

The Atoms of Space and Gravity

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T.P., arXiv:1603.08658

***SINGULARITIES: BLACK HOLES,
UNIVERSE***

COSMOLOGICAL CONSTANT

THE THERMODYNAMIC CONNECTION

These challenges involve \hbar !

$$A_{Planck} = \frac{G\hbar}{c^3}; \quad \Lambda \left(\frac{G\hbar}{c^3} \right) \approx 10^{-123}; \quad k_B T = \frac{\hbar}{c} \left(\frac{g}{2\pi} \right)$$

***HOW DO WE PUT TOGETHER
THE PRINCIPLES OF
QUANTUM THEORY AND GRAVITY?***

Everybody Wants To Quantize Gravity!

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Hathaway's Einstein theories

NO fashion magazines for Hollywood actress Anne Hathaway, she spends her free time studying books on physics to enhance her knowledge on the universe.

The Devil Wears Prada star admits she shuns fashion magazines and instead stocks up on books by scientist Albert Einstein and physics textbooks in a bid to better understand the universe, reported a website. "I'm interested in elementary particles. What I like thinking about is how time and space exist in the universe and how we understand it. Any spare time I have, I bury my head in a physics textbook. I'm reading a lot about Einstein. I like theories and I want to understand (string theory)," she said.

— IANS



... But Nobody Has Succeeded!

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- ▶ *The perturbative approach does not work*
- ▶ *Virtually every interesting question about gravity is non-perturbative by nature*
- ▶ *No guiding principle; metric is assumed to be a quantum variable*

GR: THE NEXT 100 YEARS

Needs another paradigm shift

The equations governing classical gravity has the same conceptual status as those describing elasticity/hydrodynamics.

***Related Ideas: Cai, Damour, Hu,
Jacobson, Liberati, Sakharov,
Thorne, Visser, Volovik,.... and
many others***

***I will describe the work by me +
collaborators (2004-16)***

Part 1: Review of older results

Part 2: Recent Progress (2014-16)

***Spacetime dynamics should/can
be described in a thermodynamic
language; not in a geometrical
language.***

***Evolution arises from departure from
'holographic equipartition':***

Time evolution \propto *Heating/Cooling*
 $\propto (N_{\text{sur}} - N_{\text{bulk}})$

All static geometries have

$$N_{\text{sur}} = N_{\text{bulk}}$$

*The gravity-thermodynamics connection transcends GR even when the entropy is **not** proportional to area.*

*All these results generalize to the
Lanczos-Lovelock models of gravity with:*

$$sd^2x = -\frac{1}{2} \frac{\sqrt{\sigma} d^{D-2}x}{4L_P^2} P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd}$$

*The thermodynamic connection
transcends GR!*

***You should get more than what
you put in!***

***You should get more than what
you put in!***

***Leads to significant new insights about:
(i) classical gravity (ii) the microscopic
structure of spacetime and (iii)
cosmological constant.***

The key new variable which distinguishes thermodynamics from point mechanics

$$\textit{Heat Density} = \mathcal{H} = \frac{Q}{V} = \frac{TS}{V} = \frac{1}{V}(E - F)$$

$$\frac{TS}{V} = Ts = p + \rho = T_{ab}\ell^a\ell^b$$

Normal matter has a heat density

Spacetime also has a heat density!

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One can associate a T and s with every event in spacetime just as you could with a glass of water!

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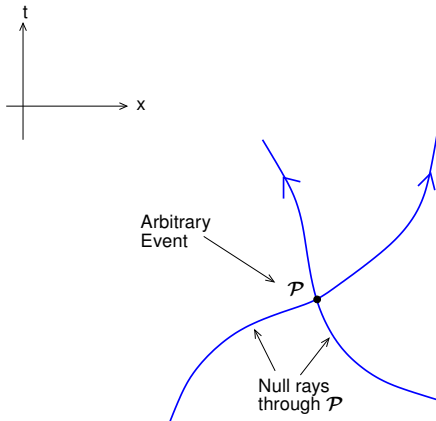
This fact transcends black hole physics and Einstein gravity.

Spacetime also has a heat density!

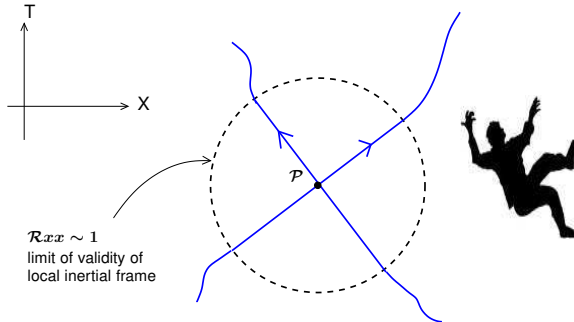
One can associate a T and s with every event in spacetime just as you could with a glass of water!

The T is independent of the theory of gravity; s depends on/determines the theory.

Spacetime in Arbitrary Coordinates



Local Inertial Observers



Validity of laws of SR \Rightarrow How gravity affects matter

$$\text{Matter equations of motion} \Leftrightarrow \nabla_a T_b^a = 0$$

***The most beautiful result in
the interface of quantum theory and gravity***

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**OBSERVERS WHO PERCEIVE A HORIZON
ATTRIBUTE A TEMPERATURE TO SPACETIME**

$$k_B T = \frac{\hbar}{c} \left(\frac{g}{2\pi} \right)$$

[Davies (1975), Unruh (1976)]

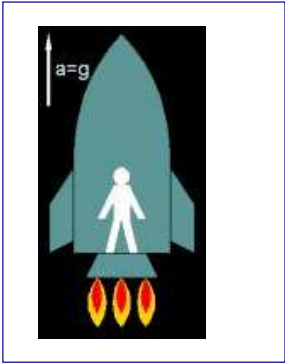
The most beautiful result in the interface of quantum theory and gravity

**OBSERVERS WHO PERCEIVE A HORIZON
ATTRIBUTE A TEMPERATURE TO SPACETIME**

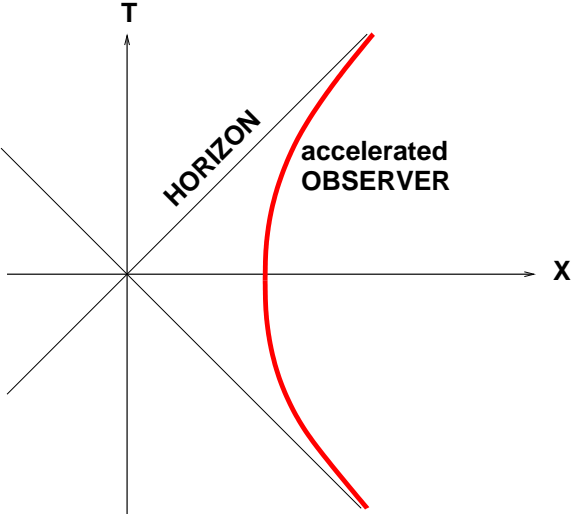
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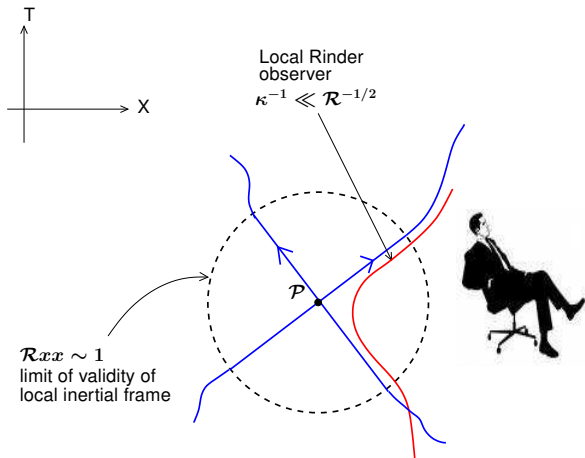
*This allows you to associate a heat density
 $\mathcal{H} = T_s$ with every event of spacetime!*

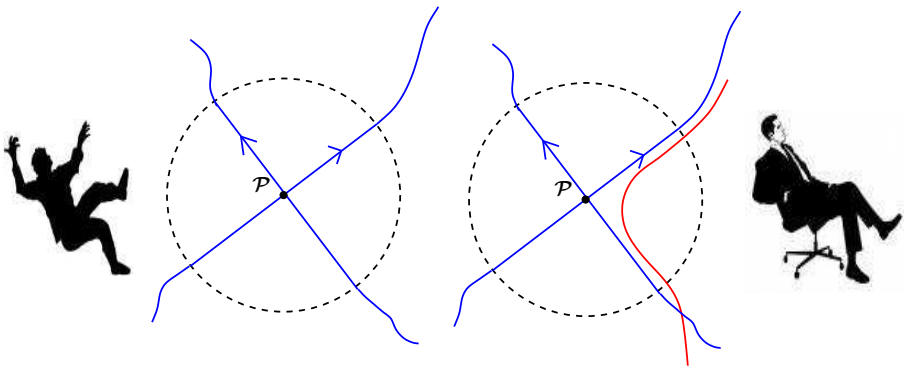


FLAT SPACETIME



Local Rindler Observers

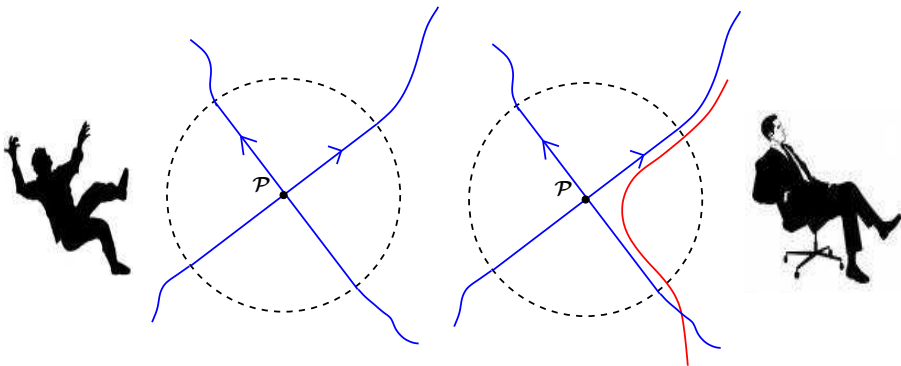




Vacuum fluctuations



Thermal fluctuations

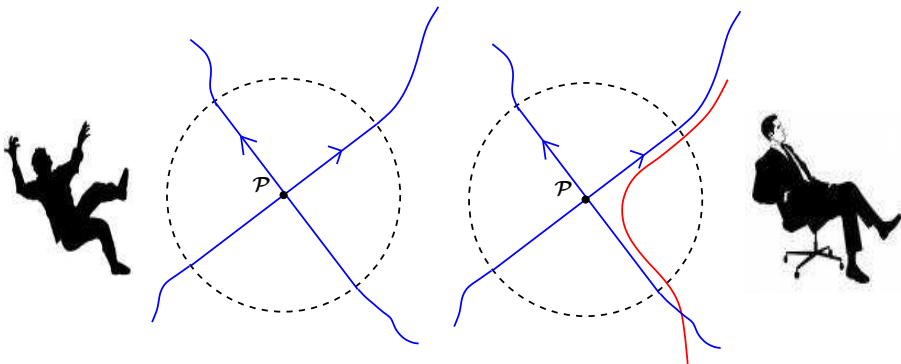


Vacuum fluctuations



Thermal fluctuations

A VERY NON-TRIVIAL EQUIVALENCE!



Vacuum fluctuations



Thermal fluctuations

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QFT in FFF introduces \hbar ; we now have (\hbar/c) in the temperature

***Regions of spacetime can be inaccessible
to certain class of observers in any
spacetime!***

Regions of spacetime can be inaccessible to certain class of observers in any spacetime!

Take non-inertial frames seriously: not “just coordinate relabeling”.

- ▶ Heat transferred due to matter crossing a null surface:

[T. Jacobson, gr-qc/9504004]

$$Q_m = \int d\mathcal{V} (T_{ab} \ell^a \ell^b); \quad \mathcal{H}_m \equiv T_{ab} \ell^a \ell^b$$

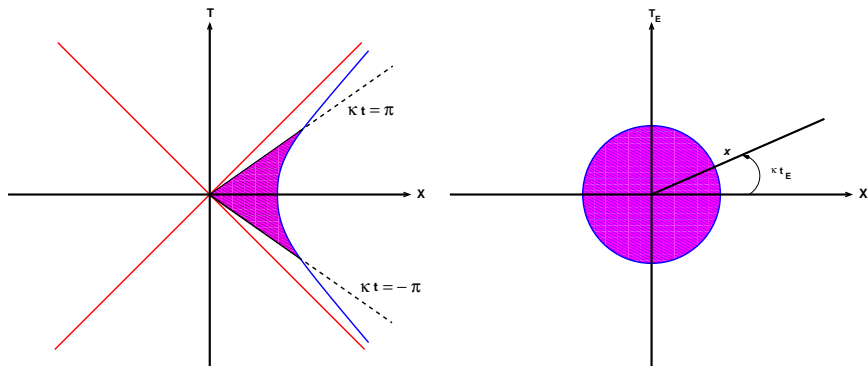
- ▶ Heat transferred due to matter crossing a null surface:

[T. Jacobson, gr-qc/9504004]

$$Q_m = \int d\mathcal{V} (T_{ab} \ell^a \ell^b); \quad \mathcal{H}_m \equiv T_{ab} \ell^a \ell^b$$

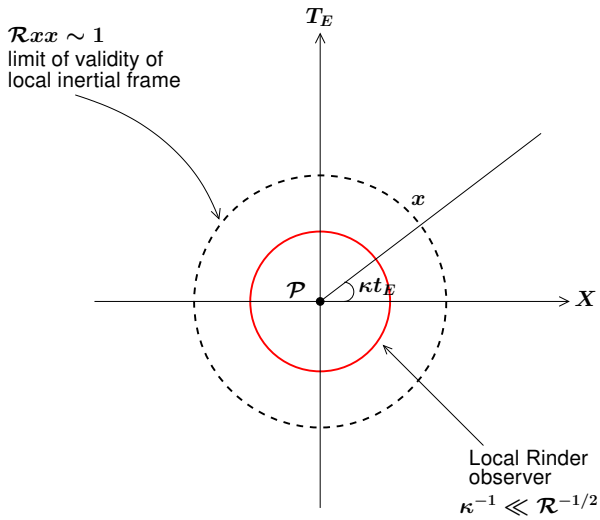
- ▶ Note: Null horizon \Leftrightarrow Euclidean origin

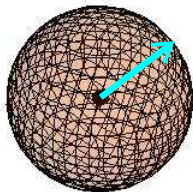
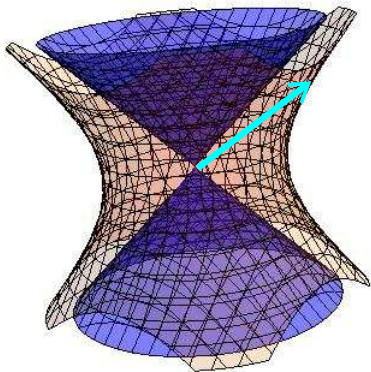
$$X^2 - T^2 = 0 \Leftrightarrow X^2 + T_E^2 = 0$$



$$T = x \sinh \kappa t, \quad X = x \cosh \kappa t \quad T_E = x \sin \kappa t_E, \quad X = x \cos \kappa t_E$$

$$X^2 - T^2 = 0 \Leftrightarrow X^2 + T_E^2 = 0$$





$$X^2 - T^2 = \sigma^2 \Leftrightarrow X^2 + T_E^2 = \sigma^2$$

$$X^2 - T^2 = 0 \Leftrightarrow X^2 + T_E^2 = 0$$

The Importance Of Being Hot

***You could have figured out that
water is made of discrete atoms
without ever probing it at
Angstrom scales!***

***Boltzmann: If you can heat it,
it must have micro-structure!***

To store energy ΔE at temperature T , you need

$$\Delta n = \frac{\Delta E}{(1/2)k_B T}$$

***degrees of freedom. Microphysics leaves its
signature at the macro-scales***

*Boltzmann: If you can heat it,
it must have micro-structure!*

You can heat up spacetime!

*Do we have an equipartition law for the
microscopic spacetime degrees of freedom?*

Can you count the atoms of space?

Equipartition with a surface-bulk correspondence

$$E_{\text{bulk}} = \int_{\partial\mathcal{V}} \frac{dA}{L_P^2} \left(\frac{1}{2} k_B T_{\text{loc}} \right) \equiv \frac{1}{2} k_B \int_{\partial\mathcal{V}} dn T_{\text{loc}}$$

Associates $dn = dA/L_P^2$ microscopic degrees of freedom ('atoms') with an area dA

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Associates $dn = dA/L_P^2$ microscopic degrees of freedom ('atoms') with an area dA

Extends to all Lanczos-Lovelock models with

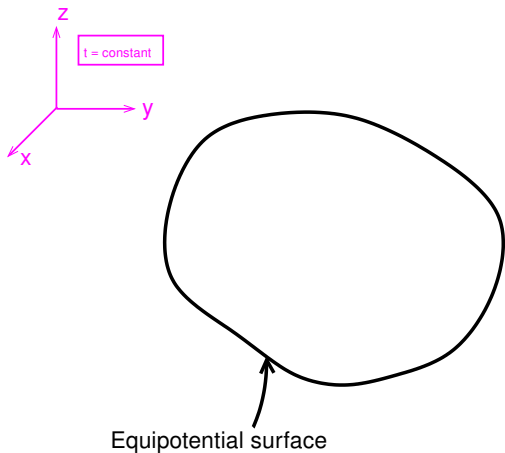
$$dn = (32\pi P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd}) dA/L_P^2$$

Holographic Equipartition

TP [gr-qc/0308070], [arXiv:0912.3165], [arXiv:1003.5665]

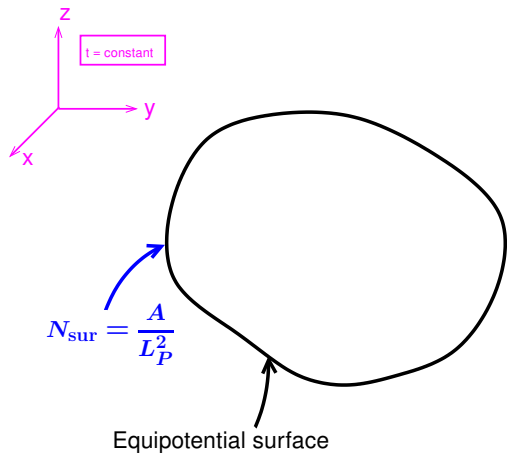
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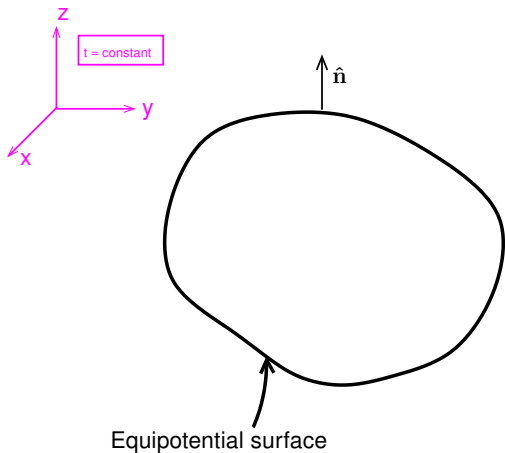
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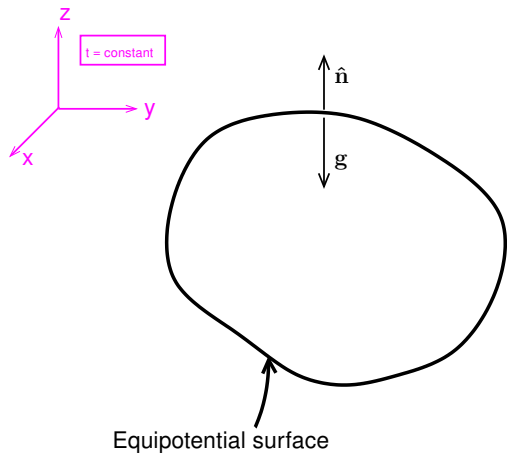
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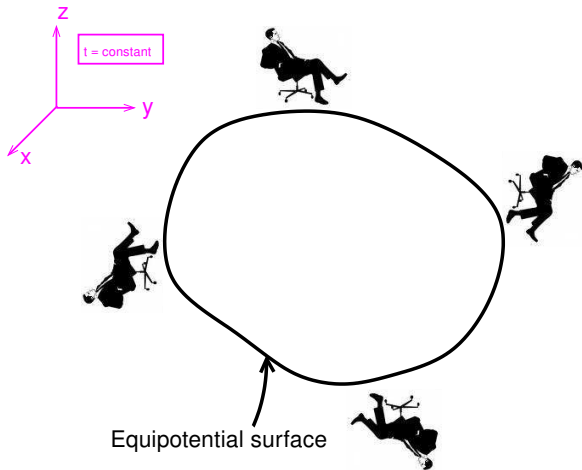
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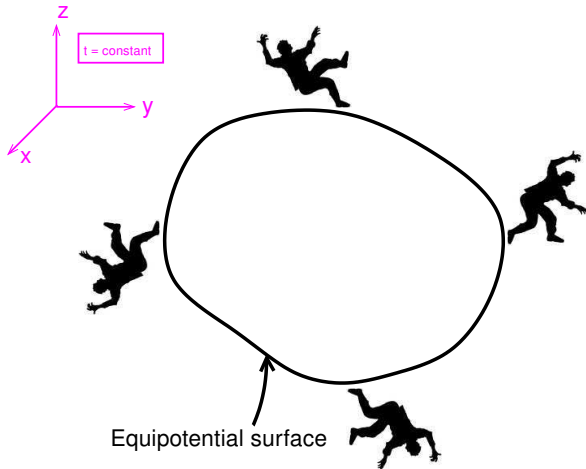
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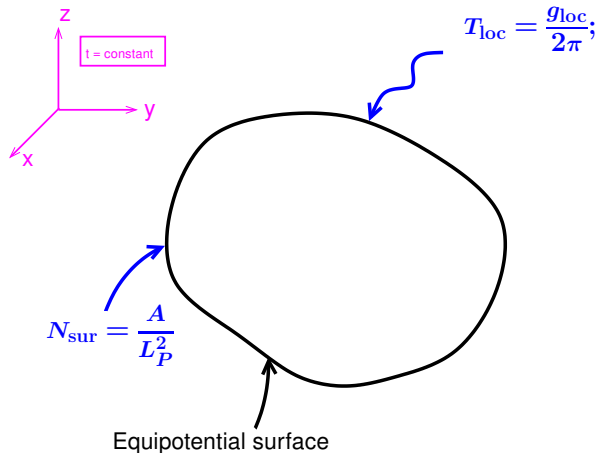
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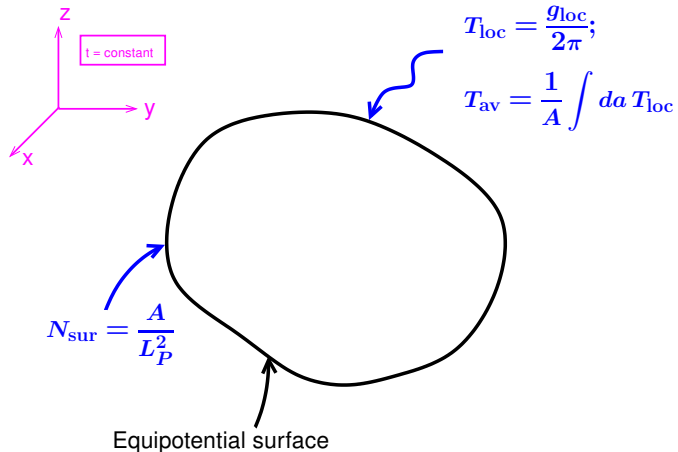
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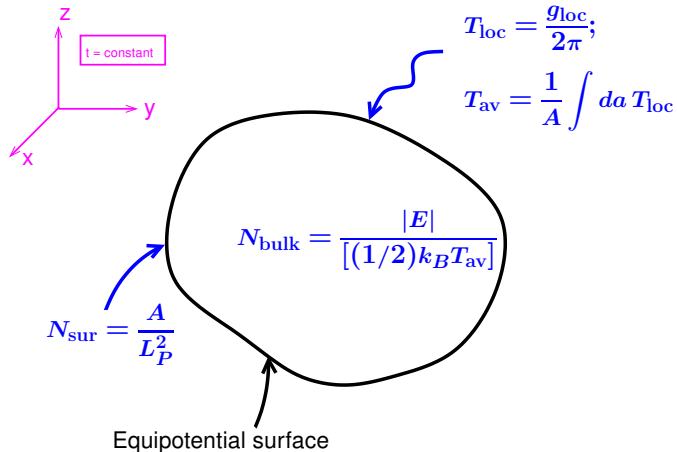
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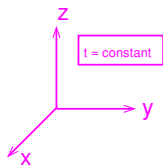
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$$T_{\text{loc}} = \frac{g_{\text{loc}}}{2\pi};$$

$$T_{\text{av}} = \frac{1}{A} \int da T_{\text{loc}}$$

$$N_{\text{bulk}} = \frac{|E|}{[(1/2)k_B T_{\text{av}}]}$$

$$N_{\text{sur}} = \frac{A}{L_P^2}$$

Equipotential surface

$$N_{\text{sur}} = N_{\text{bulk}}!$$

$$N_{\text{sur}} = \frac{A}{L_P^2} = N_{\text{bulk}} = \frac{E}{[(1/2)k_B T_{\text{av}}]}$$

$$E = \frac{1}{2L_P^2} \int da \, k_B T_{\text{loc}}$$

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$$\int \rho \, dV = \frac{1}{2L_P^2} \int da \, \left(\frac{\hbar}{c} \right) \left(\frac{g}{2\pi} \right)$$

$$E = \frac{1}{2L_P^2} \int da k_B T_{\text{loc}}$$

$$\int \rho dV = \frac{1}{2L_P^2} \int da \frac{\hbar}{c} \left(\frac{-\mathbf{g} \cdot \hat{\mathbf{n}}}{2\pi} \right)$$

$$E = \frac{1}{2L_P^2} \int da k_B T_{\text{loc}}$$

$$\begin{aligned} \int \rho dV &= \frac{1}{2L_P^2} \int da \frac{\hbar}{c} \left(\frac{-\mathbf{g} \cdot \hat{\mathbf{n}}}{2\pi} \right) \\ &= -\frac{\hbar}{4\pi c L_P^2} \int dV \nabla \cdot \mathbf{g} \end{aligned}$$

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Geometry \Leftrightarrow Thermodynamics

K. Parattu, B.R. Majhi, T.P. [arXiv:1303.1535]

$$q^{ab} \equiv \sqrt{-g} g^{ab}$$

$$q^{ab} \equiv \sqrt{-g}g^{ab}$$

$$p_{bc}^a \equiv -\Gamma_{bc}^a + \frac{1}{2}(\Gamma_{bd}^d \delta_c^a + \Gamma_{cd}^d \delta_b^a)$$

These variables have a thermodynamic interpretation

$$(q\delta p, p\delta q) \Leftrightarrow (s\delta T, T\delta s)$$

On any null surface: $(\delta p_{bc}^a, \delta q^{bc}) \Leftrightarrow (\delta T, \delta s)$

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$$\frac{1}{16\pi L_P^2} \int_{\mathcal{H}} d^2 x_{\perp} \ell_c (p_{ab}^c \delta q^{ab}) = \int_{\mathcal{H}} d^2 x_{\perp} T \delta s$$

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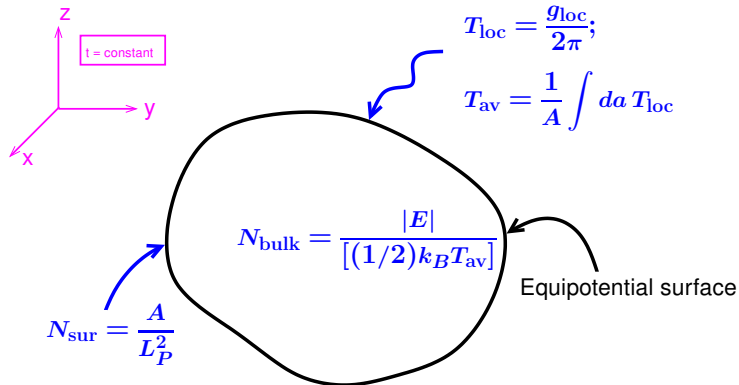
$$\frac{1}{16\pi L_P^2} \int_{\mathcal{H}} d^2 x_{\perp} \ell_c (q^{ab} \delta p_{ab}^c) = \int_{\mathcal{H}} d^2 x_{\perp} s \delta T$$

What makes Spacetime Evolve ?

TP, Gen.Rel.Grav (2014) [arXiv:1312.3253]

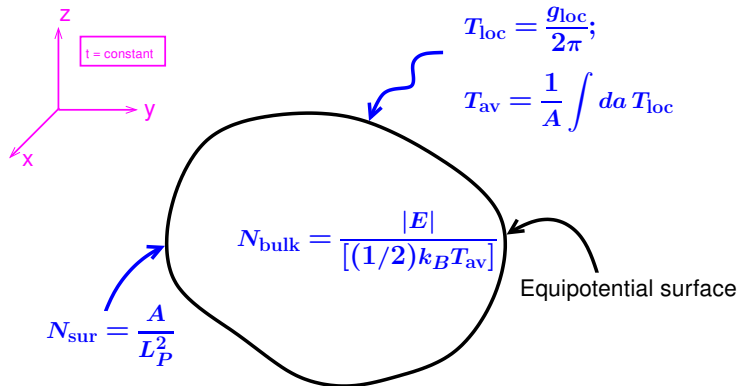
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$$\int \frac{d\sigma_a}{8\pi L_P^2} q^{\ell m} \mathcal{L}_\xi P_{\ell m}^a = -\frac{1}{2} k_B T_{\text{av}} (N_{\text{sur}} - N_{\text{bulk}})$$

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time evolution of spacetime

= heating of spacetime

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time evolution of spacetime

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***Evolution of spacetime is described in
thermodynamic language; not in geometric
language!***

COSMIC EXPANSION: A QUEST FOR HOLOGRAPHIC EQUIPARTITION

T.P., [1207.0505]

$$\frac{dR_H}{dt} = \left(1 - \frac{\epsilon N_{\text{bulk}}}{N_{\text{sur}}}\right) \quad \epsilon = \pm 1$$

$$N_{\text{sur}} = 4\pi \frac{R_H^2}{L_P^2}; \quad N_{\text{bulk}} = -\epsilon \frac{E}{(1/2)k_B T}; \quad T = \frac{H}{2\pi}$$

Remarkably enough, this leads to the standard FRW dynamics!

Provides insights into classical GR!

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For matter, $\nabla_a T_b^a = 0$ but

$$\nabla_a P^a[v] \equiv -\nabla_a (T_b^a v^b) \neq 0.$$

Why?

Provides insights into classical GR!

For matter, $\nabla_a T_b^a = 0$ but

$$\nabla_a P^a[v] \equiv -\nabla_a (T_b^a v^b) \neq 0.$$

Why?

Because you did not add the momentum density of spacetime!

$$\nabla_a (P^a[v] + \mathcal{G}^a[v]) = 0$$

Momentum of Gravity

T.P. [arXiv:1506.03814]

$$\sqrt{-g}P^a[v] \equiv -\sqrt{-g}Rv^a - q^{ij}\mathcal{L}_v p_{ij}^a$$

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Restores momentum conservation to nature!

$\nabla_a(P^a + M^a) = 0$ for all observers imply field equations

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Restores momentum conservation to nature!

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Variational principle has a physical meaning:

$$Q_{\text{tot}} = - \int d\mathcal{V} \ell_a [P^a(\xi) + M^a(\xi)]$$

Fluid Mechanics Of Spacetime

S. Chakraborty, K. Parattu, and T.P. [arXiv:1505.05297]; S. Chakraborty, T.P. [arXiv:1508.04060]

Three projections $P^a \ell_a$, $P^a k_a$, $P^a q_a^b$ on a null surface give

▶ Navier-Stokes equation

[T.P., arXiv:1012.0119]

▶ $TdS = dE + PdV$

[T.P., gr-qc/0204019; D. Kothawala, T.P., arXiv:0904.0215]

▶ Evolution equation for the null surface

Thermodynamic variational principle

T.P., A. Paranjape [gr-qc/0701003]; T.P. [arXiv:0705.2533]

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Field equations arise from maximizing entropy/heat density of gravity plus matter on all null surfaces.

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$$Q = \int d\mathcal{V} (\mathcal{H}_g + \mathcal{H}_m)$$

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$$Q = \int d\mathcal{V} (\mathcal{H}_g + \mathcal{H}_m)$$

Works for a wide class gravitational theories; entropy decides the theory.

Thermodynamic variational principle

T.P., A. Paranjape [gr-qc/0701003]; T.P. [arXiv:0705.2533]

$$\mathcal{H}_m = T_b^a \ell_a \ell^b$$

$$\mathcal{H}_g = - \left(\frac{1}{16\pi L_P^2} \right) (4P_{cd}^{ab} \nabla_a \ell^c \nabla_b \ell^d)$$

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$$\mathcal{H}_m = T_b^a \ell_a \ell^b$$

$$\mathcal{H}_g = - \left(\frac{1}{16\pi L_P^2} \right) (4P_{cd}^{ab} \nabla_a \ell^c \nabla_b \ell^d)$$

$$P_{cd}^{ab} \propto \delta_{cdc_2d_2\dots c_md_m}^{aba_2b_2\dots a_mb_m} R_{a_2b_2}^{c_2d_2} \dots R_{a_mb_m}^{c_md_m}$$

- ▶ **The P_{cd}^{ab} is the entropy tensor of the spacetime which determines the theory**

[Iyer and Wald (1994)]

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- ▶ **The demand $\delta Q / \delta \ell_a = 0$ for all null ℓ_a leads to:**

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- ▶ **On-shell value of Q_{tot}**

$$Q_{\text{tot}}^{\text{on-shell}} = \int d^2x (T_{\text{loc}} s) \Big|_{\lambda_1}^{\lambda_2}$$

- ▶ *Interestingly enough:*

$$2P_{cd}^{ab}\nabla_a n^c\nabla_b n^d = \mathcal{R}_{ab}n^a n^b + \left\{ \begin{array}{l} \text{ignorable} \\ \text{total divergence} \end{array} \right.$$

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- ▶ *Alternative, dimensionless, form in GR:*

$$\mathcal{K}_g \equiv -\frac{1}{8\pi}(L_P^2 R_{ab}n^a n^b)$$

Newton's Law of Gravitation

T.P. [hep-th/0205278]

Three constants: \hbar, c, L_P^2

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Gravity, like matter, is intrinsically quantum and cannot exist in the limit of $\hbar \rightarrow 0$!

WHY?

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Can we understand these results from a deeper level? Can we get something new?

$$\begin{aligned}\Omega_{\text{tot}} &= \prod_{\phi_A} \prod_x \rho_g(\mathcal{G}_N, \phi_A) \rho_m(\mathcal{T}_{ab}, \phi_A) \\ &\equiv \prod_{n_a} \exp \sum_x (\ln \rho_g + \ln \rho_m)\end{aligned}$$

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 \end{aligned}$$

$$\ln \rho_m \equiv L_P^4 \mathcal{H}_m = L_P^4 \mathcal{T}_{ab} \ell^a \ell^b$$

$$\ln \rho_g \equiv L_P^4 \mathcal{H}_g \approx -\frac{L_P^2}{8\pi} R_{ab} \ell^a \ell^b$$

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$$R_b^a \ell_a \ell^b = (8\pi L_P^2) T_b^a \ell_a \ell^b$$

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$$G_b^a = (8\pi L_P^2) T_b^a + (\text{const}) \delta_b^a$$

- ▶ *Variational principle must remain invariant under $T_b^a \rightarrow T_b^a + (\text{const})\delta_b^a$.*
- ▶ *Quantum spacetime has a zero-point length.*

Guiding Principle For Dynamics

Matter equations of motion remain invariant when a constant is added to the Lagrangian

Gravity must respect this symmetry

The variational principle for the dynamics of spacetime must be invariant under

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The variational principle cannot have metric as the dynamical variable!

- ▶ *Minimal possibility: The matter should enter ρ_m through*

$$\mathcal{H}_m \equiv T_{ab} \ell^a \ell^b$$

- ▶ *This has a natural interpretation of heat density contributed by matter on null surfaces*

- ▶ *Heat density (energy per unit area per unit affine time) of the null surface, contributed by matter crossing a local Rindler horizon:*

$$\mathcal{H}_m \equiv \frac{dQ_m}{\sqrt{\gamma}d^2x d\lambda} = T_{ab}\ell^a\ell^b$$

- ▶ *This fixes the matter sector and leads to:*

$$\Omega_{\text{tot}} = \prod_{\ell} \exp \int d\mathcal{V} (\mathcal{H}_g + T_{ab}\ell^a\ell^b)$$

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 $(Ts = p + \rho)$ — not energy density!

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***Cosmological constant arises as an
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***Its value is determined by a new
conserved quantity for the universe!***

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We need to recognize discreteness and yet use continuum mathematics!

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 - **Many atoms with different p_i can exist at same x^i**

The Atoms Of Space

$$Q = \int d\mathcal{V} (\mathcal{H}_g + \mathcal{H}_m)$$

The distribution function for ‘atoms of space’ provides the **microscopic** origin for the variational principle

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The distribution function for ‘atoms of space’ provides the **microscopic** origin for the variational principle

Points in a renormalized spacetime has zero volume but finite area!

The \mathcal{H}_g is proportional to the number of “atoms of space at” x^i with “momentum” n_i .

We expect \mathcal{H}_g to be proportional to the volume or the area measure “associated with” the event x^i in the spacetime

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Use equi-geodesic surfaces to make this idea precise

Geodesic Interval

D. Kothawala, T.P. [arXiv:1405.4967]; [arXiv:1408.3963]

The geodesic interval $\sigma^2(x, x')$ and metric g_{ab} has same information about geometry:

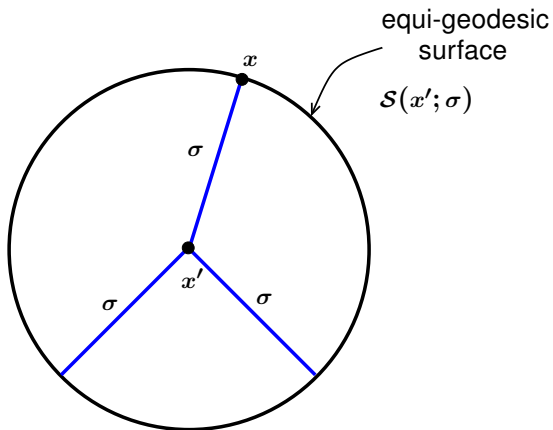
$$\sigma(x, x') = \int_x^{x'} \sqrt{g_{ab} n^a n^b} d\lambda$$

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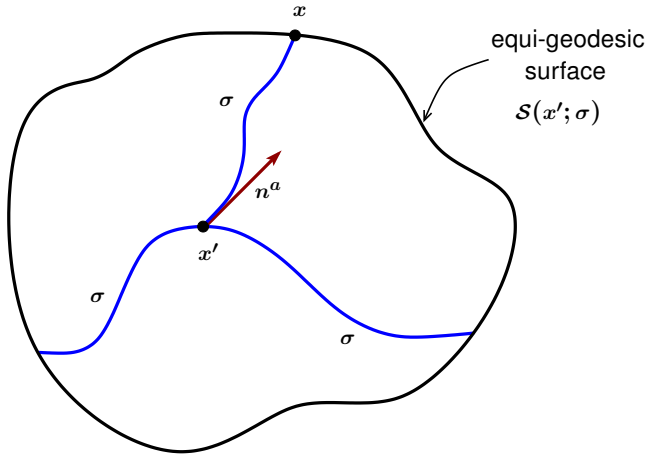
$$\frac{1}{2} \nabla_a \nabla_b \sigma^2 = g_{ab} - \frac{\lambda^2}{3} \mathcal{E}_{ab} + \frac{\lambda^2}{12} n^i \nabla_i \mathcal{E}_{ab} + \mathcal{O}(\lambda^4)$$

$$n_j = \nabla_j \sigma, \quad \mathcal{E}_{ab} \equiv R_{akbj} n^k n^j$$



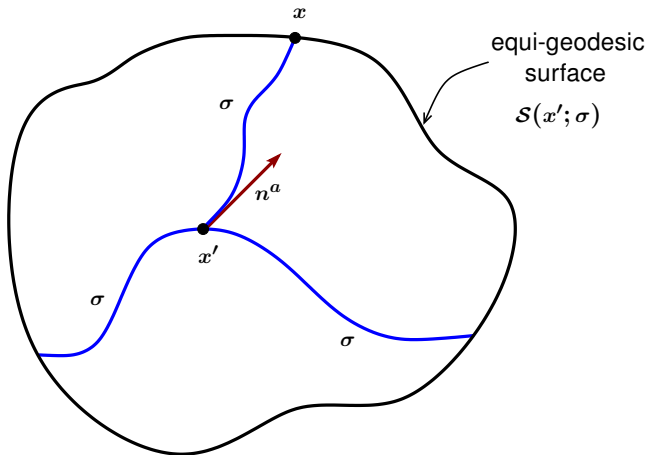
$$ds^2 = d\sigma^2 + \sigma^2 d\Omega_{(S^3)}^2$$

$$\sqrt{g} \propto \sigma^3 \quad \sqrt{h} \propto \sigma^3$$



$$ds^2 = d\sigma^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

The $\sqrt{g} = \sqrt{h}$ will pick up curvature corrections



$$ds^2 = d\sigma^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

$$\sqrt{h}(x, x') = \sqrt{g}(x, x') = \sigma^3 \left(1 - \frac{\sigma^2}{6} \mathcal{E} \right) \sqrt{h_\Omega}; \quad \mathcal{E} \equiv R_{ab} n^a n^b$$

Zero-Point Length

T.P. Ann.Phys. (1985), 165, 38; PRL (1997), 78, 1854

***Discreteness arises through a quantum of area,
which is a QG effect***

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Quantum spacetime has a zero-point length:

D. Kothawala, T.P. [arXiv:1405.4967]; [arXiv:1408.3963]

$$\begin{aligned}\sigma^2(x, x') &\rightarrow S(\sigma^2) = \sigma^2(x, x') + L_0^2 \\ g_{ab}(x) &\rightarrow q_{ab}(x, x'; L_0^2)\end{aligned}$$

In conventional units, $L_0^2 = (3/4\pi)L_P^2$.

Area Of A Point

T.P. [arXiv:1508.06286]

$$\sqrt{q} = \sigma (\sigma^2 + L_0^2) \left[1 - \frac{1}{6} \mathcal{E} (\sigma^2 + L_0^2) \right] \sqrt{h_\Omega}$$

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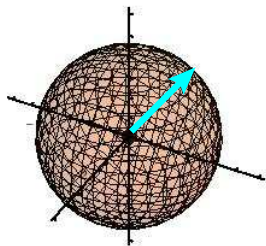
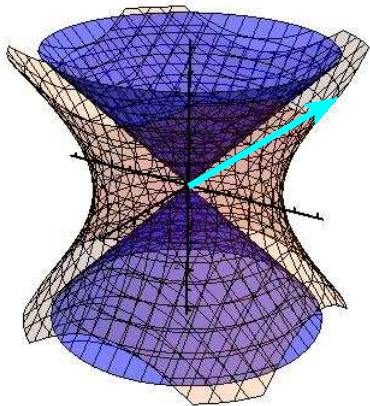
- ▶ **The number of atoms of space at x^i scales as area measure of the equigeodesic surface when $x' \rightarrow x$**

$$\rho_g(x^i, \ell_a) \propto \lim_{\sigma \rightarrow 0} \sqrt{h(x, \sigma)} \approx 1 - \frac{1}{8\pi} L_P^2 R_{ab} \ell^a \ell^b$$

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Null horizon \Leftrightarrow Euclidean origin

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- ▶ **The variational principle leads to**

$$G_{ab} = \kappa T_{ab} + \Lambda g_{ab}$$

- ▶ ***Spacetime becomes two-dimensional close to Planck scales!***

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See e.g., Carlip et al., [arXiv:1103.5993]; [arXiv:1009.1136]; G. Calcagni et al, [arXiv:1208.0354]; [arXiv:1311.3340]; J. Ambjorn, et al. [arXiv:hep-th/0505113]; L. Modesto, [arXiv:0812.2214]; V. Husain et al., [arXiv:1305.2814] etc.

- ▶ ***Spacetime becomes two-dimensional close to Planck scales!***
- ▶ ***Basic quantum of information to count spacetime degrees of freedom is***

$$I_{QG} = \frac{4\pi L_P^2}{L_P^2} = 4\pi$$

***May be we should not describe the
cosmos as a specific solution to
gravitational field equations.***

Our strange Universe

Three phases, three energy-density scales, with no relation to each other!

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$$\rho_{\Lambda} = [(2.26 \pm 0.05) \times 10^{-3} \text{ eV}]^4$$

Our strange Universe

Hope: High energy physics will (eventually!) fix $\rho_{inf}^{1/4} \approx 10^{15} \text{ GeV}$ and

$$\rho_{eq}^{1/4} \propto \left[\frac{n_{DM}}{n_\gamma} m_{DM} + \frac{n_B}{n_\gamma} m_B \right] \approx 0.86 \text{ eV}$$

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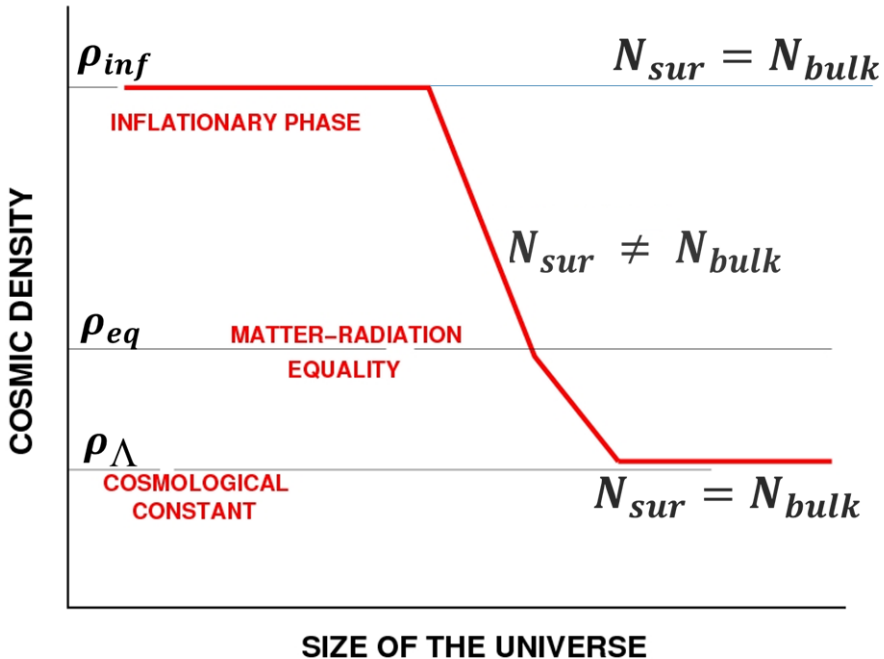
But we have no clue why

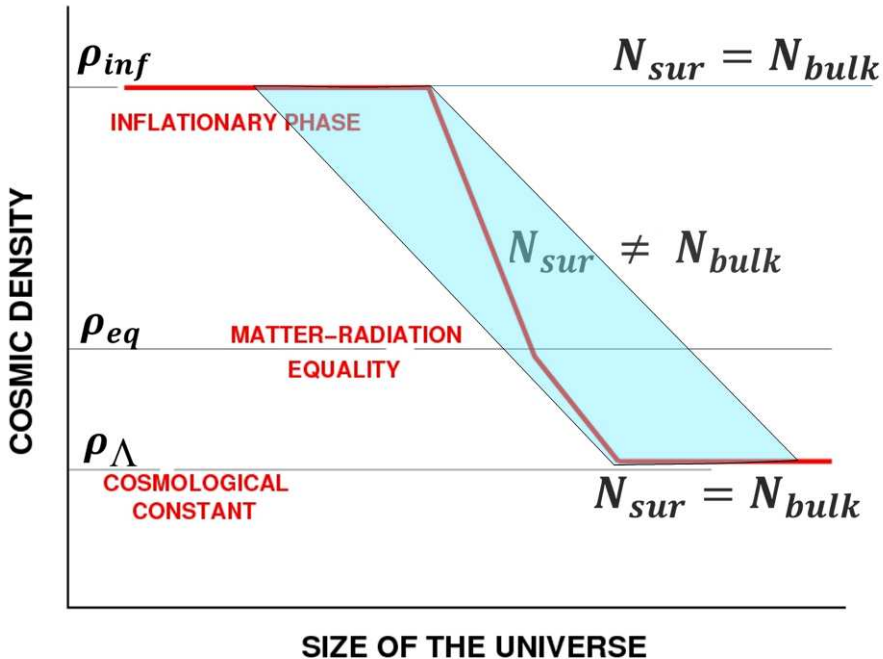
$$\rho_\Lambda L_P^4 \approx 1.4 \times 10^{-123} \approx 1.1 \times e^{-283}.$$

The FRW universe is described by the two equations (with $T = H/2\pi$)

$$\frac{dV_H}{dt} = L_P^2 (N_{sur} - N_{bulk})$$

$$\mathcal{U}_H \equiv \rho V_H = TS$$





Accessibility of Cosmic Information

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$$x(a_2, a_1) = \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{a_1}^{a_2} \frac{da}{a^2 H(a)}$$

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Boundary of accessible cosmic information for eternal observer

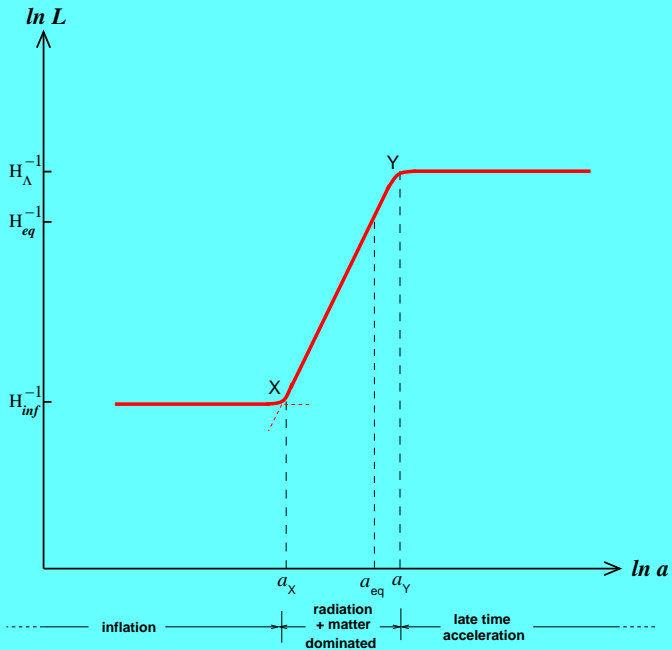
$$x(\infty, a) \equiv x_\infty(a) = \int_a^\infty \frac{d\bar{a}}{\bar{a}^2 H(\bar{a})}$$

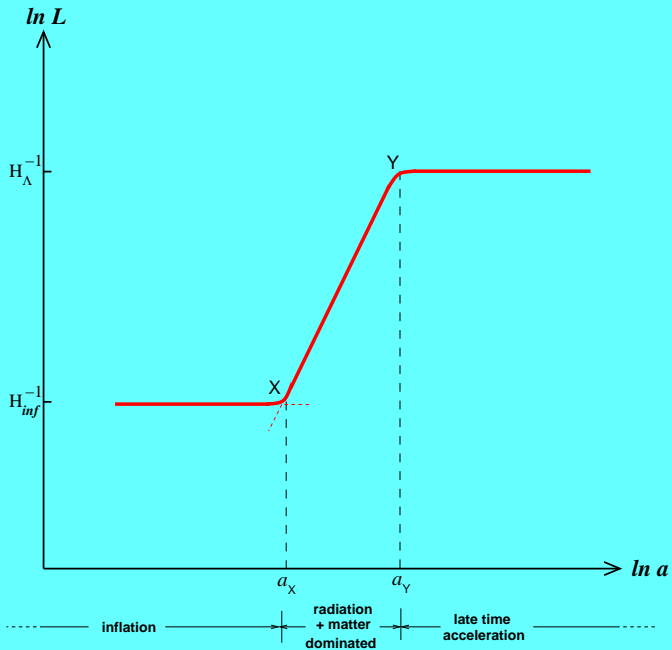
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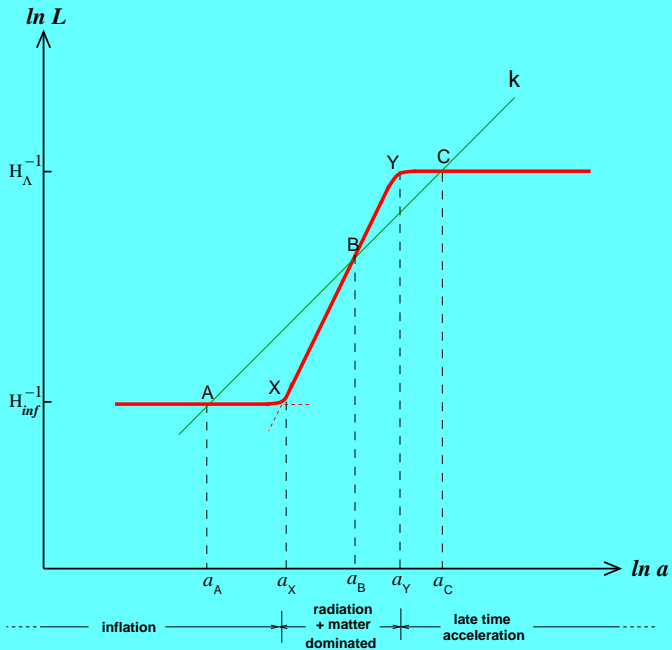
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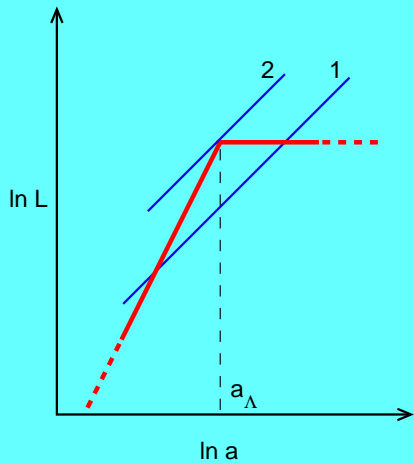
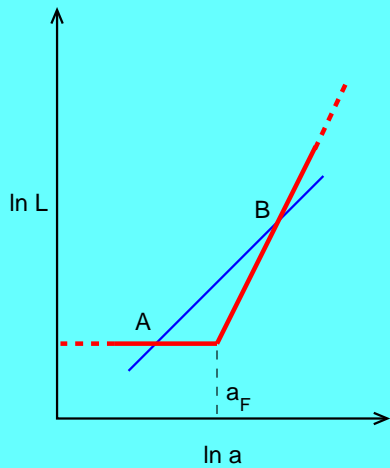
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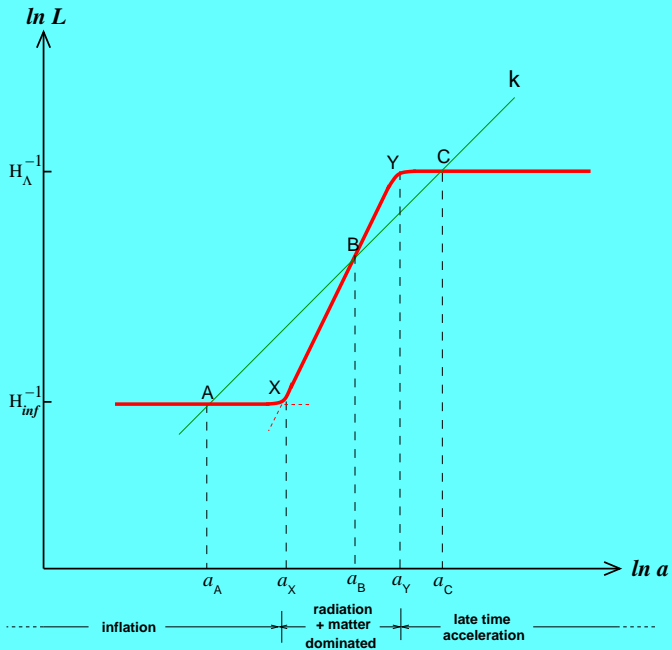
This is infinite if $\Lambda = 0$; finite if $\Lambda \neq 0$

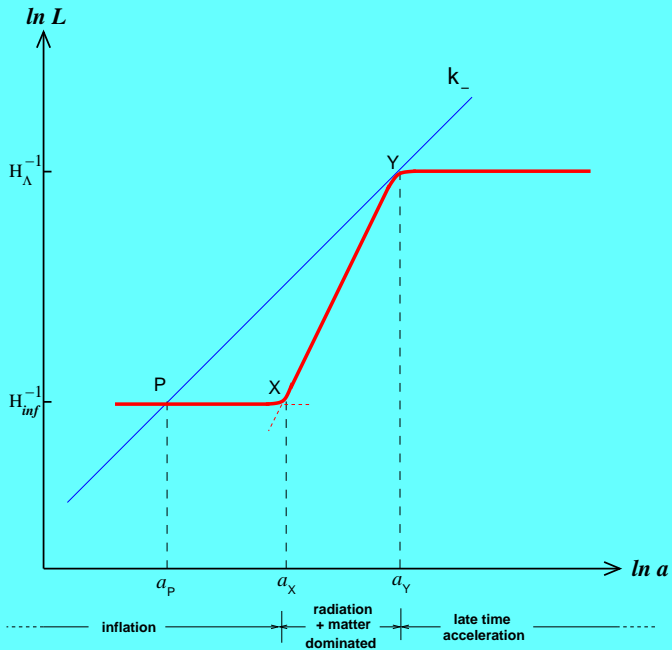


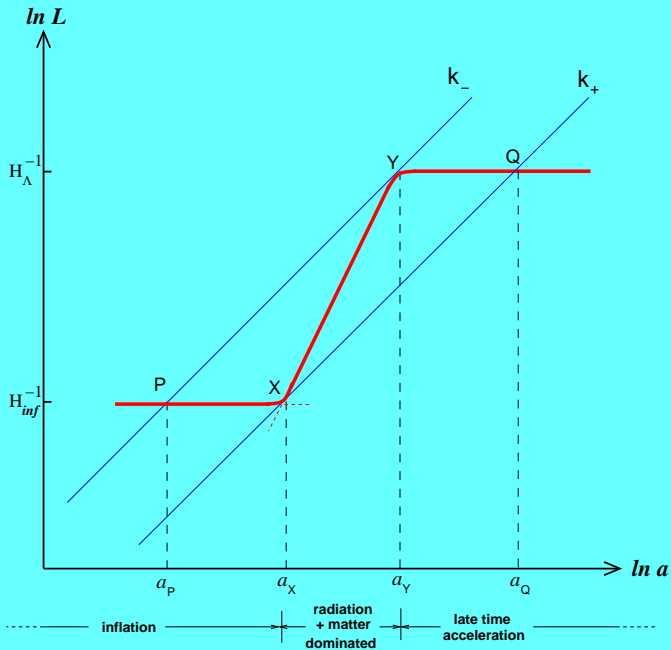




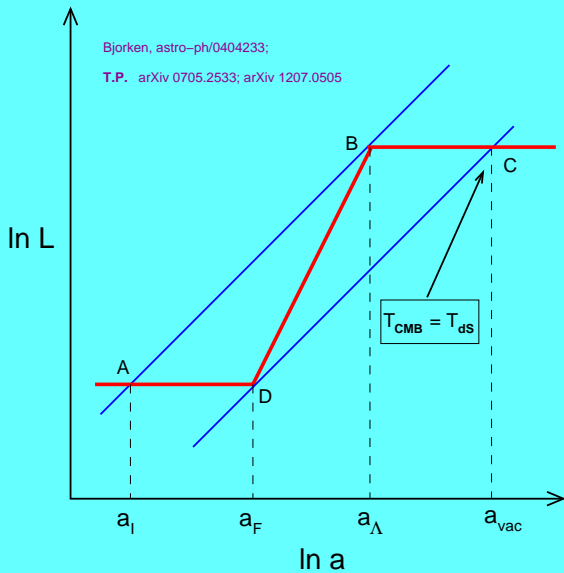




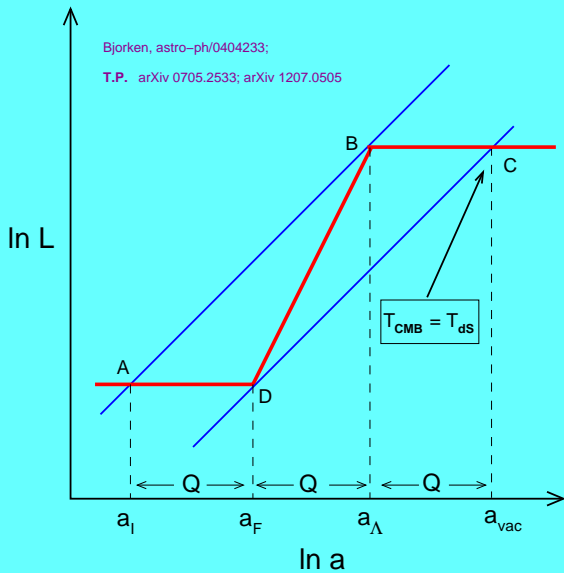




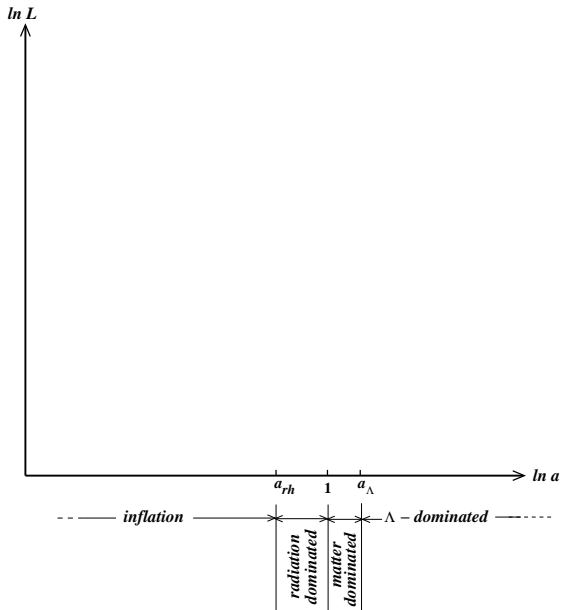
COSMIC PARALLELOGRAM



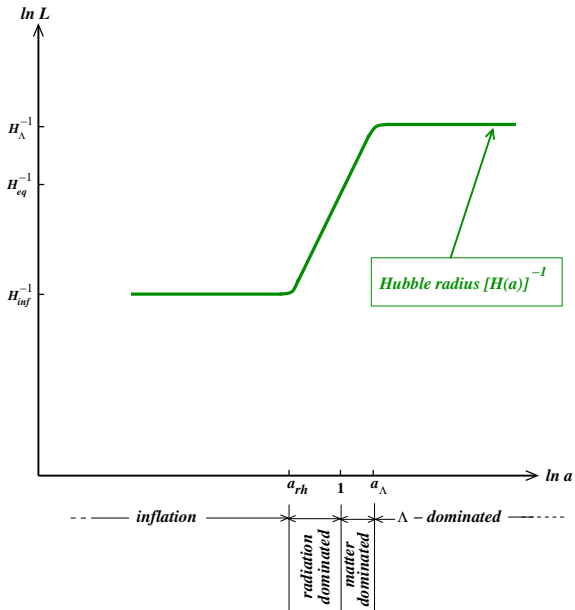
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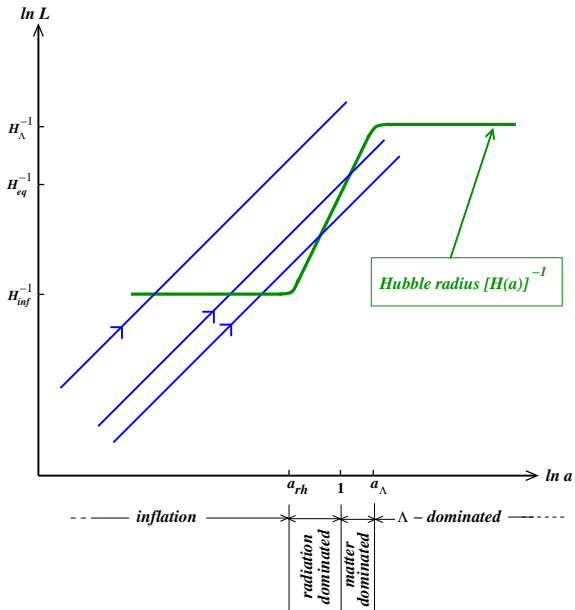
Cosmic Information: CosmIn



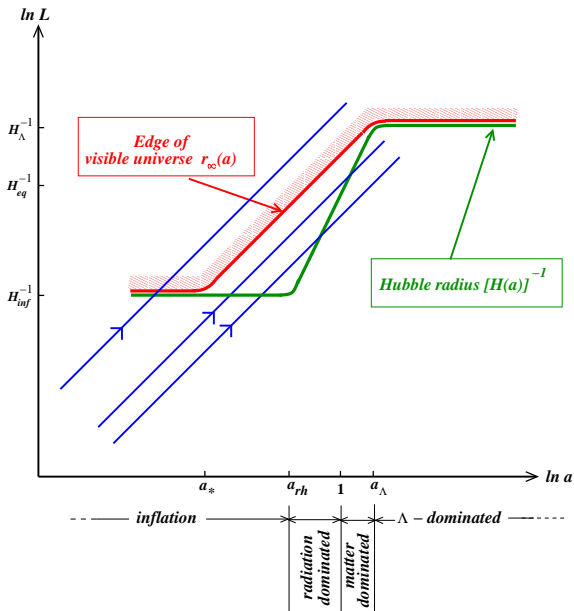
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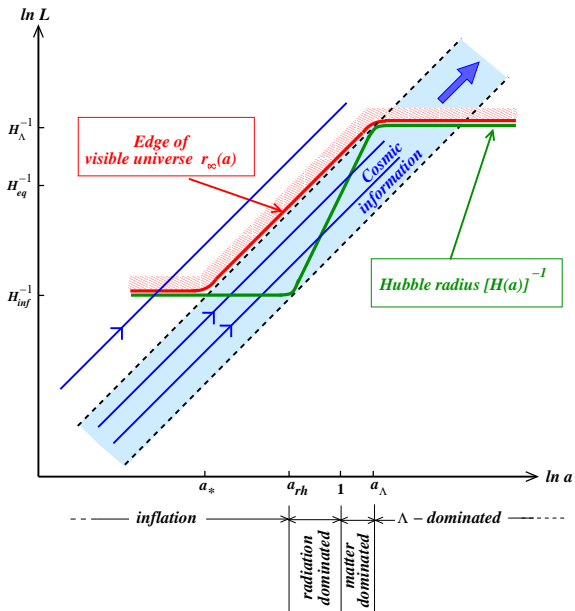
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Cosmic Information: CosmIn

T.P, Hamsa Padmanabhan [arXiv:1404.2284]

A measure of cosmic information accessible to eternal observer (“Cosmln”)

I_c = Number of modes (geodesics) which cross the Hubble radius during the radiation + matter dominated phase .

$$I_c = \frac{2}{3\pi} \ln \left(\frac{a_{\text{rh}}}{a_*} \right)$$

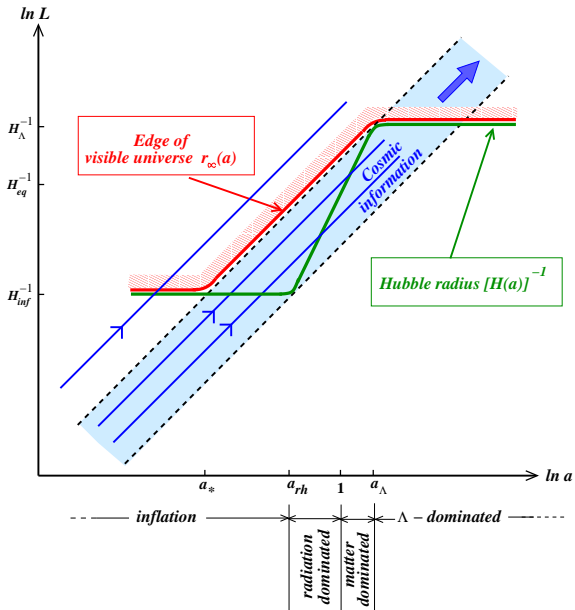
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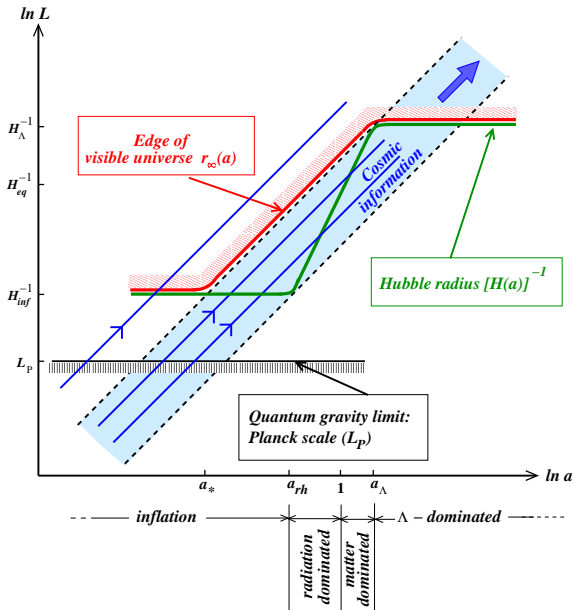
$$I_c = \frac{1}{9\pi} \ln \left(\frac{4}{27} \frac{\rho_{\text{inf}}^{3/2}}{\rho_{\Lambda} \rho_{\text{eq}}^{1/2}} \right)$$

$$\rho_{\Lambda} = \frac{4}{27} \frac{\rho_{\text{inf}}^{3/2}}{\rho_{\text{eq}}^{1/2}} \exp(-9\pi I_c)$$

Cosmic Information: CosmIn



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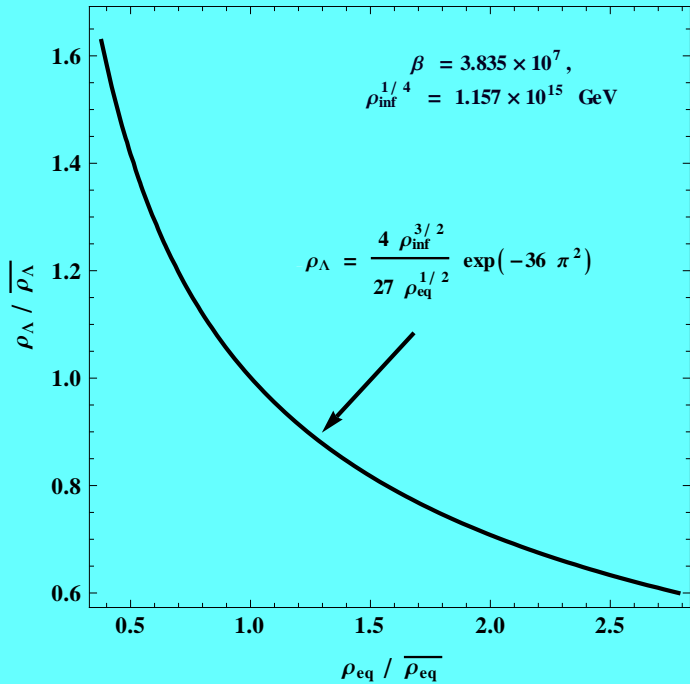
$$\rho_{\Lambda} = \frac{4}{27} \frac{\rho_{\text{inf}}^{3/2}}{\rho_{\text{eq}}^{1/2}} \exp(-9\pi I_c)$$

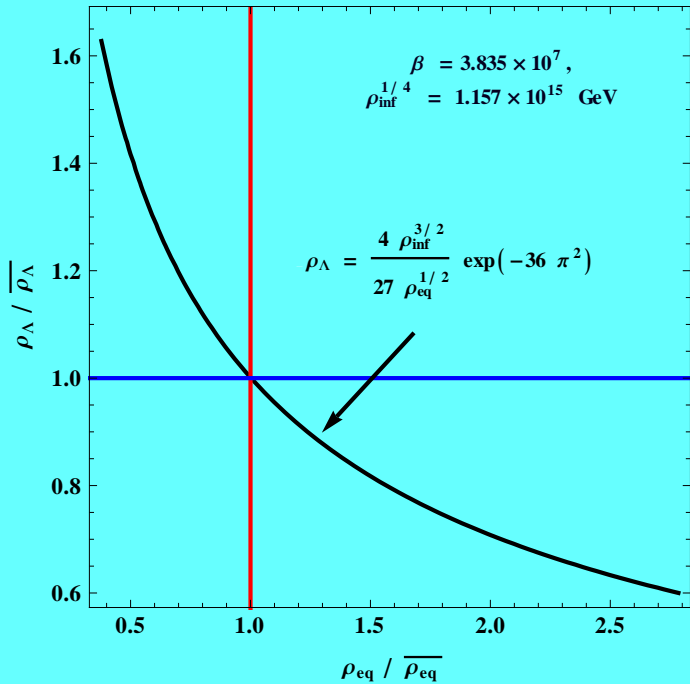
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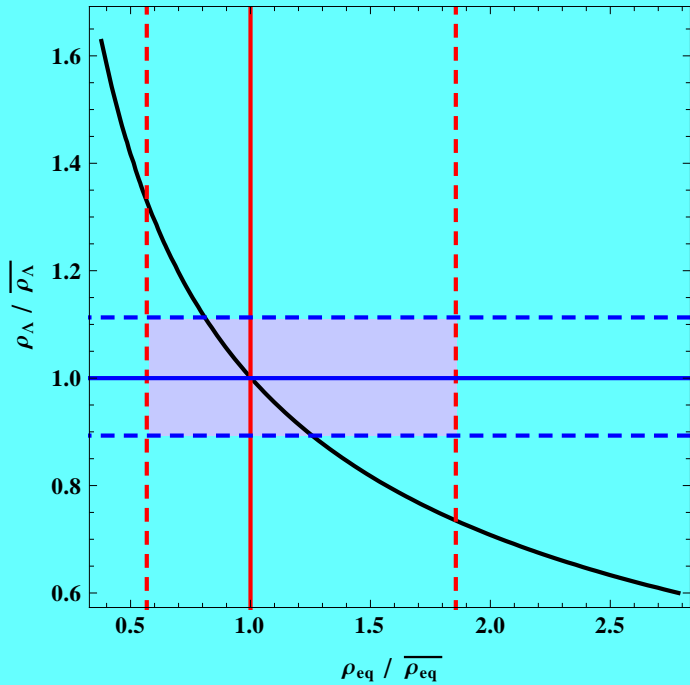
Using $I_c = I_{QG} = 4\pi$ gives the numerical value of ρ_{Λ}

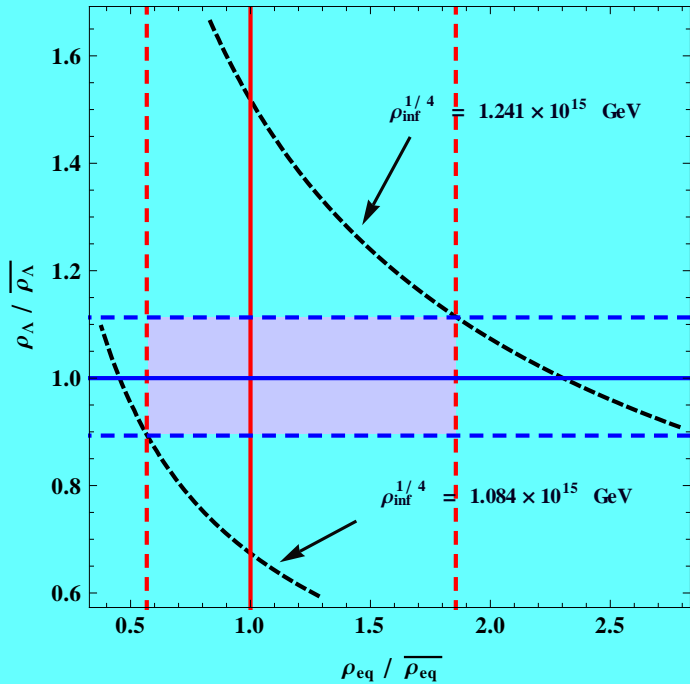
The Magical Relation!

$$\rho_{\Lambda} = \frac{4}{27} \frac{\rho_{inf}^{3/2}}{\rho_{eq}^{1/2}} \exp(-36\pi^2)$$









Key Open Question

Possibility for Matter Sector

Matter and Geometry need to emerge together for proper interpretation of $T^{ab}n_a n_b$ at the microscopic scale. How do we do this?

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$$2\mu L_P^4 \langle \bar{T}_{ab} n^a n^b \rangle \approx 2\mu L_P^4 \langle \bar{T}_{ab} \rangle \langle n^a n^b \rangle = 1$$

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- ▶ ***At Planck scales spacetime is 2-dimensional with 4π units of information; this allows the determination of the value of the cosmological constant.***

References

T.P, *General Relativity from a Thermodynamic Perspective*, *Gen. Rel. Grav.*, **46**, 1673 (2014) [arXiv:1312.3253].

Review: T.P, *The Atoms Of Space, Gravity and the Cosmological Constant*, *IJMPD*, **25**, 1630020 (2016) [arXiv:1603.08658].

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Hamsa Padmanabhan

Donald Lynden-Bell

THANK YOU FOR YOUR TIME!

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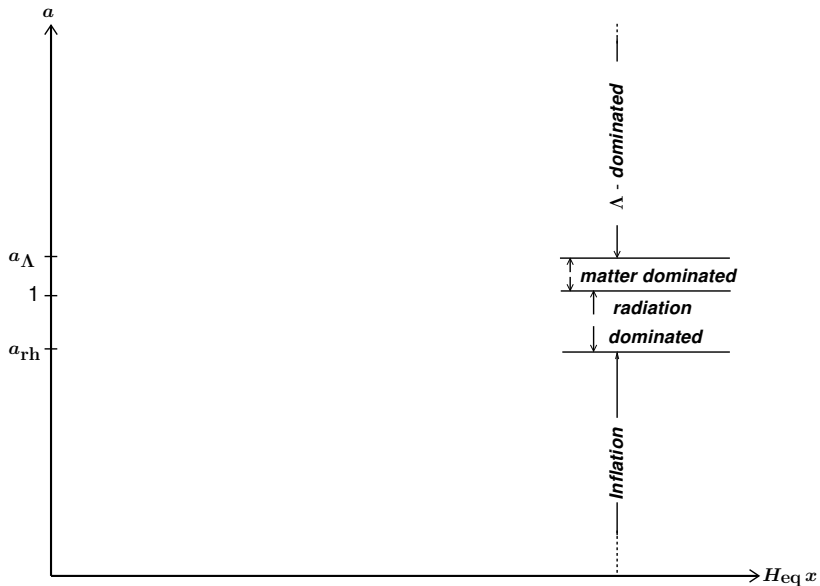
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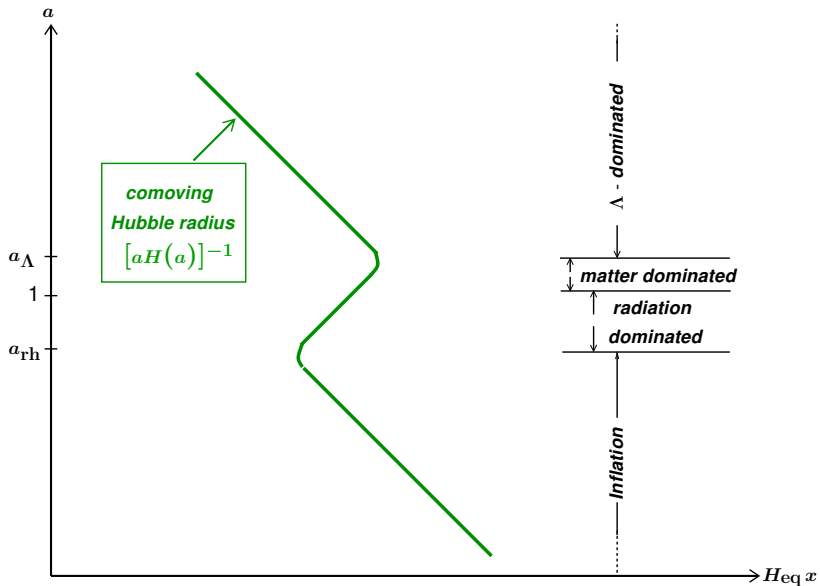
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Why?

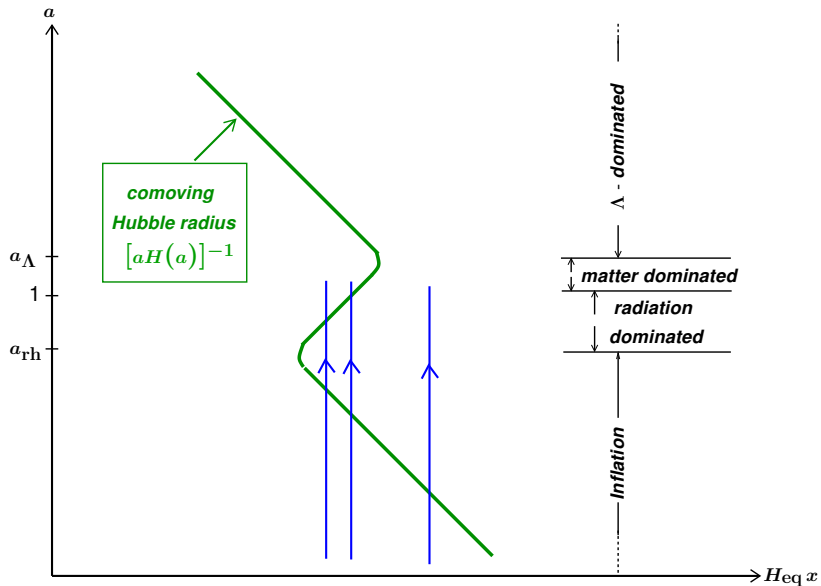
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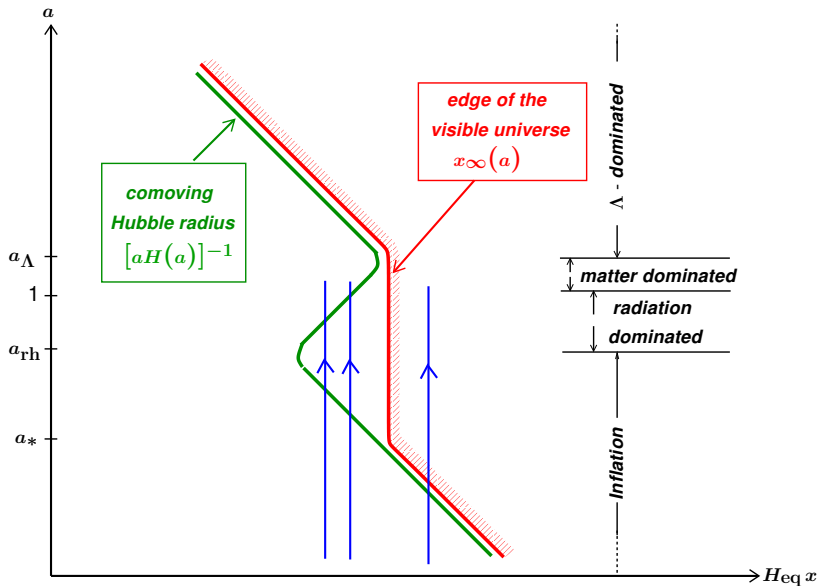
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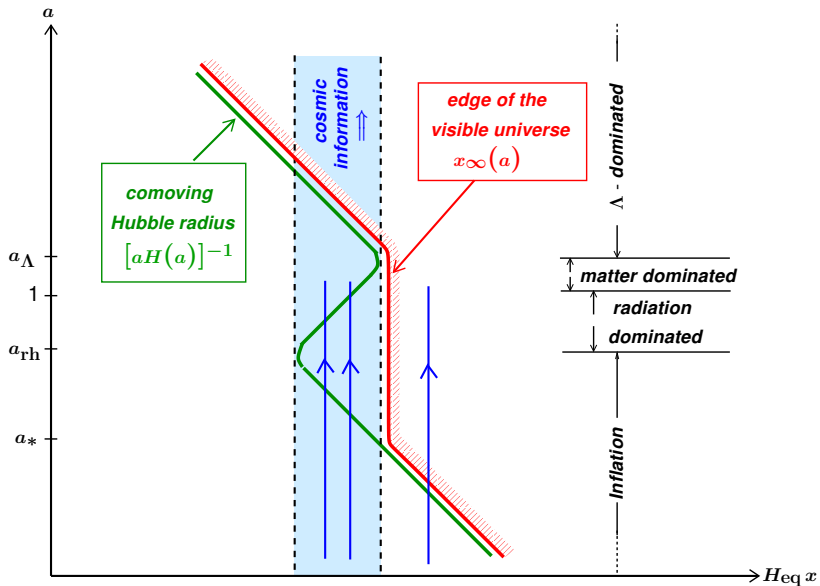
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