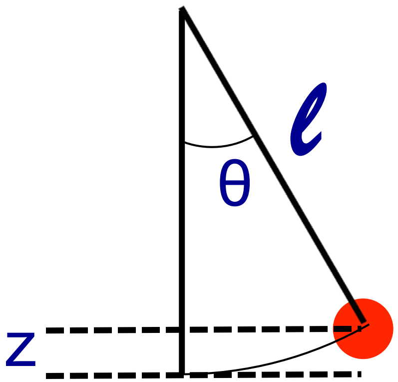
Example of application of Lagrangian and Hamiltonian formalism: the pendulum



$$U = mgz = mg\ell(1-\cos\theta) \qquad T = \frac{1}{2}mv^2 = \frac{1}{2}m\ell^2\dot{\theta}^2$$

$$L = T - U = m\ell\left[\frac{\ell\dot{\theta}^2}{2} - g\left(1-\cos\theta\right)\right] \qquad P_\theta = \frac{\partial L}{\partial\dot{\theta}} = m\ell^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0 \quad \Rightarrow \quad m\ell\left[\frac{d}{dt}\left(\ell\dot{\theta}\right) + g\sin\theta\right] = 0$$

$$\ell\ddot{\theta} + g\sin\theta = 0$$

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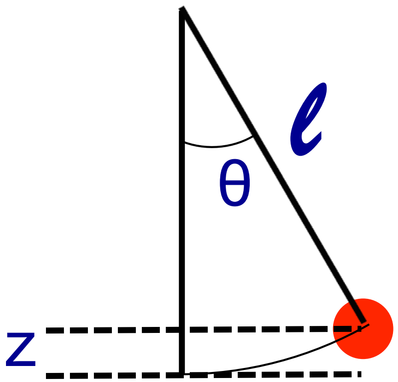

$$U = mgz = mg\ell(1-\cos\theta) \qquad T = \frac{1}{2}mv^2 = \frac{1}{2}m\ell^2\dot{\theta}^2$$

$$H = T + U = mg\ell\left(1-\cos\theta\right) + \frac{1}{2}m\ell^2\dot{\theta}^2$$

As variables let's consider θ and its canonical momentum $P_\theta = m\ell^2\dot{\theta}$ **(see previous page)**

$$H = mg\ell\left(1-\cos\theta\right) + \frac{P_\theta^2}{2m\ell^2} \qquad \dot{\theta} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{m\ell^2} \qquad \dot{P}_\theta = -\frac{\partial H}{\partial \theta} = -mg\ell\sin\theta$$

$$\dot{P}_\theta = m\ell^2\ddot{\theta} = -mg\ell\sin\theta$$

$$\ell\ddot{\theta} + g\sin\theta = 0$$