Varius, Multiplex, Multiformis
The Neutrino-Nucleus Cross Section

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OUTLINE

- Preamble: Motivation
- Lecture I
  - Basic elements of Nuclear Theory
  - The neutrino-nucleus cross section
- Detour: what we have learned from electron-nucleus scattering experiments
- Lecture II
  - Impulse approximation regime: reconstruction of neutrino energy in accelerator-based searches of neutrino oscillations
  - Emergence of collective excitation: mean free path of low-energy neutrinos in nuclear matter

MOTIVATION

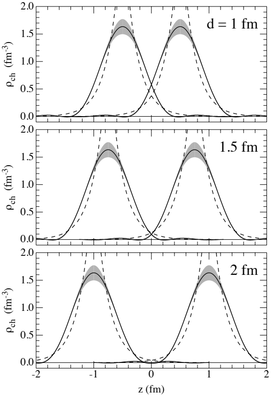
- ⋆ Nuclear weak interactions are the driving factor of a number of important astrophysical processes, such as the evolution of proto-neutron stars and neutron star cooling
- ⋆ Atomic nuclei—e.g., carbon, oxygen and argon—are used as detectors in experimental searches of neutrino oscillations
- ⋆ Neutrino interactions can be exploited to study aspects of nuclear dynamics that can not be probed by charged lepton

Lecture I

Disclaimer

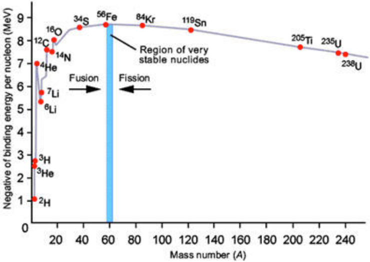
- ⋆ Bottom line: there is no such thing as an *ab initio* method to describe the properties of atomic nuclei
- ⋆ In the low-energy regime, the fundamental theory of strong interactions (QCD) becomes nearly intractable already at the level required for the description of isolated hadrons, let alone nuclei

- ⋆ Nuclei are described in terms of *effective degrees of freedom*, protons and neutrons, and *effective interactions*, mainly meson exchange processes
- ⋆ As long as their size is small compared to the relative distance, treating nucleons as individual particles appears to be reasonable

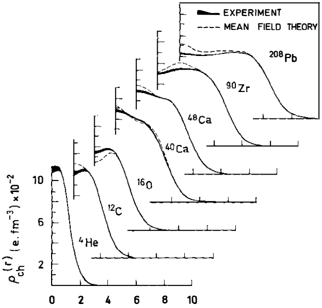


BINDING ENERGIES AND CHARGE-DENSITY DISTRIBUTIONS

- ⋆ The observation that the nuclear binding energy per nucleon is roughly the same for $A > 20$, its value being ~ 8.5 MeV, suggests that the range of the NN interaction is short compared to the nuclear radius.

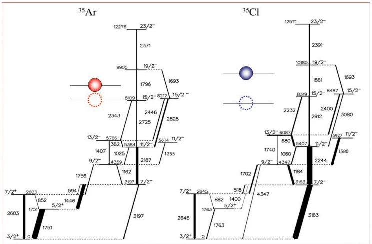


- ⋆ The observation that the charge-density in the nuclear interior is constant and independent of A indicates that the NN forces become strongly repulsive at short distance



Isotopic Invariance

- The spectra of mirror nuclei, e.g. ${}^{35}_{18}\mathrm{Ar}$ and ${}^{35}_{17}\mathrm{Cl}$, are identical up to small electromagnetic corrections
- Nuclear forces exhibit *charge independence*, which is a manifestation of a more general property: *isotopic invariance*



  - Neglecting the small mass difference, nucleons can be seen as two states of the same particle, the nucleon, specified by their *isospin*, $\tau_3 = \pm 1/2$
  - The force acting between two nucleons depends on the total isospin of the pair, $T = 0$ or 1, but not on its projection T_3

The Paradigm of Nuclear Many-Body Theory

- Nuclear matter is described as a collection of pointlike protons and neutrons interacting through the hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}}$$

- The mean field approximation, underlying the nuclear shell model, amounts to replacing

$$\sum_{j>i} \mathrm{v}_{ij} + \sum_{k>j>i} \mathrm{V}_{\mathrm{ijk}} \to \sum_i U_i \ ,$$

- While being able to explain a number of nuclear properties, the mean field approximation fails to take into account correlations, which have long being recognized to play a significant role. More on this later.

THE *bottom up* APPROACH

- Phenomenological potentials are determined by fitting the properties of the *exactly solvable* two- and three-nucleon systems, as well as the equilibrium density of isospin-symmetric matter
  - v_{ij} is strongly constrained by deuteron properties and nucleon-nucleon (NN) scattering data, and reduces to Yukawa's one-pion exchange potential at large distances.

$$\mathrm{v}_{ij} = \sum_p \mathrm{v}^p(r_{ij}) O^p_{ij}$$

$$O^p_{ij} = [\mathbb{1}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}, (\boldsymbol{\ell} \cdot \mathbf{S}), \ldots] \otimes [\mathbb{1}, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)] \ , \ \ldots$$

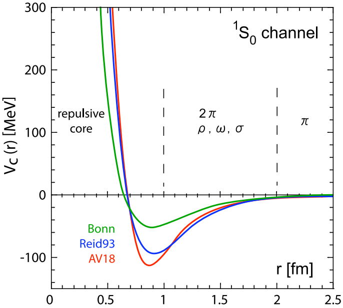
  - The three-nucleon potential consists of an attractive part arising from two-pion exchange and a purely phenomenological repulsive component

$$V_{ijk} = V^{2\pi}_{ijk} + V^{\mathrm{R}}_{ijk}$$

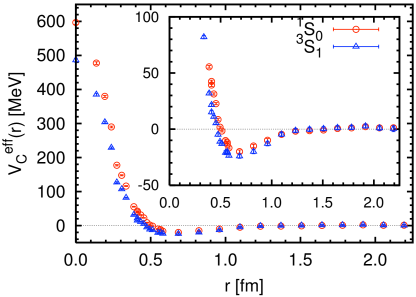
- Recently, consistent models of v_{ij} and V_{ijk} have been also derived within a formalism inspired by chiral perturbation theory

THE NN POTENTIAL IN THE $^1S_0$ CHANNEL

★ Phenomenological models

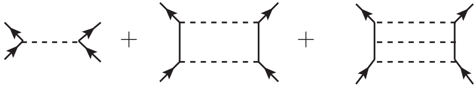


★ Lattice QCD, $m_\pi = 530$ MeV



Many-Body Theory of Nuclear Matter

- Owing to the presence of a strong repulsive core, the matrix elements of the nuclear Hamiltonian between eigenstates of the Hamiltonian describing the non-interacting system are large. Perturbation theory *in this basis* is not applicable.
- Alternate avenues
  - Replace the bare NN potential with a well behaved *effective interaction*, that can be used in perturbation theory using the Fermi gas basis
    - G-matrix perturbation theory

$\mathcal{G} =$ 

    - Renormalization group evolution of the bare interaction to low momentum
  - Modify the basis states in such a way as to mitigate the effects of the repulsive core

Correlated Basis Function (CBF) Formalism

- The eigenstates of the nuclear hamiltonian are approximated by the set of correlated states, obtained from the eigenstates of the Fermi Gas (FG) model

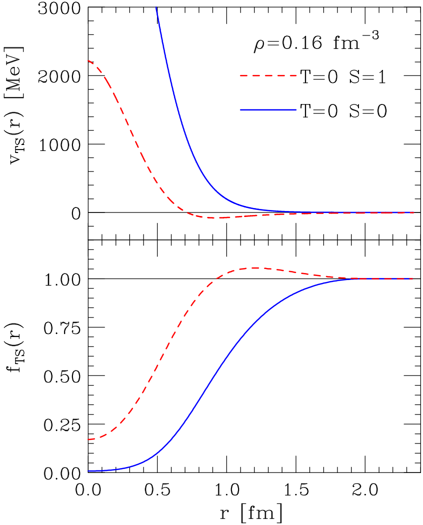
$$|n\rangle = \frac{F|n_{FG}\rangle}{\langle n_{FG}|F^{\dagger}F|n_{FG}\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}}\, F\,|n_{FG}\rangle \quad , \quad F = \mathcal{S}\prod_{j>i} f_{ij}$$

- the structure of the two-nucleon correlation operator reflects the complexity of interaction

$$f_{ij} = \sum_p f_p(r_{ij}) O^p_{ij}$$

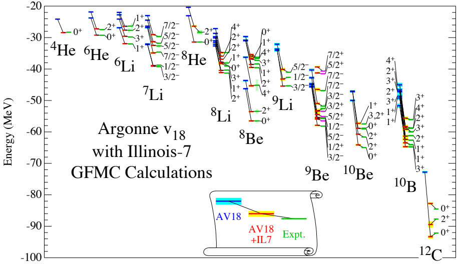
- the operators O^n_{ij} are the same as those entering the definition of the NN potential v_{ij}
- the radial shape of the $f_p(r)$ is determined through functional minimization of the ground-state energy

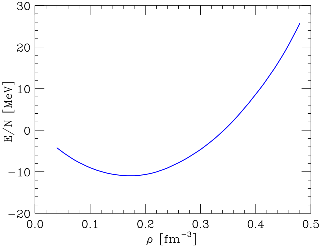
NN Potential and Correlation Functions



RESULTS OF NUCLEAR MANY-BODY THEORY

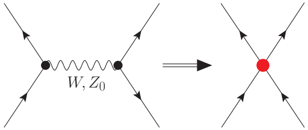
- ⋆ Quantum Monte Carlo and variational calculations performed using phenomenological nuclear Hamiltonians explain the energies of the ground- and low-lying excited states of nuclei with mass $A \leq 12$, as well as saturation of the equation of state of cold isospin-symmetric nuclear matter





WARM-UP: NEUTRINO-NUCLEON X-SECTION

- ⋆ In the regime of momentum transfer (q) discussed in this Lectures, Fermi theory of weak interaction works just fine



- ⋆ Consider, for example, the x-section of the charged-current process $\nu_\ell + n \to \ell^- + X$

$$d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}$$

  - ▸ $L_{\lambda\mu}$ is determined by the lepton kinematical variables (more on this later)
  - ▸ under very general assumptions $W^{\lambda\mu}$ can be written in the form

$$W^{\lambda\mu} = -g^{\lambda\mu}\, W_1 + p^\lambda\, p^\mu\, \frac{W_2}{m_N^2} + i\, \varepsilon^{\lambda\mu\alpha\beta}\, q_\alpha\, p_\beta + \frac{W_3}{m_N^2} + q^\lambda\, q^\mu\, \frac{W_4}{m_N^2}$$
$$+ (p^\lambda\, q^\mu + p^\mu\, q^\lambda)\, \frac{W_5}{m_N^2}$$

- ⋆ In principle, the structure functions W_i can be extracted from the measured cross sections
- ⋆ In the elastic sector $\nu_\ell + n \to \ell^- + p$ they can be expressed in terms of vector ($F_1(q^2)$ and $F_2(q^2)$), axial ($F_A(q^2)$) and pseudoscalar ($F_P(q^2)$) *form factors*

$$W_1 = 2\left[-\frac{q^2}{2}\,(F_1+F_2)^2 + \left(2\,m_N^2 - \frac{q^2}{2}\right)\,{F_A}^2\right]$$

$$W_2 = 4\left[{F_1}^2 - \left(\frac{q^2}{4\,m_N^2}\right)\,{F_2}^2 + {F_A}^2\right] = 2W_5$$

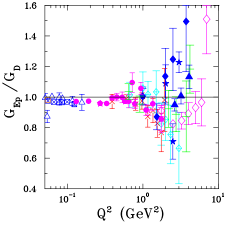
$$W_3 = -4\,(F_1+F_2)\,F_A$$

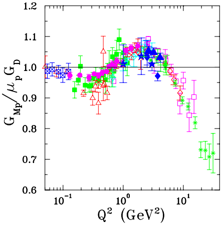
$$W_4 = -2\left[F_1\,F_2 + \left(2\,m_N^2 + \frac{q^2}{2}\right)\,\frac{{F_2}^2}{4\,m_N^2} + \frac{q^2}{2}\,{F_P}^2 - 2\,m_N\,F_P\,F_A\right]$$

- ⋆ according to the CVC hypothesis, F_1 and F_2 can be related to the electromagnetic form factors, measured by electron-nucleon scattering, while PCAC allows one to express F_P in terms of the axial form factor

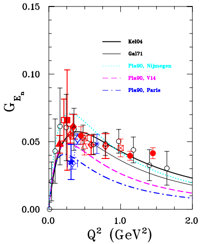
Vector Form Factors

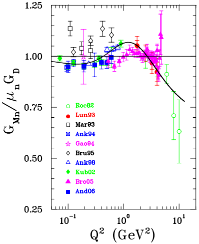
- ⋆ Proton data





- ⋆ Neutron (deuteron) data

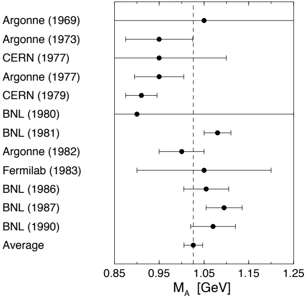




Axial Form Factor

⋆ Dipole parametrization

$$F_A(Q^2) = \frac{g_A}{\left[1 + (Q^2/M_A^2)\right]^2}$$



- ▷ g_A from neutron β-decay
- ▷ axial mass M_A from (quasi) elastic ν- and $\bar{\nu}$-deuteron experiment

NEUTRINO-NUCLEUS X-SECTION

- Consider again a charged current process

$$\nu_\ell + A \to \ell^- + X$$

- Nuclear response tensor

$$W_{\lambda\mu} = \sum_N \langle 0|J_\lambda^\dagger|n\rangle\langle n|J_\mu|0\rangle\delta^{(4)}(p + k - p_N - k')$$

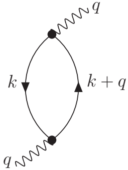
- To take into account all relevant reaction processes one needs to:
  - Model nuclear dynamics
  - Solve the many-body Schrödinger equation $H|n\rangle = E_n|n\rangle$
  - Determine the nuclear weak current (Are the nucleon weak structure functions modified by the nuclear medium? Are there additional contributions to the current?)

The One-Particle–One-Hole ($1p1h$) Sector

- Consider a $^{12}\mathrm{C}$ target as an example

$$|N\rangle = |p, {}^{11}\mathrm{C}\rangle \ , \ |n, {}^{11}\mathrm{B}\rangle$$

  - The infamous Relativistic Fermi Gas Model (RFGM): the nucleus is described as a degenerate system at constant density ρ

$L_{\mu\nu} W_A^{\mu\nu} \propto$ 

No nucleon-nucleon interaction, mean field described by a constant binding energy ϵ. Oriented lines represent the Green's functions

$$G_h(k,E) = \frac{\theta(k-k_F)}{E - e_0(k) + i\eta} \quad , \quad G_p = \frac{\theta(k_F - k)}{E - e_0(k) - i\eta}$$

where $\eta = 0^+$, $k_F = (3\pi^2\rho/2)^{1/3}$ is the Fermi momentum and

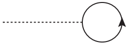
$$e_0(k) = \sqrt{k^2 + m^2} + \epsilon$$

Interacting Nucleons

- In the presence of interactions

$$e_k^0 = \frac{k^2}{2m} \to e_k^0 + \Sigma(\mathbf{k}, E) = e_k^0 + \mathrm{Re}\Sigma(\mathbf{k}, E) + i\mathrm{Im}\Sigma(\mathbf{k}, E)$$

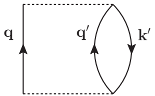
- Self energy: $\Sigma(\mathbf{k}, E)$



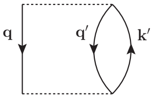
$\Sigma_{HF}(\mathbf{k})$
(a)

- Green's function

$$G_h(\mathbf{k}, E) = \frac{1}{E - e_k^0 - \Sigma(\mathbf{k}, E)}$$



$\Sigma_p(\mathbf{k}, E)$
(b)



$\Sigma_h(\mathbf{k}, E)$
(c)

- $\Sigma(\mathbf{k}, E)$ can be computed using eithr CFB or G-matrix perturbation theory

Quasiparticles and Spectral Representation

- ⋆ The identification of single particle properties in interacting many-body systems is a non trivial issue, addressed by Landau's theory of normal Fermi liquids
- ⋆ According to Landau, there is a one-to-one correspondence between the elementary excitations of a Fermi liquid, dubbed quasiparticles, and those of the non interacting Fermi gas. Quasiparticle states of momentum $\mathbf{k}$ are specified by their energy, e_k and lifetime τ_k

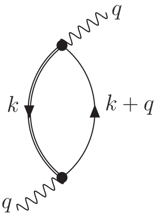
$$G_h(\mathbf{k},E) = \frac{Z_k}{E - e_k - i\tau_k^{-1}} + G_h^B(\mathbf{k},E)$$

$$e_k = e_k^0 + \Sigma(\mathbf{k},e_k),\ \tau_k^{-1} = Z_k \mathrm{Im}\Sigma(\mathbf{k},e_k),\ Z_k = \left[1 - \frac{\partial}{\partial E}\mathrm{Re}\,\Sigma(k,E)\right]^{-1}_{E=e_k}$$

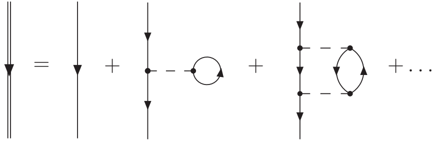
- ⋆ $G_h^B(\mathbf{k},E)$ is a smooth contribution associated with multiparticle-multihole excitations

- Including nucleon-nucleon interactions in the initial state

$W_A^{\mu\nu} =$

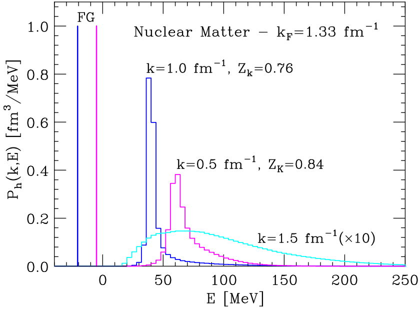


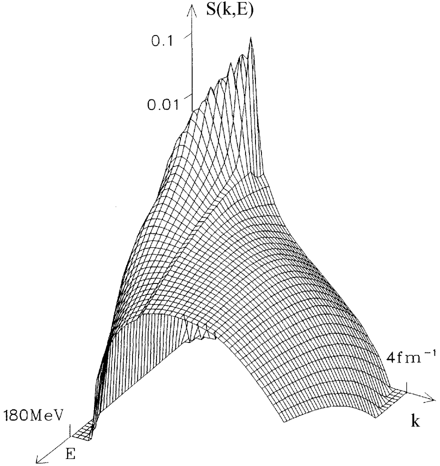
$G_h(k, E) =$



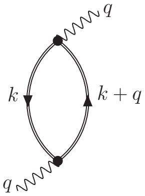
- the *bare* nucleon-nucleon interaction cannot be used for perturbation theory in the basis of eigenstates of the non-interacting system. Eiher the interaction or the basis states need to be "renormalized" using G-matrix or Correlated Basis Function (CBF) perturbation theory.

Hole States in Isospin-Symmetric Nuclear Matter





- In principle, the effects of nucleon-nucleon interactions in the final state may be taken into account in a consistent fashion, using

$W_A^{\mu\nu} =$ 

However, the propagation of the outgoing nucleon, described by the Green's function $G_p(\mathbf{k}+\mathbf{q}, E)$, requires either a relativistic model of nuclear dynamics or an approximation scheme based on nucleon-nucleon and nucleon-nucleus scattering data

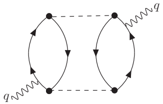
THE TWO-PARTICLE–TWO-HOLE (2P2H) SECTOR

- Interactions couple the 1h (1p) states of the residual nucleon to 2h1p (2p1h) states, in which one of the spectator nucleons is excited to the continuum. This mechanism leads to the appearance of 2p2h final states

$$|N\rangle = |pp, {}^{10}\mathrm{B}\rangle \ , \ |np^{10}\mathrm{C}\rangle \ldots$$

- In addition, 2p2h states appear through their couplig to the ground state

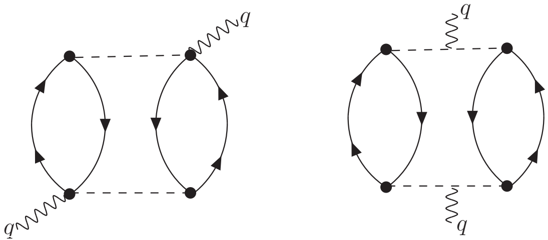
$$W_A^{\mu\nu} =$$



- These contributions exhibit a specific energy dependence, and give rise to a characteristic event geometry
- Note: in interacting many body systems the excitation of 2p2h states *does not* require a two-nucleon current

Meson-Exchange Currents (MEC)

- Two-nucleon currents naturally couple the nuclear ground state to 2p2h final states, e.g. through the processes

$W_A^{\mu\nu} =$ 

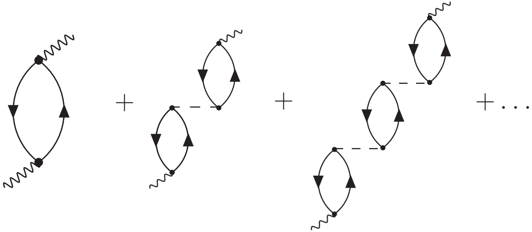
as well as through similar processes involving the excitation of a Δ-resonance

- Note: amplitudes involving one- and two-body currents and the same 2p2h state give rise to interference

Long-Range Correlations

- At low momentum transfer, processes involving many nucleons may become important. Within the Tamm-Dancoff (ring) approximation the nuclear final state is written in the form

$$|N\rangle = \sum_i C_i |p_i h_i\rangle$$

$W_A^{\mu\nu} =$ 

Note: the Random-Phase-Approximation (RPA) is a generalization of the above scheme

Summary of Lecture I

- ⋆ In spite of the severe difficulties associated with both the nature of nuclear interactions and the complexity of the nuclear many-body problem, nearly exact calculations of many properties of nuclei with mass number $A \leq 12$ are now possible.

- ⋆ Extended modeling is still needed to describe the reaction mechanisms contributing to the neutrino-nucleus cross sections, and clarify the role of different dynamical effects (mean field effects, short- and long-range correlations, $\ldots$)

- ⋆ The proposed model must be validated through comparison to external data set. In this context, a critical role is played by the large database of precisely measured electron-nucleus cross section.

Detour

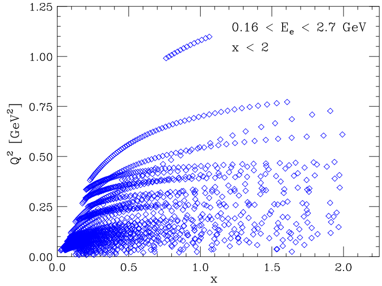
What we have learned from electron-nucleus scattering experiments

Information from Electron Scattering

- Vast supply of precise data available

$$Q^2 = 4E_e E_{e'} \sin^2 \frac{\theta_e}{2} \quad , \quad x = \frac{Q^2}{2M\omega}$$

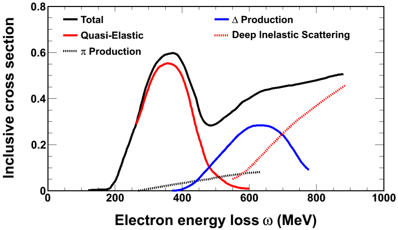
- Carbon target



- Different rection mechanisms contributimg to the mesured cross sections can be readily identified

$$e + A \to e' + X$$

$$E_e \sim 1 \text{ GeV}$$



THE MEAN-FIELD APPROXIMATION

- Nuclear systematics offers ample evidence supporting the further assumption, underlying the nuclear shell model, that the potentials appearing in the Hamiltonian can be eliminated in favour of a mean field

$$H \to H_{MF} = \sum_i \left[\frac{\mathbf{p}_i^2}{2m} + U_i \right]$$

$$\left[\frac{\mathbf{p}_i^2}{2m} + U_i \right] \phi_{\alpha_i} = \epsilon_{\alpha_i} \phi_{\alpha_i} \quad , \quad \alpha \equiv \{n, \ell, j\}$$

- For proposing and developing the nuclear shell model, E. Wigner, M. Goeppert Mayer and J.H.D. Jensen have been awarded the 1963 Nobel Prize in Physics
- A warning from Blatt & Weiskopf (AD 1952): "The limitation of any independent particle model lies in its inability to encompass the correlation between the positions and spins of the various particles in the system"

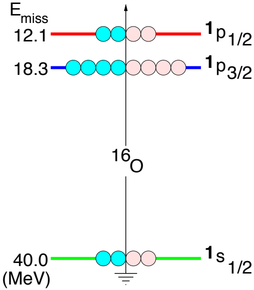
The Shell-Model Ground State

- According to the shell model, in the nuclear groud state protons and neutrons occupy the A lowest energy eigenstates of the mean field Hamiltonian

$$H_{MF}\Psi_0 = E_0\Psi_0 \quad , \quad \Psi_0 = \frac{1}{A!}\det\{\phi_\alpha\} \quad , \quad E_0 = \sum_{\alpha\in\{F\}} \epsilon_\alpha$$

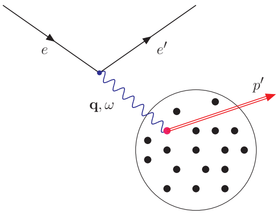
- Ground state of ^{16}O: $Z = N = 8$

$$(1S_{1/2})^2 \;\; , \;\; (1P_{3/2})^4 \;\; , \;\; (1P_{1/2})^2$$



THE $(e, e'p)$ REACTION

- Consider the process $e + A \to e' + p + (A-1)$ in which both the outgoing electron and the proton, carrying momentum p', are detected in coincidence



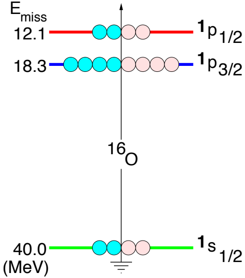
- Assuming that there are no final state interactions (FSI), the initial energy and momentum of the knocked out nucleon can be identified with the *measured* missing momentum and energy, respectively

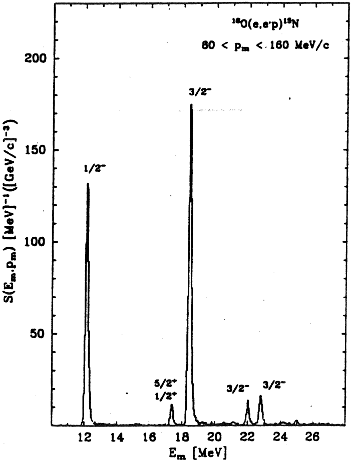
$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q} \quad , \quad E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

Proton Knockout from Shell-Model States

- The spectral lines corresponding to the shell model states clearly seen in the missing energy spectra of measured by

$$e + \mathrm{A} \rightarrow e' + p + X$$





Exposing the Limits of the Independent Particle Model

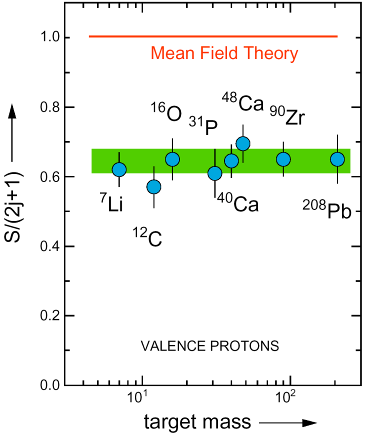
- The measured missing energy spectra, while exhibiting the lines predicted by the nuclear shell model, provide unambiguous evidence of its limitations

		η_α	ΔT_α	N_α	$\langle E \rangle_\alpha$	$\langle T \rangle_\alpha$
^{12}C	1p	0.66	2.1	2.5	17.5 ± 0.4	18.3
	1s	0.52	1.9	1.0	38.1 ± 1.0	12.7
^{28}Si	2s	0.46	3.2	0.4	13.8 ± 0.5	18.6
	1d	0.46	2.2	5.5	16.1 ± 0.8	19.5
	1p	0.39	2.0	2.9	32	14.1
	1s	0.28	1.1	0.9	(51)	8.5
^{40}Ca	2s	0.38	3.2	1.3	11.2 ± 0.3	19.7
	1d	0.38	2.1	7.7	14.9 ± 0.8	19.6
	1p	0.32	2.4	5.7	41	14.0
	1s	0.23	1.2	1.5	(56)	8.0
^{58}Ni	1f	0.32	2.4	7.6	9.3 ± 0.3	23.4
	2s	0.31	3.2	1.9	14.7 ± 0.5	18.6
	1d	0.32	2.2	8.9	21	19.4
	1p	0.27	2.0	6.8	45	14.4
	1s	0.19	1.1	1.0	(62)	9.1

- The systematic deviation of the spectroscopic factors from the shell model prediction $Z_\alpha = 2j_\alpha + 1$ is a clear signature of strong correlation effects

Spectroscopic Factors of Valence States

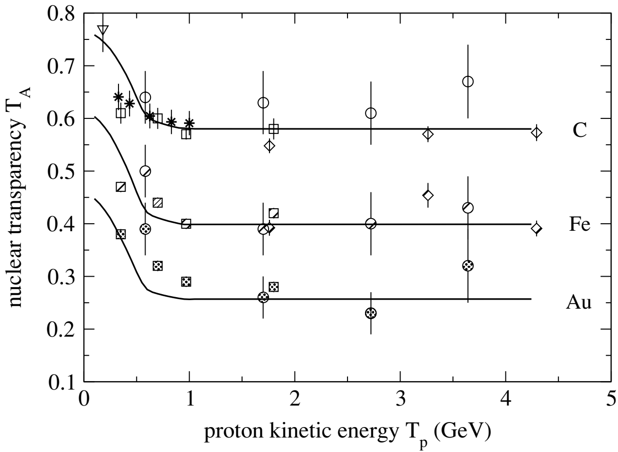
- The quenching of the spectroscopic factor of valence states has been confirmed by a number of high-resolution $(e, e'p)$ experiments carried out at NIKHEF-K using a 700 MeV high duty factor electron beam



- quenching is large and independent of target mass
- both short- and long-range correlations contribute

Gauging FSI: Nuclear Transparency from $(e, e'p)$

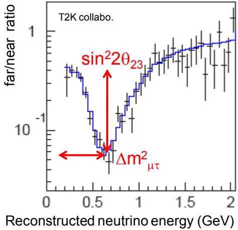
- Nuclear transparency, measured by the ratio $\sigma_{\mathrm{exp}}/\sigma_{\mathrm{PWIA}}$



Lecture II

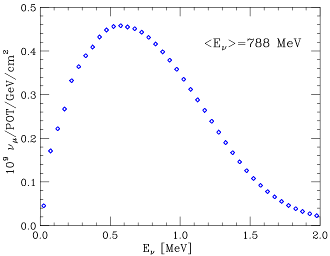
Determination of the Oscillation Parameters

⋆ The oscillation parameters Δm^2 and $\sin^2 2\theta$ are extracted from the energy-dependence of the oscillation probability



$$P_{\alpha\to\beta} = \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

⋆ The incoming neutrino energy E_ν is not known, its value being distributed according to a broad flux (MiniBeeNE as an example)



Neutrino Energy Reconstruction

- Consider the charged current quasi elastic (CCQE) process

$$\nu_\mu + (A, Z) \to \mu^- + p + (A-1, Z)^\star$$

- Assuming single-nucleon knock out, the incoming neutrino energy can be *recosntructed* from

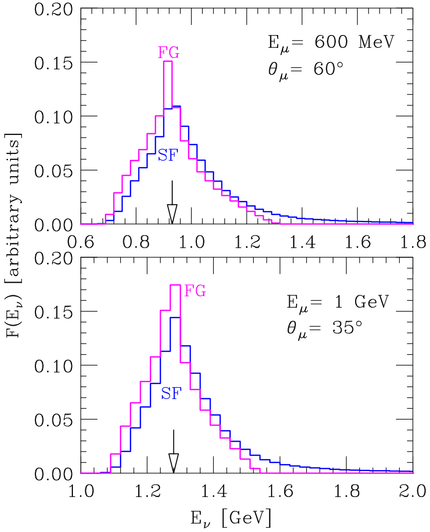
$$E_\nu = \frac{m_p^2 - m_\mu^2 - {E_n}^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |{\mathbf{p}_n}^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu - |\mathbf{p}_n| \cos\theta_n)} ,$$

  where $|\mathbf{k}_\mu|$ and θ_μ are measured, while $\mathbf{p}_n$ and E_n are the *unknown* momentum and energy of the interacting neutron

- Existing simulation codes routinely use $\mathbf{p}_n = 0$, $E_n = m_n - \epsilon$, with $\epsilon \sim 20$ MeV for carbon and oxygen, or the Fermi gas (FG) model with $p_F \sim 220$ MeV

RECONSTRUCTED NEUTRINO ENERGY IN $\nu_\mu + {}^{16}\mathrm{O} \to \mu^- + X$

- Neutrino energy reconstructed using 2×10^4 pairs of $\{|\mathbf{p}|, E\}$ values sampled from the nucleon energy-momentum distribution in the oxygen ground state, obtained from a realistic dynamical model (SF) and the Fermi Gas model (FG)
- The average value $\langle E_\nu \rangle$ obtained from the realistic model turns out to be shifted towards larger energy by ~ 70 MeV



The Neutrino-Nucleus Cross Section

⋆ Double differential cross section of the process

$$\nu_\ell(k) + A \to \ell^-(k^{'}) + X$$

$$\frac{d^2\sigma}{d\Omega_\ell dE_\ell} = \frac{G_F^2\, V_{ud}^2}{16\,\pi^2} \frac{|\mathbf{k}^{'}|}{|\mathbf{k}|} L_{\mu\nu}\, W_A^{\mu\nu} \,,$$

$$L_{\mu\nu} = 8\left[k^{'}_\mu\, k_\nu + k^{'}_\nu\, k_\mu - g_{\mu\nu}(k\cdot k^{'}) - i\,\varepsilon_{\mu\nu\alpha\beta}\, k^{'\,\beta}\, k^\alpha\right]$$

- The determination of the nuclear response tensor

$$W_A^{\mu\nu} = \sum_N \langle 0|J_A^{\mu\,\dagger}|N\rangle\langle N|J_A^\nu|0\rangle \delta^{(4)}(P_0 + k - P_N - k')$$

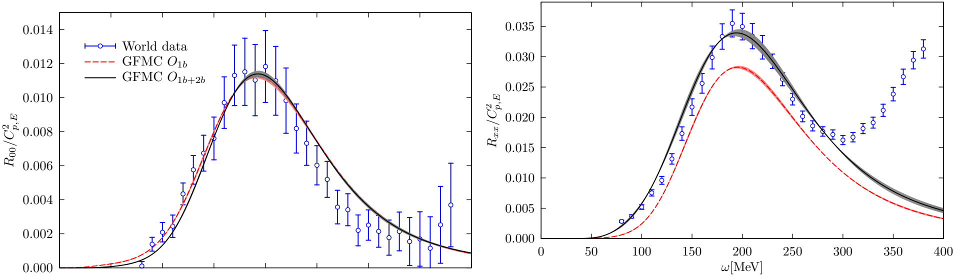
  requires a *consistent* description of the target initial and final states and the nuclear current operator

$$J_A^\mu = \sum_i j_i^\mu + \sum_{j>i} j_{ij}^\mu$$

- Accurate Quantum Monte Carlo calculations are feasible for light nuclear targets ($A \leq 4$) in the non relativistic regime ($|\mathbf{q}| \lesssim 500$ MeV)

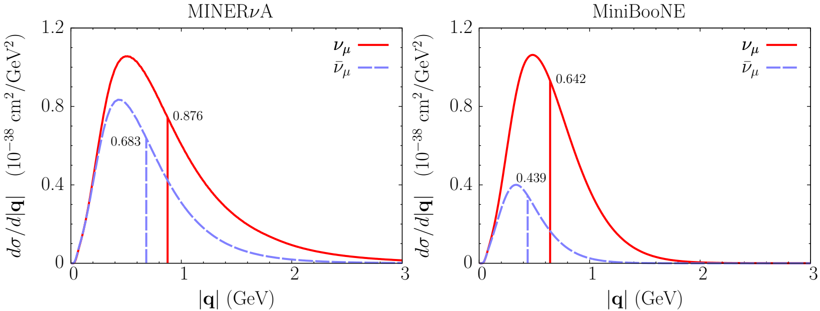
GREEN'S FUNCTION MONTE CARLO (GFMC)

- Longitudinal (left) and transverse (right) electromagnetic responses of ^{12}C at $|\mathbf{q}| = 570$ MeV



- Full "Exact" calculation based on a realistic nuclear Hamiltonian

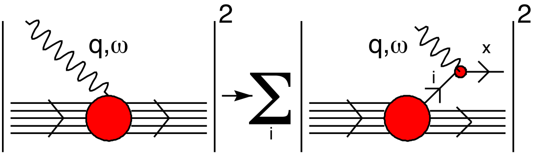
WHY WORRY ABOUT RELATIVITY



- $|\mathbf{q}|$-dependence of the CCQE cross section averaged with the Minerνa (left) and MiniBooNE (right) fluxes
- In the kinematical regime in which relativistic effects become important, approximations—involving both the reaction mechanism and the underlying dynamics—are required

Impulse Approximation (IA) & Factorization

- At $\lambda = \pi/|\mathbf{q}| \ll d_{\rm NN}$, the average NN distance in the target nucleus



- Basic assumptions
  - $|N\rangle \approx |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$
  - $J_A^\mu(q) = \sum_i j_i^\mu(q) + \sum_{j>i} j_{ij}^\mu(q) \approx \sum_i j_i^\mu(q)$

- As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected (will be added as corrections)

FACTORIZATION (CONTINUED)

- ⋆ Within the factorization *ansatz* the target tensor reduces to

$$W_A^{\mu\nu} = \int d^3k \, dE \, \frac{m}{E_k} P_h(\mathbf{k}, E) w^{\mu\nu}(k, k + \tilde{q}) \ ,$$

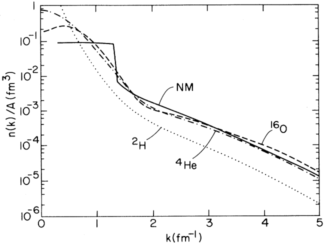
- ⋆ $w^{\mu\nu}$ is the tensor describing the interaction of a free nucleon of momentum $\mathbf{k}$ at four momentum transfer

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \quad , \quad \tilde{\omega} = E_x - E_k = \omega + m - E - E_k$$

- ⋆ The substitution $\omega \to \tilde{\omega} < \omega$ is needed to take into account the fact that an amount $\delta\omega = \omega - \tilde{\omega}$ of the energy transfer goes into excitation energy of the spectator system.
- ⋆ The probability distribution is given by the spectral function describing hole states, $P_h(\mathbf{k}, E)$

$P_h(\mathbf{k}, E)$ Within The Local Density Approximation

$$n(k) = \int dE\, P(\mathbf{k}, E)$$



- ⋆ Bottom line: the tail of the momentum distribution, arising from the continuum contribution to the spectral function, turns out to be largely A-independent for $A > 2$

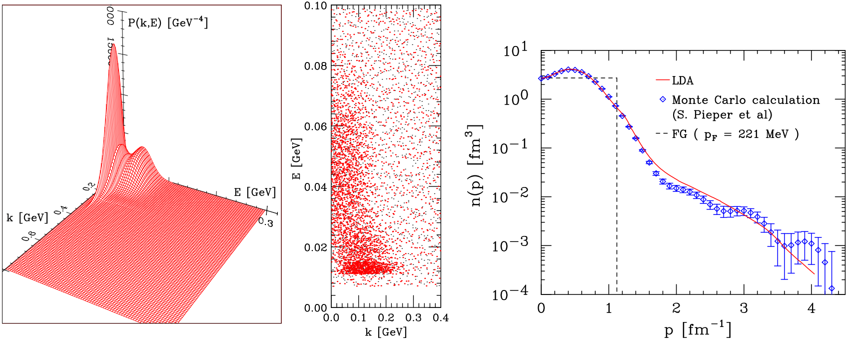
- ⋆ Spectral functions of complex (isospin symmetric) nuclei have been obtained within the local density approximation (LDA)

$$P_{\rm LDA}(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + \int d^3r\, \rho_A(r)\, P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

using the MF contributions extracted from $(e, e'p)$ data

- ⋆ The continuum contribution $P^{NM}_{corr}(\mathbf{k}, E)$ can be accurately computed in uniform nuclear matter at different densities

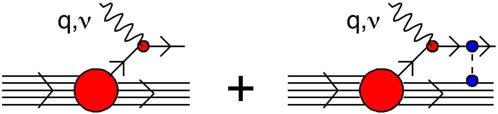
Oxygen Spectral Function



- FG model: $P(\mathbf{p}, E) \propto \theta(p_F - |\mathbf{p}|)\, \delta(E - \sqrt{|\mathbf{p}|^2 + m^2} + \epsilon)$
- shell model states account for $\sim 80\%$ of the strenght
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum *and* large removal energy ($|\mathbf{p}| \gg p_F \sim 220$ MeV$, E \gg \epsilon$)

Correctione to the IA: Final State Interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the importance of FSI



$$d\sigma_A = \int d^3k dE \, d\sigma_N \; P_h(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing the particle spectral function with the one of the non interacting systemai, i.e. setting

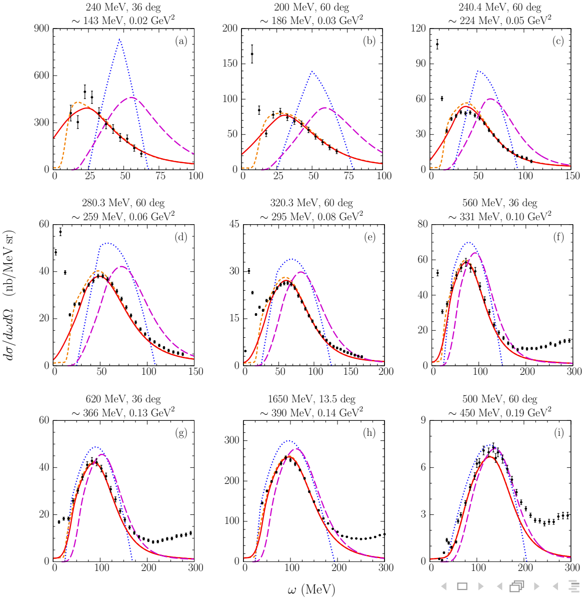
$$P_p(|\mathbf{k}+\mathbf{q}|, \omega - E) \sim \delta(\omega - E - e_{|\mathbf{k}+\mathbf{q}|})$$

- effects of FSI on the inclusive cross section
  (i) shift in energy transfer, $\omega \to \omega + U(\mathbf{k}+\mathbf{q})$, arising from interactions with the mean field of the spectators
  (ii) redistributions of the strenght, arising from the coupling of $1p1h$ final state to $2p2h$ final states
- high-energy approximation
  (i) the struck nucleon moves along a straight trajectory with constant velocity
  (ii) the fast struck nucleon “sees” the spectator system as a collection of fixed scattering centers.

$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T}\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$
$$+(1-\sqrt{T})f(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

- the nuclear transparency T and the folding function f can be computed within nuclear many-body theory using the *measured* nucleon-nucleon scattering amplitude

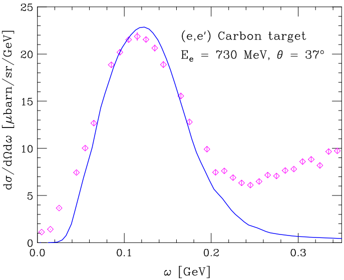
$e + {}^{12}\mathrm{C} \to e' + X$ IN THE QUASI ELASTIC CHANNEL (IA+FSI)

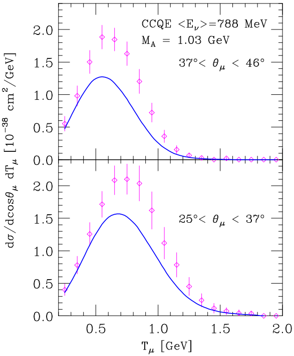


e- AND ν_μ-CARBON X-SECTIONS IN QE CHANNEL

- Double differential CCQE neutrino x-section (MiniBooNE)

$$\frac{d\sigma_A}{dT_\mu d\cos\theta_\mu} = \frac{1}{N_\Phi}\int dE_\nu \Phi(E_\nu)\frac{d\sigma_A}{dE_\nu dT_\mu d\cos\theta_\mu}$$

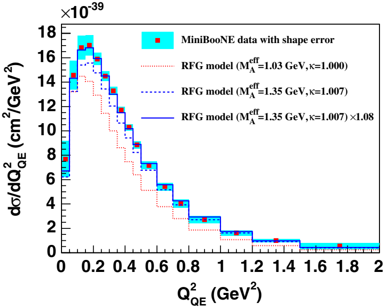


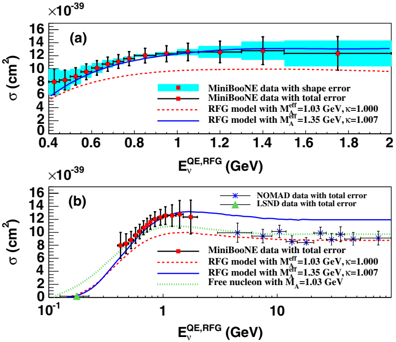


- Note that the neutrino x-section is given as a function of muon kinetic energy, not energy transfer

The Axial Mass "Puzzle"

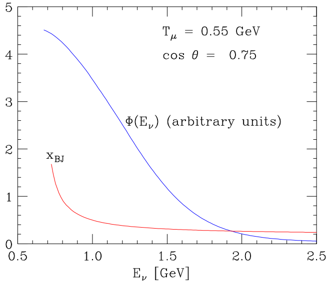
- The "excess" cross section in the CCQE channel observed by the MiniBooNE and K2K Collaborations has been initially ascribed to an increased value of the nucleon axial mass in the nuclear medium. However, this explanation is not supported by NOMAD data





Contribution of Different Reaction Mechanisms

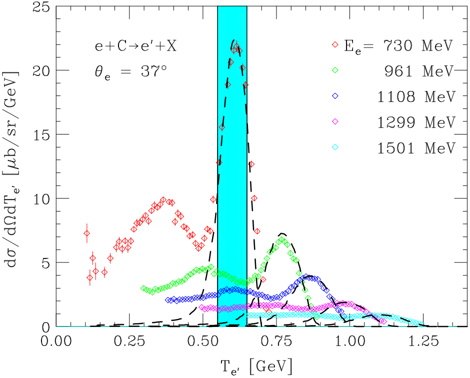
- ⋆ In neutrino interactions the lepton kinematics is *not* determined. The flux-averaged cross sections at fixed T_μ and $\cos\theta_\mu$ picks up contributions at different beam energies, corresponding to a variety of kinematical regimes in which different reaction mechanisms dominate



- ⋆ $x = 1 \to E_\nu$ 0.788 GeV , $x = 0.5 \to E_\nu$ 0.975 GeV
- ⋆ $\Phi(0.975)/\Phi(0.788)$ = 0.83

“Flux averaged” electron-nucleus x-section

- The electron scattering x-section off Carbon at θ_e= 37 deg has been measured for a number of beam energies



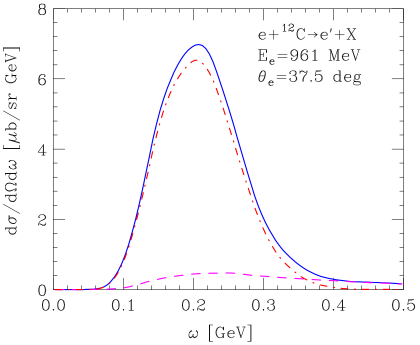
- electron-carbon data at different energies, plotted as a function of the energy of the scattered electron

Where does the "excess" strength come from?

- ⋆ It has been suggested that 2p2h (CCQE like) final states provide a large contribution to the measured neutrino cross section
- ⋆ Two particle-two hole final states may be produced through different mechanisms
  - Initial state correlations lead to the tail extending to large energy loss, clearly visible in the calculated QE cross section. The corresponding strength is consistent with the measurements of the coincidence $(e, e'p)$ x-section carried out by the JLAB E97-006 Collaboration.
  - Final state interactions lead to a redistribution of the inclusive strength, mainly affecting the region of i.e. low energy loss, where the cross section is small
  - Coupling to the two-body current leads to the appearance of strength at $x < 1$, in the *dip* region between the QE and Δ-excitation peaks
- the description of the measured neutrino cross sections requires that all the above mechanism be taken into account in a consistent fashion

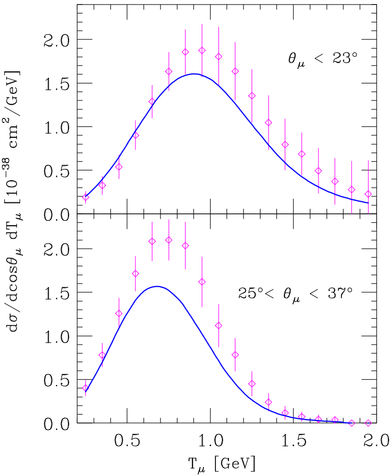
Correlation Effects on the QE Cross Section

- At IA level, correlations move strenght from the 1p1h sector (bound state left in a residual system) to the 2p2h sector (one spectator nucleon excited to the continuum), leading to a quenching of the peak and to the appearance of a tail extending to large energy loss

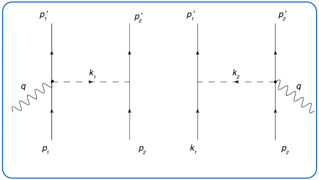


CIRCUMSTANTIAL EVIDENCE OF A TRANSVERSE MECHANISM

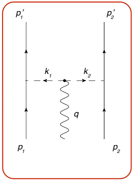
⋆ θ_μ-dependence of the CCQE excess strength



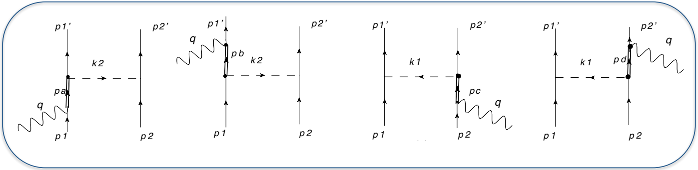
Two-Nucleon Meson-Exchange Currents



Seagull
or
contact
term



Pion
in
flight
term



THE EXTENDED FACTORIZATION *ansatz*

- Highly accurate and consistent calculations of processes involving MEC can be carried out in the non relativistic regime
- Fully relativistic MEC used within the Fermi gas model
- Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
  - Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)},\mathbf{p},\mathbf{p}'\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle \, \delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

The amplitude

$$M_n(\mathbf{k},\mathbf{k}') = \langle n_{(A-2)},\mathbf{k},\mathbf{k}'|0\rangle$$

is independent of q and can be obtained from non relativistic many-body theory

$|0\rangle \to |2p2h\rangle$ Transition Probability

- In interacting many body systems 2p2h states can be excited through the action of both one- and two-body transition operators

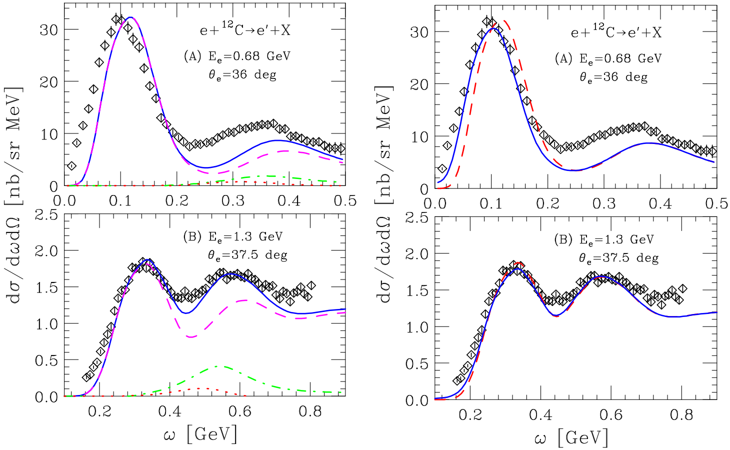
$$
\begin{aligned}
|\langle 2p2h| \, J \, |0\rangle|^2 = |\langle 2p2h| \, J_1 \, |0\rangle|^2 + |\langle 2p2h| \, J_2 \, |0\rangle|^2 \\
+ 2 \, \mathrm{Re} \, \langle 2p2h| \, J_1 \, |0\rangle^\star \langle 2p2h| \, J_2 \, |0\rangle
\end{aligned}
$$

- Within the independent particle model (either FG or shell model)

$$
\langle 2p2h| \, J_1 \, |0\rangle = 0
$$

- Strong nucleon-nucleon correations lead to the appearance of sizable interference contributions to the $|0\rangle \to |2p2h\rangle$ transition probability
- 2p2h excitations can be consistently described within a generalization of the spectral function formalism

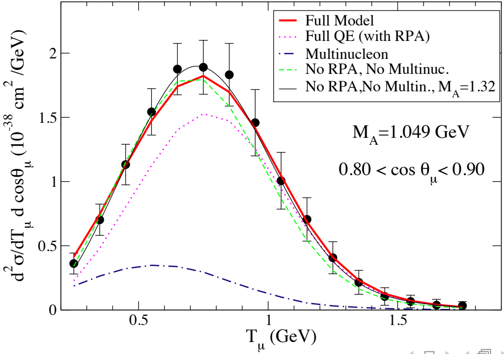
$e +^{12}\mathrm{C} \to e' + X$ (IA+FSI+MEC), Quasi Elastic + Inelastic



⋆ e-carbon x-section obtained within the extended spectral function formalism

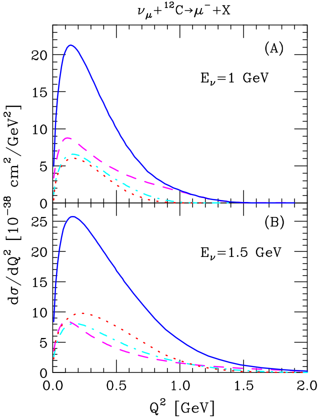
Towards a Solution of the Axial Mass Puzzle

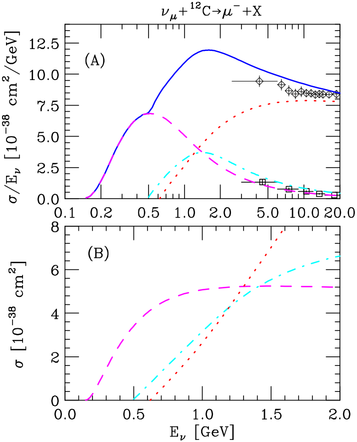
- Calculation of the Valencia group: single and multinucleon emission included. Long range correlations included within the Random Phase Approximation (RPA)
- Flux intergated double differential neutrino-carbon cross section in the CCQE channel



INELASTIC X-SECTION WITHIN THE FACTORIZATION SCHEME

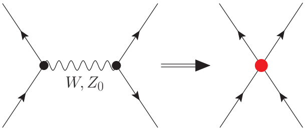
- ⋆ No conceptual problems involved: replace nucleon form factors with inelastic structure functions
- ⋆ $\nu_\mu +^{12} \mathrm{C} \to \mu^- + X$. Factorization ansatz and LDA spectral function (NOMAD data)





Low-Energy Neutrino-Nucleon Interactions

- ⋆ Neutrino interactions are mediated by the gauge bosons $W^{\pm}$ and Z_0, whose masses are in the range $\approx 80 - 90$ GeV
- ⋆ In the regime of momentum transfer discussed in this talk, $q \sim$ 10 MeV, Fermi theory of weak interactions works just fine



$$\mathcal{L}_F = \frac{G}{\sqrt{2}} J_{N\,\mu} J_\ell{}^\mu$$

$$J_\ell{}^\mu = \begin{cases} \bar{u}_{\ell^-}\gamma^\mu(1-\gamma_5)u_\nu & (CC) \\ \bar{u}_{\nu'}\gamma^\mu(1-\gamma_5)u_\nu & (NC) \end{cases}$$

- ⋆ The nucleon current can be cast in the non relativistic limit

$$J_{N\,\mu} = \begin{cases} \bar{u}_p\gamma_\mu(1-g_A\gamma_5)u_n & \rightarrow & \chi^\dagger_{s_p}(g^0_\mu + g_A g^\mu_i \sigma_i)\chi_{s_n} & (CC) \\ \bar{u}_{n'}\gamma_\mu(1-c_A\gamma_5)u_n & \rightarrow & \chi^\dagger_{s'_n}(g^0_\mu + c_A g^\mu_i \sigma_i)\chi_{s_n} & (NC) \end{cases}$$

Nuclear Response Tensor

- ⋆ Consider a neutral current process

$$\nu + A \rightarrow \nu' + X$$

- ⋆ The nuclear response tensor reads

$$W^{\lambda\mu} = \sum_n \langle 0|J^\lambda|n\rangle\langle n|J^\mu|0\rangle\delta^{(4)}(P_0 + q - P_n)$$

- ⋆ Interaction rate

$$W(\mathbf{q},\omega) \propto \frac{G_F}{4\pi^2} L_{\lambda\mu} W^{\lambda\mu} = \frac{G_F}{4\pi^2}\left[(1+\cos\theta)\mathcal{S}^\rho + \frac{{c_A}^2}{3}(3-\cos\theta)\mathcal{S}^\sigma\right]$$

where $\cos\theta = (\mathbf{k}\cdot\mathbf{k}')/(|\mathbf{k}||\mathbf{k}'|)$, while $\mathcal{S}^\rho$ and $\mathcal{S}^\rho$ are the nuclear responses in the density and spin-density channels, respectively.

Nuclear Weak Response Functions at Low Energy

- ⋆ density response

$$\mathcal{S}^{\rho} = \frac{1}{N}\sum_{n} |\langle 0|J_0|n\rangle\langle n|J_0|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- ⋆ spin-density response ($\alpha,\ \beta = 1,\ldots 3$)

$$\mathcal{S}^{\sigma} = \sum_{\alpha} \mathcal{S}^{\sigma}_{\alpha\alpha}$$

$$\mathcal{S}^{\sigma}_{\alpha\beta} = \frac{1}{N}\sum_{n} |\langle 0|J_\alpha|n\rangle\langle n|J_\beta|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- ⋆ Neutral weak current

$$J_0 = \sum_i j_i^0 = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \quad , \quad J_\alpha = \sum_i j_i^\mu = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i}\sigma_\alpha$$

Effects of NN Interactions

- Mean field effects
  - Change of nucleon energy spectrum

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | V_{\rm eff} | \mathbf{k}\mathbf{k}' \rangle_a$$

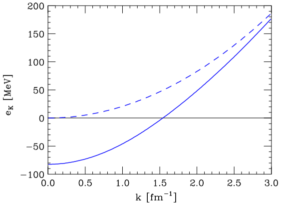
  - Effective mass

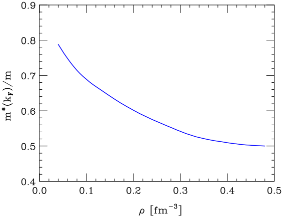
$$\frac{1}{m^\star_k} = \frac{1}{|\mathbf{k}|} \frac{de_k}{d|\mathbf{k}|}$$

- Correlation effects
  - Effective operators couple the ground state to two-particle–two-hole (2p2h) final states, thus removing strength from the 1p1h sector

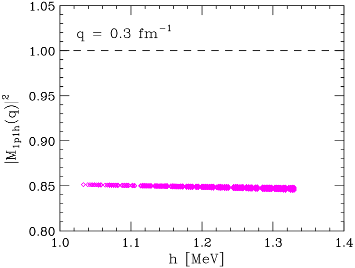
$$M_{2p2h} = \langle 2p2h | J^\mu_{\rm eff} | 0 \rangle \neq 0 \to M_{1p1h} = \langle 1p1h | J^\mu_{\rm eff} | 0 \rangle < \langle 1p1h | J^\mu | 0 \rangle$$

⋆ Nucleon energy spectrum and Effective mass in isospin-symmetric matter at equilibrium density



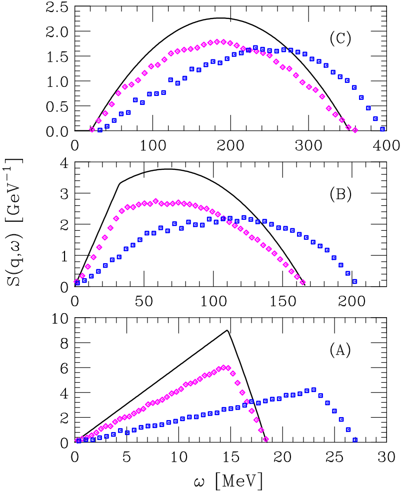


⋆ Quenching of Fermi transition strength in isospin-symmetric matter at equilibrium density



q-EVOLUTION OF INTERACTION EFFECTS

- ⋆ Density response of isospin-symmetric matter at equilibrium density



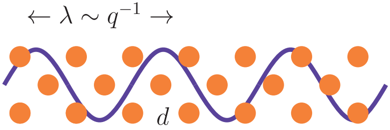
$|\mathbf{q}| = 3.0\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 1.8\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 0.3\ \mathrm{fm}^{-1}$

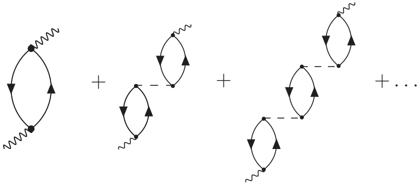
Long-Range Correlations

- At low momentum transfer the space resolution of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i\rangle$$



Tamm-Dancoff (ring) Approximation

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i|V_{\rm eff}|h_j p_j)$$

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}'|V_{\rm eff}|\mathbf{k}\mathbf{k}'\rangle_a$$

- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

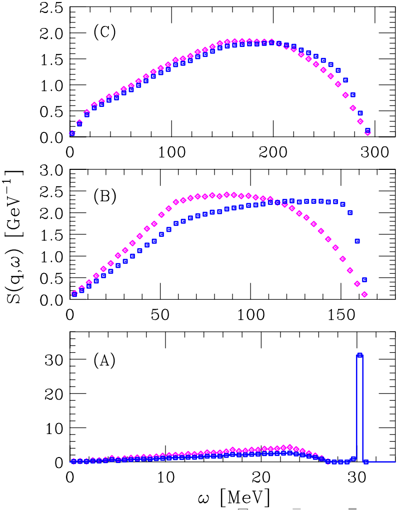
Effects of Long-Range Correlations

⋆ Density response of isospin-symmetric nuclear matter at equilibrium density

$|\mathbf{q}| \approx 480\,\text{MeV}$

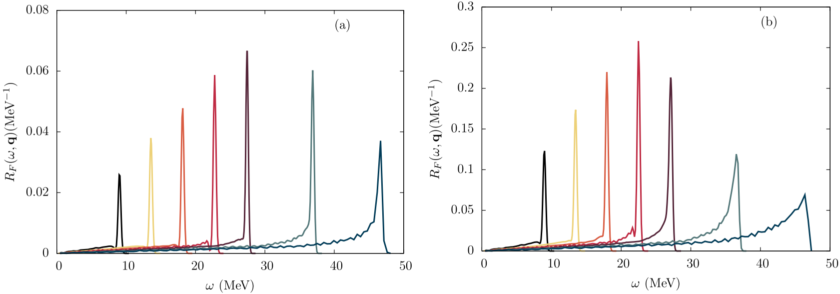
$|\mathbf{q}| \approx 300\,\text{MeV}$

$|\mathbf{q}| \approx 60\,\text{MeV}$



Excitation of collective modes

- ⋆ Density (a) and spin-density (b) responses of isospin-symmetric nuclear matter at equilibrium density



- ⋆ $|\mathbf{q}|$ = 0.1, 0.15, 0.20, 0.25, 0.30, 0.40 and 0.50 fm^{-1}

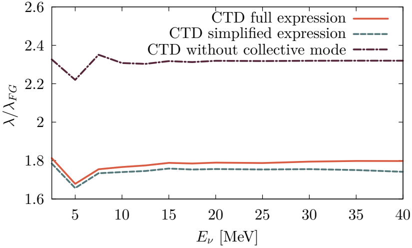
Neutrino mean free path in neutron matter

- The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\rho \int \frac{d^3q}{(2\pi)^3} \left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3}-\cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively

A Lovato et al, arXiv 1310.0510 [nucl-th]



- Both short and long range correlations important

Summary of Lecture II

- ⋆ Thanks to the significant efforrts of the past two decades, a consistent framework suitable to describe neutrino-nucleus cross sections in the broad kinematical regime corresponding to beam energies from ~ 10 MeV to several GeV is emerging

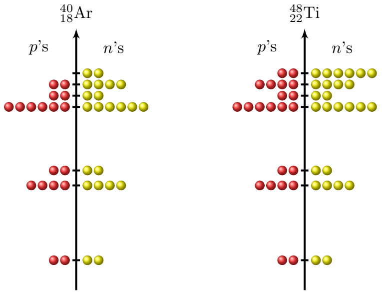
- ⋆ The main challenges to be faced in the near future are the description of exclusive channels, including those involving resonance production and deep-inelastic scattering, as well as of complex nuclear targets, such as argon

- ⋆ The factorization formalism, involving non adjustable parameters, appears to be ideally suited to achieve theses goal, as long as the spectral function describing initial state dynamics is available. More electron scattering data needed.

- ⋆ If, and to what extent, the theoretical progress will have a significant impact on the experiments remains to be seen…

The E12-14-012 Experiment at Jefferson Lab

- The reconstruction of neutrino and antineutrino energy in liquid argon detectors will require the understanding of the spectral functions describing both protons and neutrons
- The $\mathrm{Ar}(e, e'p)$ cross section only provides information on proton interactions. The information on neutrons can be obtained from the $\mathrm{Ti}(e, e'p)$, exploiting the pattern of shell model levels



Backup slides

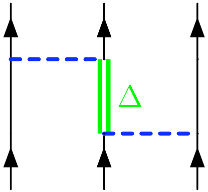
Three-Nucleon Interactions

- Interactions involving more two nucleons arise as a consequence of the internal structure of the participating particles

- The main contribution to the three nucleon forces comes from the Fujita-Miyazawa mechanism
- Phenomenological three-nucleon potentials, written in the form

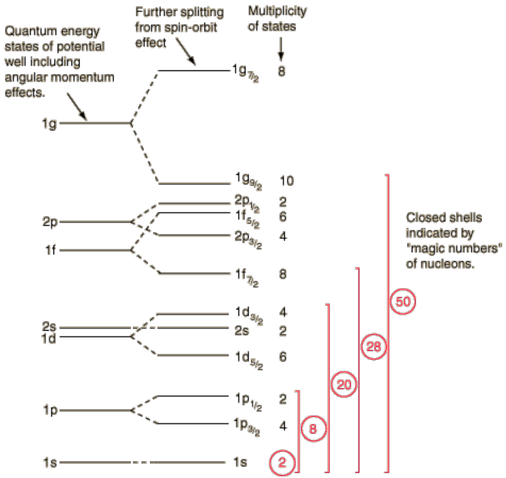
$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{N}$$

are determined through a fit to the properties of the three-nucleon system

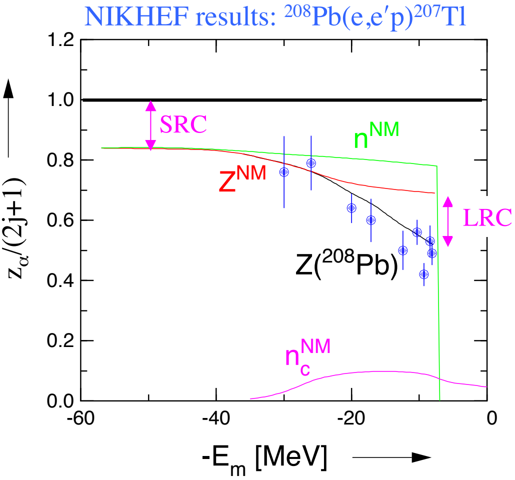


Evidence of Nuclear Shell Structure

- Energy spectra and emergence of *magic numbers*

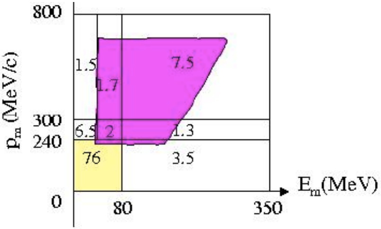


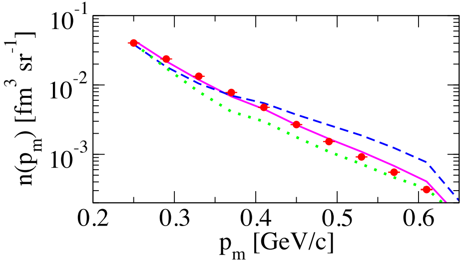
Spectroscopic Factors of ^{208}Pb



Measured Correlation Strength

- the correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1} \frac{\mathbf{k}^2}{2m}$

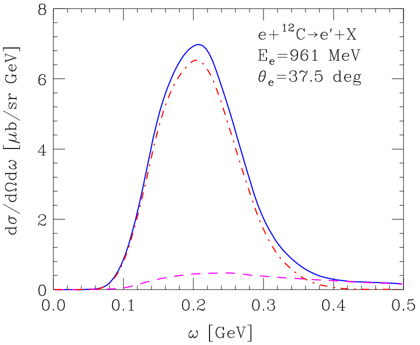




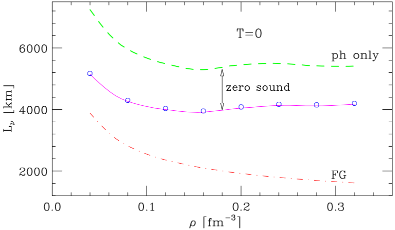
- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions of *ab initio* approaches: 0.46 (GF), 0.61 (SCGF) and 0.64 (CBF)

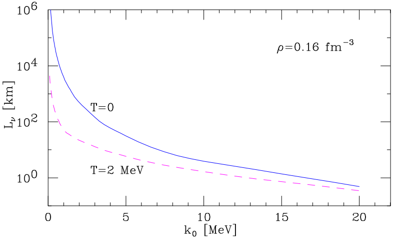
Correlation Effects on the QE Cross Section

- ⋆ At IA level, correlations move strenght from the 1p1h sector (bound state left in a residual system) to the 2p2h sector (one spectator nucleon excited to the continuum), leading to a quenching of the peak and to the appearance of a tail extending to large energy loss



⋆ Mean free path of a non degenerate neutrino in neutron matter. Left: density-dependence at $k_0 = 1$ MeV and $T = 0$; Right: energy dependence at $\rho = 0.16\ \mathrm{fm}^{-3}$ and $T = 0, 2$ MeV





⋆ Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm^{-3}