Neutrino masses and mixing angles

International School
on Astroparticle Physics

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Plan of Lectures

Extensions of the SM providing neutrino masses
1- recap of the SM
2- Dirac and Majorana masses
3- Weinberg operator
4- see-saw mechanism

I

5- Grand Unification
6- the flavour puzzle
7- the baryon asymmetry
8- neutrino masses and the Higgs boson
9- neutrino masses and lepton flavor violation

II

1, 2 see lectures by G. Ridolfi
4, see lectures by E. Lisi
3, 4, 6, 9 see lectures by J.F. Valle
7, see lectures by Melchiorri

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Neutrino Masses and Mixing Angles (F. Feruglio)

Bibliography

- on-line slides

- Lectures

W. Grimus hep-ph/0307149

P. Hernandez hep-ph/1010.4131

L. Maiani hep-ph/1406.5503

- Book

"Fundamentals of Neutrino Physics and Astrophysics" by C. Giunti and C. Kim

- PDG: Section on "Neutrino Masses, Mixing and Oscillations"

- Further readings:

General: A. Strumia and F. Vissani, hep-ph/0606054

GUTS: R. Mohapatra hep-ph/9801235

Leptogenesis: P. Di Bari hep-ph/1206.3168

Lepton Flavor Violation: F. Deppisch hep-ph/1206.5212,

Raidal et al hep-ph/0801.1826

Lecture 1
Neutrino Masses

Recap of the Standard Model

1. Gauge group: SU(3)xSU(2)xU(1)

2. Particle content: 3 copies of

$q = \begin{pmatrix} u \\ d \end{pmatrix}$	$(3,2,+1/6)$
u^c	$(\bar{3},1,-2/3)$
d^c	$(\bar{3},1,+1/3)$
$l = \begin{pmatrix} \nu \\ e \end{pmatrix}$	$(1,2,-1/2)$
e^c	$(1,1,+1)$

notation:

$$\psi_L \equiv \psi$$

$$\psi_R \to (\psi_R)^c = (\psi^c)_L \equiv \psi^c$$

all fermions come in four helicity states (ψ,ψ^c), but the neutrino which has only two.

Higgs doublet $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ $(1,2,+1/2)$

3. Renormalizability

(i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

1.+2.+3. -> Standard Model (SM)

$$L_{SM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \gamma^{\mu} D_{\mu} \Psi$$ gauge sector

$$+ \left(D_{\mu}\Phi\right)^{+} \left(D^{\mu}\Phi\right) - V\left(\Phi^{+}\Phi\right)$$ symmetry breaking sector

$$- y \bar{\Psi} \Phi \Psi$$ Yukawa sector

Symmetries of the Standard Model

- Lorentz invariance
- gauge invariance

invariance of the gauge sector: global $U(3)^5$

$$q \to \Omega_q q \quad u^c \to \Omega_{u^c} u^c \quad d^c \to \Omega_{d^c} d^c \quad l \to \Omega_l l \quad e^c \to \Omega_{e^c} e^c$$

$$q_i \to \left(\Omega_q\right)_{ij} q_j$$ etc... $\Omega s \in U(3)$

this huge invariance is broken by the Yukawa sector down to $U(1)^4$
the four – classically – conserved charges are B, L_i ($i=e,\mu,\tau$)

$$L_Y = -d^c y_d(\Phi^+ q) - u^c y_u(\tilde{\Phi}^+ q) - e^c y_e(\Phi^+ l) + h.c.$$

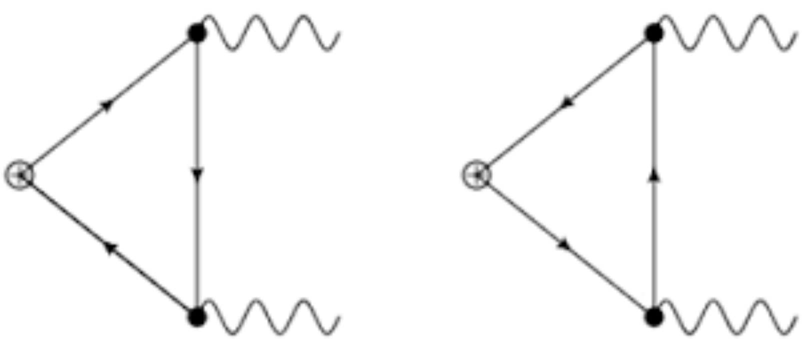
after moving in the mass basis the U(1)^4 invariance read

B $\qquad q \to e^{i\frac{\alpha}{3}} q \quad u^c \to e^{-i\frac{\alpha}{3}} u^c \quad d^c \to e^{-i\frac{\alpha}{3}} d^c$

L$_i$ $\qquad l_i \to e^{i\beta} l_i \quad e_i^c \to e^{-i\beta} e_i^c$

quantum effects break B and L$_i$ leaving three linear combinations unbroken

$$\partial_\mu j_a^\mu \propto tr[C_a \{T^A, T^B\}]$$



the conserved charges are (B/3-L$_i$)
and any combination of these, e.g. (B-L)

all gauge currents of the SM are anomaly free

Exercise 1: anomalies of B and L_i

the anomaly of the baryonic current and the individual leptonic currents are proportional to tr[Q {T^A,T^B}] and tr[Q {Y,Y}] where Q=(B,L_i) and (T^A,Y) are the generators of the electroweak gauge group

compute these traces in the SM with 3 fermion generations

$$\frac{1}{2}\mathrm{tr}[B\{T^A,T^B\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{4}(up)+\frac{1}{4}(down)\right]\delta^{AB} = \frac{3}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{T^A,T^B\}] = 1(L_i)\times\left[\frac{1}{4}(nu)+\frac{1}{4}(e)\right]\delta^{AB} = \frac{1}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[B\{Y,Y\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{18}(Doubl)-\frac{10}{18}(Singl)\right] = -\frac{3}{2}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{Y,Y\}] = 1(L_i)\times\left[\frac{1}{2}(Doubl)-1(Singl)\right] = -\frac{1}{2}$$

(B+L) is anomalous, (B/3-L_i) [and (B-L)] are anomaly-free

Fermion masses in the Standard Model

lepton sector

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} = (1,2,-1/2) \qquad e^c = (1,1,+1)$$

remember that in our basis
all fermion fields are left-handed

$$\left(\frac{1-\gamma_5}{2}\right)\psi = -\psi$$

in the massless limit each spinor comes in two helicity states

e destroys an electron of negative helicity
creates a positron of positive helicity

e^c destroys a positron of negative helicity
creates an electron of positive helicity

gauge invariance of the SM forbids a direct mass term like ($m\, e^c\, e$) in QED

Exercise 2: why?

Yukawa interactions

$$L_Y = -e^c y_e (\Phi^+ l) + \ldots + h.c.$$
$$= -\frac{h+v}{\sqrt{2}} e^c y_e e + \ldots + h.c.$$

$$\Phi = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$$ Higgs doublet in the unitary gauge

$$m_e = \frac{y_e}{\sqrt{2}} v \qquad m_e e^c e + h.c.$$ Dirac mass term

Exercise 3: express the Dirac mass term in the Weyl basis

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \psi = \begin{pmatrix} e \\ \overline{e}^c \end{pmatrix}$$

$$e_L = \frac{1-\gamma_5}{2}\psi = \begin{pmatrix} e \\ 0 \end{pmatrix} \qquad e_R = \frac{1+\gamma_5}{2}\psi = \begin{pmatrix} 0 \\ \overline{e}^c \end{pmatrix}$$

$$m_e \overline{e}_R e_L + m_e \overline{e}_L e_R = m_e e^c e + m_e \overline{e}^c \overline{e}$$

e and e^c are called Weyl spinors, each carrying two components

a Dirac mass term couples two Weyl spinors e and e^c

- if the two spinors carry opposite charges C, then the mass term conserves C. In our case the electric charge Q and total lepton number L are conserved. The hypercharge Y is not conserved, since Y(e)=-1/2 and $Y(e^c)$=+1 Y is spontaneously broken by the Higgs VEV

- the chirality L,R are just labels. Actually with Weyl spinors we are working in a basis where all fermions are left-handed.

suppose now we have an electrically neutral Weyl spinor ν

if we set $e=e^c=\nu$ in the previous mass terms we get the special case

$$m\nu\nu + m\bar{\nu}\bar{\nu}$$

this is called a Majorana mass term

- it is Lorentz invariant
- it conserves the electric charge Q
- it cannot conserved (B-L): $|\Delta(B-L)|=2$

however in the SM this is forbidden.

Exercise 4: why?

Overwhelming evidence of non-vanishing neutrino masses

[see other lectures]

Beyond the Standard Model

a non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model [SM]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

the requirement of invariance under the gauge group G=SU(3)xSU(2)xU(1) forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

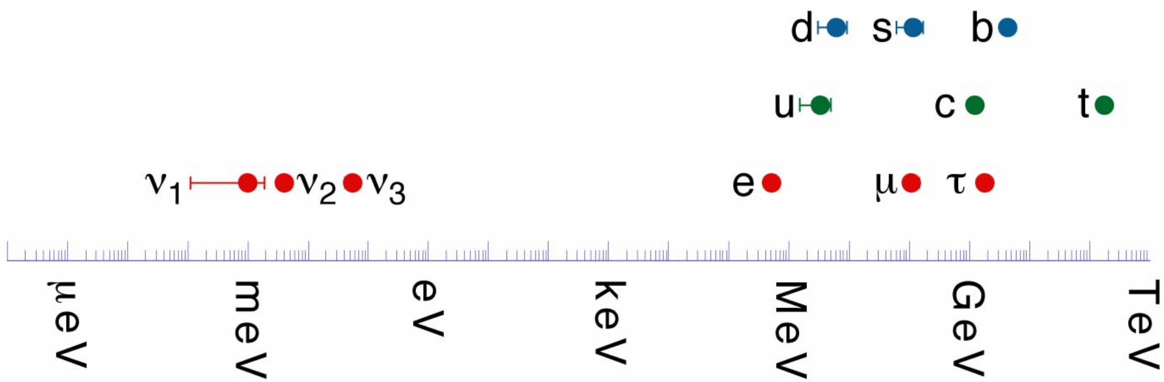
$$\Phi \quad \underbrace{\Psi\Psi'}_{\text{same helicity}}$$

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

How to modify the SM?

the SM, as a consistent QFT, is completely specified by

1. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

2. particle content — three copies of (q, u^c, d^c, l, e^c)
   one Higgs doublet Φ

3. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(1.+2.+3.) leads to the SM Lagrangian, L_{SM}, possessing additional, accidental, global symmetries: $(B/3-L_i)$

1. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 1. (and/or 2.) anyway…

First possibility: modify (2), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

- add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)
- ask for (global) invariance under B-L (no more automatically conserved as in the SM) $(\nu_R)^c = \nu^c$

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = -d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) - e^c y_e (\Phi^+ l) - \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

Exercise 5: count the number of physical parameters in the lepton sector of example 1

y_e, y_ν depend on (18+18)=36 parameters, 18 moduli and 18 phases

we are free to choose any basis leaving the kinetic terms canonical (and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases) and effectively modify y_e, y_ν

$$y_e \to \Omega^T_{e^c} y_e \Omega_l \qquad y_\nu \to \Omega^T_{\nu^c} y_\nu \Omega_l$$

one of these transformation is B-L, a symmetry of the Lagrangian so that we can remove 26=(27-1) parameters from y_e, y_ν

we remain with 10 parameters: 9 moduli and 1 phases
the moduli are 6 physical masses and 3 mixing angles

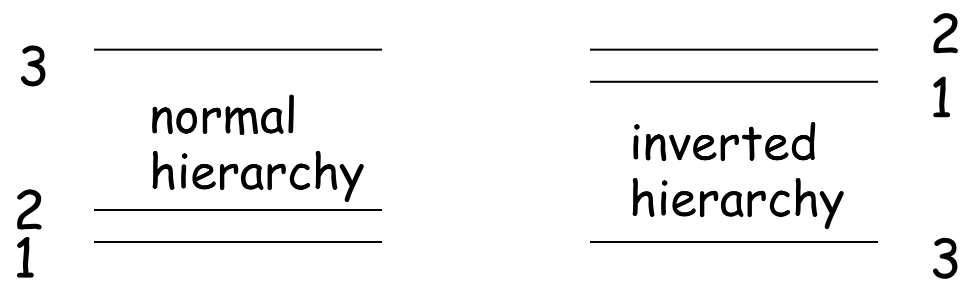
this is exactly the same count as in the quark sector and Example 1 replicates for leptons what occurs for quarks

conventions for neutrino masses:

$$m_1 < m_2 \qquad [\Delta m^2_{ij} \equiv m^2_i - m^2_j]$$

$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



Mixing matrix U_{PMNS} (Pontecorvo,Maki,Nakagawa,Sakata)

$0 \le \vartheta_{ij} \le \pi/2$

$0 \le \delta < 2\pi$

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i \quad (f = e, \mu, \tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

not yet the most general possibility!

$$U_{PMNS} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{13}\, s_{23} \\ -c_{12}\, s_{13}\, c_{23}\, e^{-i\delta} + s_{12}\, s_{23} & -s_{12}\, s_{13}\, c_{23}\, e^{-i\delta} - c_{12}\, s_{23} & c_{13}\, c_{23} \end{pmatrix}$$

Summary of data

$m_\nu < 2.2\ eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

$$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = (2.525^{+0.042}_{-0.030}) \times 10^{-3}\ \text{eV}^2 & \text{NO} \\ \Delta m^2_{32} = -(2.505^{+0.034}_{-0.032}) \times 10^{-3}\ \text{eV}^2 & \text{IO} \end{cases}$$

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.37^{+0.17}_{-0.16}) \times 10^{-5}\ \text{eV}^2$$

$$\sin^2\vartheta_{13} = 0.0215 \pm 0.0007 \qquad \delta_{CP}/\pi = 1.38^{+0.23}_{-0.20}$$

$$\sin^2\vartheta_{23} = \begin{cases} 0.425^{+0.021}_{-0.015} & \text{NO} \\ [0.433^{+0.015}_{-0.016}] \oplus [0.589^{+0.016}_{-0.022}] & \text{IO} \end{cases}$$

$$\sin^2\vartheta_{12} = 0.297^{+0.017}_{-0.016}$$

[Capozzi et al. 1703.04471]

violation of individual lepton number implied by neutrino oscillations

Summary of unknown

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

a generic problem of this approach

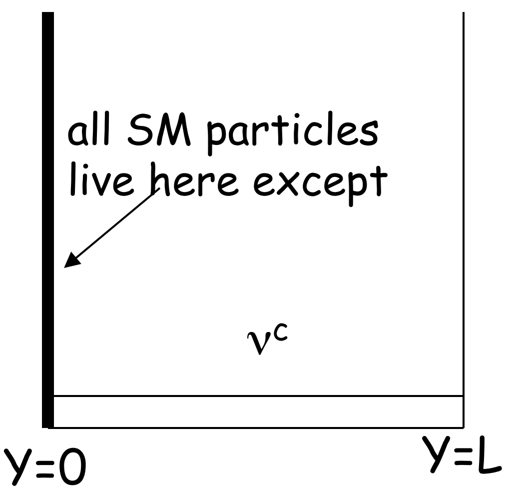
the particle content can be modified in several different ways in order to account for non-vanishing neutrino masses (additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,...). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \leq 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$
$$= \frac{1}{\sqrt{L}}\nu_0^c(\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale) then neutrino Yukawa coupling is suppressed

Second possibility: abandon (3) renormalizability

A disaster?

$$L = L^{SM}_{d\le 4} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,… contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \rightarrow \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \rightarrow \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\, GeV}{10^{15}\, GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top} !

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} = $$
$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a Majorana mass term

Weinberg operator: a unique operator [up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

Exercise 6: count the number of physical parameters in the low-energy theory described by the Weinberg operator

$$\frac{(\tilde{\Phi}^+ l)\, w\, (\tilde{\Phi}^+ l)}{\Lambda}$$

y_e and w depend on (18+12)=30 parameters, 15 moduli and 15 phases

we are free to choose any basis leaving the kinetic terms canonical (and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad l \to \Omega_l l \qquad [U(3)^2]$$

these transformations contain 18 parameters (6 angles and 12 phases) and effectively modify y_e and w

$$y_e \to \Omega_{e^c}^T y_e \Omega_l \qquad w \to \Omega_l^T w \Omega_l$$

so that we can remove 18 parameters from y_e and w

we remain with 12 parameters: 9 moduli and 3 phases
the moduli are 6 physical masses and 3 mixing angles

Majorana phases

$$U_{PMNS} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{-i\delta} & c_{13}\, s_{23} \\ -c_{12}\, s_{13}\, c_{23}\, e^{-i\delta} + s_{12}\, s_{23} & -s_{12}\, s_{13}\, c_{23}\, e^{-i\delta} - c_{12}\, s_{23} & c_{13}\, c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = -\nu^c y_\nu (\tilde{\Phi}^+ l) - \frac{1}{2}\nu^c M \nu^c + h.c.$$

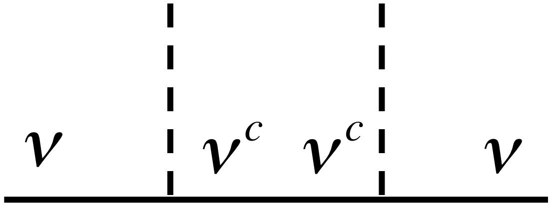
mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out" the field ν^c

$$L_{eff}(l) = \frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + \ldots$$

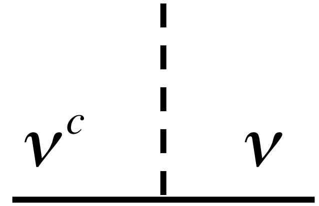
terms suppressed by more powers of M^{-1}

this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.



$$\left(-i\frac{y_\nu^T}{\sqrt{2}}\right)\left(\frac{i}{-M}\right)\left(-i\frac{y_\nu}{\sqrt{2}}\right)$$

$$=\frac{i}{2}\, y_\nu^T\, M^{-1}\, y_\nu$$



$$-i\frac{y_\nu}{\sqrt{2}}$$

vertex

$$\frac{i}{-M}$$

propagator
in heavy mass
limit

Exercise 7

derive the see-saw relation by integrating out the fields ν^c through their e.o.m. in the heavy M limit. Compute the 1st order corrections in p/M

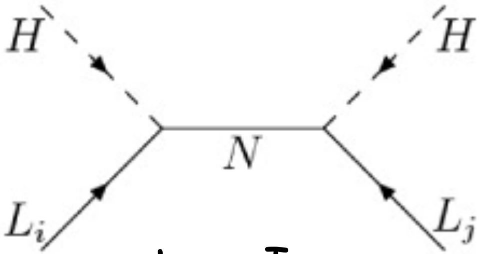
equations of motion of ν^c

$$\begin{pmatrix} \nu^c \\ \bar{\nu}^c \end{pmatrix} = \begin{pmatrix} i\bar{\sigma}^\mu\partial_\mu & -M^+ \\ -M & i\sigma^\mu\partial_\mu \end{pmatrix}^{-1} \begin{pmatrix} y_\nu^*\bar{\omega} \\ y_\nu\omega \end{pmatrix} = \begin{pmatrix} -M^{-1}y_\nu\omega \\ -M^{*-1}y_\nu^*\bar{\omega} \end{pmatrix} + \dots \qquad \omega \equiv (\tilde{\Phi}^+ l)$$

$$L_{eff} = i\bar{l}\,\bar{\sigma}^\mu\partial_\mu l + \frac{1}{2}\Big[\underbrace{\omega(y_\nu^T M^{-1} y_\nu)\omega + h.c.}_{\text{d-=5}}\Big] + \underbrace{i\bar{\omega}(y_\nu^+ M^{+-1} M^{-1} y_\nu)\bar{\sigma}^\mu\partial_\mu\omega}_{\text{d-=6}} + O(M^{-3})$$

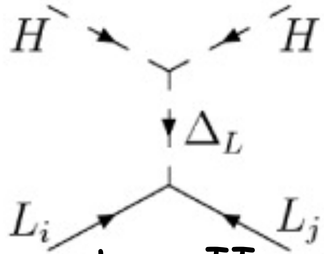
renormalizes the KE of ν by v^2/M^2

there are 3 types of see-saw depending on the particle we integrate out
they all give rise to the same d=5 operator



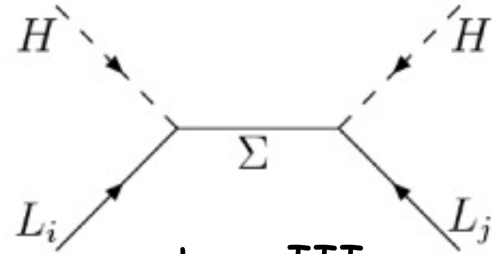
type I

$$y_N^T (M_N)^{-1} y_N$$



type II

$$y_\Delta \frac{\mu}{M_\Delta^2}$$



type III

$$y_\Sigma^T (M_\Sigma)^{-1} y_\Sigma$$

Exercise 8
find the quantum numbers of the three type of particles that can be exchanged in this diagram

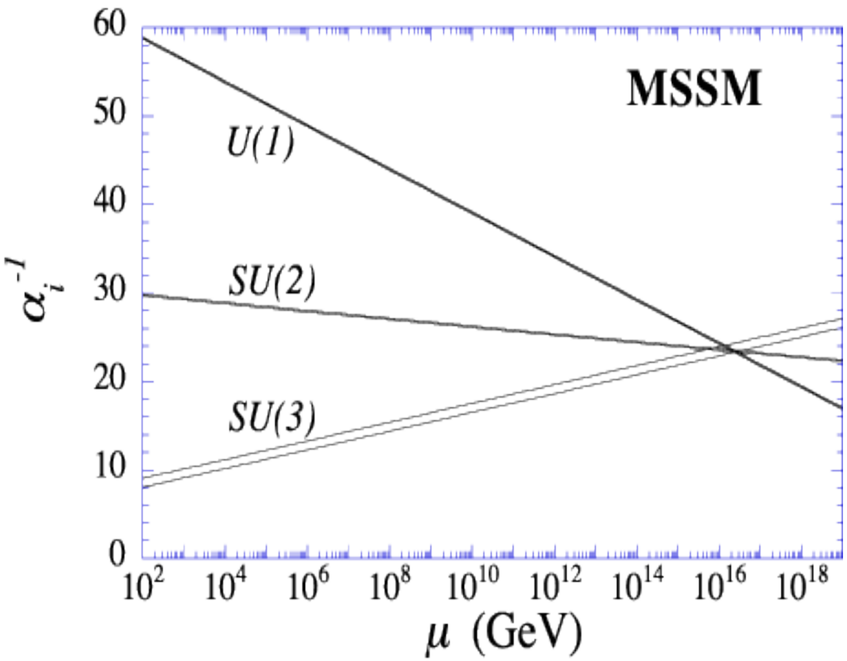
Type I (1,1,0)
Type II (1,3,0)
Type III (1,3,±1)

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),…



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

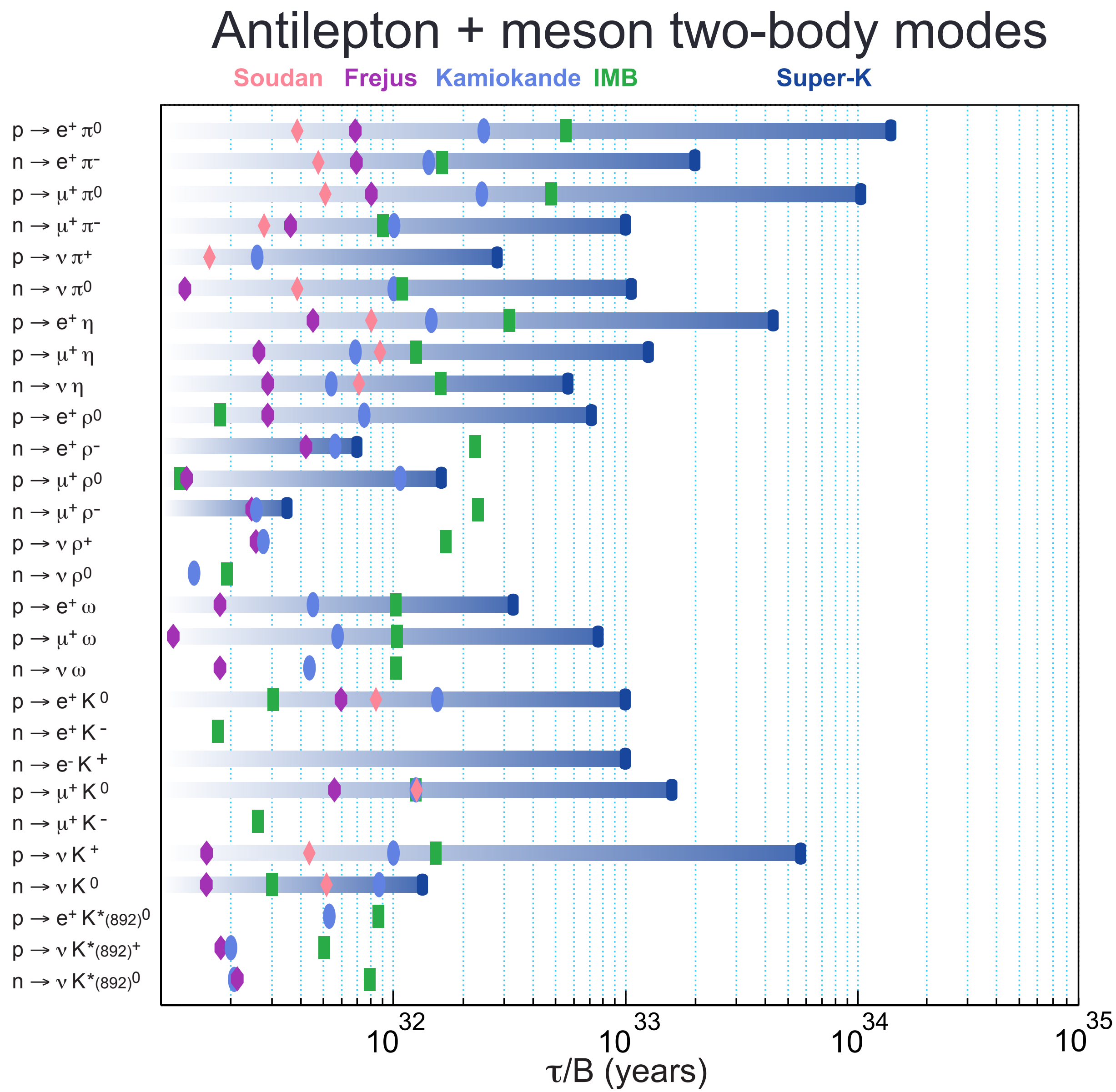
a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

Unity of All Elementary-Particle Forces
Phys. Rev. Lett. 32, (1974) 438
Howard Georgi and S. L. Glashow

Georgi, H.; Quinn, H.R. and Weinberg, S.
Hierarchy of interactions in unified gauge theories.
Phys. Rev. Lett. 33 (1974) 451

nucleon lifetime lower bounds



Exercise 9: gauge coupling unification

0^{th} order approximation

justify this $\sqrt{\frac{5}{3}} g_Y = g_2 = g_3$ $\sin^2\vartheta_W = \frac{g_Y^2}{g_Y^2 + g_2^2} = \frac{3}{8} \approx 0.375$

include 1-loop running

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(m_Z)} + \frac{b_i}{2\pi}\log\frac{Q}{m_Z}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{MSSM} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{SM} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}$$

knowledge of b.c. M_{GUT} and $\alpha_U = \alpha(M_{GUT})$ would allow to predict $\alpha_i(m_Z)$
in practice, we use as inputs

$\alpha_{em}^{-1}(m_Z)\Big|_{\overline{MS}} = 127.934$ $\sin^2\vartheta(m_Z)\Big|_{\overline{MS}} = 0.231$

to predict
[MSSM]

$$\alpha_3(m_Z)\Big|_{\overline{MS}} = \frac{7\alpha_{em}(m_Z)}{15\sin^2\vartheta(m_Z) - 3} \approx 0.118$$

$$\alpha_U = \frac{28\alpha_{em}(m_Z)}{36\sin^2\vartheta(m_Z) - 3} \approx \frac{1}{25}$$

$$\log\left(\frac{M_{GUT}}{m_Z}\right) = \pi\frac{3 - 8\sin^2\vartheta(m_Z)}{14\alpha_{em}(m_Z)} \quad \Rightarrow \quad M_{GUT} \approx 2\times 10^{16}\,\text{GeV}$$

[corrections from 2-loop RGE,
threshold corrections at M_{SUSY},
threshold corrections at M_{GUT}]

Exercise 10: effective lagrangian for nucleon decay

recognize that, the with the SM particle content, the lowest dimensional operators violating B occur at d=6. Make a list of them

$$\frac{1}{\Lambda_B^2} \times \begin{cases} q q u^{c+} e^{c+} & q q q l \\ q l u^{c+} d^{c+} & u^c u^c d^c e^c \end{cases}$$

color and SU(2) indices contracted

notice that they respect $\Delta B=\Delta L$: nucleon decay into antileptons e.g. $p \to e^+ \pi^0$, $n \to e^+ \pi^-$ [$n \to e^- \pi^+$ suppressed by further powers of Λ_B]

naïve estimate

$$\tau_p \approx \frac{\Lambda_B^4}{m_p^5}$$

assuming

$$\tau_p(p \to e^+ \pi^0) > 1.4 \times 10^{34}\ ys \qquad \text{[SK]}$$

we get

$$\Lambda_B > 2.6 \times 10^{16} \quad \text{GeV}$$

in GUTs Λ_B is related to the scale M_{GUT} at which the grand unified symmetry is broken down to SM gauge group

the observed proton stability is guaranteed by the largeness of M_{GUT}

In SUSY extensions of the SM the lowest dimensional operators violating B occur at d=5: why?

SU(5) GUT in one slide

1. gauge group SU(5)

it contains 24 generators: 12 <-> SM gauge bosons
remaining 12 = (3,2,-5/6) and conjugate <-> (X,Y) gauge bosons

at the superheavy scale M_{GUT}
(X,Y) gauge bosons become massive

SU(5) → SU(3)xSU(2)xU(1)

2. (minimal) particle content

Higgs

$$\bar{5} = (l, d^c) \qquad 10 = (q, u^c, e^c) \qquad 1 = \nu^c \qquad \Phi_5 = (\Phi_D, \Phi_T)$$

$$L_Y = -10\, y_u\, 10\, \Phi_5 - \bar{5}\, y_d\, 10\, \Phi_5^+ - 1\, y_\nu\, \bar{5}\, \Phi_5 - \frac{1}{2} 1 M 1 + h.c.$$

$$y_d = y_e^T$$

$$m_b = m_\tau$$ O.K.

$$m_s = m_\mu \qquad m_d = m_e$$

wrong, but not by orders of magnitude
can be fixed with additional Higgs

$$m_s \approx m_\mu / 3 \qquad m_d \approx 3\, m_e$$

Exercise 11. (X,Y) mediate nucleon decay.
Which of the 4 operators arises from their exchange?

flavor puzzle made simpler in SU(5) ?

suppose that y_u, y_e, y_ν and M/Λ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to a rescaling of matter multiplets (there are many mechanism that can produce this)

$$\begin{array}{ccc} 10 & \rightarrow & F_{10}\,10 \\ \bar{5} & \rightarrow & F_{\bar{5}}\,\bar{5} \\ 1 & \rightarrow & F_1\,1 \end{array} \qquad F_X = \begin{pmatrix} \lambda^{Q_{X_1}} & 0 & 0 \\ 0 & \lambda^{Q_{X_2}} & 0 \\ 0 & 0 & \lambda^{Q_{X_3}} \end{pmatrix} \qquad \begin{array}{c} \lambda \approx 0.22 \\ Q_{X_1} \geq Q_{X_2} \geq Q_{X_3} \end{array}$$

$$\boxed{\Upsilon_u = F_{10} y_u F_{10}} \qquad \boxed{\Upsilon_d = F_{\bar{5}} y_d F_{10}} \qquad \boxed{\Upsilon_e = F_{10} y_d^T F_{\bar{5}}} \qquad \boxed{m_\nu \propto F_{\bar{5}} y_\nu^T M^{-1} y_\nu F_{\bar{5}}}$$

F_1 dependence cancels in m_ν

large mixing in lepton sector suggests $F_{\bar{5}} \approx \mathrm{diag}(1,1,1)$

hierarchy mostly due to F_{10} $\quad m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$

large l mixing corresponds to a large d^c mixing: unobservable in weak int. of quarks

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

example

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

$\delta \ll 1$
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\bar{B}})}{s} \approx 6 \times 10^{-10}$$

Sakharov conditions met by the see-saw theory

1. (B-L) violation at high-temperature and (B+L) violation by pure SM interactions
2. C and CP violation by additional phases in see-saw Lagrangian (more on this later)
3. out-of-equilibrium condition

restrictions imposed by leptogenesis on neutrinos

here: thermal leptogenesis
dominated by lightest ν^c
no flavour effects]

active neutrinos should be light

out-of-equilibrium controlled
by rate of RH neutrino decays

$$\frac{M_1}{8\pi}(y_\nu y_\nu^+)_{11} < \left.\frac{T^2}{M_{Pl}}\right|_{T\approx M_1}$$

$$\frac{(y_\nu y_\nu^+)_{11} v^2}{M_1} \equiv \tilde{m}_1 < 10^{-3}\ \text{eV}$$

Exercise 12; compute this

more accurate estimate $m_i < 0.15$ eV

RH neutrinos should be heavy

[efficiency factor ≤1
washout effects]

$$\eta_B \approx 10^{-2}\,\varepsilon_1\,\eta$$

$$\varepsilon_1 = \frac{\Gamma(\nu_1^c \to l\Phi) - \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)}{\Gamma(\nu_1^c \to l\Phi) + \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)} = -\frac{3}{16\pi}\sum_{j=2,3}\frac{M_1}{M_j}\frac{\mathrm{Im}\{[(yy^+)_{1j}]^2\}}{(yy^+)_{11}} \approx 0.1\times\frac{M_1 m_i}{v^2}$$

[Yukawas y in mass eigenstate basis for ν^c_i]

$M_1 > 6\times10^8$ GeV

more refined bound [Davidson and Ibarra 0202239]

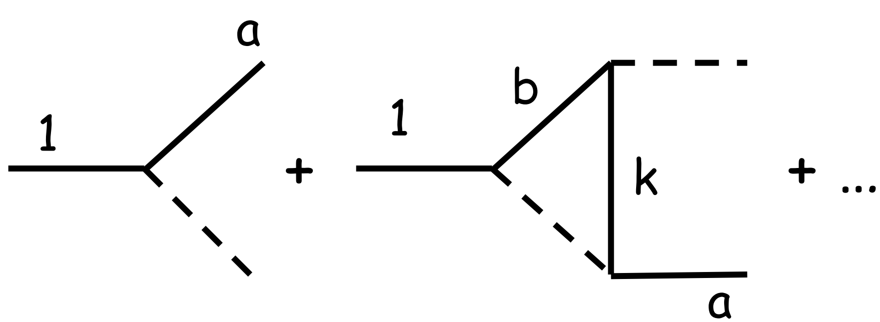
$$\left|\varepsilon_1^\infty\right| \le \varepsilon_1^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_3 - m_1)$$

$$T_R \approx M_1 > (4\times 10^8 \div 2\times 10^9)\ GeV$$

in conflict with the bound on T_R in SUSY models
to avoid overproduction of gravitinos

$$T_R^{SUSY} < 10^{7-9}\ GeV$$

Exercise 13: reconstruct the flavour structure of ε_1



$$\mathcal{A}(\nu_1^c \to l_a \Phi) \propto y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}$$

$$\mathcal{A}(\nu_1^c \to \bar{l}_a \Phi^*) \propto y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}$$

$$\varepsilon_1 \propto \frac{\left|y^+_{a1} + W\ y_{1b}y^+_{bk}y^+_{ak}\right|^2 - \left|y_{1a} + W\ y^+_{b1}y_{kb}y_{ka}\right|^2}{\left|y^+_{a1} + W\ y_{1b}y^+_{bk}y^+_{ak}\right|^2 + \left|y_{1a} + W\ y^+_{b1}y_{kb}y_{ka}\right|^2} \approx \frac{\mathrm{Im}(W)\,\mathrm{Im}\{[(yy^+)_{1k}]^2\}}{(yy^+)_{11}}$$

[sums understood]

$$\mathrm{Im}(W) \approx \frac{M_1}{M_k}$$

Exercise 14: count the number of physical parameters in the type I see-saw model distinguish between moduli and phases

y_e, y_ν and M depend on (18+18+12)=48 parameters, 24 moduli and 24 phases

we are free to choose any basis leaving the kinetic terms canonical
(and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases)
and effectively modify y_e, y_ν and M

$$y_e \to \Omega_{e^c}^T y_e \Omega_l \qquad y_\nu \to \Omega_{\nu^c}^T y_\nu \Omega_l \qquad M \to \Omega_{\nu^c}^T M \Omega_{\nu^c}$$

so that we can remove 27 parameters from y_e, y_ν and M

we remain with 21 parameters: 15 moduli and 6 phases
the moduli are 9 physical masses and 6 mixing angles

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2}\nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing (L_{SM})+L_5:
3 masses, 3 mixing angles and 3 phases, as in lecture 1

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is

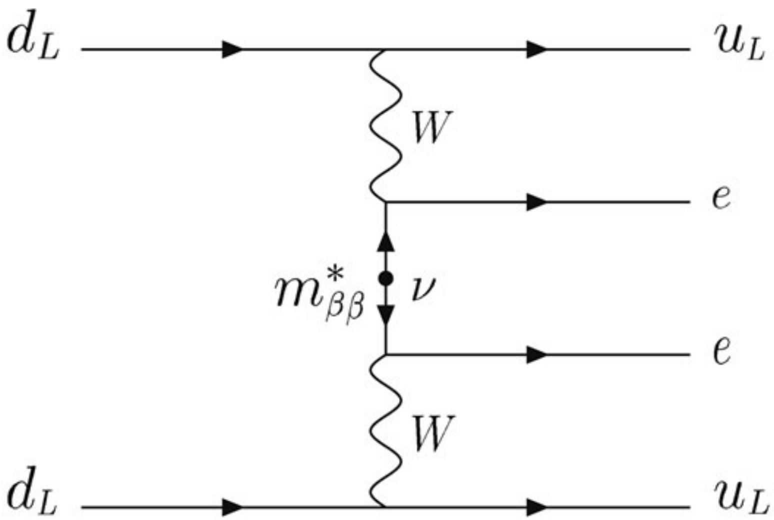
$0\nu\beta\beta$ decay: $(A,Z)\to(A,Z+2)+2e^-$

this would discriminate L_5 from other possibilities, such as Example 1.

[see Valle's lectures]

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e}\,\sigma^\mu U_{PMNS}\,\nu + h.c.$$

Feynman diagram for 0νββ decay



U_{ei}

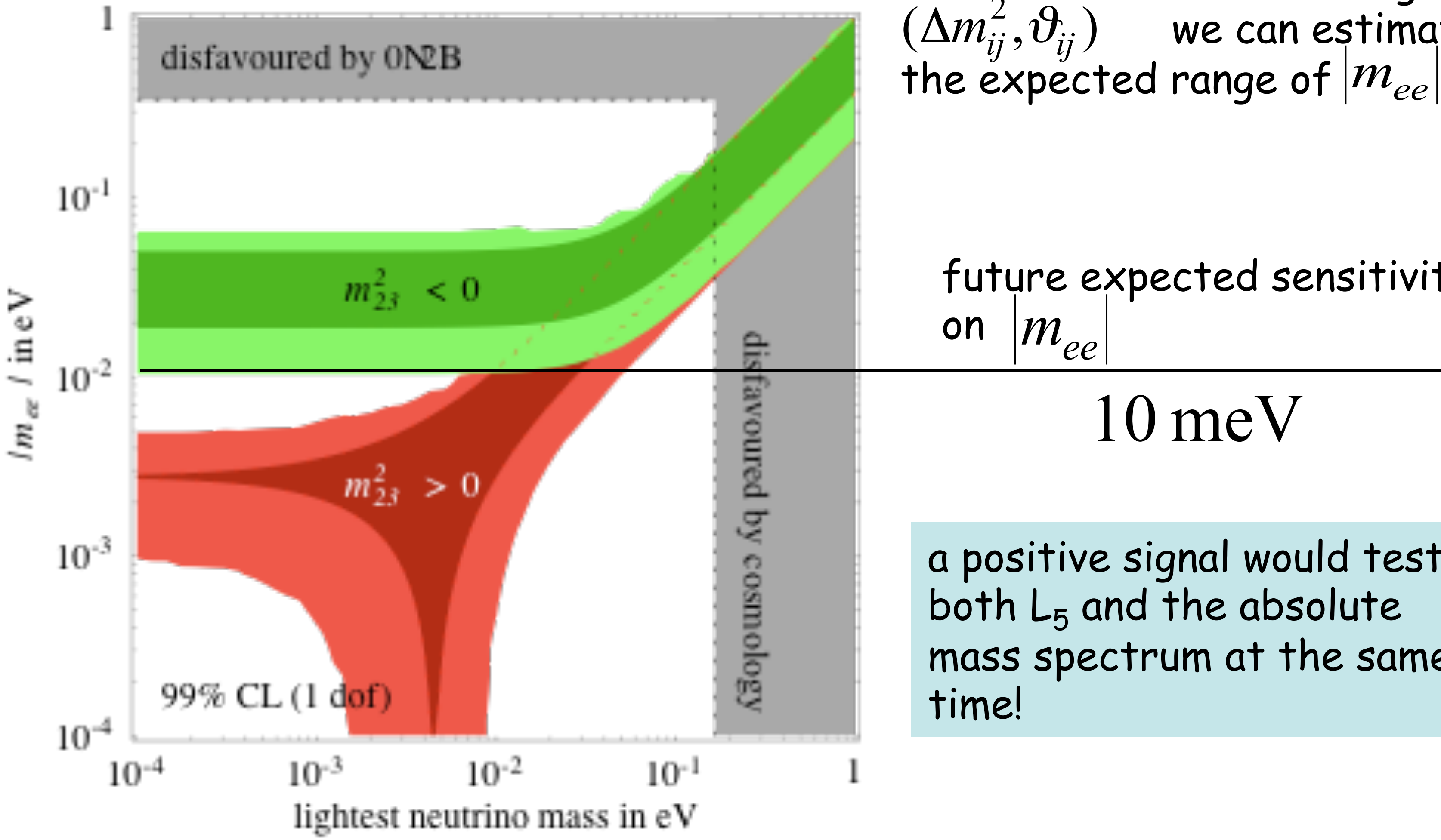
m_i $\quad = \quad \sum_i m_i U_{ei}^2 = m_{ee}$

U_{ei}

the decay in 0νββ rates depend on the combination $\quad |m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\; m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\; m_2) + \sin^2\vartheta_{13} e^{2i\beta}\; m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

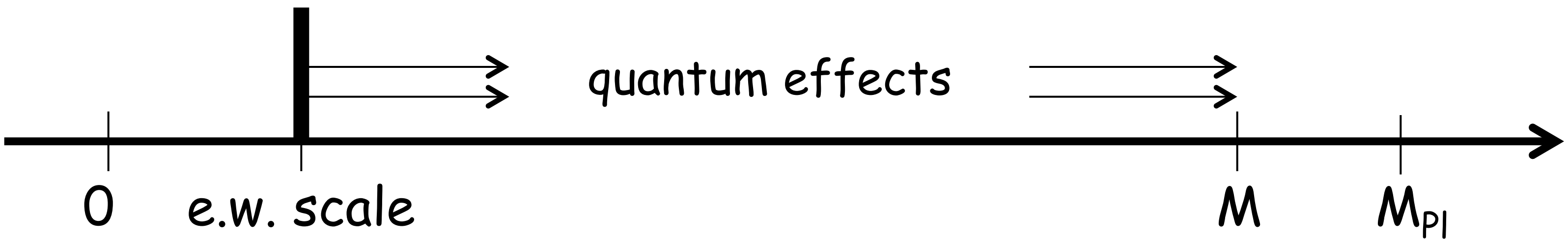
a positive signal would test both L_5 and the absolute mass spectrum at the same time!

Neutrinos and the Hierarchy Problem

Why

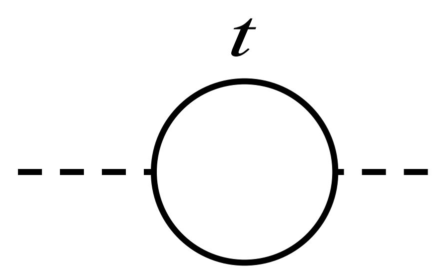
any new particle threshold: $M_{GUT,...}$

e.w. scale << ..., M_{Pl} ? ⟺ sensitivity of m_h to UV physics



often discussed in terms of quadratic divergences

$$\delta m_h^2 \propto \frac{y_t^2}{16\pi^2}\Lambda^2$$



but

-- what represents exactly Λ ? Any evidence from experiment?

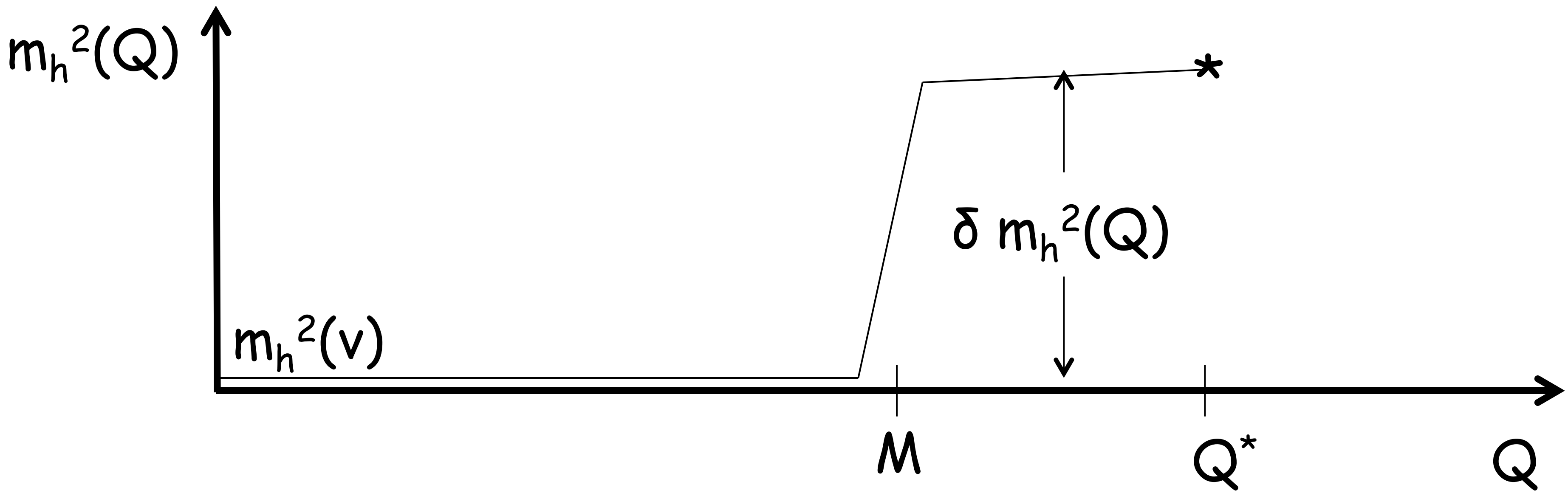
-- can we get rid of Λ in some suitable scheme ?

-- technical aspect obscure physics

hierarchy problem can be formulated entirely in terms of renormalized quantities with no reference to regulators

assumption: coupling y of Higgs particle to an heavy state of mass M

running Higgs mass $$\delta m_h^2(Q) \approx \frac{y^2}{16\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$



fine-tune the initial conditions at Q^* such that

$$m_h^2(v) \approx m_h^2(Q^*) - \frac{y^2}{16\pi^2} M^2 \log\frac{Q^*}{M}$$

consider type I see-saw

heavy state ν^c mass M
Yukawa coupling y_ν

$$\delta m_h^2(Q) \approx -\frac{y_\nu^2}{4\pi^2} M^2 \log\frac{Q}{M} \qquad Q > M$$

by using $m_\nu \approx \frac{y_\nu^2 v^2}{M}$ to eliminate the y^2 dependence

$$\left|\delta m_h^2(Q)\right| \approx \frac{1}{4\pi^2} \frac{m_\nu M^3}{v^2} \log\frac{Q}{M} < v^2$$

$$M < 1.4 \times 10^7 \quad \text{GeV}$$

$$\left[\begin{array}{c} \log\frac{Q}{M} \approx 1 \\ m_\nu \approx 0.05\ \text{eV} \end{array}\right]$$

$$y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}} < 10^{-4}$$

too small for thermal leptogenesis ?

similar conclusions in type II and type III see-saw where threshold corrections are dominated by 2-loop gauge interactions

type III $\delta m_h^2(Q) \approx -\frac{72 g^4}{(4\pi)^4} M^2 \log\frac{Q}{M}$ $\quad Q > M$ $\quad M < 940$ GeV

type II $M < 200$ GeV

ways out

the initial conditions at the scale Q^* are fine-tuned to an accuracy of order (e.w. scale)/M

the threshold correction at the scale M is almost cancelled by an other contribution, as e.g. in supersymmetry with a splitting between neutrinos and sneutrinos of order 4π x (e.w. scale)

the Higgs is not an elementary particle and dissolves above a compositness scale ~ TeV

Neutrinos
and
Lepton Flavour Violation

LFV expected at some level

neutrino masses
and $U_{PMNS} \neq 1$ ➡ L_i violated ($i=e,\mu,\tau$)

evidence for lepton flavor conversion

direct $\nu_e \to \nu_\mu, \nu_\tau$ sol, LBL exp

indirect $\nu_\mu \to \nu_\tau$ atm

should show up in processes with charged leptons

Process	Relative probability	Present Limit	Experiment	Year
$\mu \to e\gamma$	1	5.7×10^{-13}	MEG	2012
$\mu^- \mathrm{Ti} \to e^- \mathrm{Ti}$	$Z\alpha/\pi$	4.3×10^{-12}	SINDRUM II	2006
$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$	$Z\alpha/\pi$	7×10^{-13}	SINDRUM II	2006
$\mu \to eee$	α/π	4.3×10^{-12}	SINDRUM	1988
$\tau \to \mu\gamma$	$(m_\tau/m_\mu)^{2\div 4}$	3.3×10^{-8}	B-factories	2011
$\tau \to e\gamma$	$(m_\tau/m_\mu)^{2\div 4}$	4.5×10^{-8}	B-factories	2011

prospects
6×10^{-14}

$10^{-15} \div 10^{-16}$

Table 1: Relative sensitivities and experimental limits of the main CLFV processes.

here: focus on radiative decays of charged leptons in the SM, minimally extended to accommodate e.g. Dirac neutrinos

$$BR(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi} \left| U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2} \right|^2 \approx 10^{-53}$$

Exercise 15: reproduce this

[solution in Cheng and Li]

[unobservable also within type I see-saw] $m_i \approx 0.05\, eV \quad U_{fi} \approx O(1)$

depleted by
-- weak interactions
-- loop factor
-- GIM mechanism (mixing angle large, but neutrino masses tiny)

<->

GIM suppression for quarks:
small mixing angles
large top mass

a good place to look for BSM physics

LFV probes physics beyond the νSM [=SM minimally extended to accommodate ν masses]
observable rates for LFV require new physics at a scale Λ_{LFV} well below the GUT or the L-violation scales

can Λ_{LFV} be close to the TeV scale <-> explorable at the LHC?

low-energy effective Lagrangian in the lepton sector

$$L = L_{SM} + i\frac{e}{\Lambda^2} e^c\left(\sigma^{\mu\nu}F_{\mu\nu}\right)\mathcal{Z}(\Phi^+ l) + \frac{1}{\Lambda^2}[\text{4-fermion}] + h.c. + \ldots$$

[relation between the scale Λ and new particle masses M' can be non-trivial in a weakly interacting theory g Λ/4π≈M']

$\mathcal{Z}_{ij}$ a matrix in flavour space

$$L_Y = -e^c y_e(\Phi^+ l) + h.c. + \ldots$$

in the basis where charged leptons are diagonal

$\mathrm{Im}\left[\mathcal{Z}\right]_{ii}$	d_i	electric dipole moments
$\mathrm{Re}\left[\mathcal{Z}\right]_{ii}$	$a_i = \dfrac{(g-2)_i}{2}$	anomalous magnetic moments
$\left\|\left[\mathcal{Z}\right]_{ij}\right\|^2 \quad (i \neq j)$	$R_{ij} = \dfrac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)}$	radiative decays $\mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma$
[4-fermion operators]	other LFV transitions	$\mu \to eee \quad \tau \to \mu\mu\mu \quad \tau \to eee \quad \ldots$

$$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$$ [MEG 1605.05081]

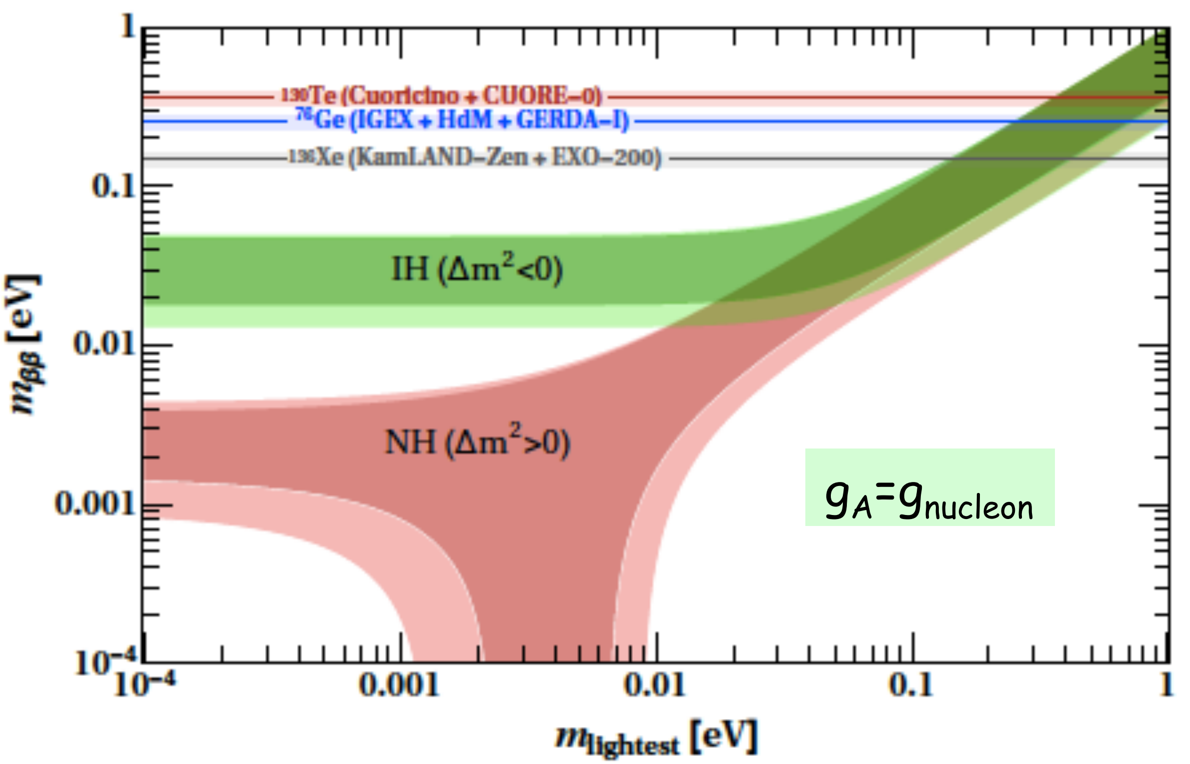
$$\frac{\mathcal{Z}_{\mu e}}{\Lambda^2} < 2 \times 10^{-9}\ \text{TeV}^{-2}$$

either the scale of new physics is very large or flavour violation from New Physics is highly non-generic

$$\Lambda > 2 \times 10^4 \left[\sqrt{\mathcal{Z}_{\mu e}}\right] TeV$$

Back up slides

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

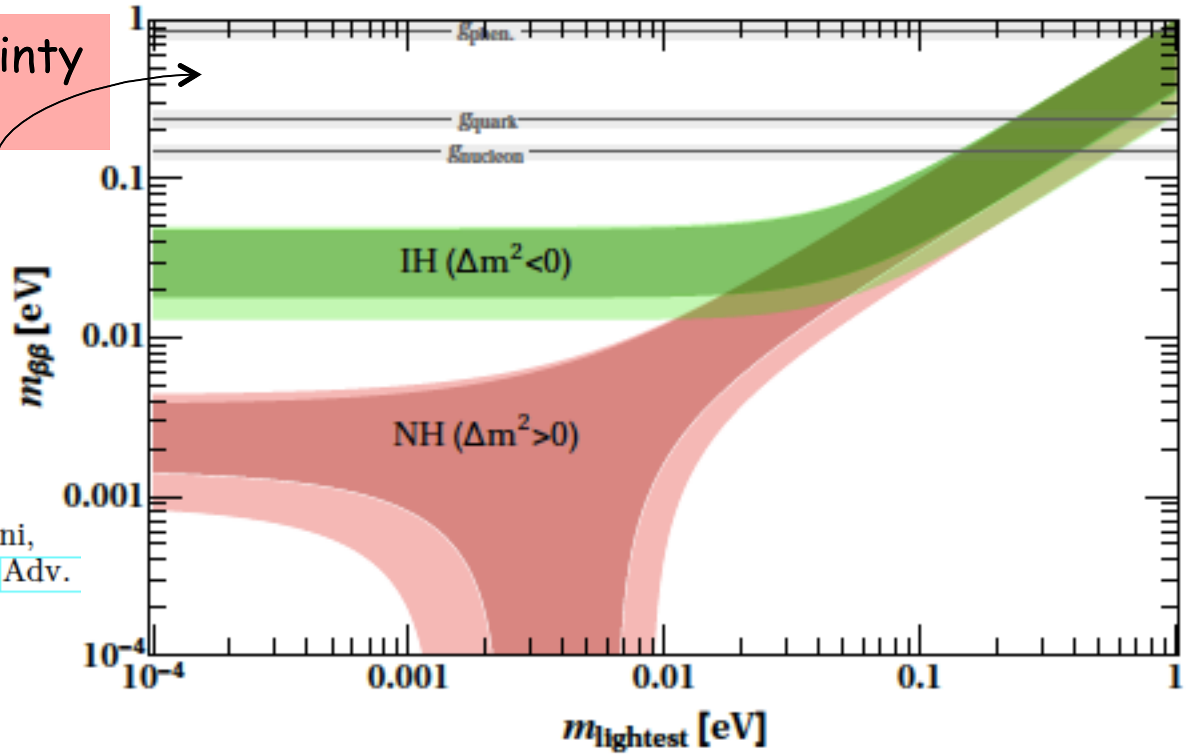


from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

Experiment	Isotope	$S^{0\nu}$ (90% C. L.) $[10^{25}\text{ yr}]$	Lower bound for $m_{\beta\beta}$ [eV]		
			g_{nucleon}	g_{quark}	$g_{\text{phen.}}$
IGEX + HdM + GERDA-I, [174]	^{76}Ge	3.0	0.25 ± 0.02	0.40 ± 0.04	1.21 ± 0.11
Cuoricino + CUORE-0, [180]	^{130}Te	0.4	0.36 ± 0.03	0.58 ± 0.05	2.07 ± 1.05
EXO-200 + KamLAND-ZEN, [187]	^{136}Xe	3.4	0.15 ± 0.02	0.24 ± 0.03	0.87 ± 0.10

largest theoretical uncertainty is from g_A

limits from ^{136}Xe



S. Dell'Oro, S. Marcocci, M. Viel, and F. Vissani, "*Neutrinoless double beta decay: 2015 review*," Adv. High Energy Phys. **2016** (2016) 2162659, arXiv:1601.07512.

Experiment	Isotope	$S^{0\nu}$ (90% C. L.) [10^{25} yr]	Lower bound for $m_{\beta\beta}$ [eV]		
			g_{nucleon}	g_{quark}	$g_{\text{phen.}}$
CUORE, [189]	^{130}Te	9.5	0.073 ± 0.008	0.14 ± 0.01	0.44 ± 0.04
GERDA-II, [174]	^{76}Ge	15	0.11 ± 0.01	0.18 ± 0.02	0.54 ± 0.05
LUCIFER, [190]	^{82}Se	1.8	0.20 ± 0.02	0.32 ± 0.03	0.97 ± 0.09
MAJORANA D., [191]	^{76}Ge	12	0.13 ± 0.01	0.20 ± 0.02	0.61 ± 0.06
NEXT, [193]	^{136}Xe	5	0.12 ± 0.01	0.20 ± 0.02	0.71 ± 0.08
AMoRE, [194]	^{100}Mo	5	0.084 ± 0.008	0.14 ± 0.01	0.44 ± 0.04
nEXO, [195]	^{136}Xe	660	0.011 ± 0.001	0.017 ± 0.002	0.062 ± 0.007
PandaX-III, [196]	^{136}Xe	11	0.082 ± 0.009	0.13 ± 0.01	0.48 ± 0.05
SNO+, [197]	^{130}Te	9	0.076 ± 0.007	0.12 ± 0.01	0.44 ± 0.04
SuperNEMO, [198]	^{82}Se	10	0.084 ± 0.008	0.14 ± 0.01	0.41 ± 0.04

neutrinos and the stability of the electroweak vacuum

for the current values

$$m_h = (125.66 \pm 0.34) \quad \text{GeV}$$

$$m_t = (173.2 \pm 0.9) \quad \text{GeV}$$

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

the Higgs potential develops an instability at

$$10^9\,\text{GeV} < \Lambda < 10^{15}\,\text{GeV}$$

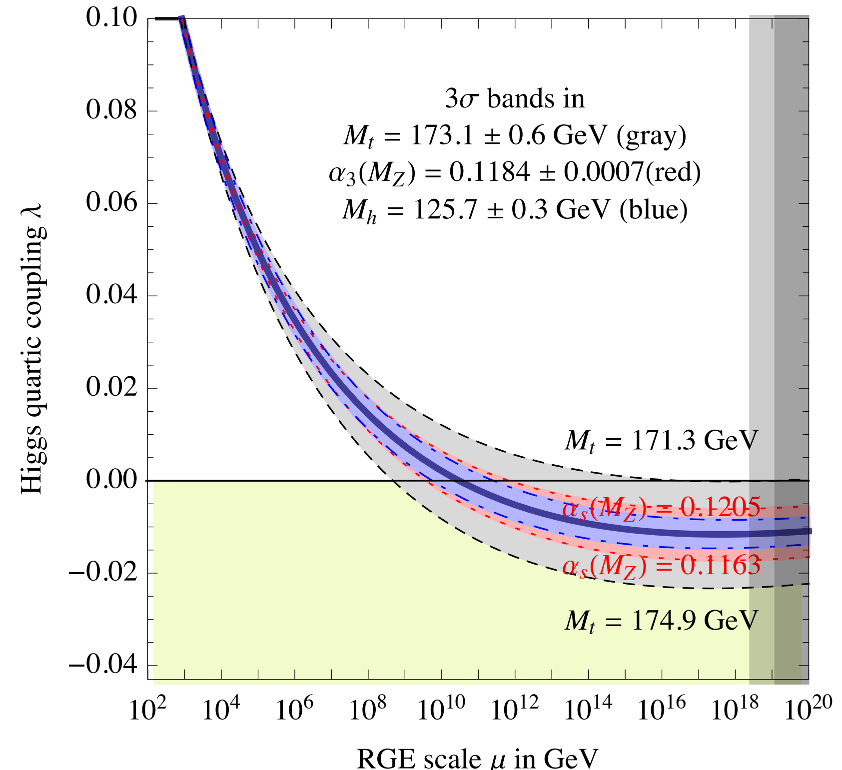
assumption: only SM all the way up to the scale Λ

for large values of the field h

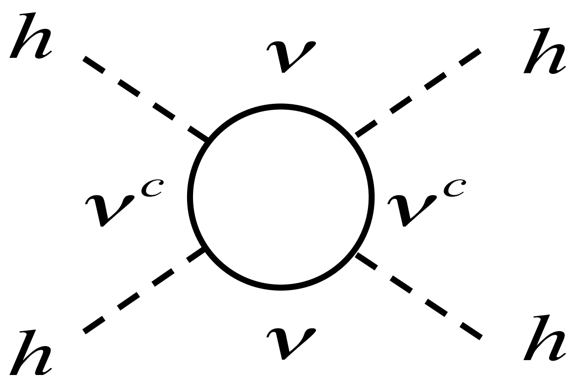
$$V(h) \approx \frac{\lambda}{4} h^4$$

$$(4\pi)^2 \frac{d\lambda}{dt} = -6y_t^4 + \frac{3}{8}[2g^4 + (g^2 + g'^2)]$$

$$+\underbrace{12\lambda y_t^2 - 3\lambda(g^2 + 3g'^2)}_{O(\lambda)} + \underbrace{24\lambda^2}_{O(\lambda^2)} + \ldots$$

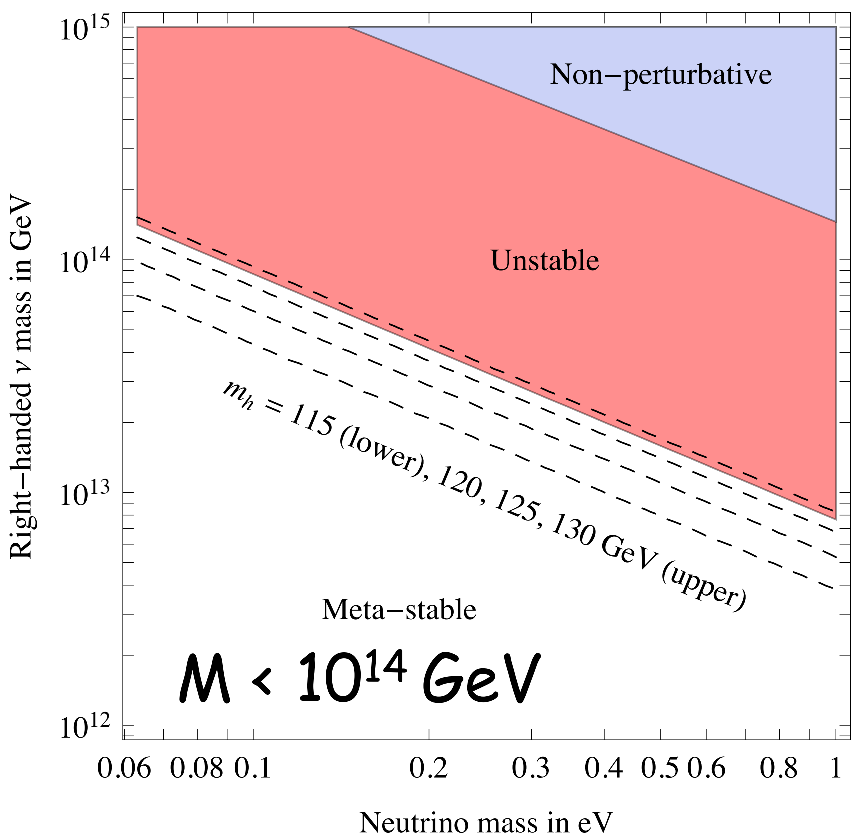


above the scale M a new contribution to β_λ arises from neutrino Yukawa couplings



$$\delta\beta_\lambda = -2\,\mathrm{tr}(y_\nu y_\nu^+ y_\nu y_\nu^+) < 0$$

contributes to instability above M



the larger M,
the larger the contribution

$$y_\nu \approx \sqrt{\frac{m_\nu M}{v^2}}$$

the bound applies only to the portion of SM parameter space that guarantees a stable vacuum in the limit y_ν=0
(m_t on the lower side
α_s on the higher side)

how can a wave function renormalization (effectively) arise?

several possibilities

here (Exercise 5): bulk fermions in a compact extra dimension S^1/Z_2

$$\mathcal{L} = i\bar{\Psi}_1\Gamma^M\partial_M\Psi_1 + i\bar{\Psi}_2\Gamma^M\partial_M\Psi_2 - m_1\varepsilon(y)\bar{\Psi}_1\Psi_1 + m_2\varepsilon(y)\bar{\Psi}_2\Psi_2 - \left[\delta(y)\frac{y}{\Lambda}\bar{f}_1(h+v)f_2 + h.c.\right]$$

$$\Psi_1 = \begin{pmatrix} E_1 \\ \bar{f}_1 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_2 \\ \bar{E}_2 \end{pmatrix}$$

solve the e.o.m. for the fermion zero modes with the b.c.

$$-\gamma_5\partial_y\Psi^0_{1,2} \pm m_{1,2}\,\varepsilon(y)\Psi^0_{1,2} = 0$$

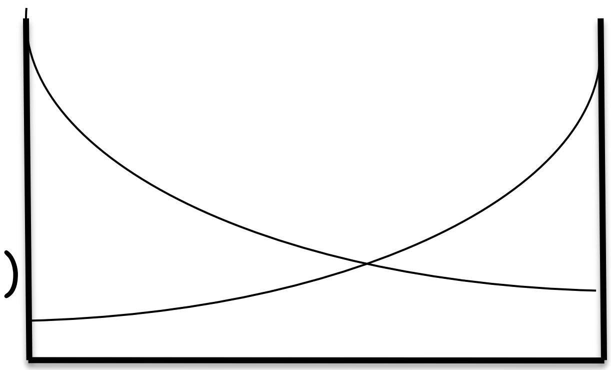
$$\Psi_1(-y) = +\gamma_5\Psi_1(y)$$

$$\Psi_2(-y) = -\gamma_5\Psi_2(y)$$

$$f_i^0(y) = \sqrt{\frac{2m_i}{1-e^{-2m_i\pi R}}}e^{-m_i y}$$

vanishing zero-modes for $(E_1, \bar{E}_2)$

Y≈O(1)



$$\mathcal{L}_Y = -\frac{1}{\Lambda\pi R}\bar{f}_1(F_1 y F_2)(h+v)f_2$$

$$F_i = \sqrt{\frac{x_i}{1-e^{-x_i}}} \approx \begin{cases} e^{-x_i/2} & x_i \gg 1 \\ 1 & x_i \approx 0 \\ \sqrt{-x_i} & x_i \ll -1 \end{cases}$$

Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why ξ_i<<1 consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When ξ_i=0 the theory becomes invariant under a flavour symmetry F

Example: why y_e<<y_{top}? Assume F=U(1)$_F$

F(t)=F(t^c)=F(h)=0 $\qquad y_{top}(h+v)t^c t \qquad$ allowed

F(e^c)=p>0 F(e)=q>0 $\qquad y_e(h+v)e^c e \qquad$ breaks U(1)$_F$ by (p+q) units

if ξ=<φ>/Λ<1 breaks U(1) by one negative unit $\qquad y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$

provides a qualitative picture of the existing hierarchies in the fermion spectrum