Fisica ed applicazioni degli acceleratori ad alta brillanza 1

Massimo Ferrario

Introduzione

Acceleratori di particelle lineari e circolari.
Luminosita' e Brillanza.
Sorgenti di particelle.
Fasci di particelle cariche ad alta brillanza e loro applicazioni.
Fasci di particelle cariche ad alta luminosita' e Colliders.
Sorgenti di radiazione, laser ad elettroni liberi, sorgenti Compton.
Nuove tecniche di accelerazione, laser e plasmi.
Impiego della Superconduttivita'.

Moto di particelle cariche in campi elettromagnetici esterni [1]

Richiami di dinamica relativistica.
Moto in strutture acceleranti a radiofrequenza.
Moto in campi magnetici dipolari, quadrupolari e solenoidali.
Moto in ondulatori magnetici.

Dinamica Longitudinale

Dinamica longitudinale e spazio delle fasi
Stabilita' di fase
Emittanza longitudinale
Phase Space Matching
Criteri di Stabilita'

Compressione longitudinale dei fasci di particelle [5,6]

Oscillazioni di densita' longitudinale dei fasci di particelle.
Compressione magnetica.
Instabilita' dei fasci in compressori magnetici.
Compressione mediante onda a radiofrequenza.
Generazione di fasci a pettine.
Misure di lunghezza del fascio.

Dinamica trasversa senza carica spaziale. [1,2]

Teorema di Liouville.
Emittanza, Brillanza e Luminosita'.
Equazioni di trasporto per sistemi a simmetria assiale.
Analogie con l'ottica geometrica, focalizzazione.
Trasporto del fascio in sistemi di focalizzazione periodici.
FODO Lattice.
Moto di Betatrone in strutture periodiche.
Equazione di Hill.

Dinamica trasversa con carica spaziale, effetti collettivi. [1,2]

Lunghezza di Debye.
Modelli di fascio di particelle con carica spaziale.
Dalle equazioni di Vlasov alle equazioni di inviluppo.
Oscillazioni di plasma ed oscillazioni di inviluppo.
Effetti delle cariche imagine.

Degradazione di emittanza [3,4]

Emittanza ed entropia.
Cause di degradazione dell'emittanza.
Oscillazioni di plasma ed oscillazioni di emittanza.
Compensazione di emittanza in un linac ad alta brillanza.
Effetti cromatici.
Metodi di misura dell'emittanza.

Wake fields ed instabilita' [7]

Campi e potenziali di scia prodotti da una distribuzione di particelle cariche.
Effetti a corto raggio.
Instabilita' prodotta dai campi di scia.
Cure alla instabilita' prodotta dai campi di scia.
Landau damping.
Effetti cumulativi di multi-bunch.

Radiazione di Sincrotrone

Sorgenti di radiazione
Radiazione da magneti curvanti
Radiazione da ondulatore magnetico
Sorgenti Compton

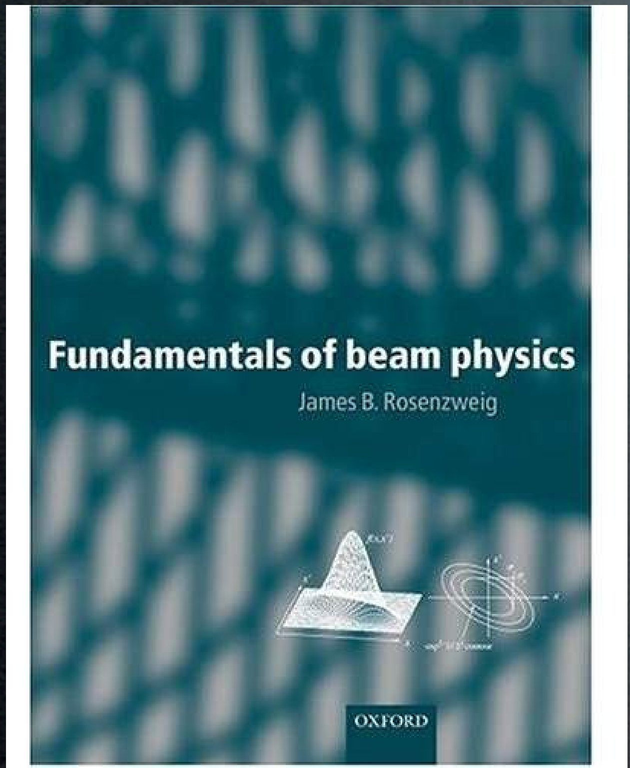
Il laser ad elettroni liberi [10,11]

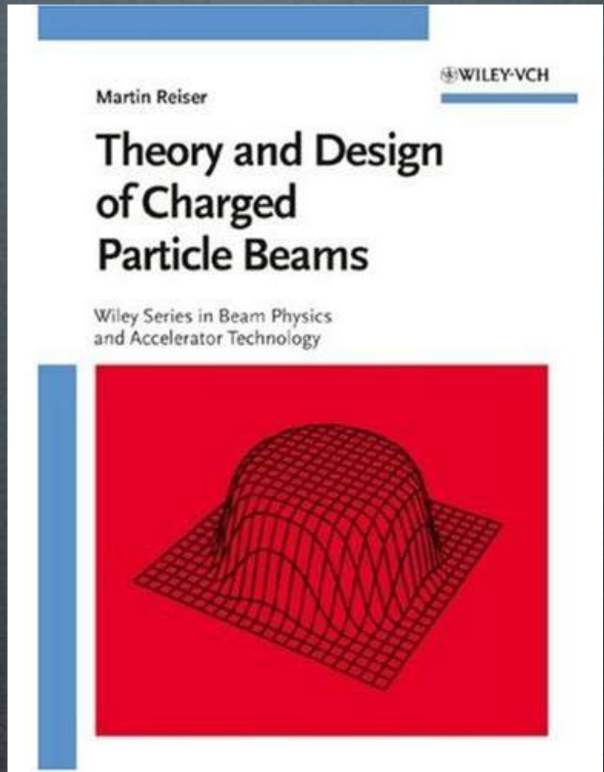
Radiazione coerente da fasci di particelle.
Radiazione di ondulatore.
Teoria del laser ad elettroni liberi.
Regimi di Amplificazione, Seeding, Self amplified spontaneous emission e singola spike.
Esperimenti esitenti.
Applicazioni.

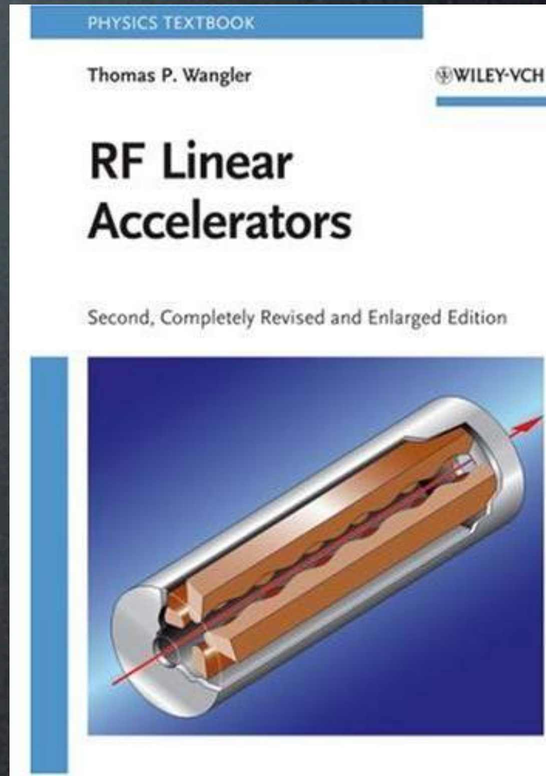
Acceleratori a plasma [8,9]

Eccitazione di onde di plasma.
Acceleratori a plasma pilotati da laser.
Regime di autoiniezione ed iniezione esterna.
Acceleratori a plasma pilotati da fasci intensi di paricelle.
Eccitazione risonante mediante fasci a pettine.

Visita a SPARC-Lab presso INFN-LNF
(Sources for Plasma Accelerators and Radiation Compton with Lasers and Beams)


Fundamentals of beam physics
James B. Rosenzweig
OXFORD


WILEY-VCH
Martin Reiser
Theory and Design of Charged Particle Beams
Wiley Series in Beam Physics and Accelerator Technology


PHYSICS TEXTBOOK
Thomas P. Wangler
WILEY-VCH
RF Linear Accelerators
Second, Completely Revised and Enlarged Edition


WALT DISNEY
TOPOLINO
E
L'ACCELERATORE
NUCLEARE


...MA STIAMO PER INAUGURARNE UNO MOLTO PIU' GRANDE! ECCOLO!
OH!


ARRIVATI A DESTINAZIONE...
C.E.R.N
C.E.R.N
QUANTO VI DOBBIAMO?
NIENTE, NIENTE! MI BASTA QUELLO CHE AVRO' DA RACCONTARE AGLI AMICI!


CHE MACCHINARI IMPRESSIONANTI!
DISPONIAMO DI APPARECCHIATURE SOFISTICATISSIME!


ORA FACCIO MARCIARE I DUE TRENINI IN SENSO OPPOSTO!
ESSI SI SCONTRERANNO NEL PUNTO DI INTERSEZIONE DEI BINARI! BIP!
EHI, FERMO! COSA VUOI FARE?


CRAASCHH
VISTO, LE CARAMELLE?! SE FOSSERO STATI ATOMI SI SA, REBBERO FRANTUMATI...
...PERMETTENDOCI DI SCOPRIRE LE PARTICELLE PIU' PICCOLE CHE COMPONGONO LA MATERIA!
GRUNT! FORTUNA CHE LA SVIZZERA E' LA PATRIA DEL MODELLISMO...

Modern accelerators require high quality beams:
==> High Luminosity & High Brightness

$$L = \frac{N_{e+} N_{e-} f_r}{4\pi\sigma_x\sigma_y}$$

-N of particles per pulse => 10^9

-High rep. rate f_r => bunch trains

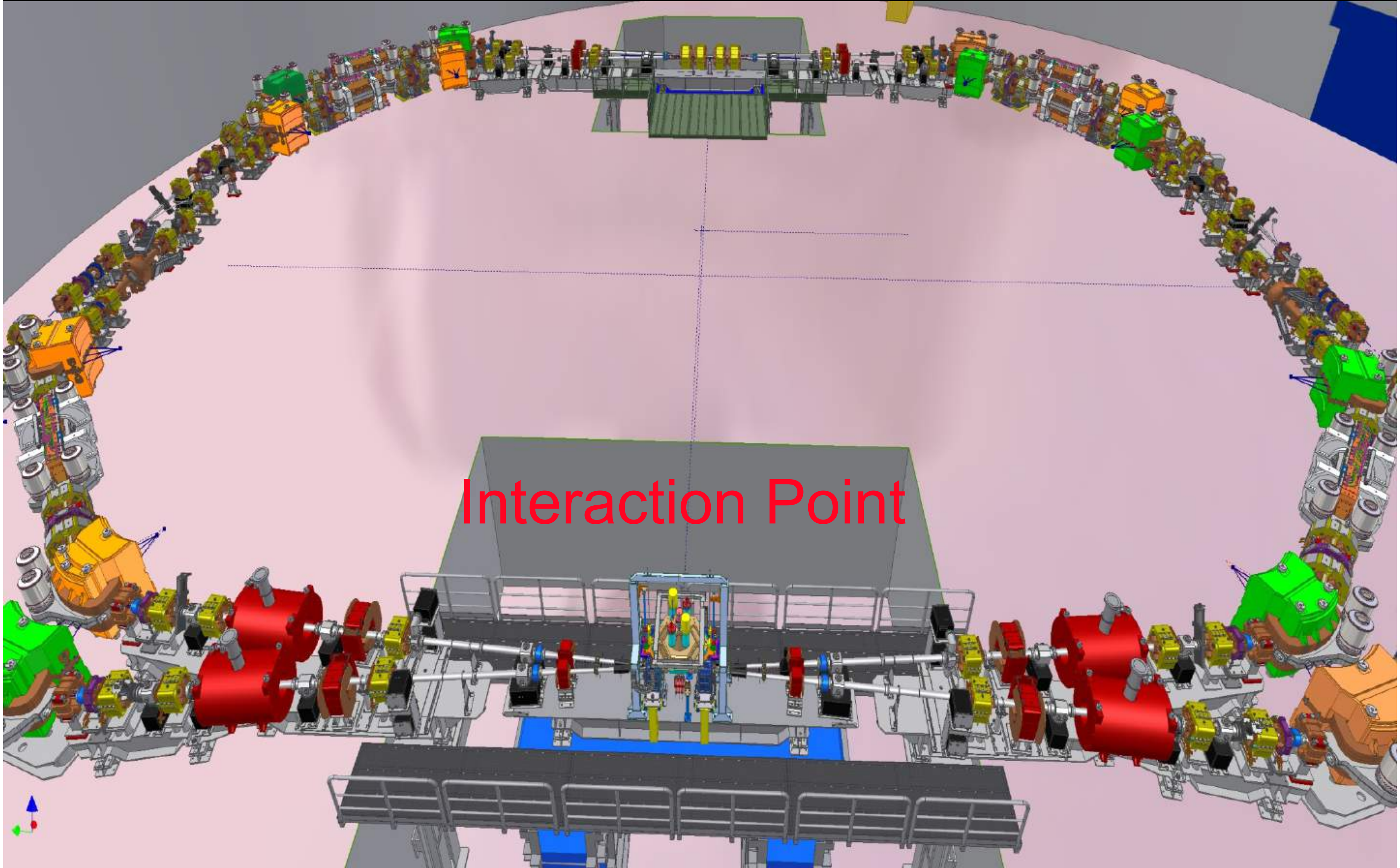
-Small spot size => low emittance

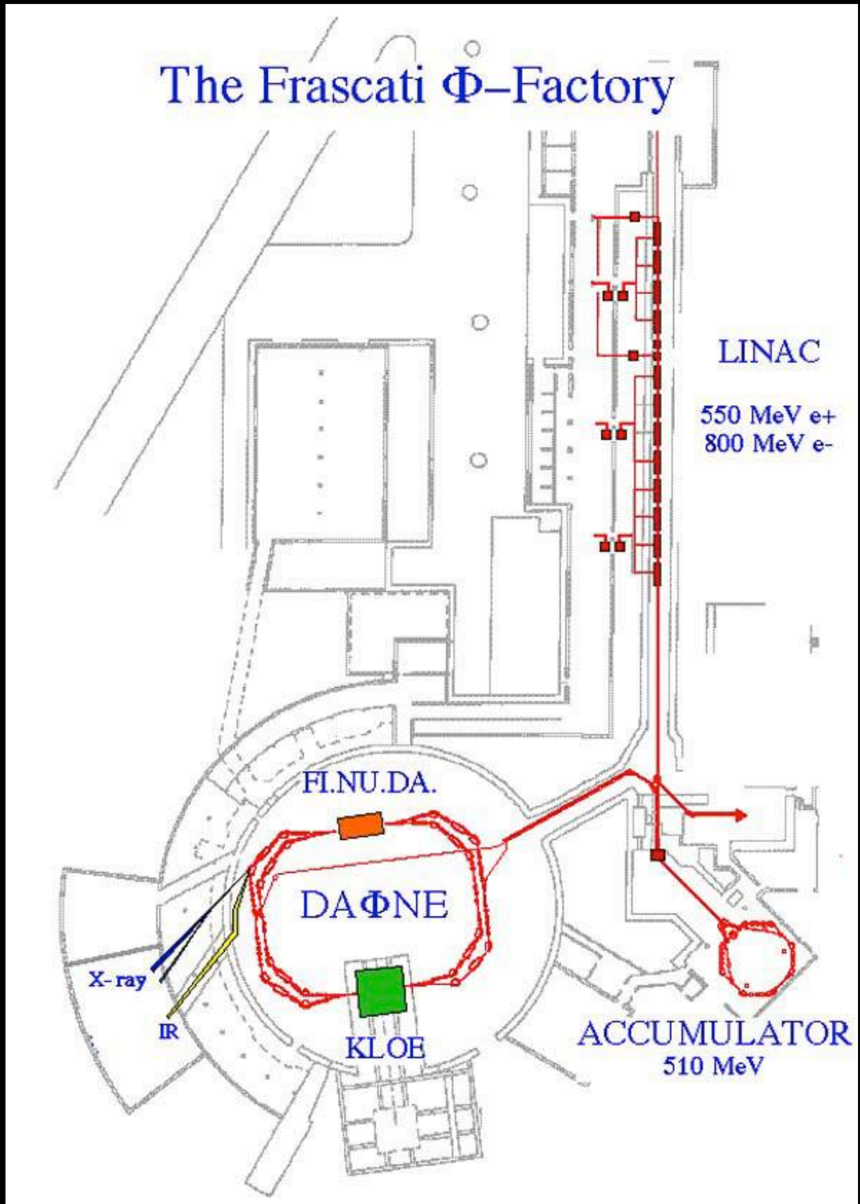
$$B_n \approx \frac{2I}{\varepsilon_n^2}$$

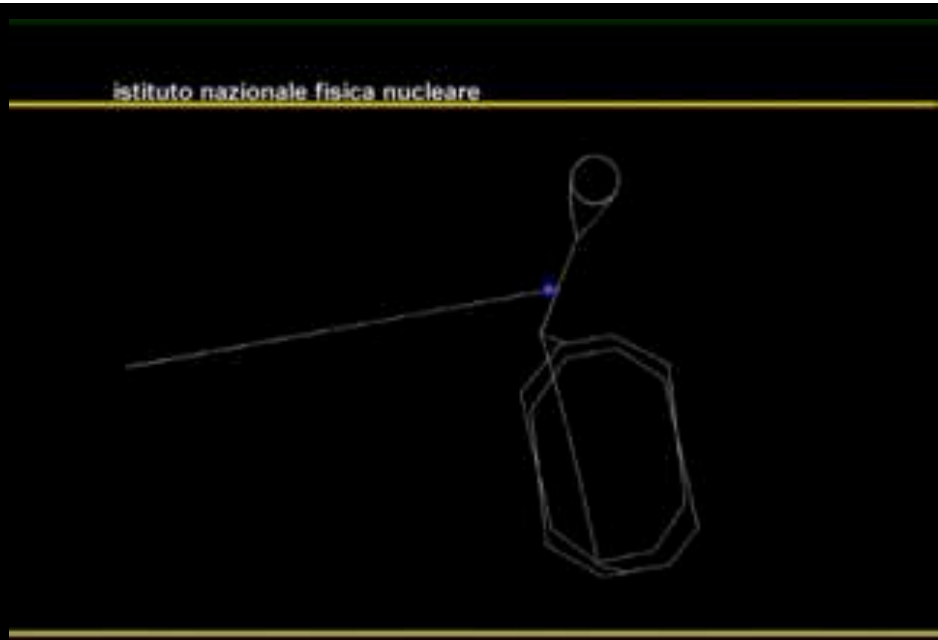
-Short pulse (ps to fs)

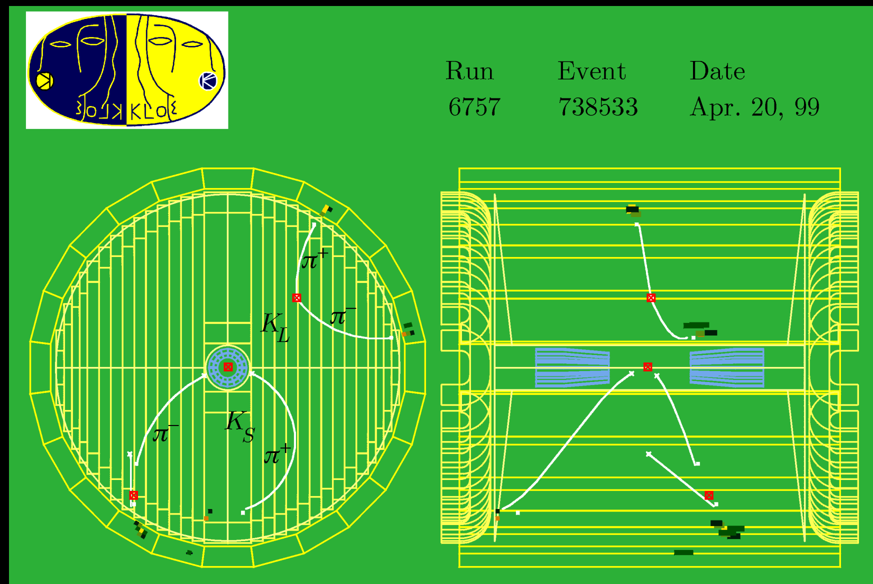
-Little spread in transverse momentum and angle => low emittance

Collider e+ e- DAFNE (INFN)

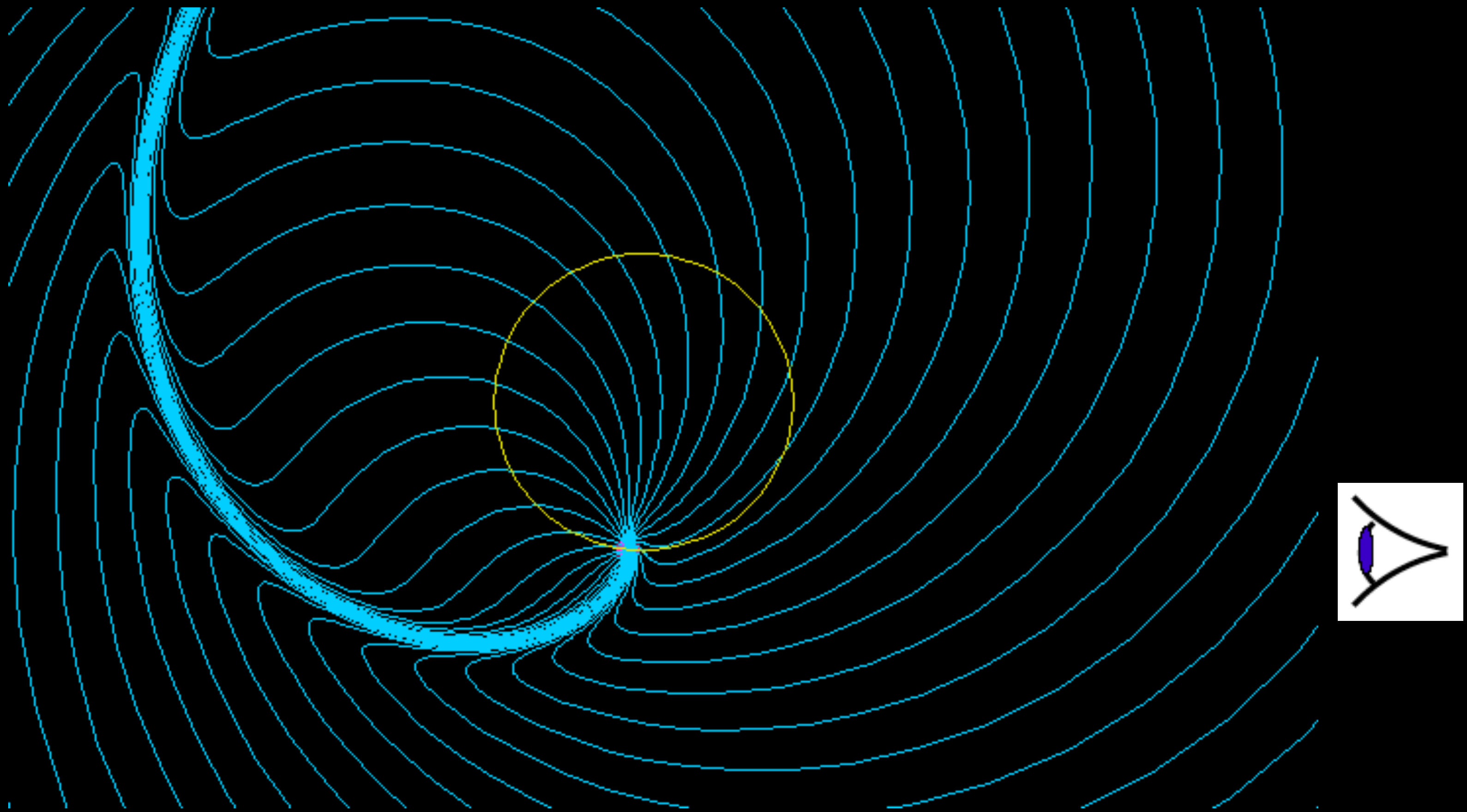



The Frascati Φ–Factory
LINAC
550 MeV e+
800 MeV e-
FI.NU.DA.
DAΦNE
X-ray
IR
KLOE
ACCUMULATOR
510 MeV


istituto nazionale fisica nucleare

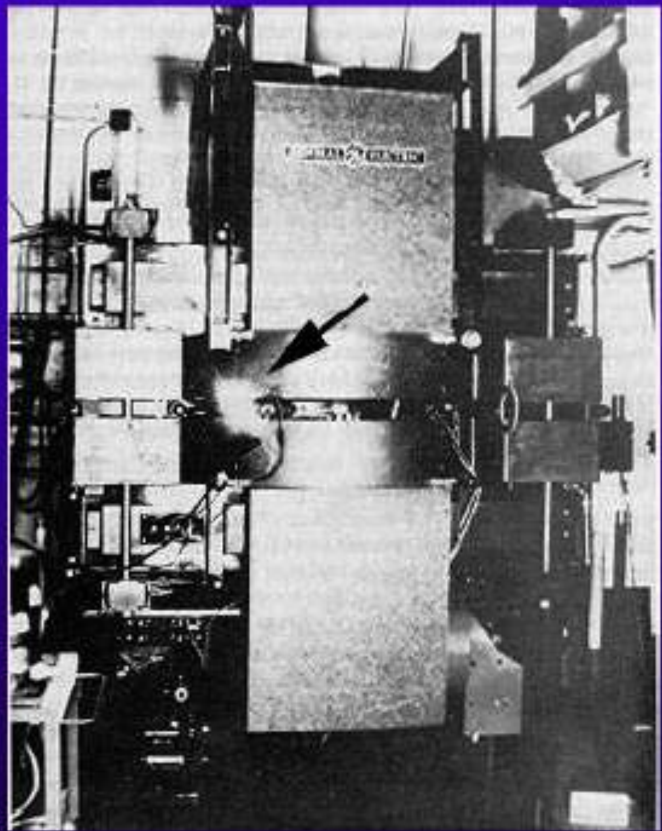

Run
6757
Event
738533
Date
Apr. 20, 99
π^+
π^-
K_L
K_S
π^-
π^+

Particella carica in moto circolare



Radiation Simulator – T. Shintake, @ http://www-xfel.spring8.or.jp/Index.htm

GE Synchrotron
New York State



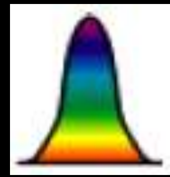
First light observed
1947

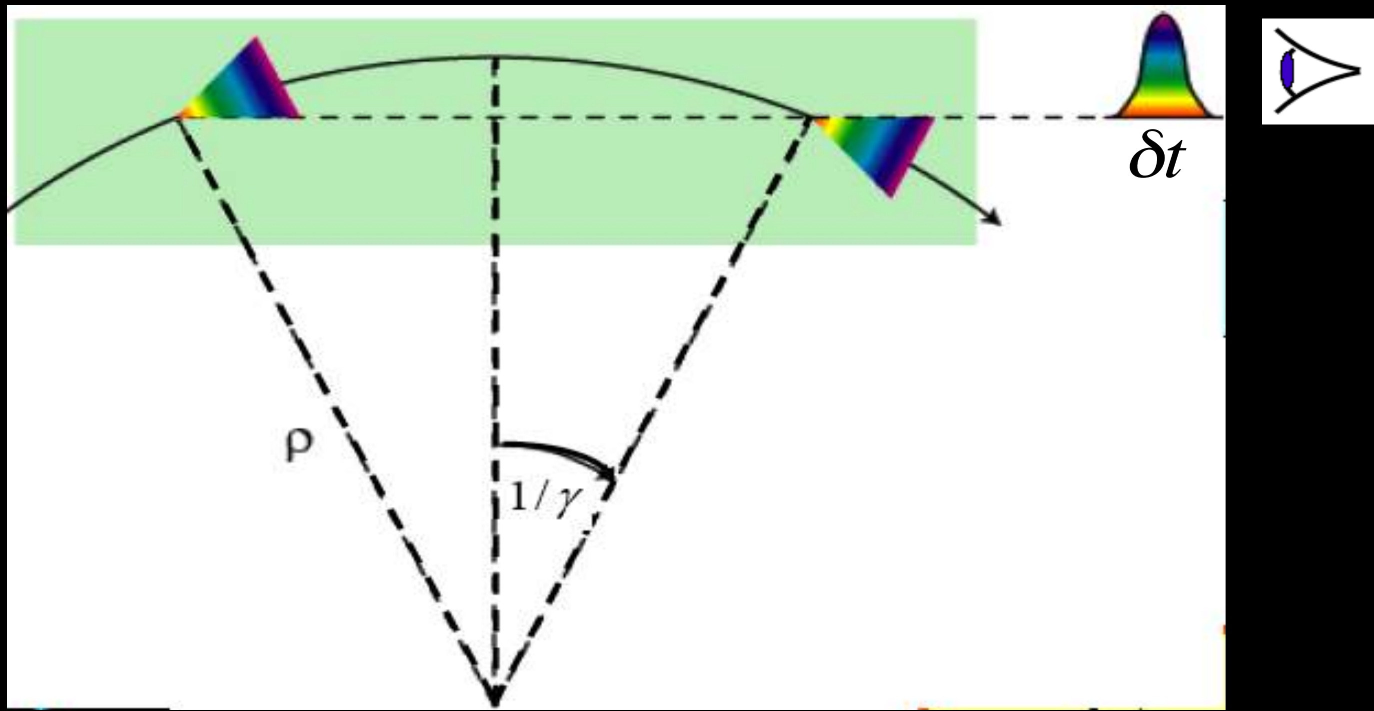
Elettra (Trieste)

ESRF (Francia)

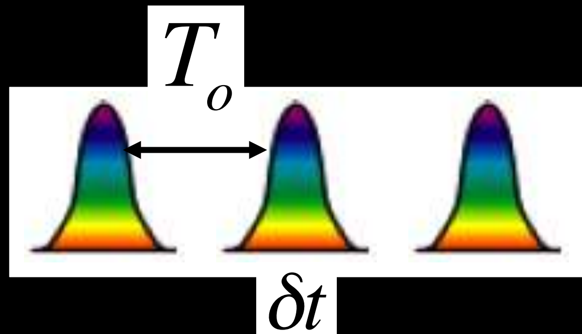
SLS (Svizzera)

Durata dell' impulso $\delta t \approx \frac{\rho}{E^3} \approx 100\,ps$

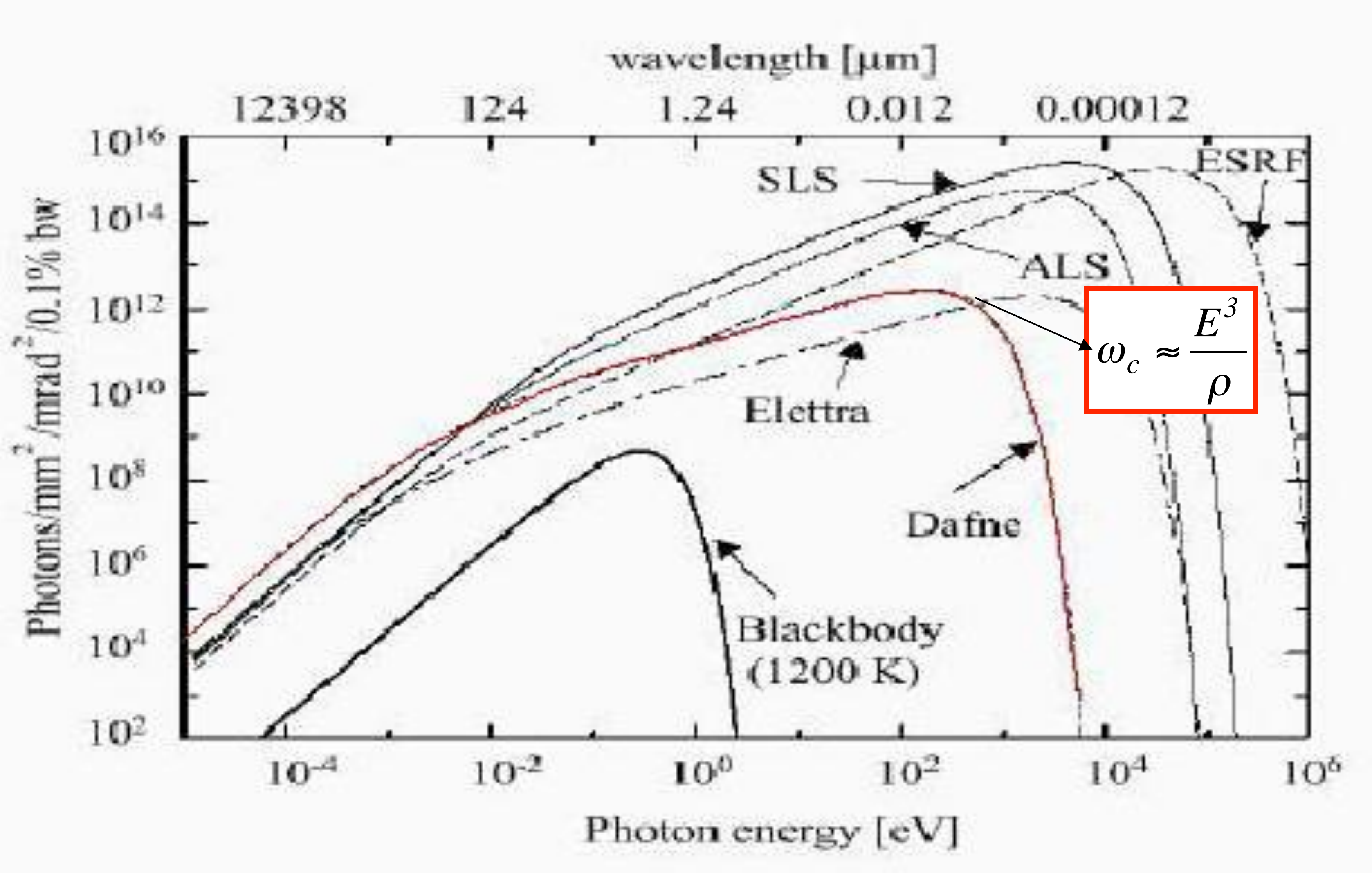


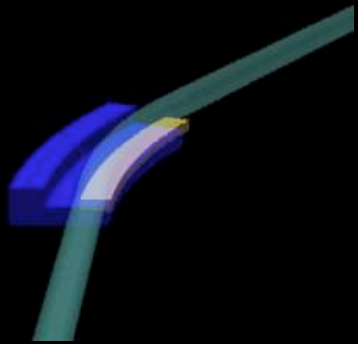


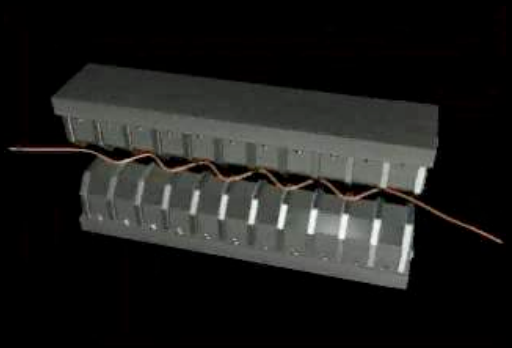
Ripetizione dell' impulso $\omega_o = \frac{2\pi}{T_o}$

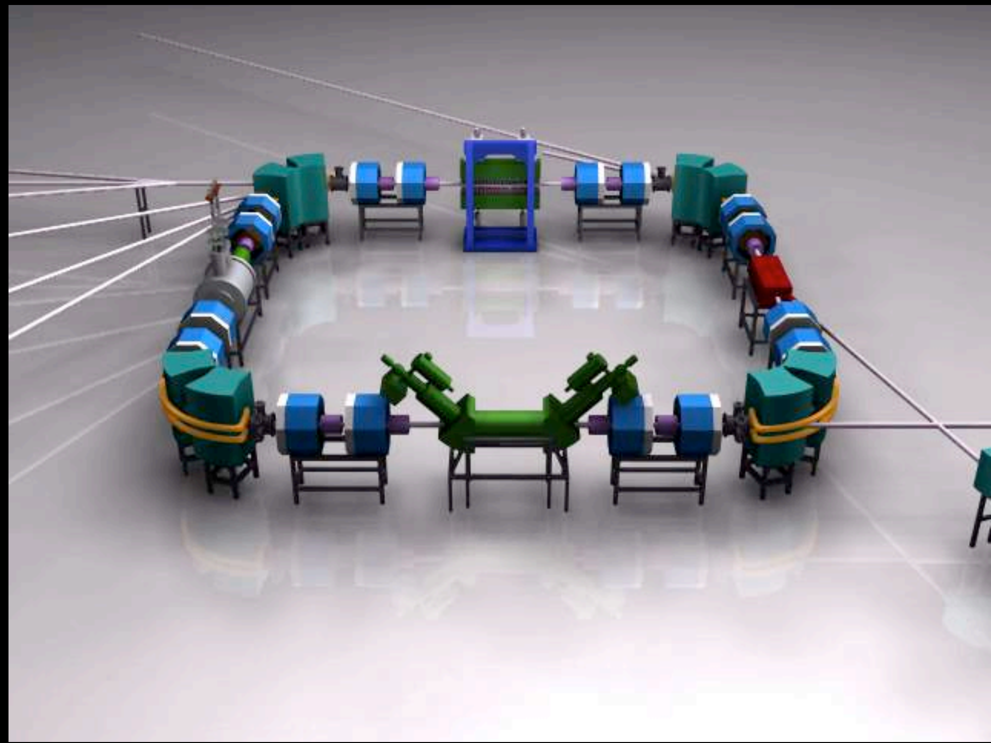


$$\langle P_s\,(\mathrm{MW})\rangle_{\mathrm{iso}} = 0.088463\,\frac{E^4\,(\mathrm{GeV})}{\rho\,(\mathrm{m})}\,I\,(\mathrm{A})\ .$$



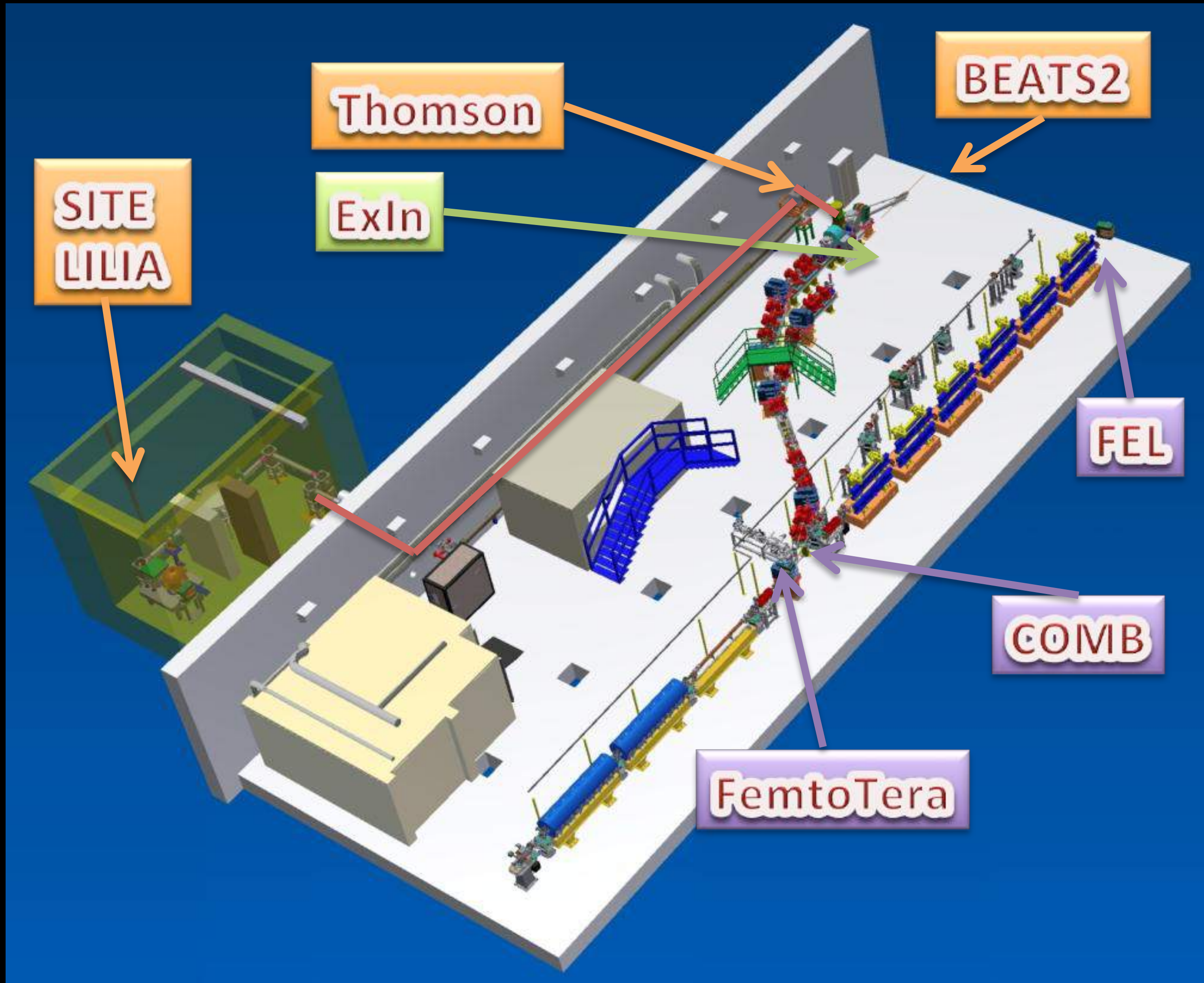


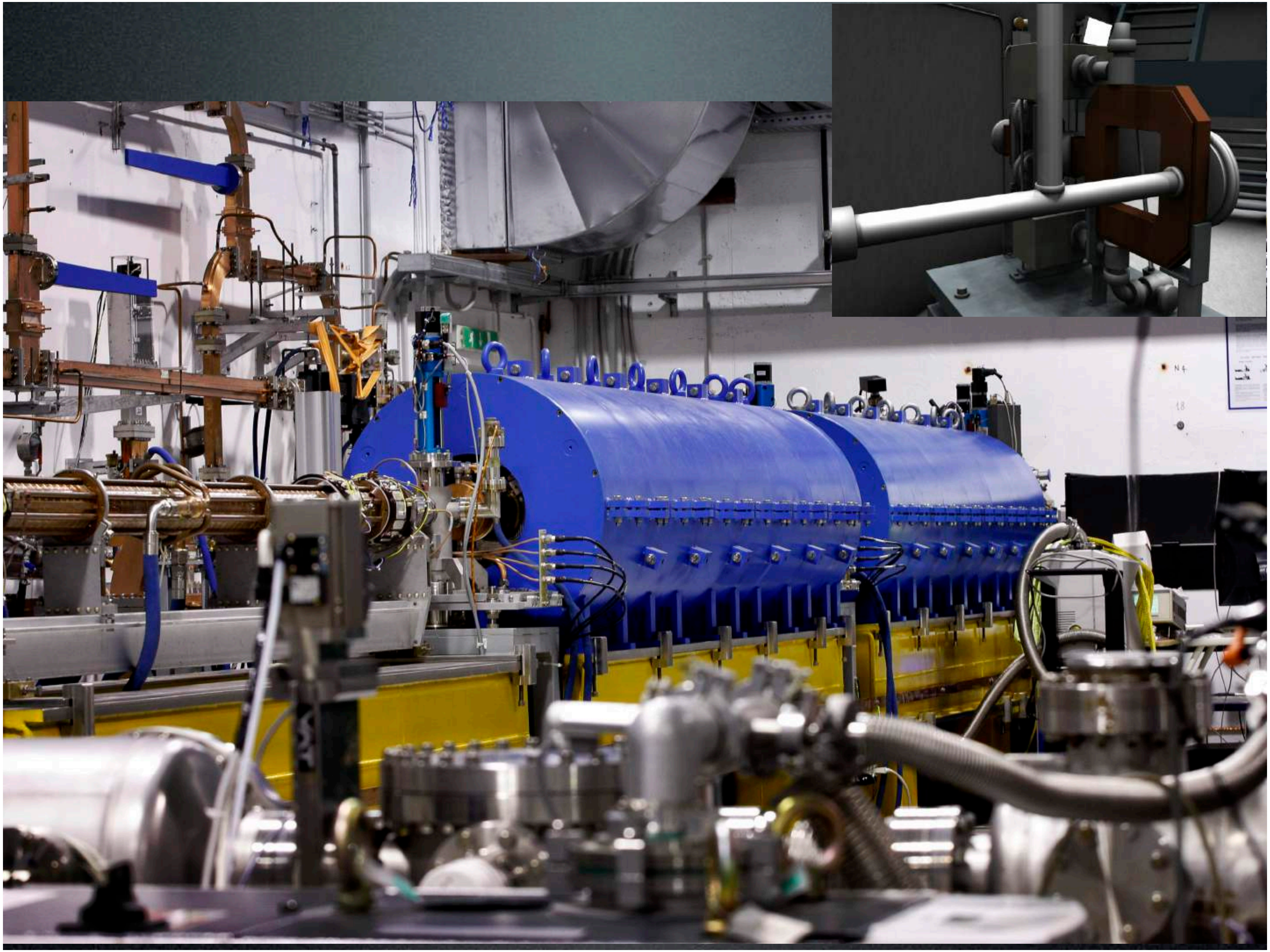


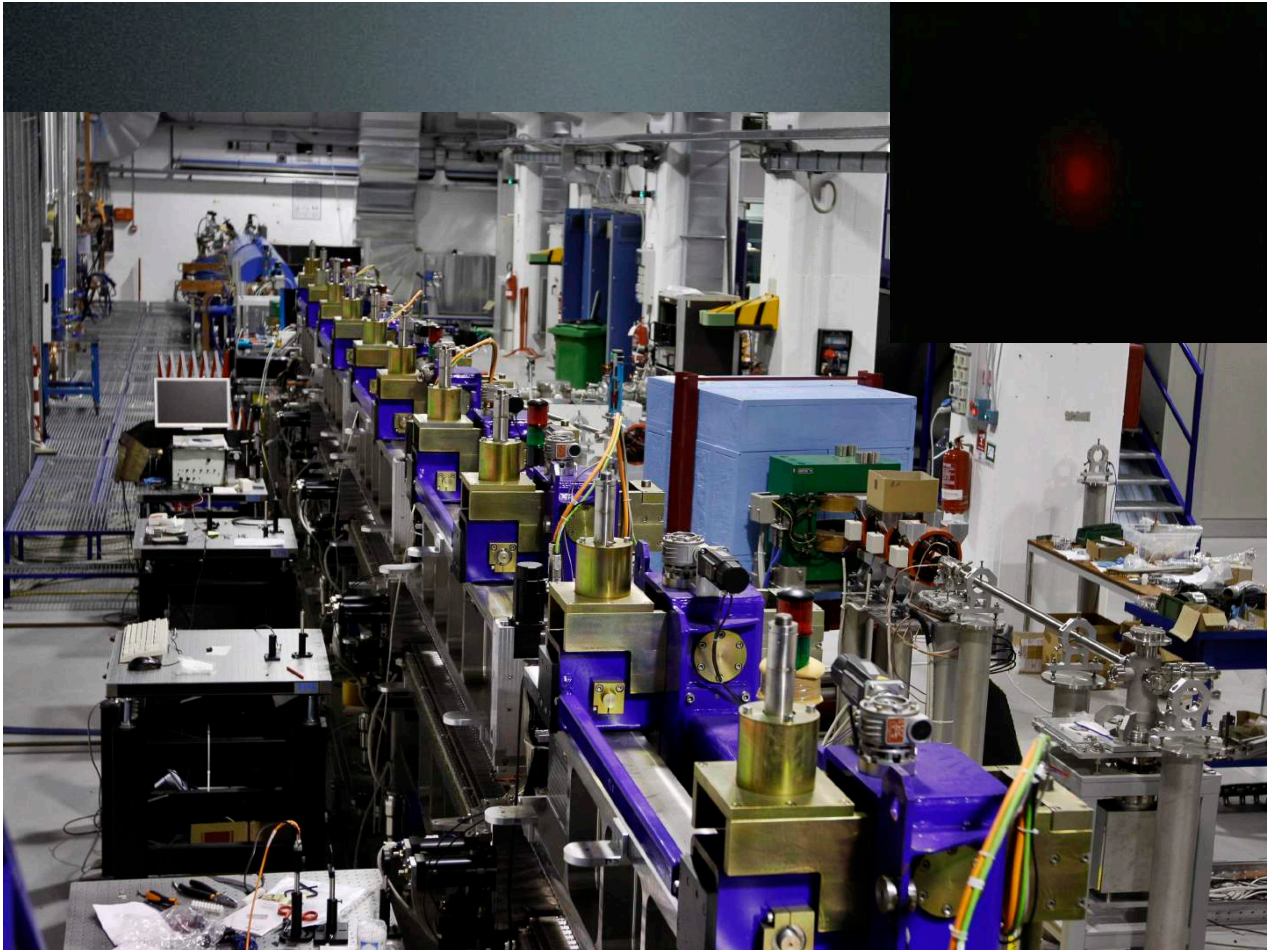


SPARC_LAB

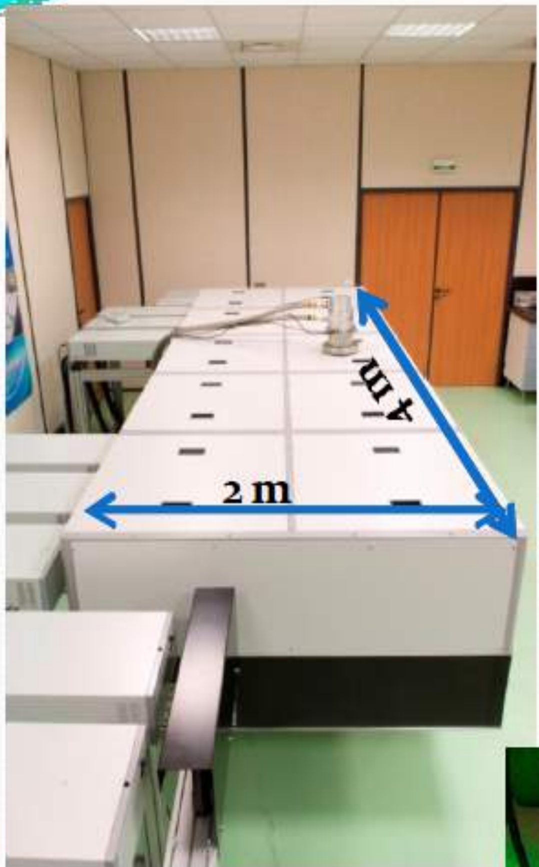
Sources for Plasma Accelerators and Radiation Compton with Lasers And Beams

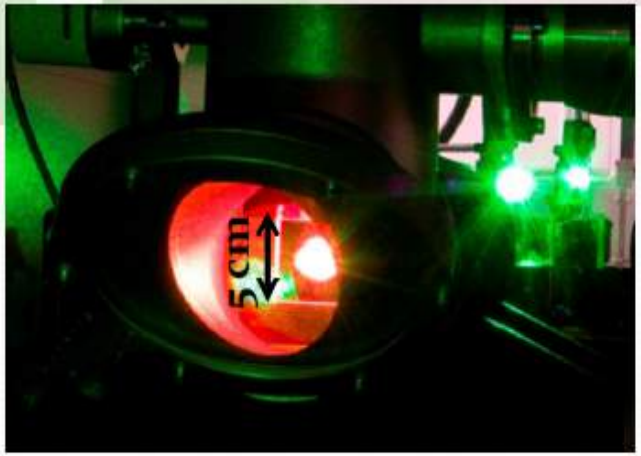






Il laser FLAME





Energia massima: 7J

Energia massima sul target: ~5J

Durata minima: 23 fs

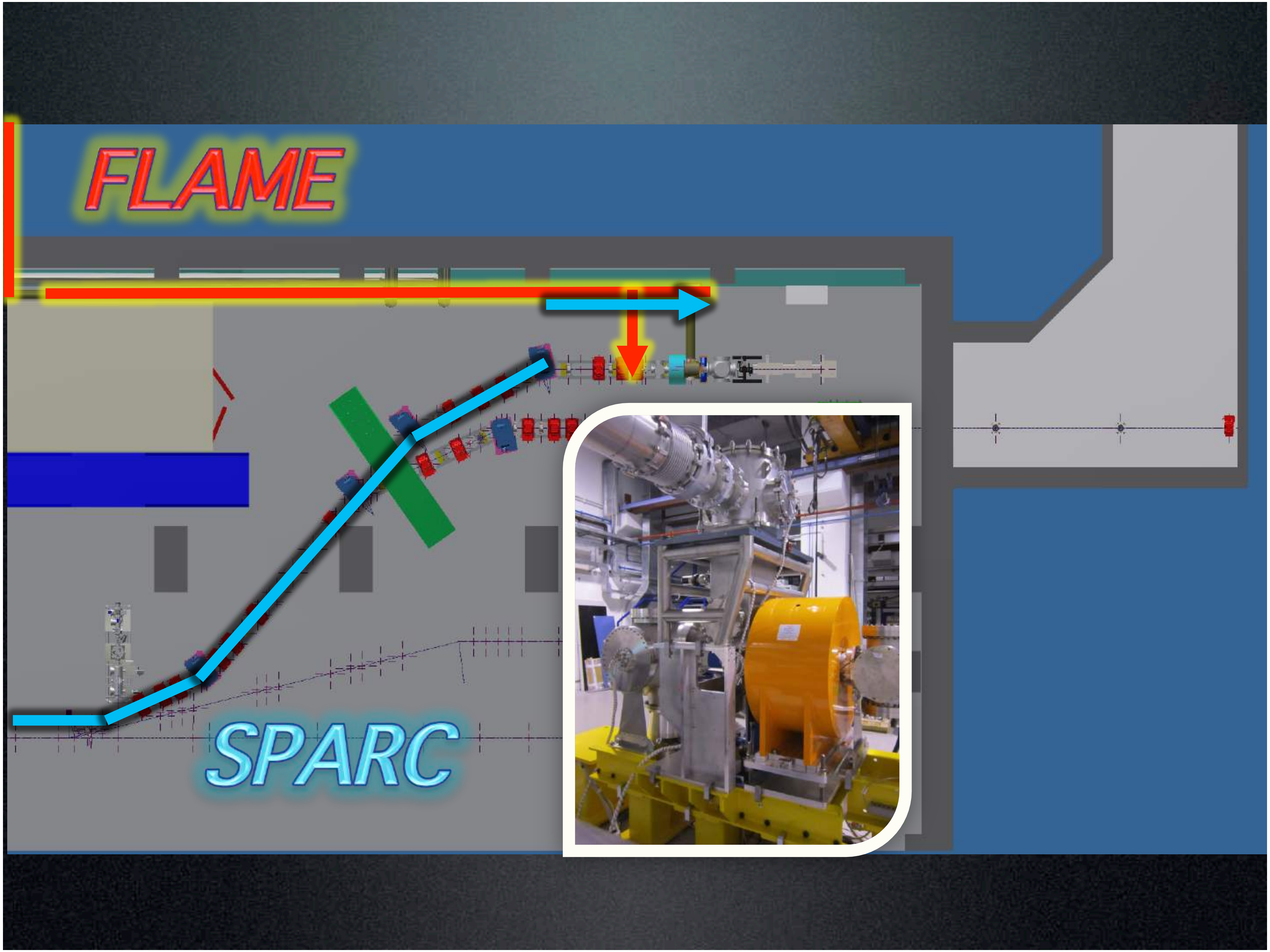
Lunghezza d'onda: 800 nm

Larghezza di banda: 60/80 nm

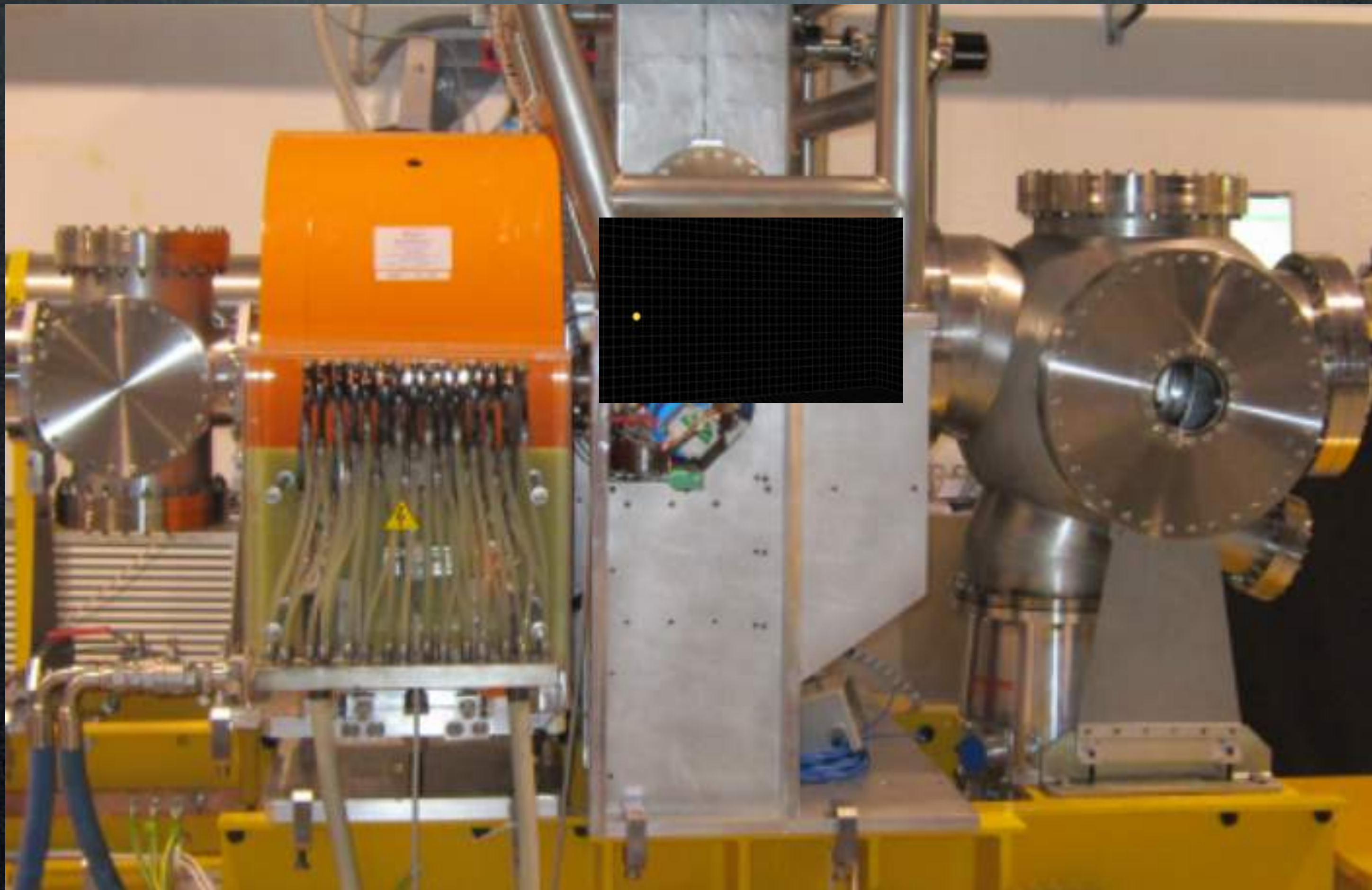
Spot-size @ focus: 10 μm

Potenza massima: ~300 TW

Contrasto: 10^{10}

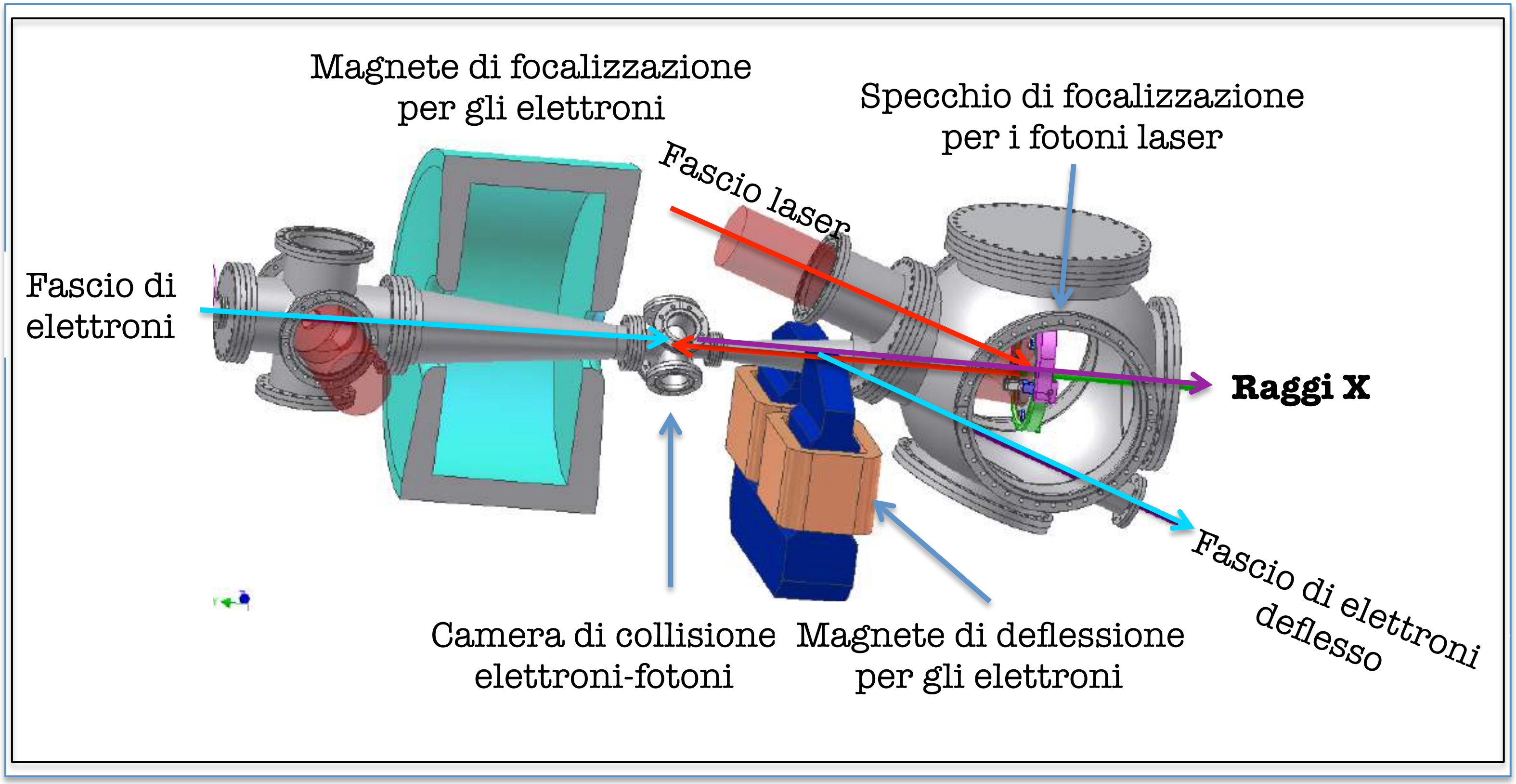

FLAME
SPARC

Thomson backscattering



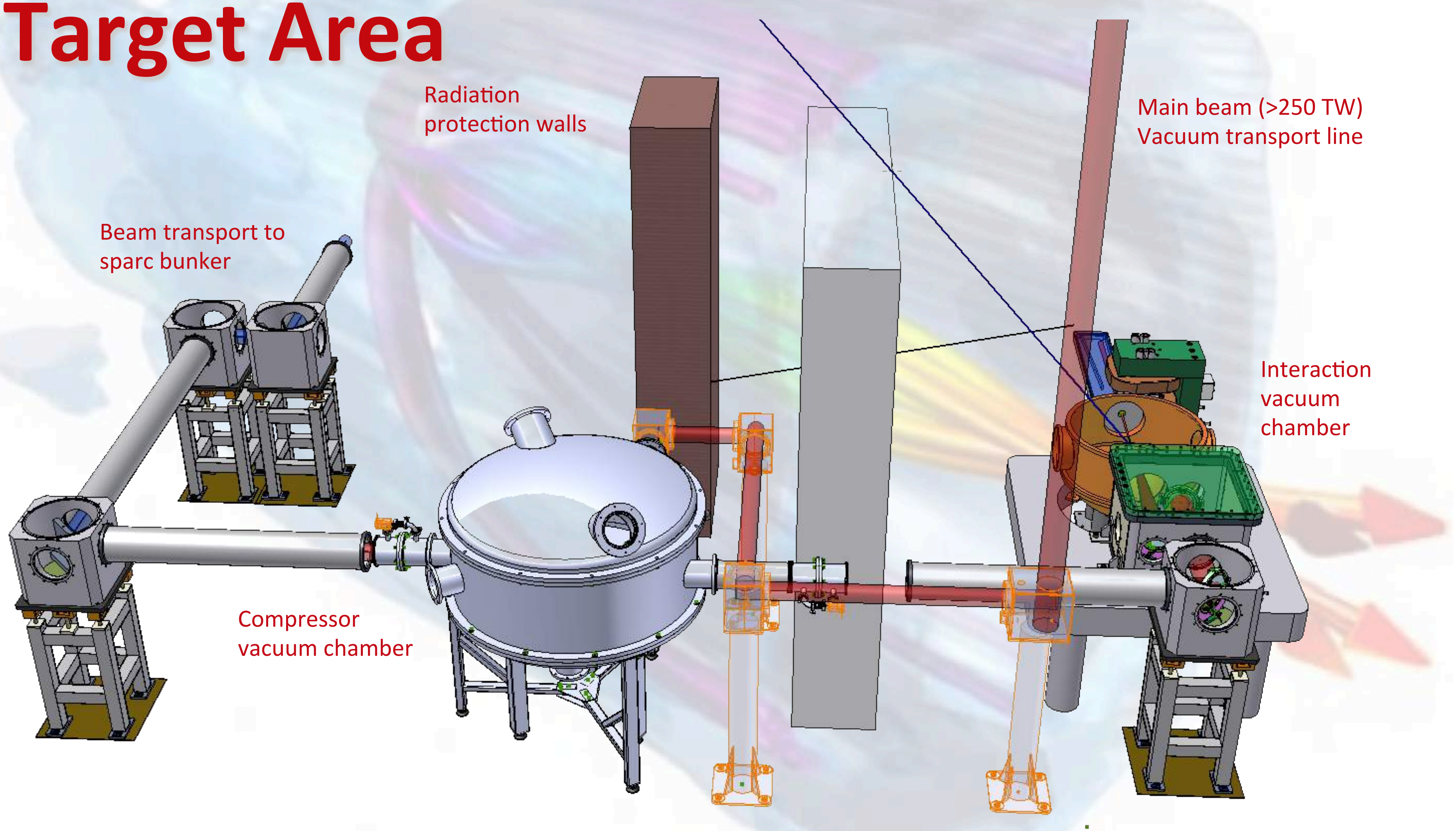
Courtesy C. Vaccarezza

Thomson Interaction region (20-550 keV)

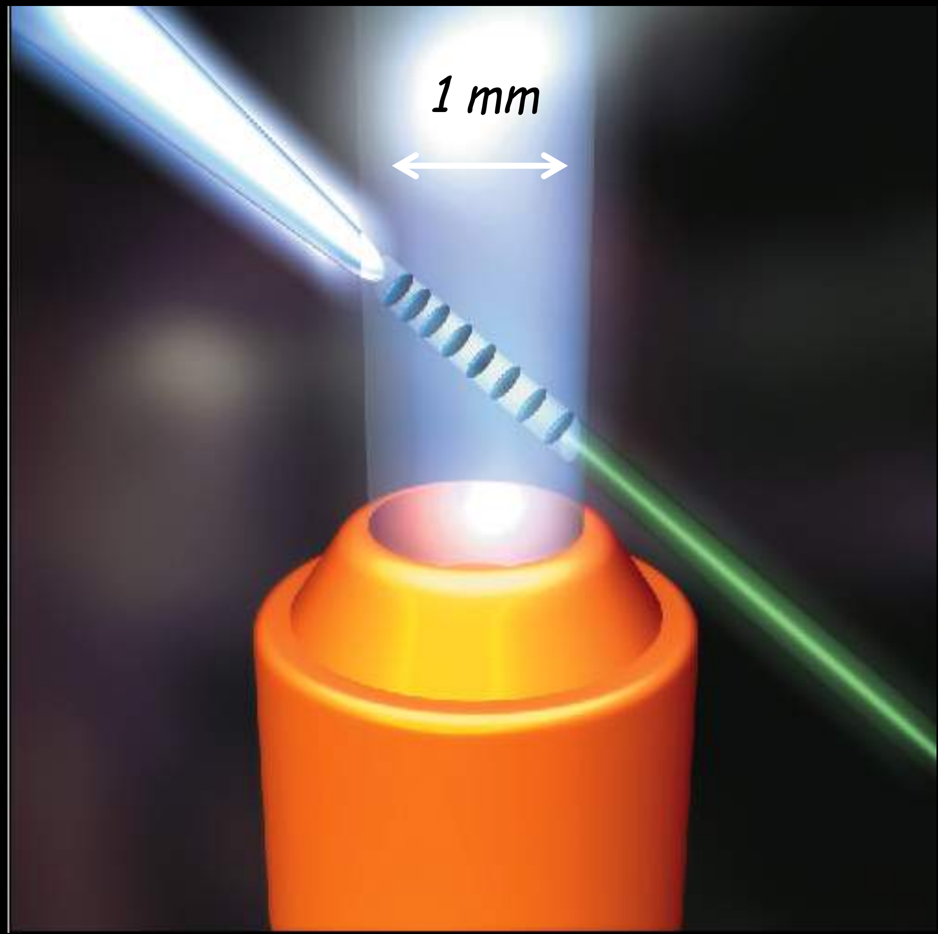


Esperimenti di auto-iniezione

Target Area

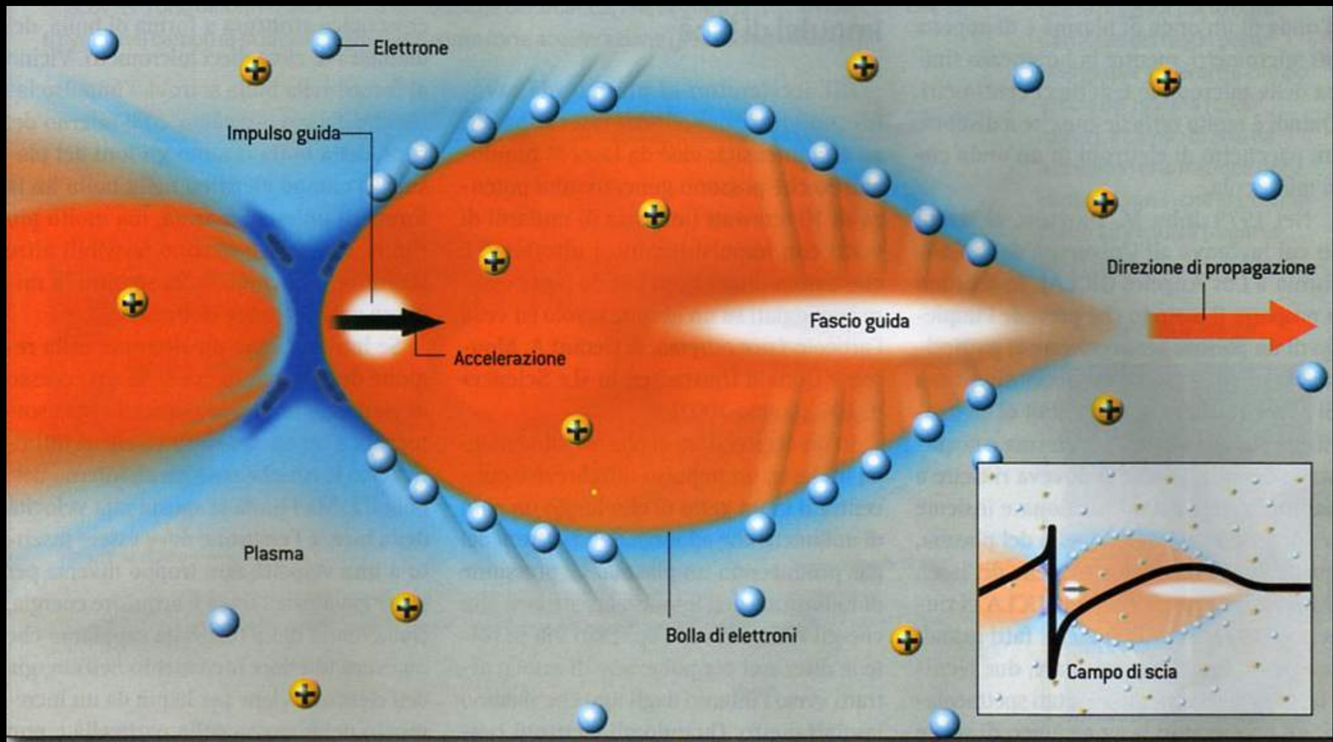


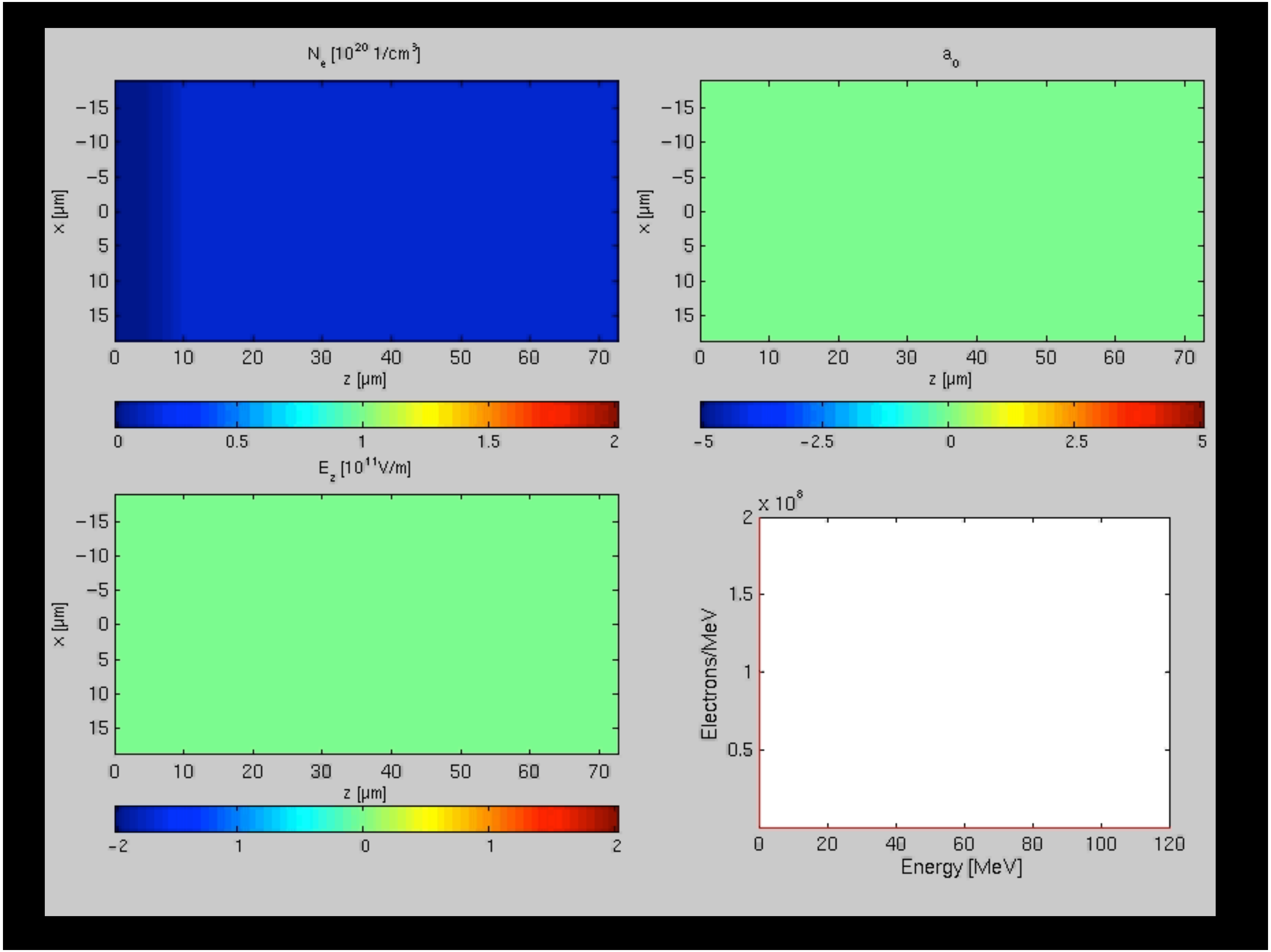
Direct production of e-beam



Electron beam

High quality beam Plasma Accelerattion





N_e [10^{20} 1/cm^3]
x [µm]
z [µm]
a_0
E_z [10^{11}V/m]
x 10^8
Electrons/MeV
Energy [MeV]

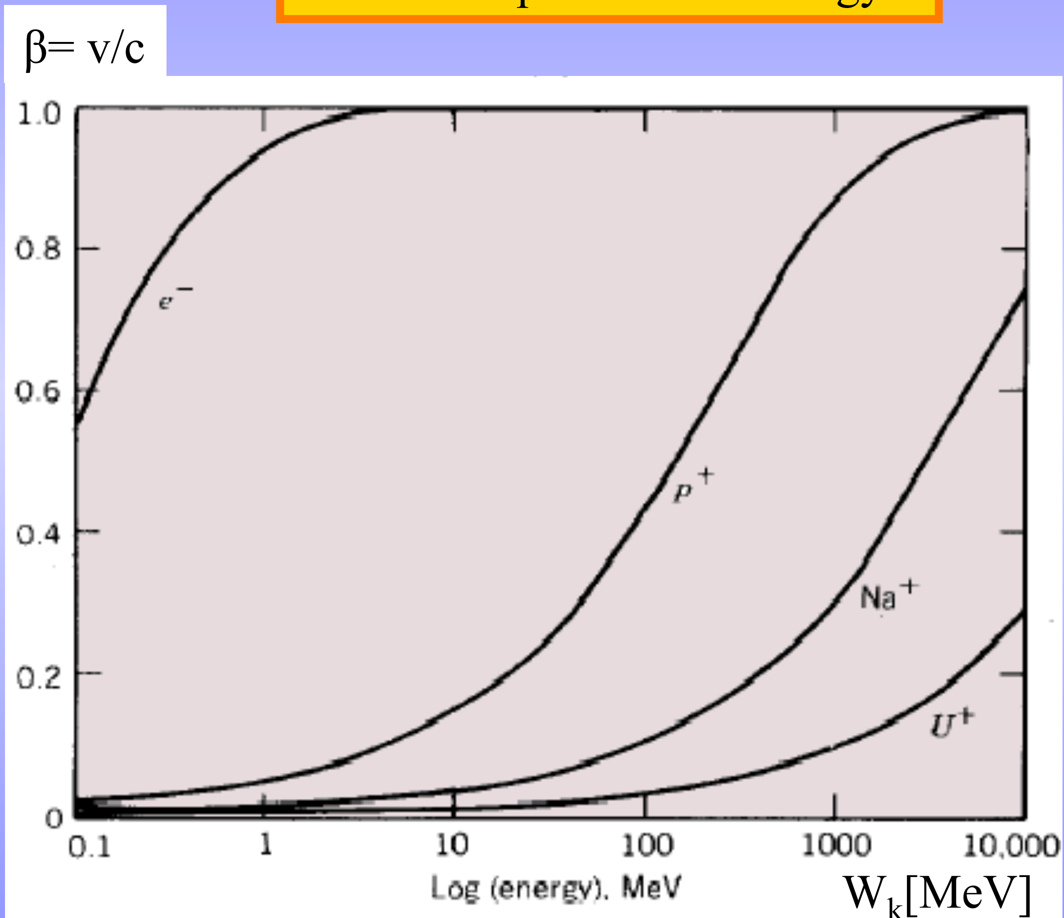
Fundamental relations of the relativistic dynamics

Rest Energy	Relativistic β-factor	Relativistic γ-factor	Total Energy	Kinetic Energy
$W_0 = m_0 c^2$	$\beta = v/c$, $\beta < 1$ *always*!	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\gamma \geq 1$ *always*! $m = \gamma\, m_0$	$W = \gamma\, m_0 c^2 = \gamma\, W_0$ $W^2 = W_0^2 + p^2 c^2$	$W_k = W - W_0 =$ $= (\gamma - 1) m_0 c^2 \approx$ $\approx \frac{1}{2} m_0 v^2 \quad se \quad \beta << 1$

Newton's 2nd Law	Lorentz Force
$\vec{F} = \frac{d}{dt}\vec{p} = \frac{d}{dt}(m\vec{v})$	$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$

Energy-velocity plot

Velocity variations are negligible at energies well above the particle rest energy!



e^- relativistic ($v \cong c$) at W>1MeV (W_0=511keV)
p relativistic at W>1000 MeV (W_0=938MeV)

Leptons (light particles) are pratically **fully relativistic** in any existing dedicated accelerators ($W_k >> W_0$, with the exception of the very first acceleration stage) while **protons** and **ions** are typically **weakly relativistic** ($W_k < W_0$ – but not always, see high energy hadron colliders such as the LHC).

For leptons the accelerating process occours at **constant particle velocity** (***v ≈ c***), while protons and ions velocity may **change a lot** during acceleration. This implies major important differences in the technical characteristics of the dedicated accelerating structures.

Particle energies are tipically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:
$1\ eV = 1.6 \times 10^{-19}$ J

Fundamental equation of the particle motion

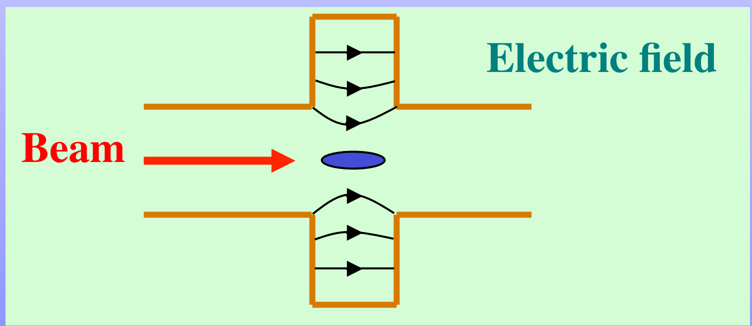
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{p}$ = *momentum*
m = *mass*
$\vec{v}$ = *velocity*
q = *charge*

ACCELERATION

Longitudinal Dynamics

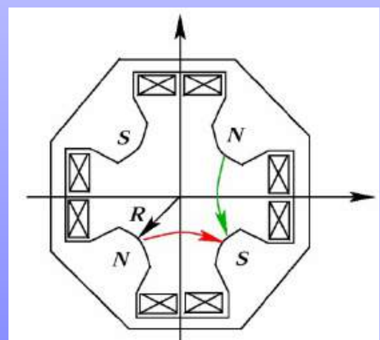
$\vec{E}$ = electric field

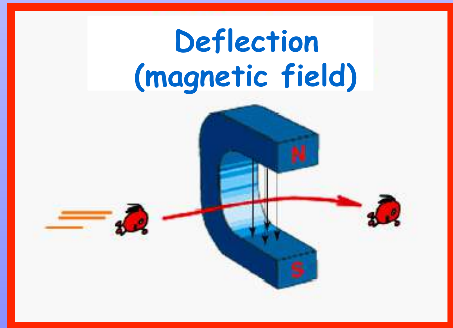


BENDING AND FOCUSSING

Transverse Dynamics

$\vec{B}$ = magnetic field





3. RELATIVISTIC RELATIONS

Some relativistic expressions (the symbols have the usual meaning):

$$E_0 = mc^2 \; ; \; E = E_0\gamma = mc^2\gamma \; ; \; p = mc\beta\gamma \; ; \; cp = mc^2\beta\gamma = E_0\beta\gamma \; ; \; E^2 = E_0^2 + p^2c^2$$

$$\beta\gamma = \frac{cp}{E_0} \; ; \; \gamma = (1-\beta^2)^{-\frac{1}{2}} \; ; \; \beta^2\gamma^2 = \gamma^2 - 1 \; ; \; W = E - E_0 \; ; \; \frac{mc\beta\gamma}{q} = B\rho \; .$$

Table 1. *Analytic relations between β, γ, W, cp*

	β	γ	W	cp
β	β	$\frac{\sqrt{\gamma^2-1}}{\gamma}$	$\frac{\sqrt{(1+W/E_0)^2-1}}{1+W/E_0}$	$\frac{cp/(mc^2)}{\sqrt{1+[cp/(mc^2)]^2}}$
γ	$\frac{1}{\sqrt{1-\beta^2}}$	γ	$1+W/E_0$	$\sqrt{1+\left(\frac{cp}{mc^2}\right)^2}$
W	$\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)E_0$	$E_0(\gamma - 1)$	W	$mc^2\left[\sqrt{1+\left(\frac{cp}{mc^2}\right)^2}-1\right]$
cp	$mc^2\frac{\beta}{\sqrt{1-\beta^2}}$	$E_0(\gamma^2 - 1)^{1/2}$	$[W(2E_0+W)]^{1/2}$	cp

Some relations concerning first derivatives of relativistic factors:

$$\frac{d\beta}{d\gamma} = \frac{1}{\beta\gamma^3} \; ; \; \frac{d(1/\beta)}{d\gamma} = -\frac{1}{\beta^3\gamma^3} \; ; \; \frac{d(\beta\gamma)}{d\beta} = \gamma^3 \; ; \; \frac{d(\beta\gamma)}{d\gamma} = \frac{1}{\beta} \; ;$$

Logarithmic first derivatives:

$$\frac{d\beta}{\beta} = \frac{1}{\beta^2\gamma^2}\frac{d\gamma}{\gamma} = \frac{1}{\gamma(\gamma+1)}\frac{dW}{W} = \frac{1}{\gamma^2}\frac{dp}{p} \; ; \; \frac{d\gamma}{\gamma} = (\gamma^2-1)\frac{d\beta}{\beta} = \left(1-\frac{1}{\gamma}\right)\frac{dW}{W} = \beta^2\frac{dp}{p} .$$

Relativistic equation of motion

$$\mathbf{P} = m\mathbf{v} = m_0\,\gamma(v)\,\mathbf{v}$$

$$\boxed{\mathbf{f} = \frac{d\mathbf{P}}{dt}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

$$\boxed{1 + \beta^2\gamma^2 \equiv \gamma^2}$$

$$\mathbf{f} = m_0\,\frac{d}{dt}\,\mathbf{v}\,\gamma(v) = m_0\left[\frac{d\mathbf{v}}{dt}\cdot\gamma(v) + \mathbf{v}\,\frac{d}{dt}\,\gamma(v)\right]$$

$$\frac{d}{dt}\,\gamma(v) = \frac{d}{dt}\left(1-\frac{\mathbf{v}^2}{c^2}\right)^{-1/2} = -\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\cdot\left(-2\,\frac{\mathbf{v}}{c^2}\,\frac{d\mathbf{v}}{dt}\right) = \gamma^3(v)\,\frac{\mathbf{a}\mathbf{v}}{c^2}$$

$$\boxed{\mathbf{f} = m_0\,\gamma(v)\left[\mathbf{a} + \gamma^2(v)\,\frac{\mathbf{a}\mathbf{v}}{c^2}\cdot\mathbf{v}\right]}$$

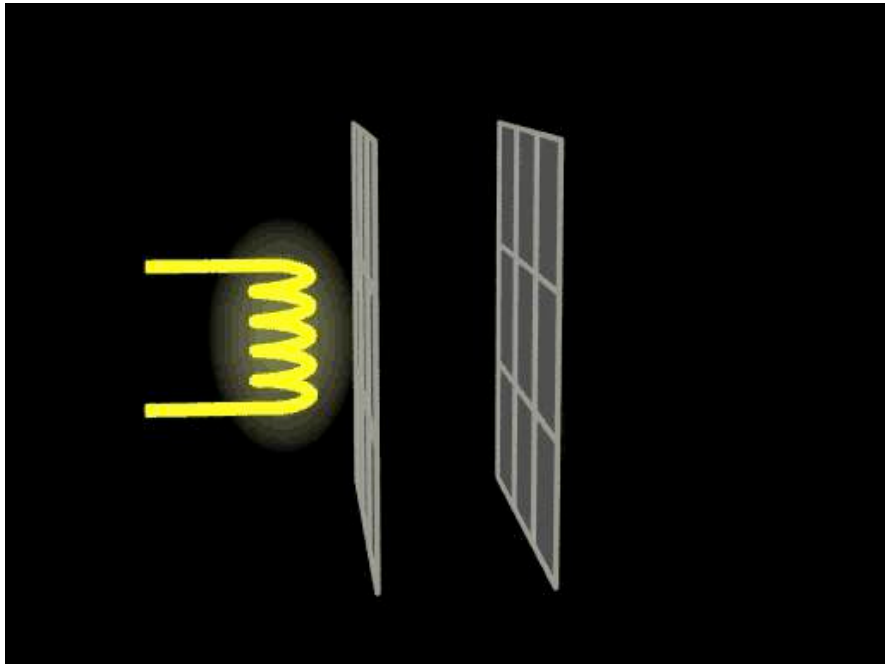
Acceleration does not generally point in the direction of the applied force

$a \perp v$ $\qquad \mathbf{f} = m_0\,\gamma(v)\,\mathbf{a} \qquad \boxed{m_\perp = m_0\,\gamma(v)}$

$a \,//\, v$ $\qquad \mathbf{f} = m_0\,\gamma(v)\left[\mathbf{a} + \gamma^2(v)\,\frac{v^2}{c^2}\cdot\mathbf{a}\right] = m_0\,\gamma^3(v)\,\mathbf{a} \qquad \boxed{m_{//} = m_o\gamma^3(v)}$

A moving body is more inert in the longitudinal direction than in the transverse direction

Longitudinal motion in the laoratory frame
==> ex: beam dynamics in a relativistic capacitor



Consider longitudinal motion only :

$$\gamma^3 \frac{d\beta}{dt} = \frac{a_o}{c} \qquad a_o = \frac{eE_z}{m_o}$$

$$\int_{\beta_o}^{\beta} \frac{d\beta}{\left(1-\beta^2\right)^{3/2}} = \frac{a_o}{c} \int_{t_o}^{t} dt$$

$$\frac{\beta}{\sqrt{1-\beta^2}} - \beta_o \gamma_o = \frac{a_o}{c}\left(t - t_o\right)$$

Solving explicitly for β one can find:

$$\beta(t) = \frac{a_o(t-t_o) + c\beta_o\gamma_o}{\sqrt{c^2 + \left(c\beta_o\gamma_o + a_o(t-t_o)\right)^2}}$$

After separating the variables one can integrate once more to obtain the position as a function of time :

$$z(t) - z_o = \frac{c^2}{a_o}\left(\sqrt{1 + \left(\beta_o\gamma_o + \frac{a_o}{c}(t-t_o)\right)^2} - \gamma_o\right) = h(t)$$

In the non relativistic limit: $z(t) - z_o = \beta_o c(t-t_o) + \frac{1}{2}a_o(t-t_o)^2$

The previous solution can be written also in the form:

$$\left(z(t) - z_o + \gamma_o\frac{c^2}{a_o}\right)^2 - \left(\frac{c^2}{a_o}\beta_o\gamma_o + c(t-t_o)\right)^2 = \left(\frac{c^2}{a_o}\right)^2$$

the corresponding world line in the Minkowsky space-time (ct,z) is an hyperbola

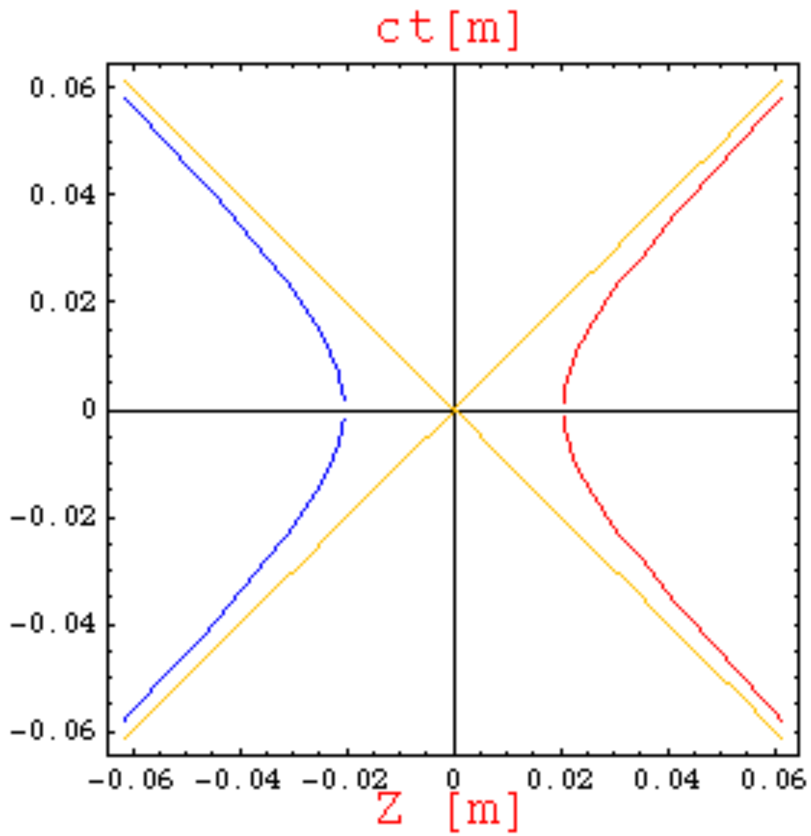
==> hyperbolic motion

in the simpler case with initial conditions:

$$\begin{cases} \beta_o = 0 \\ \gamma_o = 1 \\ z_o = 0 \end{cases}$$

and shifted variable: $Z(t) = z(t) + \dfrac{c^2}{a_o}$

$$\boxed{Z(t)^2 - (ct)^2 = \left(\frac{c^2}{a_o}\right)^2}$$



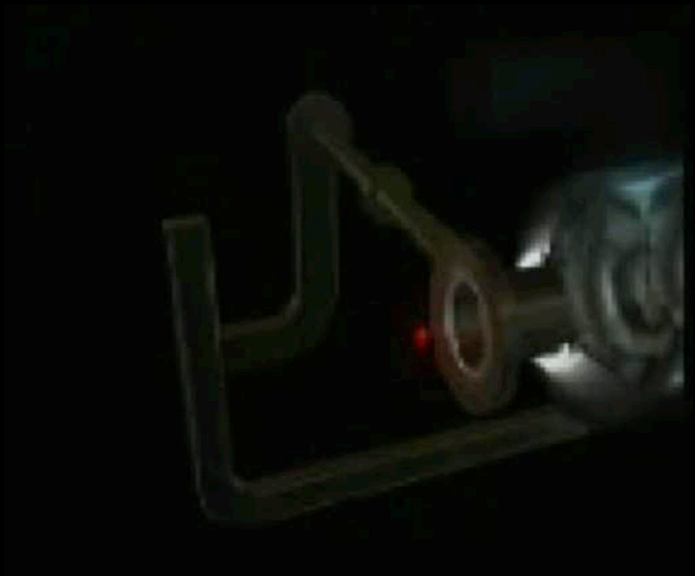
Therefore such motion is called hyperbolic motion.
It describes the motion of a particle that arrives from large positive z , slows down and stops at turning point $Z_t = c^2/a_o$
then it accelerates back up the z axis.
The world-line is asymptotic to the light cones, and obviously, it will never reach the speed of light.

The paradox of relativistic bunch compression

Low energy electron bunch injected in a linac:

$$\gamma \approx 1$$

$$L_b = 3mm \approx L_b'$$



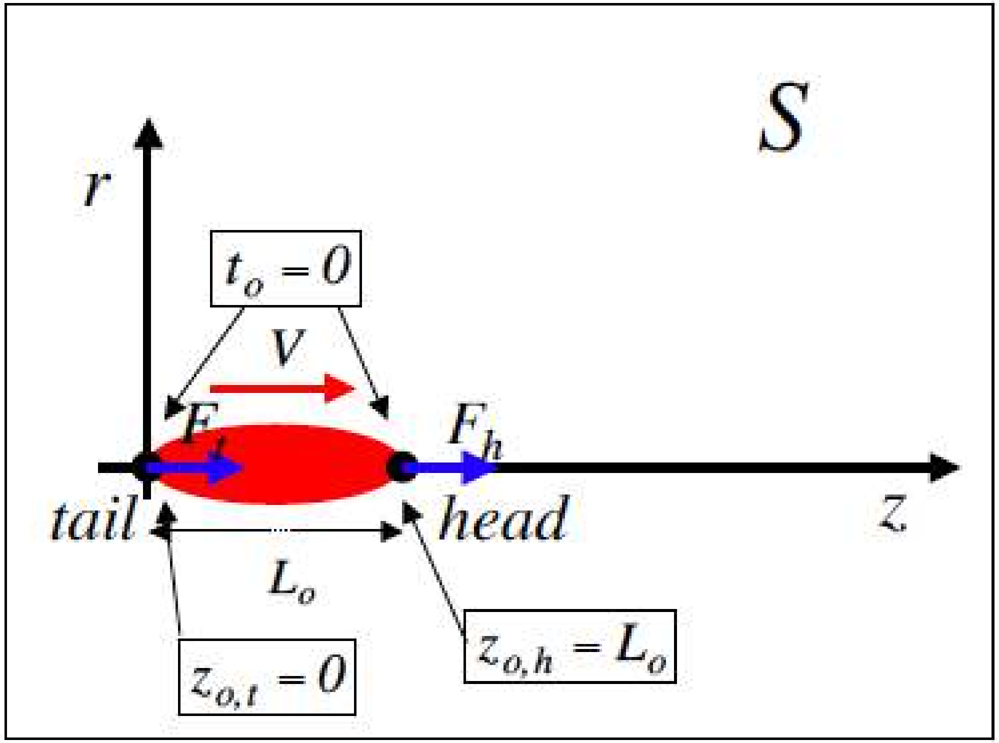
Length contraction?

$$\gamma = 1000$$

$$L_b = \frac{L_b'}{\gamma} = 3\mu m$$

Bunch length in the laboratory frame S

Let consider an electron bunch of initial length L_o inside a capacitor when the field is suddenly switched on at the time t_o.



$$L(t) = z_h(t) - z_t(t)$$

$$L(t) = \left(L_o + h(t)\right) - h(t) = L_o$$

Thus a simple computation show that no observable contraction occurs in the laboratory frame, as should be expected since both ends are subject to the same acceleration at the same time.

Bunch length in the moving frame S'

More interesting is the bunch dynamics as seen by a moving reference frame S', that we assume it has a relative velocity V with respect to S such that at the end of the process the accelerated bunch will be at rest in the moving frame S'. It is actually a deceleration process as seen by S'

Inverse Lorentz transformations:

$$\begin{cases} ct' = \gamma\left(ct - \frac{V}{c}z\right) \\ z' = \gamma(z - Vt) \end{cases}$$

leading for the tail particle to:

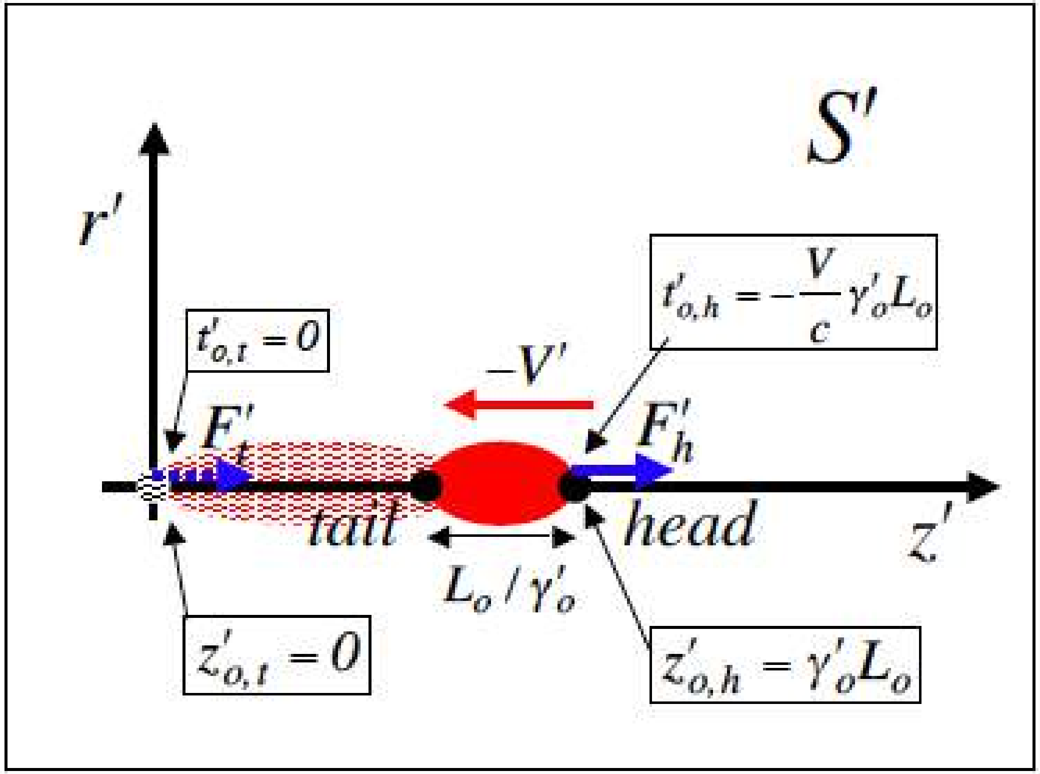
$$\begin{cases} t'_{o,t} = t_o = 0 \\ z'_{o,t} = z_{o,t} = 0 \end{cases}$$

and for the head particle to:

$$\begin{cases} t'_{o,h} = -\frac{V}{c}\gamma'_o L_o < t_o \\ z'_{o,h} = \gamma'_o L_o > z_{o,h} \end{cases}$$

The key point is that as seen from S' the decelerating force is not applied *simultaneously* along the bunch but with a *delay* given by:

$$\Delta t'_o = t'_{o,h} - t'_{o,t} = -\frac{V}{c}\gamma'_o L_o < 0$$



At the end of the process when both particle have been subject to the same decelerating field for the same amount of time the bunch length results to be:

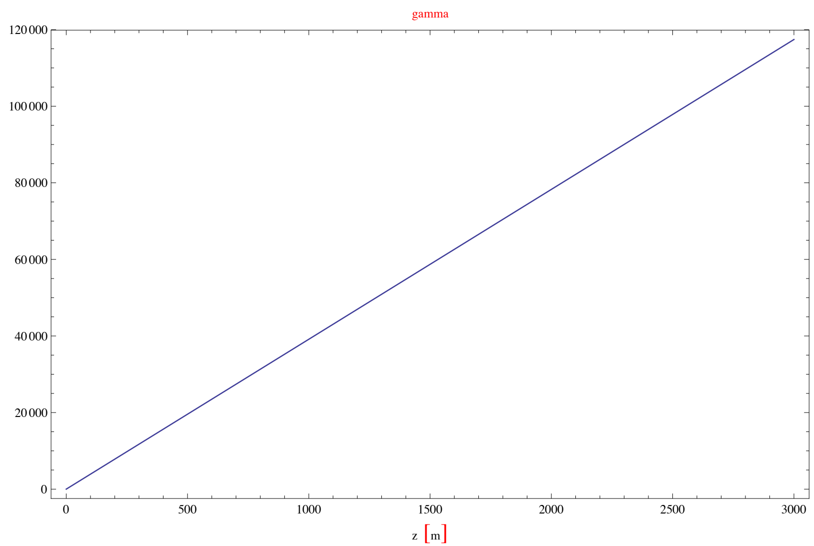
$$L'(t')=\left(\gamma' L_o+h'(t')\right)-h'(t')=\gamma' L_o$$

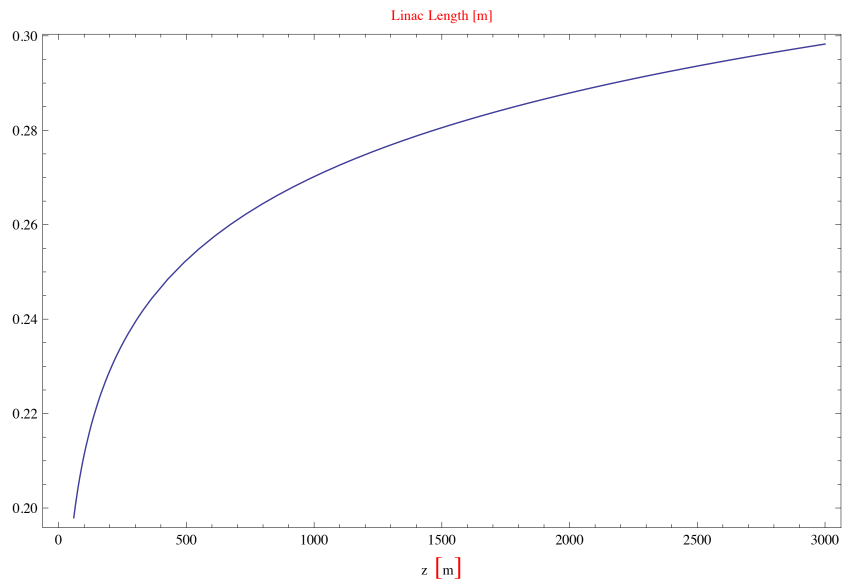
$$z'(t')-z'_o=\frac{c^2}{a_o}\left(\sqrt{1+\left(\beta'_o\gamma'_o+\frac{a_o}{c}\left(t'-t'_{o,h}\right)\right)^2}-\gamma'_o\right)=h'(t')$$

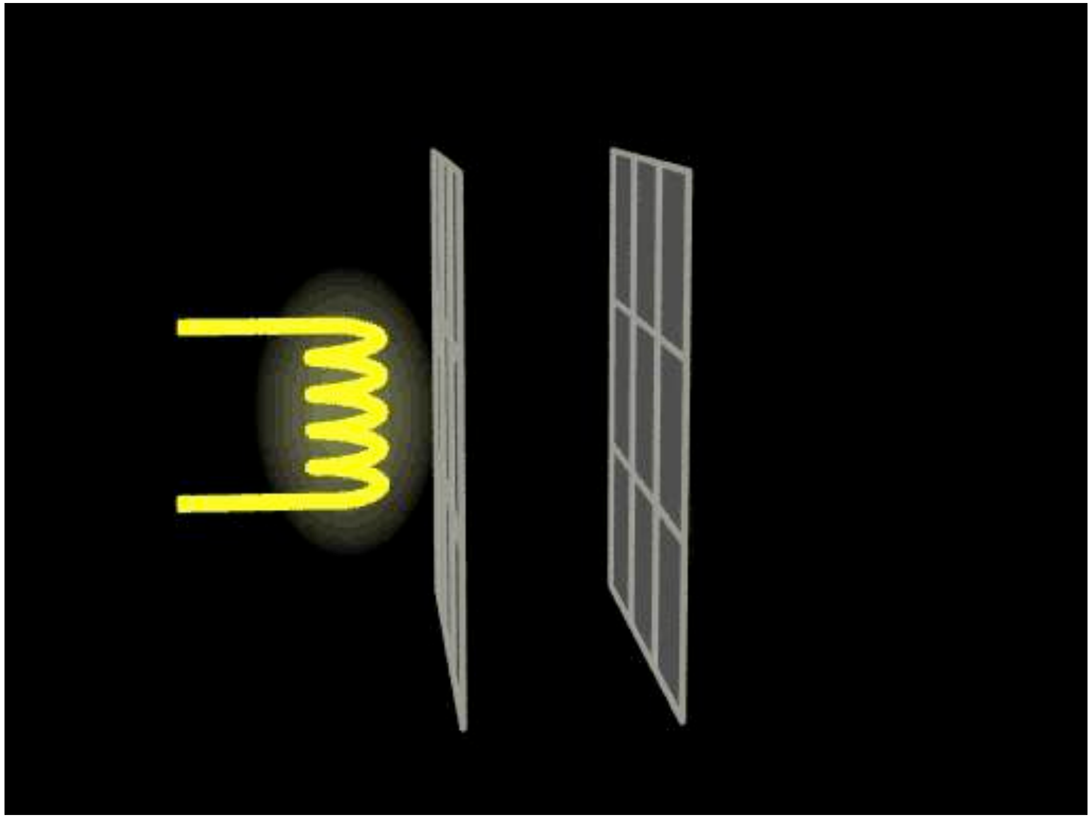
Accelerator length in the moving frame $\tilde{\Sigma}$

$$\tilde{L}_{linac} = \int_0^{L_{linac}} d\tilde{z} = \int_0^{L_{linac}} \frac{dz}{\gamma} = \int_0^{L_{linac}} \frac{dz}{\gamma' z + \gamma_o} = \left[\frac{1}{\gamma'} \ln\left(\gamma' z + \gamma_o\right)\right]_o^{L_{linac}} = \frac{1}{\gamma'} \ln\left(\frac{\gamma_f}{\gamma_o}\right)$$

$$\gamma = \frac{d\gamma}{dz} z + \gamma_o \qquad \frac{d\gamma}{dz} = \gamma' = \frac{eE_{acc}}{mc^2}$$

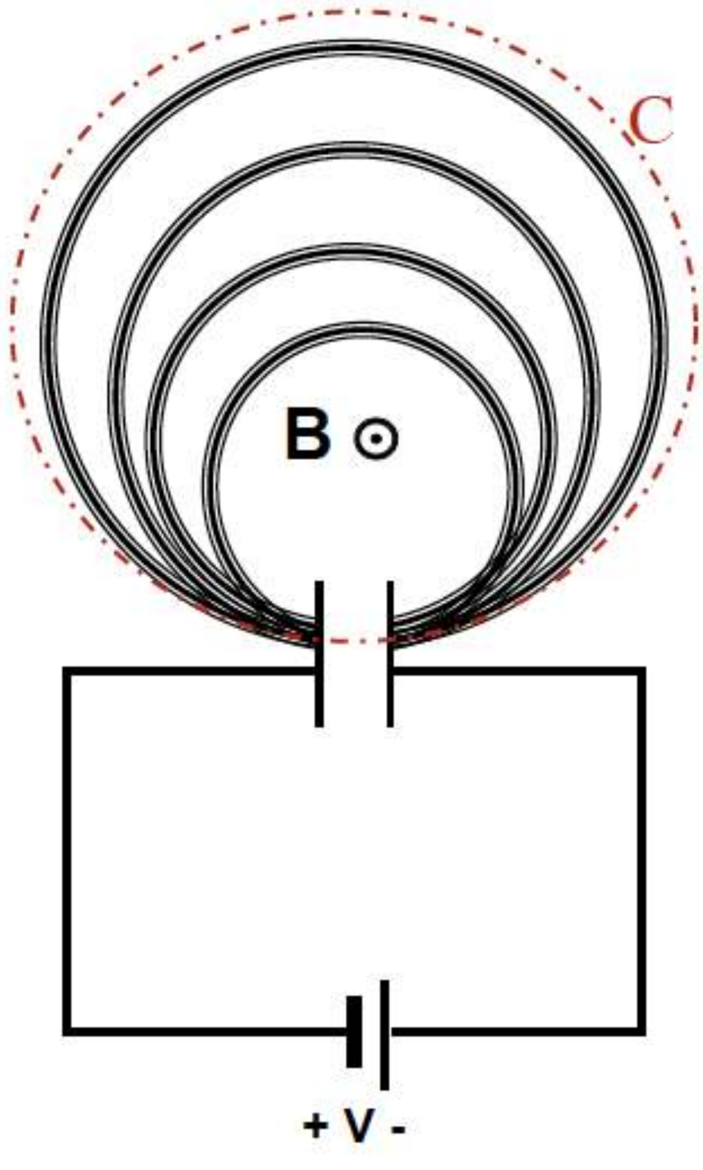




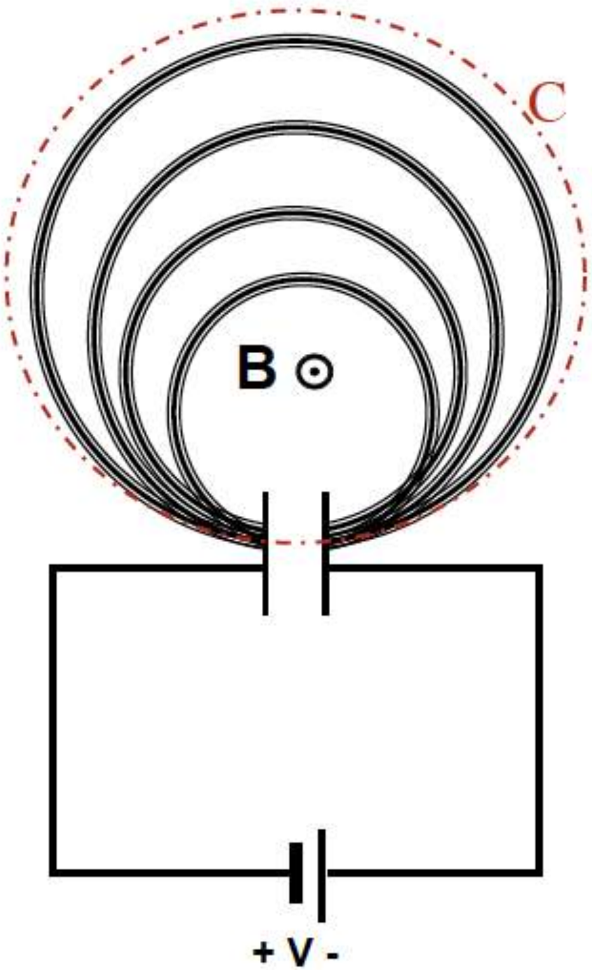


$T = q\Delta V$

Possible DC accelerator?



Maxwell forbids this!



$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

or in integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} \, da$$

∴ **There is no acceleration without time-varying magnetic flux**

$\Delta V_T = 0$

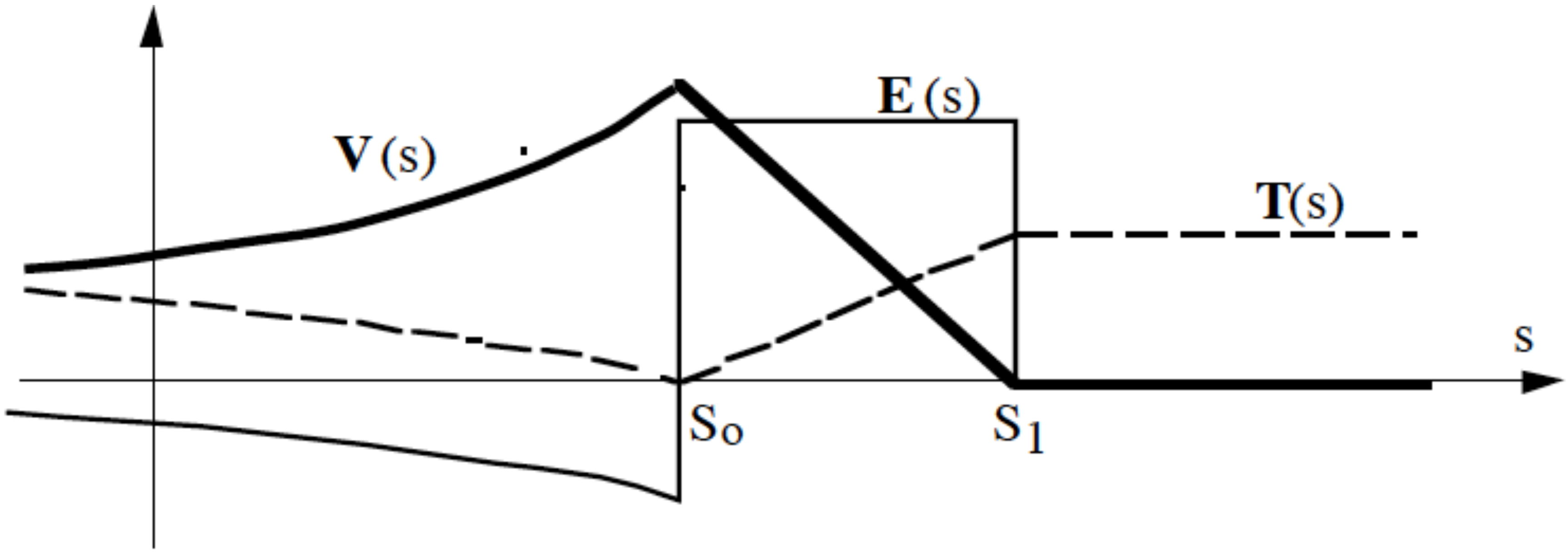
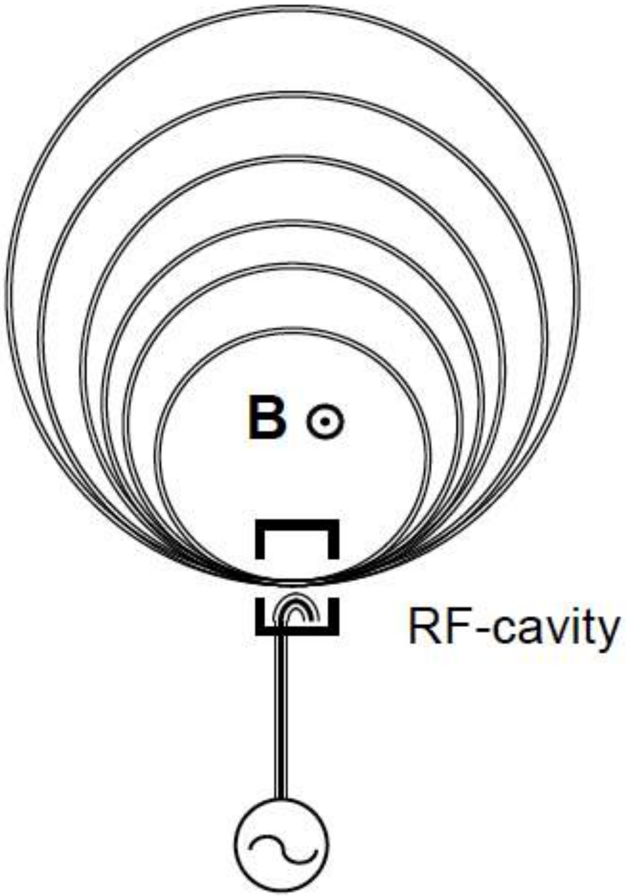
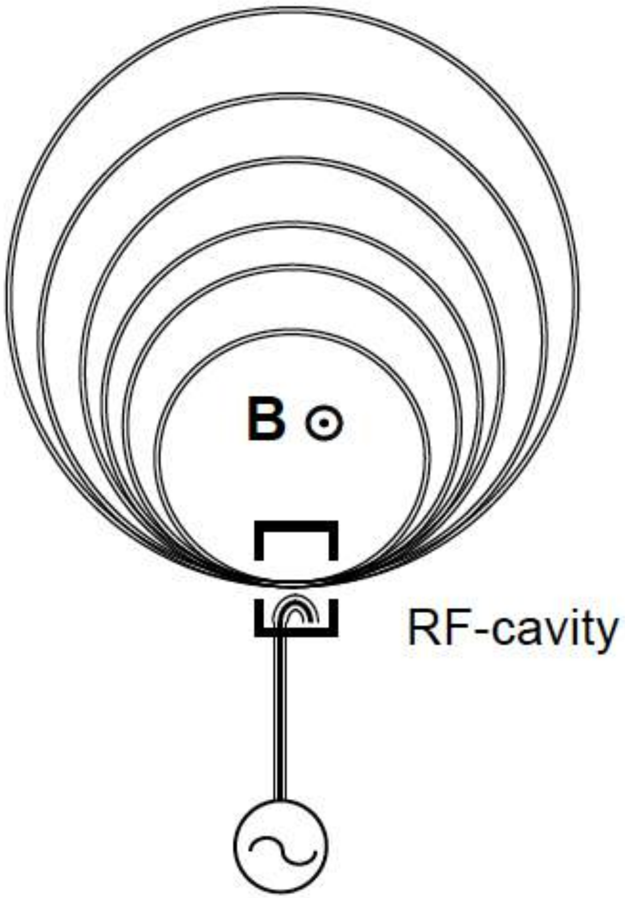


Fig.2.2. Campo e.s.(E), potenziale e.s.(V), energia cinetica(T)

Will this work?



We can vary B in an RF cavity



Note that inside the cavity

dB/dt ≠ 0

$$E_z = E_0 J_0(k_r r) \cos \omega t$$

$$B_\theta = -\frac{E_0}{c} J_1(k_r r) \sin \omega t$$

RF-cativties for acceleration