Gain Fluctuations Summary Studies for the Muon g-2 Experiment.

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Gain Fluctuations Outline

- Experiment - Basics concepts.
- The ideal wiggle plot (no gain fluctuations).
- Effect of a hypothetical/theoretical gain fluctuations on the uncertainties in ω_a.
- Correction to theoretical gain fluctuations using laser simulated gain fluctuations of the SiPMs.

Experiment Basics: Muons in a storage ring

1. Start with polarized muon beam (from pion decay)

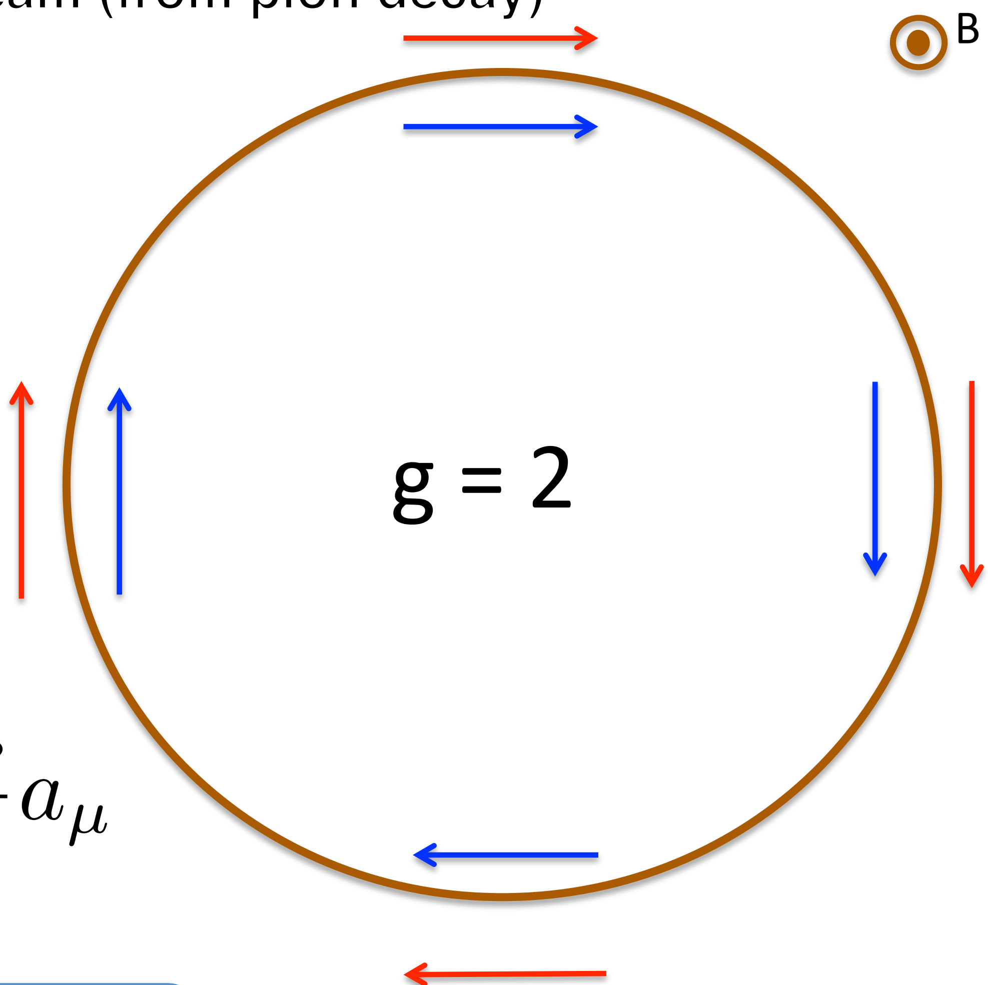
2. Cyclotron frequency :

$$\omega_c = \frac{e}{m\gamma} B$$

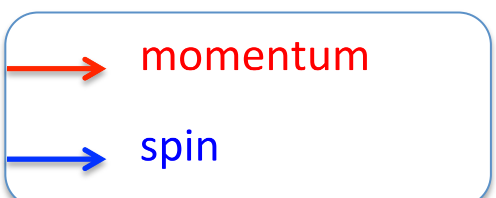
3. Spin precession frequency :

$$\omega_S = \frac{e}{m\gamma} B \left(1 + \gamma a_\mu\right)$$

Larmor + Thomas precession



$$\omega_a = \omega_S - \omega_c = \frac{eB}{m} a_\mu$$



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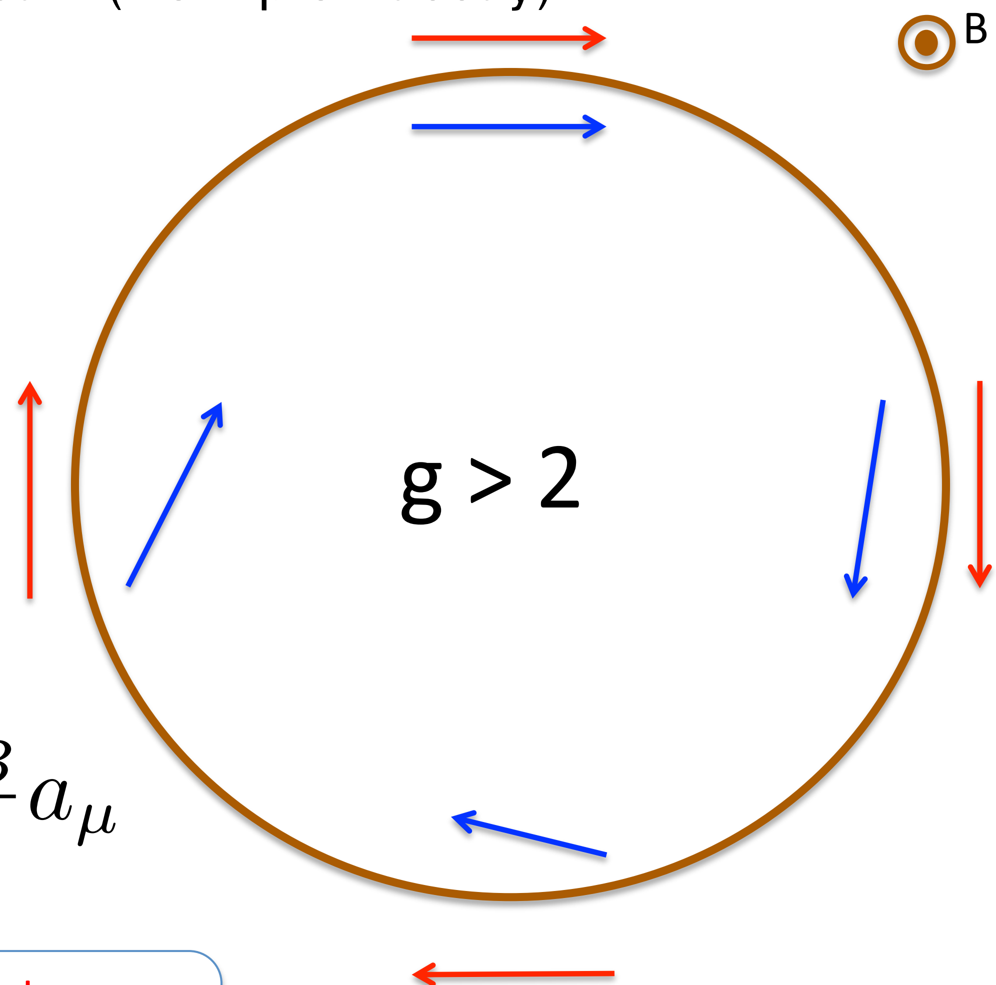

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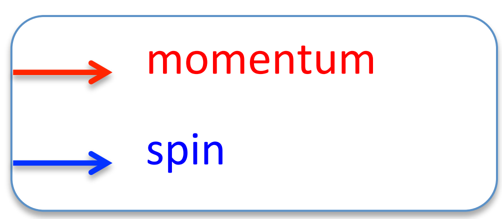
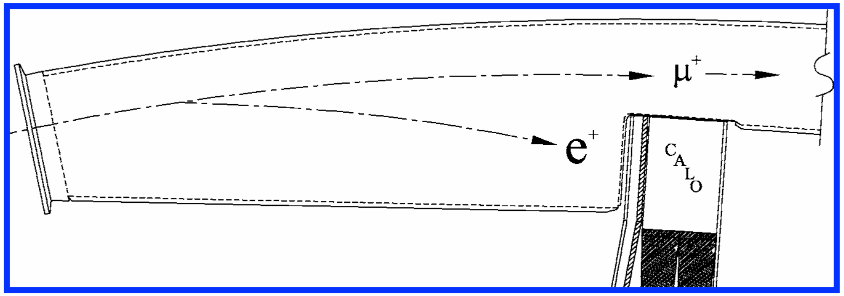
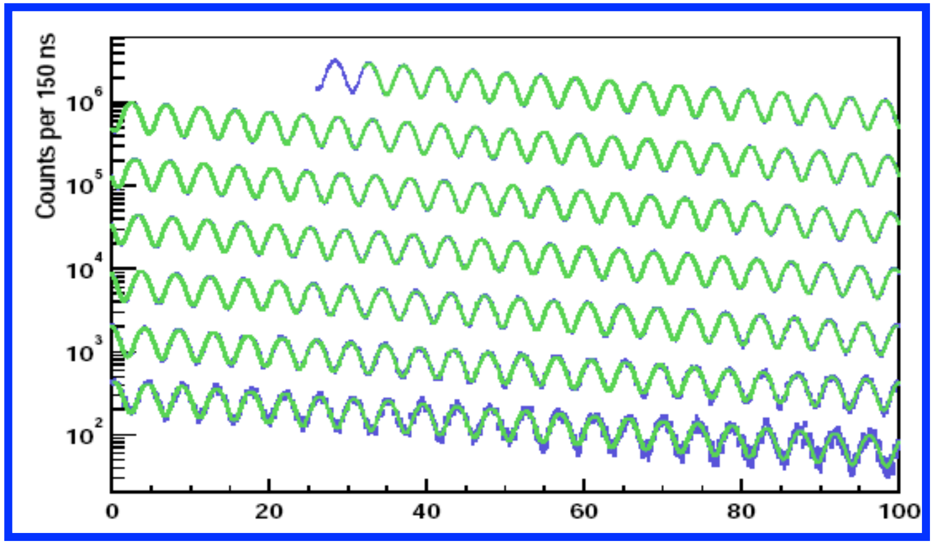


Muon spin precession frequency



E821 data: e^+ with E > 1.8 GeV



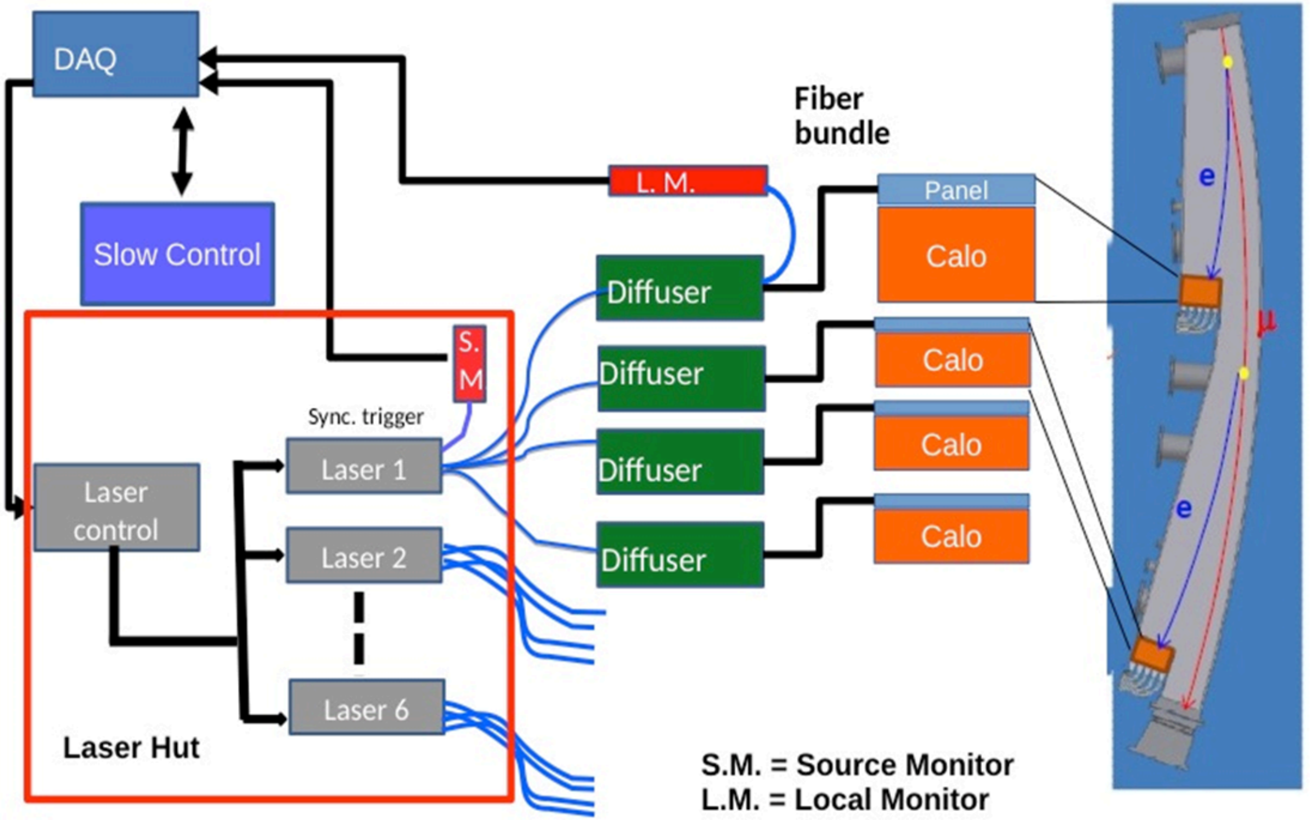
$$\omega_a = \omega_S - \omega_c = \frac{eB}{m} a_\mu$$

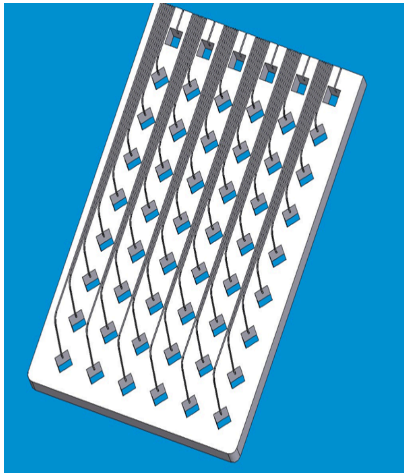
- Decay self-analyzing:
  - Higher energy positrons emitted preferentially in direction of muon spin

$$N(t) = N_0 e^{-t/\tau} \left(1 + Acos(\omega_a t + \phi)\right)$$

- Spectrum distortions from
  - Pileup, gain stability
  - Beam Effects, Losses

Laser Calibration System





Front Panel
9x6 crystals

The big picture – Extent for 24 calorimeters

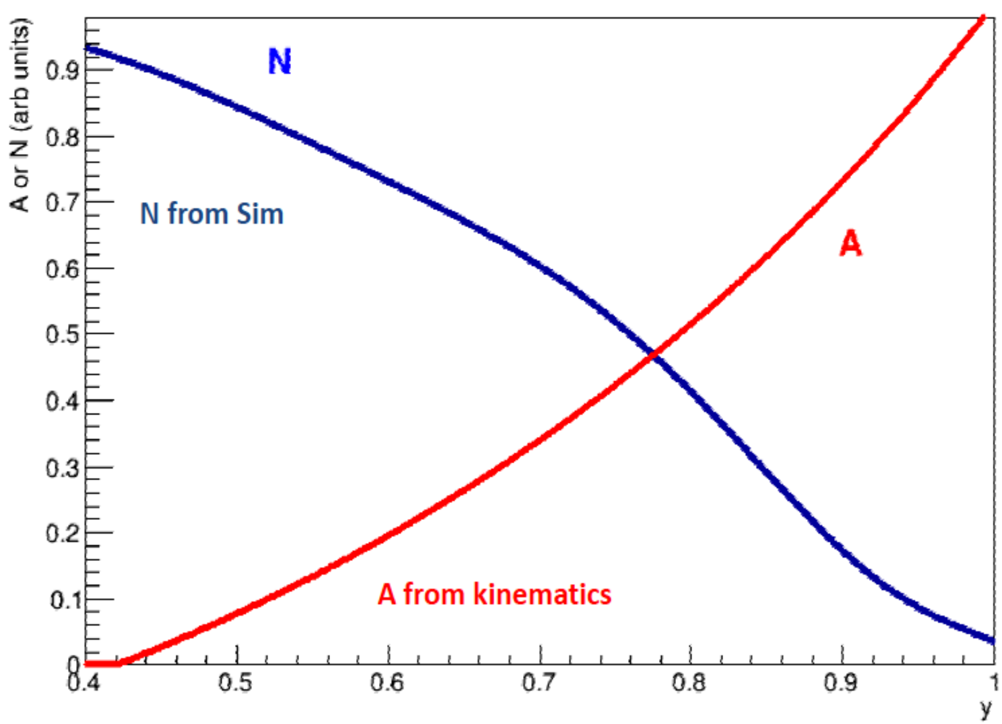
Gain Fluctuations – Why study?

- The goal of the experiment is to measure ω_a precisely.
- The goal of the laser calibration system is to measure the gain of the calorimeters and if there are inconsistencies in measuring the gain how would ω_a be effected.
- Thus we apply a fluctuation / perturbation in the gain function G(t) and see how that effects ω_a.
- We begin by simulating an ideal wiggle plot (shown in slide 5) which is a distribution of the events collected by the calorimeter, study the effect of a perturbed gain function on this plot and finally apply a correction simulating the laser calibration.

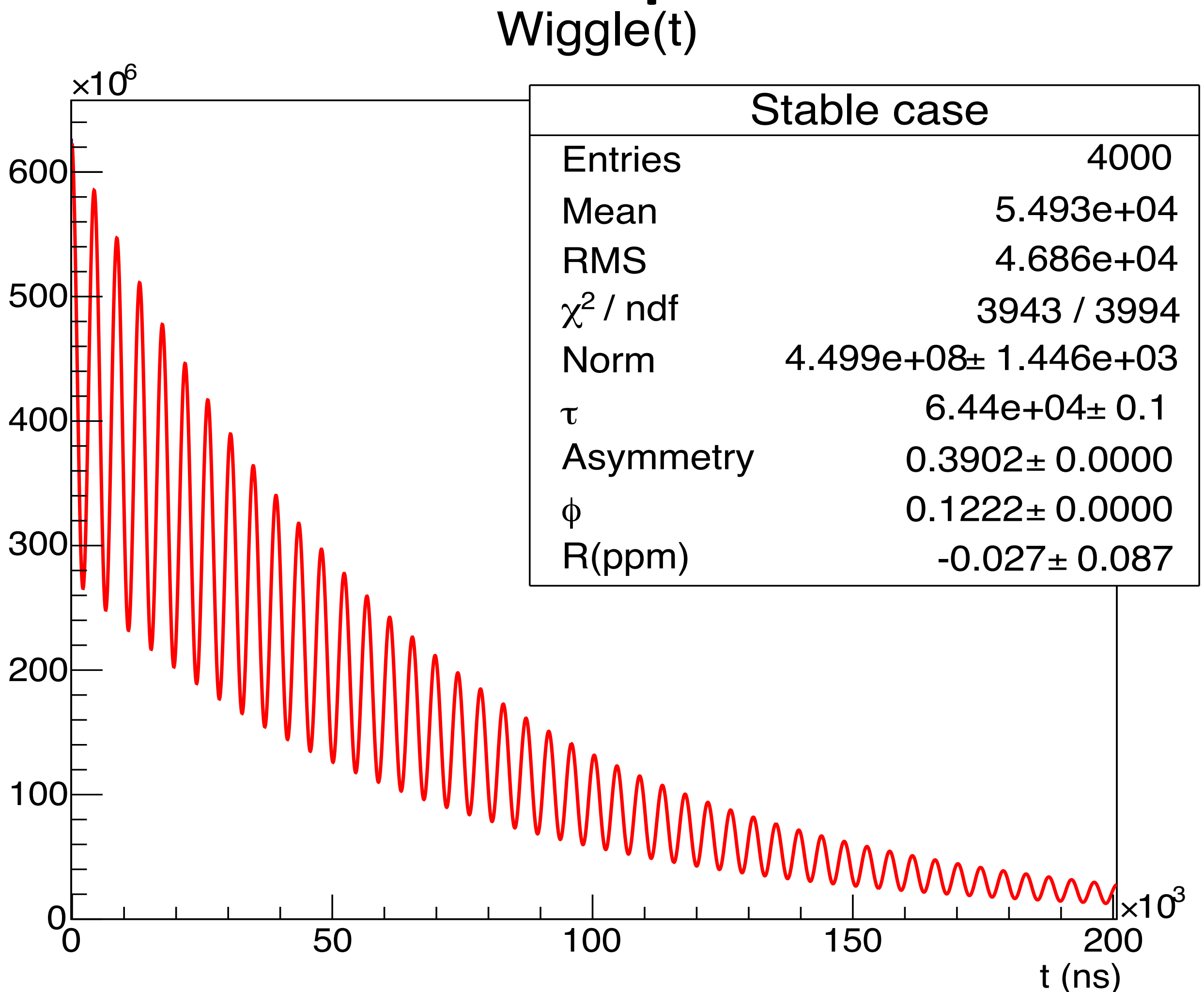
More about the wiggle plot in the next slides………

Simulating the Ideal Wiggle Plot

- Effect on the beam/events due to drifts in gain. The frequency of events follow the wiggle plot as,
  $N(y)[1+A(y)\cos(\omega_a(1+R)t+\phi(y))]\exp(-t/\tau)$ => R is change in ω_a, and y is E/E_{max}
- N(y), A(y) and $\phi(y)$ obtained from simulations / kinematics.



Ideal case without perturbation



R is change in ω_a – our desired goal is 0.02 ppm – but since from the unperturbed wiggle plot fit, gives R =-0.027 ppm its just an offset in R (say R_0)– we consider this as the ideal case.

Effect of Theoretical Gain Fluctuations on Uncertainties in ω_a

Effect of Gain Changes on Uncertainties of ω_a

- Reduce error due to gain changes to 20 ppb
- Study / simulate systematic hardware gain drifts by introducing a perturbation in gain function G(t)
- Note: G(t) is the correction in gain from the above i.e.

$$G(t) = (G'-G_0)/G_0$$

  where G_0 is the ideal gain and G' is true gain vs. time due to detectors, readouts etc.
- A very stable laser calibration system used which monitors the source for stability/fluctuation before calibration which gives G_0.

Perturbation

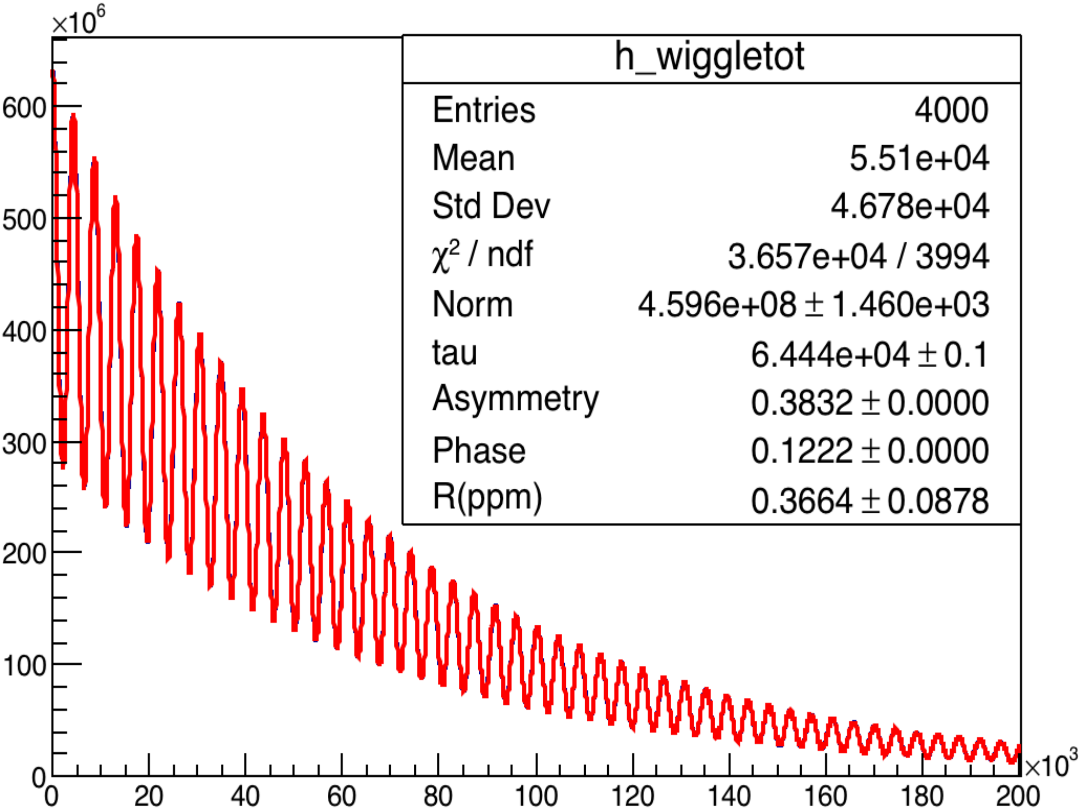
There can be various functional forms of perturbations. We use **ε = 0.001** (unless mentioned) for all types which are:

- Linear: $1+ \varepsilon(\text{endtime} - t)/\text{endtime}$ (endtime = 700 μs)
- Exponential: $1 + \varepsilon\, e^{-t/\tau}$
- Phase: $1 + \varepsilon \cos(\omega_a t + \phi)$
- Mixed exponential and phase:

  $1 + \varepsilon\, e^{-t/\tau} * \cos(\omega_a t + \phi)$

We assume an exponential perturbation for this study (in principle it could of any form – even different from the ones mentioned above)

Exponential perturbation $1 + \varepsilon \exp(-t/\tau)$



The perturbation in this case is a theoretical perturbation in gain i.e. a mathematical exponential function for gain of the form $G_T(t)=\mathbf{1 + \varepsilon\, e^{-t/\tau}}$ was assumed.

With an exponential perturbation $\Delta\omega_a$ is $R - R_0 \sim 0.393$ ppm, which exceeds our error budget. Thus we need to apply a correction to get back the nominal value (back to R_0)

Correction to Theoretical Gain Fluctuations using Simulated Gain Fluctuations of SiPMs only

Correction to the perturbed wiggle plot using simulations

Goal

- to simulate an exponential perturbation plot
- fit this simulated plot and extract the corrected values of τ and ε of the fit results and apply it to the wiggle plot with an exponential perturbation (i.e. the plot of slide 8) in gain.
- Check if get back the nominal ω_a

Procedure / Problems

- How many simulation points or cycles to use? Depends on our error budget (next slide explains this)
- How many fills we need to sample data to achieve our desired goal? Depends on the laser frequency, number of cycles etc. (ref. slide 17)

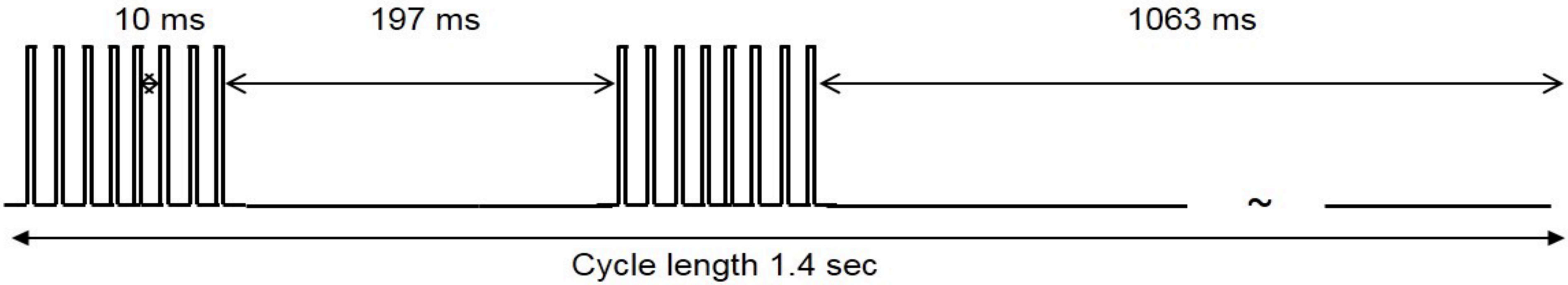
Correction to the perturbed wiggle plot using simulations

Number of simulation points/cycles:

- Depends on the number of laser cycles required to achieve our goal. Our error budget for $\Delta\omega_a/\omega_a$ due to a gain changes is 0.02 ppm.
- **Rule**: $\Delta G/G$ ~ 0.2% gives $\Delta\omega_a/\omega_a$~0.1 ppm (F. Gray's thesis). Thus for a $\Delta\omega_a/\omega_a$ 0.02 ppm we should have $\Delta G/G$ ~ 0.04%.
- This can be obtained by statistical fluctuations arising from the phtostats of SiPM given by $\frac{\sigma}{\sqrt{\mathbf{N}}}$ with σ ~ 2% having about

  2000 points (or N) gives our required 0.04%
- Thus N ~ 2000 cycles or laser calibration point per time bin

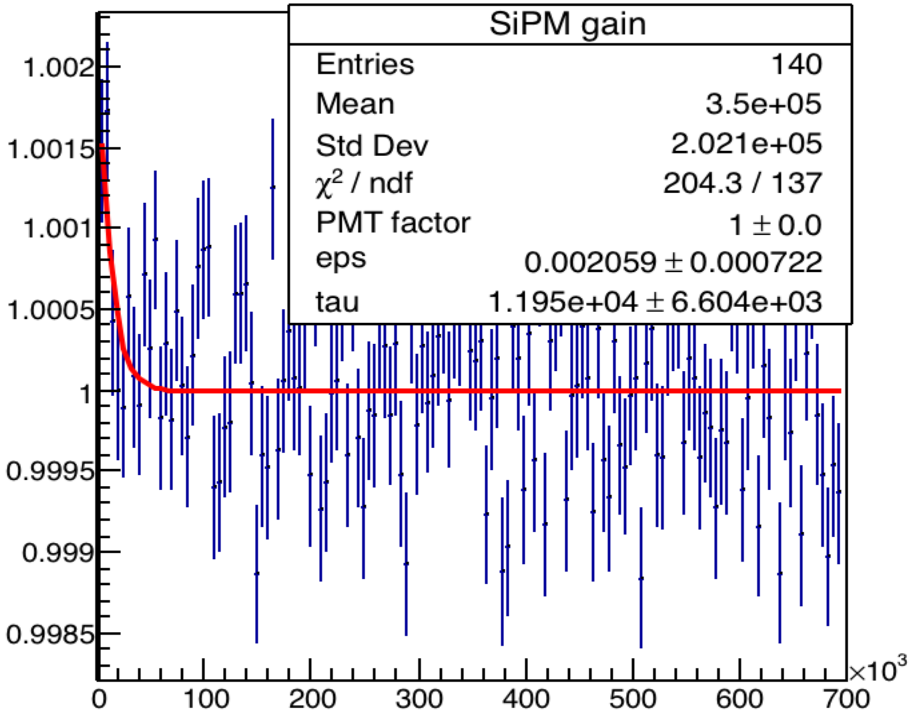
Correction to the perturbed wiggle plot using simulations



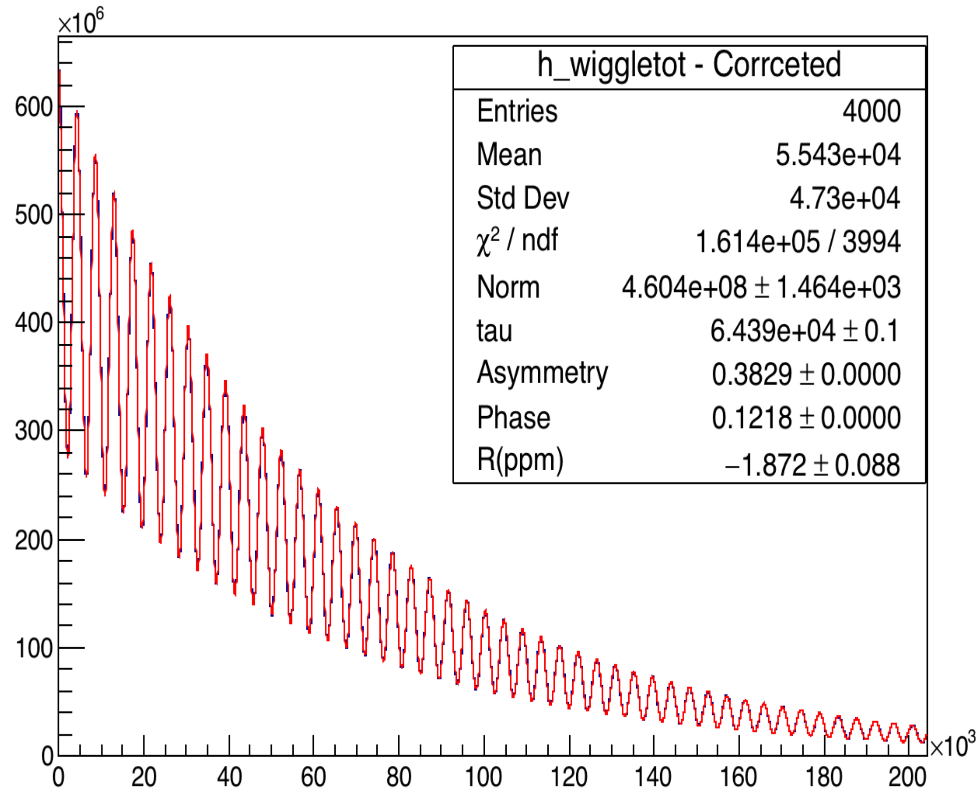
Number of fills required:

- In case of 12.5 kHz laser (80 μs) we get ~ 8 points in a fill (700 μs)
- After each subsequent fill, move offset by 5 μs => 16 fills for a calibration cycle/event = one beam cycle i.e. 1.4 s.
- Accuracy for the 140 points separated by 5 μs (time bin) – our goal with 2000 cycles / points. This defines a **calibration run (~1h or 46 min).**

Correction to the perturbed wiggle plot using simulations

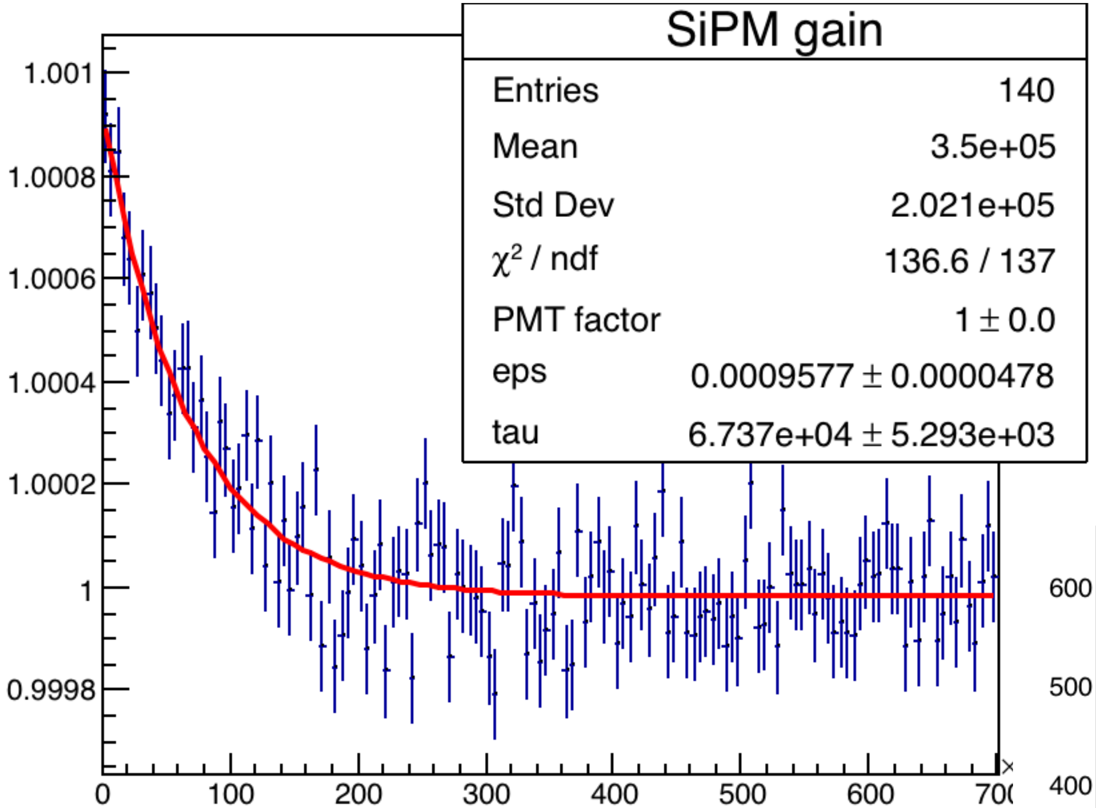


2000 events or points

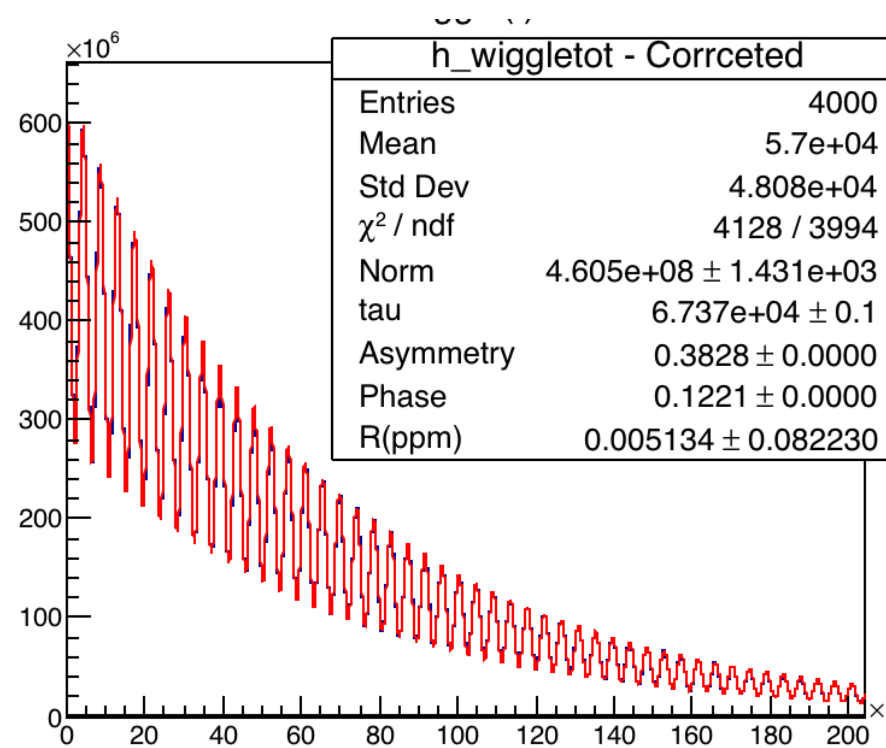


We fit the wiggle plot (bottom) using τ and ε of the fit results of the simulated plot (top) as explained before.

Correction to the perturbed wiggle plot using simulations



50000 events or points



Correction to the perturbed wiggle plot using simulations

Conclusions:

- Evident from slide 18 that 2000 cycles are not enough to simulate a desirable exponential gain function.
- Thus we tested with more cycles (shown in subsequent slides) and found **50000 cycles (add 25 runs) pretty good** as seen in slide 19. Thus a day is good for a dataset.

Note: We checked the results of the wiggle plot with $\tau \pm \Delta\tau$ for each case. We also checked the code by reproducing the stable case with correct value of τ and ε.

Thank you for listenting !!!

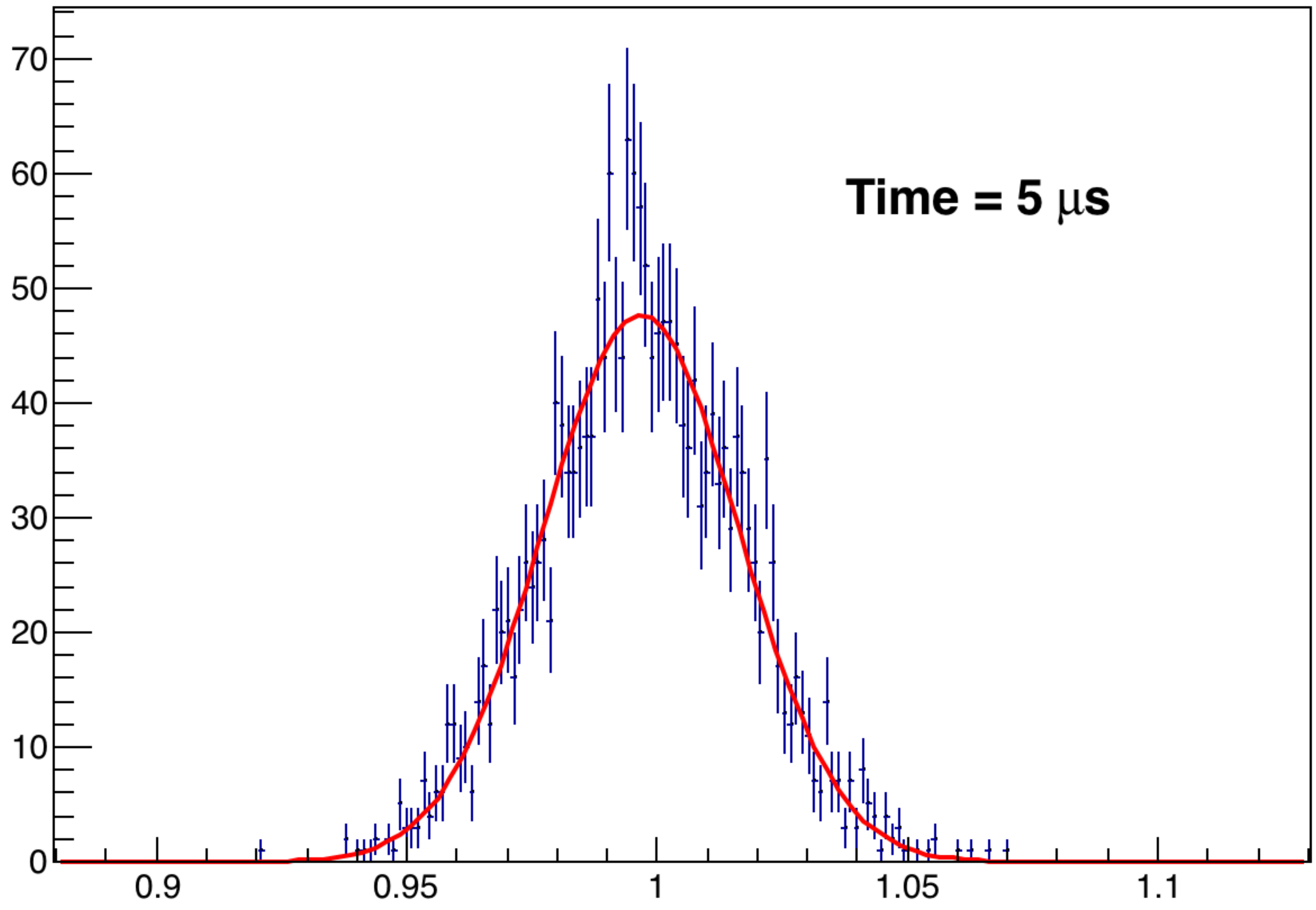
Back Up Slides

Algorithm: For each time bin (5 μs) simulated a Gaussian of 2000 event obeying our exponential perturbation function with a sigma of 2%. Fitted the Gaussian and extracted the fitted mean and plotted it in a histogram. This histogram gives the stat distribution of perturbation

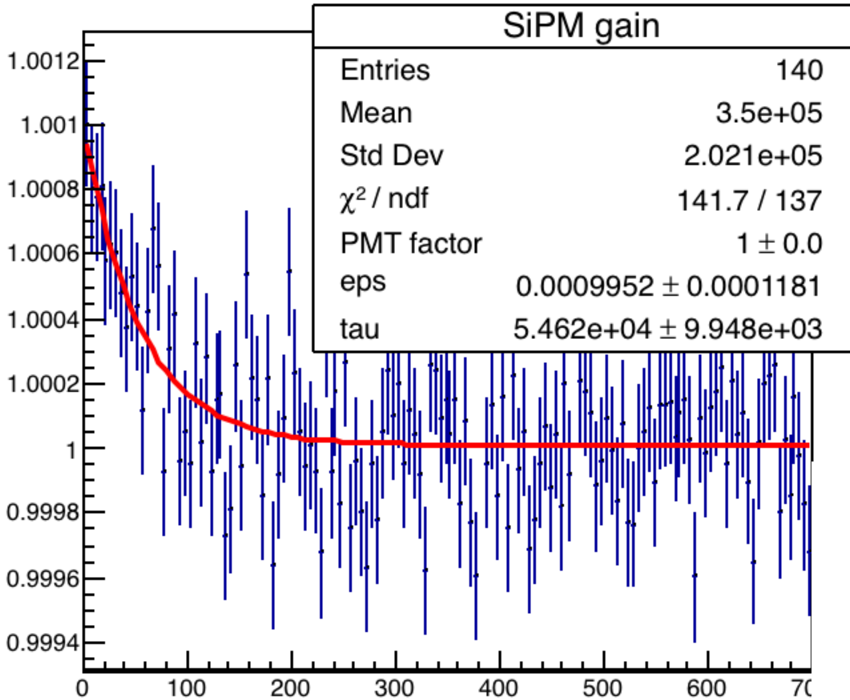
Correction to the perturbed wiggle plot using simulations

Simulation: SiPM gain - Simulated a Gaussian for 2000 events, $\varepsilon = 0.01$ with mean $G_T(t)=\mathbf{1 + \varepsilon\ exp(-t/\tau)}$

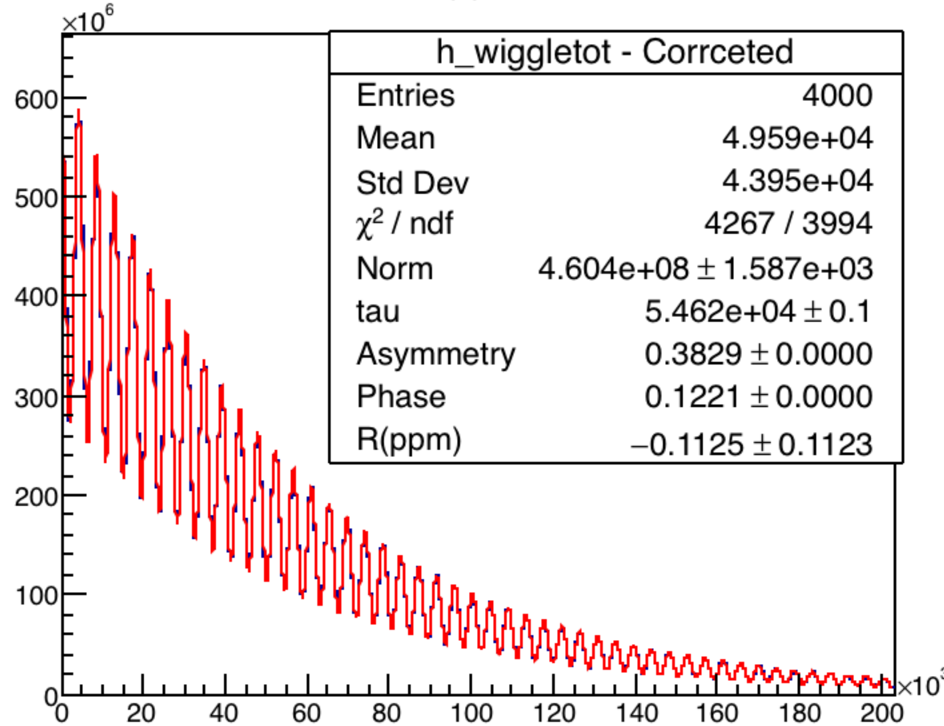
and sigma 2% of the mean for a point.



Correction to the perturbed wiggle plot using simulations



10000 events or points



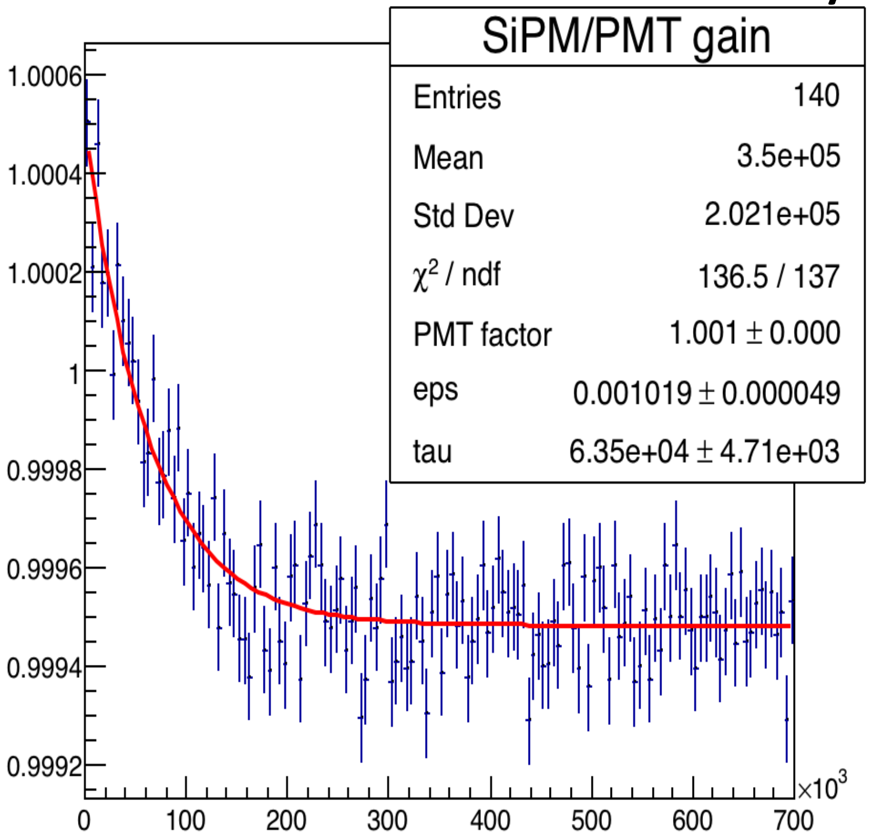
Simulating Short Term Folded with Long Term PMT Gains

We perform a few tests and checks. A brief outline of these and their motivation is listed below:

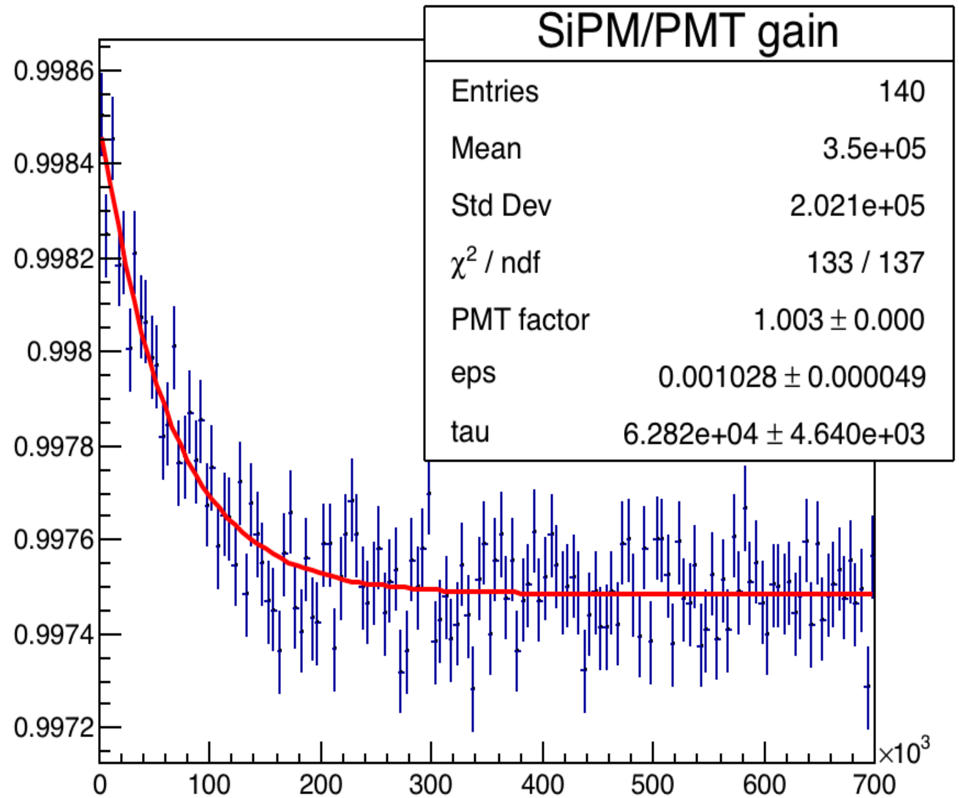
- Assume PMT drift of 0.1% in an hour for 2000, 10000 and 50000 events i.e. 1, 5 and 25 runs.
- Fit the wiggle plot with τ and ε obtained from the fit results of the simulated plots – call this uncorrected.
- Fit the wiggle plot after applying a correction with these values relative to the theoretical values.
- Perform more checks - like check with offsets in step function of PMT gains, check effect of PMT only with no SiPM gains.

Compare PMT gains 0.1% skipping 1 cycle – 50000 events

PMT = 1 + 0.1% for each cycle



PMT = 1.001 + 0.1% for each cycle



Not much difference in both cases