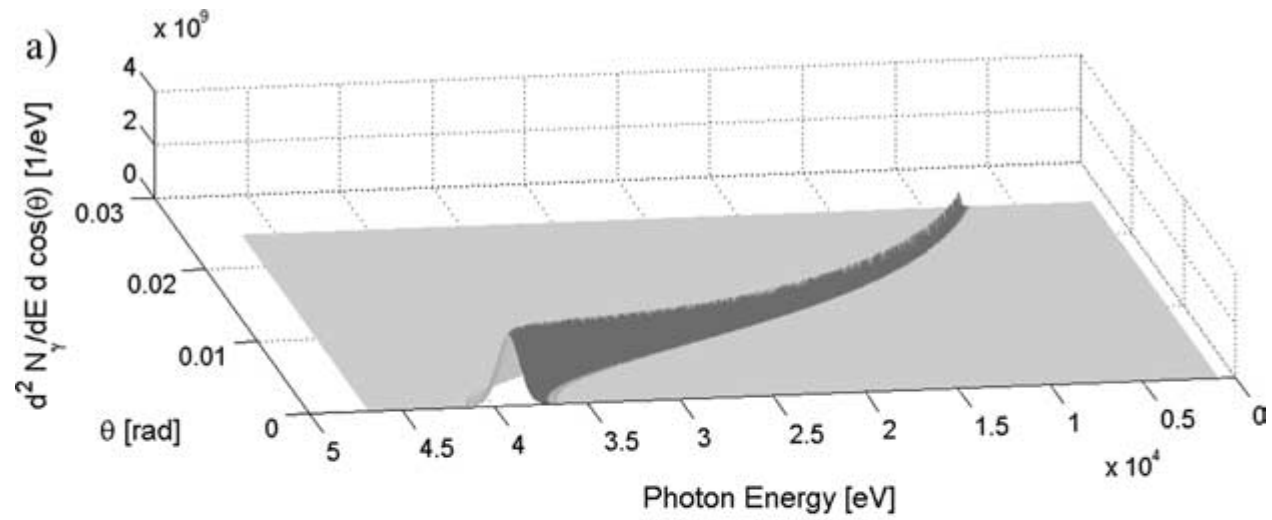
enhancement of the background

superposition, shift and merging of the harmonics



The non-linear shift is subdominant with respect to the quantum one.





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