Neutrino Masses and Proton Decay in SO(10).

Franco Buccella

INFN, Sezione di Napoli

NAPOLI, September 13, 2016

General Program

with M. Abud, D. Falcone and L. Oliver

!) THE EXPERIMENTAL STATUS OF FOUR SO(10) MODELS WITH INTERMEDIATE SYMMETRY $SU(2)_R$ IS STUDIED WITH THE ACTUAL VALUES OF THE GAUGE COUPLINGS AT M_Z AND LOWER LIMIT FOR PROTON LIFETIME

2) IN THE FRAMEWORK OF SEE-SAW MODEL FOR NEUTRINO MASSES WITH REASONABLE ASSUMPTIONS ON DIRAC NEUTRINO MASS MATRIX, WE DEDUCE A LOWER LIMIT FOR THE SCALE OF SPONTANEOUS SYMMETRY BREAKING OF B - L AND RESTRICTIONS FOR PHYSICAL OBSERVABLES RELATED TO THE LEFT-HANDED NEUTRINO MASS MATRIX (Σm_{Li}, m_{ee} AND m_{ν_e})

3) IMPLICATIONS FOR LEPTOGENESIS SCENARIO (C. S. Fong, E. Nardi and G. Ricciardi)

4) PERSPECTIVES OPEN BY THE ALTARELLI MELONI MODEL

5) CONCLUSIONS

Gauge Unification

THE STRONG MOTIVATIONS FOR GAUGE UNIFICATION

The quantum numbers of the three families of the fundamental fermions (quark and leptons) are a motivation stronger than generally believed for gauge unification beyond the standard model group:

SU(3) x SU(2) x U(1) Trace	Tr Y	Tr Y^2	Tr C_2	Tr C_3
(3, 2, +1/6) quantum numbers	+1	1/6	9/2	8
($\bar{3}$, 1, -2/3) quantum numbers	-2	4/3	0	4
(1, 1, +1) quantum numbers	+1	1	0	0
	-	-	-	-
The sum of the traces is	0	5/2	9/2	12
(1, 2 -1/2) quantum numbers	-1	1/2	3/2	0
($\bar{3}$, 1, +1/3) quantum numbers	+1	1/3	0	4
	-	-	-	-
The sum of the traces is	0	5/6	3/2	4

Not only Tr Y vanishes on two sets of states, which is a peculiar property of the generators of a non-abelian Lie algebra, but also the sums on the last three columns are in the same ratio (3), which is a peculiar property of two representations of the same algebra, the 10 and the $\bar{5}$ of $SU(5)$.

From SU(5) to SO(10)

The elegant minimal gauge unified $SU(5)$ model proposed by Georgi and Glashow almost 40 years ago has been disproved by the values of the three gauge couplings, which meet at scales slightly different and only the scale corresponding to the meeting point of g_2 e g_3 is sufficiently high to be consistent with the lower limit for the lifetime of the proton decay into $e^+\pi_0$, actually 8×10^{33} years.

The extension to $SO(10)$ is very promising, but did not receive great attention by the majority of the theorist.

In fact the weak hypercharge is given by the combination of two $SO(10)$ generators :

$Y = T_{3R} + \frac{B-L}{2}$

If the intermediate symmetry in the spontaneous breaking of $SO(10)$ into the gauge group of the standard model

$SU(3) \times SU(2) \times U(1)$

contains $SU(2)_R$ and (or) $SU(4)_{PS}$, the three costants may meet at a scale consisting with the lower limit for the lifetime of the proton.

In particular Mohapatra and Parida pointed out the phenomenological interest of models, where the discrete symmetry D for the exchange $SU(2)_L \to SU(2)_R$ is broken at the scale of $SO(10)$ breaking.

An Upper Limit for the Proton Lifetime

In this framework about 30 years ago we studied in Naples the spontaneous breaking of $SO(10)$ in four cases with intermediate gauge symmetry

$SU(4) \times SU(2) \times SU(2)$

or

$SU(3) \times SU(2) \times SU(2) \times U(1)$

with or without the symmetry D conserved at the scale of the breaking of SO(10). For the case with intermediate symmetry $SU(4) \times SU(2) \times SU(2) \times D$ the highest VEV belongs to the 54, while in the other cases it is a combination of the two singlets with respect to the standard group of the 210.
The VEV with $\Delta(B-L) = 2$, which reduces the gauge symmetry down to the standard group, belongs to the 126 .

The highest value for the unification scale, M_X is found in the case with intermediate symmetry:

$SU(3) \times SU(2) \times SU(2) \times U(1)$

and the ratio $\frac{M_X}{M_Z}$ is:

$\exp \frac{\pi}{2}(\frac{\sin^2(\theta_W)}{\alpha} - \frac{1}{\alpha_s})$.

The increase of the values of $\sin^2(\theta_W)$ and α_s gives rise to a higher upper limit for the proton lifetime.

We stated that the best "smoking gun" for $SO(10)$ should be neutrino oscillations, for which at that time there was a certain skepticism, despite the deficit of solar neutrinos found in the experiment at Homestake.

Neutrino Oscillations

The best signal of physics beyond the standard model is solar and atmospheric neutrino oscillations with the values for the square mass differences

$\Delta m_s^2 = 8 \times 10^{-5}(eV)^2$

for solar and

$\Delta m_a^2 = 2.5 \times 10^{-3}(eV)^2$

for atmospheric

and the mixing angles for the $PMNS$ matrix :

$\sin^2(\theta_{12}) = .307$

$\sin^2(\theta_{13}) = .0241$

$\sin^2(\theta_{23}) = .386$

$\delta = 1.08\pi$

An upper limit for the sum of the neutrino masses is given by cosmology

$\Sigma|m_i| \leq 0.46 - 0.78 eV$

Upper limits for the effective neutrino mass, which appears in tritium decay, m_{ν_e} and, if neutrinos are Majorana particlles, for the factor m_{ee} in the amplitude of double neutrinoless beta decay:

$m_{\nu_e} \leq 2.2 eV$

$m_{ee} \leq 0.4 eV$

See-Saw Model for Neutrino Masses

The most acttractive explanation of the fact that neutrino masses are several orders of magnitude smaller than the masses of the charged fermionis is the "see-saw model", based on the fact that right-handed neutrinos, not necessary in the standard model, but to construct for them a Dirac mass , are singlets of the gauge group of the standard model and may get a Majorana mass allowed by the symmetry and therefore their mass is expected to be many orders of magnitude bigger than the scale of the spontaneous symmetry breaking of the electro-weak unified hteory. As a consequence left-handed neutrinos get the effective mass:

$$m_L = -m_t^D (M_R)^{-1} m^D \tag{1}$$

many orders of magnitude smaller than the other fundamental fermions.

The most natural framework for the "see-saw" model are the gauge unified models based on SO(10), which require the existence of right-handed neutrinos and where B - L is a generator of the group spontaneously broken at a scale orders of magnitude bigger than the electro-weak one, which is welcome to get a similar value for the elements of the right-handed neutrinos mass matrix.

SO(10) unification had been proposed before with strong motivationis.

Motivations to Choose SO(10) as the Group of Gauge Unification

SO(10) has two importanti properies :

1) The opposite anomalies of the representations 10 and $\bar{5}$ of SU(5) are a consequence of the absence of anomalies in SO(10) (The cubic Casimir operator $T_{ab}T_{bc}T_{ca}$ vanishes!)

2) SO(10) contains SU(5), obvious generalization of SU(3) x SU(2) x U(1), but also $SU(4) \times SU(2)_L \times SU(2)_R$ with quarks and leptons, transforming as a (4, 2, 1) and their antiparticles as a ($\bar{4}$, 1, 2).

The most interesting feature of $SO(10)$ is its natural explanation of the values extremely low of the masses of the left-handed neutrinos.

To correct the predictions of $SU(5)$ for the coupling costants at the electro-weak scale the symmetry B - L should be spontaneously broken at a lower scale than $SO(10)$ and this implies an upper limit to the elements of the mass matrix of the right-handed neutrinos.

The Scales of Spontaneous Symmetry Breaking of SO(10) and B - L, M_X and M_{B-L}

The present values of $\sin^2(\theta_W) = .23116 +(-) .00013$ and $\alpha_s = .1184 +(-) .0003$ imply for the four models considered the following values (in GeV) for the scales of spontaneous symmetry breaking of $SO(10)$ and $B - L$:

Intermediate Simmetry M_X and M_{B-L} in GeV

SU(4) x SU(2) x SU(2) x D .955 10^{15} and 4.47 10^{13}

SU(4) x SU(2) x SU(2) 1.15 10^{16} and 1.2 10^{11}

SU(3) x SU(2) x SU(2) x U(1) x D 3.24 10^{15} and 1.91 10^{10}

SU(3) x SU(2) x SU(2) x U(1) 3.02 10^{16} and 1.07 10^{9}

The present lower limit for the lifetime of the proton decay into $e^+\pi_0$ is 8×10^{33} years, which corresponds to a lower limit for M_X of 2.66×10^{15} GeV.

We may conclude that the first model is excluded, for the third one the increase of the lower limit for the proton lifetime might be a problem, while the second one and still more the fourth one predict lifetimes longer than the present limit.

With reasonable assumption one may establish a lower limit also for M_{B-L}.

Lower Limit for the Scale for the Breaking of M_{B-L}

To get a lower limit for M_{B-L} we assume:

1) The see-saw model, which implies:

$Det(M_R)\ Det(m_L) = Det(M_D)^2$

2) $Det(M_D) = \frac{Det(m_l)Det(m_u)}{Det(m_d)} = 2 \times 10^7\ (GeV)^3$

From the upper limit :

$Det(m_L) \leq 8 \times 10^{-3}\ (eV)^3$

we get the lower limit :

$5 \times 10^{25} (GeV)^3 \leq Det(M_R)$

which implies

$3.68 \times 10^8 GeV \leq M_{B-L}$

limit obeyed by the four models with a value slightly larger than the limit for the fourth model, which implies for the sum of the neutrino masses the lower limit .24 eV.

Conseguences of the Assumption that the Matrix V^L, which Appears in the Diagonalization of M_D is Similar to V_{CKM}

In general M_D is diagonalized by the by the biunitary transformation

$$M_D = V^R M_D^{diag} V^{L+}$$

If we assume that V^L is similar to V_{CKM}, more precisely that it may be approximated by a matrix, where on can neglect the mixing angle of the third family with the first two, by inverting the see-saw formula, one finds that the terms proportional to the square of the highest eigenvalue of M_D are proportional to:

$$\frac{U_{\tau 1}^2}{m_1} + \frac{U_{\tau 2}^2}{m_2} + \frac{U_{\tau 3}^2}{m_3}$$

and a strong cancellation between the three terms is necessary to avoid that a value about 100 GeV for the highest eigenvalue of M_D implies right-handed neutrinos with a mass larger than the scale of breaking of M_{B-L}.

Conseguences of the Assumption that the Matrix V^L, which Appears in the Diagonalization of M_D is Similar to V_{CKM}

The assumption:

$\frac{U_{\tau 1}^2}{m_1} + \frac{U_{\tau 2}^2}{m_2} + \frac{U_{\tau 3}^2}{m_3} = 0$

with the experimental values of the elements of the matrix $PMNS$ and of the differences of the square masses coming from the atmospheric and solar neutrinos one has as a consequence that $|m_1|$ is in the range

$6 \times 10^{-3}, 6 \times 10^{-2}$ eV

and consequently the sum of neutrino masses is in the range:

$7 \times 10^{-2}, 2 \times 10^{-1}$ eV and finally $Det(m_L)$ is in the range:

$3 \times 10^{-6}, 3 \times 10^{-4}$ $(eV)^3$.

and a lower limit :

$1.33 \times 10^{27} \leq Det(M_R)$

which implies

$1.1 \times 10^9 \leq GeV M_{B-L}$

just around the value found in the fourth model.

As long as for the masses, which appear in tritium decay, m_{ν_e}, one gets the same range of $|m_1|$, while for m_{ee}, proportional to the amplitudes for neutrinoless double beta decay, one has the upper bound :

6×10^{-2} eV

Status of the four SO(10) Models

In the figures 1-4 we plot the allowed zones for the values of α_s in ascisse and $\sin^2(\theta_W)$ in ordinate for the various models compared with the experimental values in correspondance with the lower limit 8×10^{33} years for the proton lifetime and 3.68×10^8 GeV for the scales of breaking of $B - L$, while in figures 5-7 we consider the lower limits :

2×10^{34} and 1.1×10^9 GeV

The considerationis from the upper limit for M_{B-L} may be applied to the general study by

Bertolini, Di Luzio e Malinsky,

who consider symmetry breaking patterns from $SO(10)$ to the gauge group of the standard model $SU(3) \times SU(2) \times U(1)$ with two intermediate scales.
In particular models with a too low scale of nreaking of rottura di $B - L$ are disfavoured as well as the ones, where the breaking of this symmetry is realized with a VEV of the 16 with $\Delta(B - L) = 1$ and therefore do not contribute to the Majorana masses of the right-handed neutrinos at the lowest order and give rise to masses many orders of magnitude smaller than the scale of breaking .

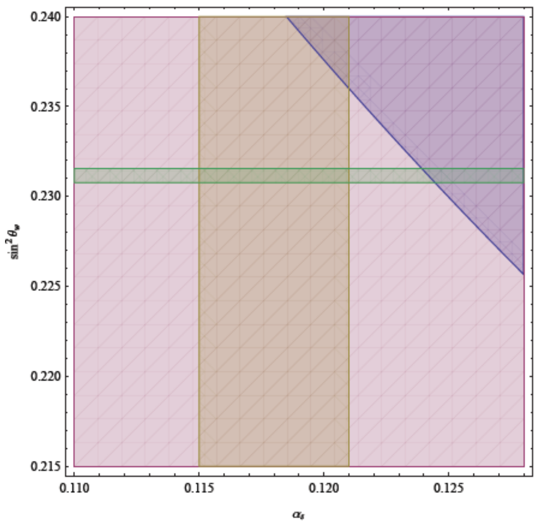

Fig. 1

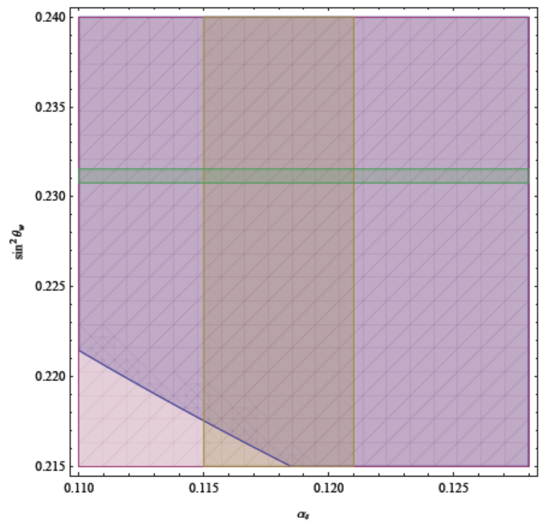

Fig. 2

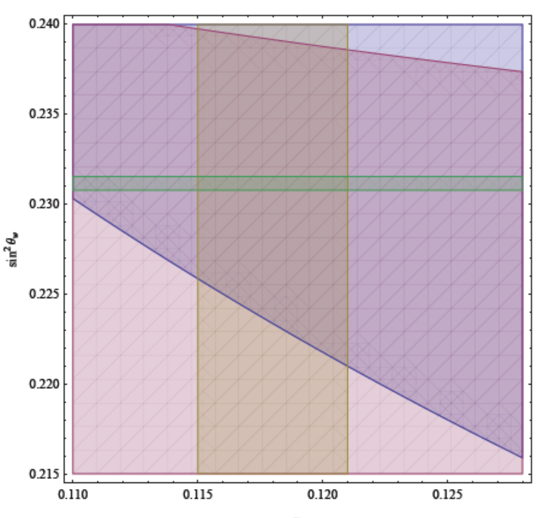

Fig. 3

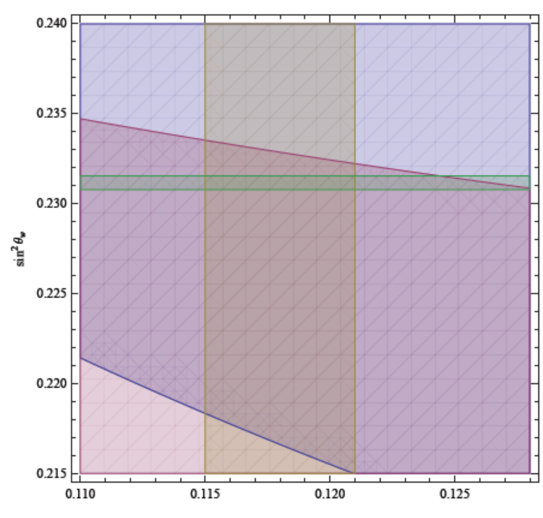

Fig. 4

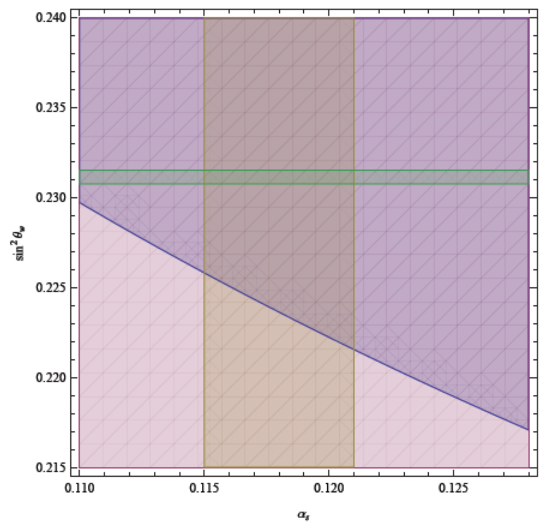

Fig. 5

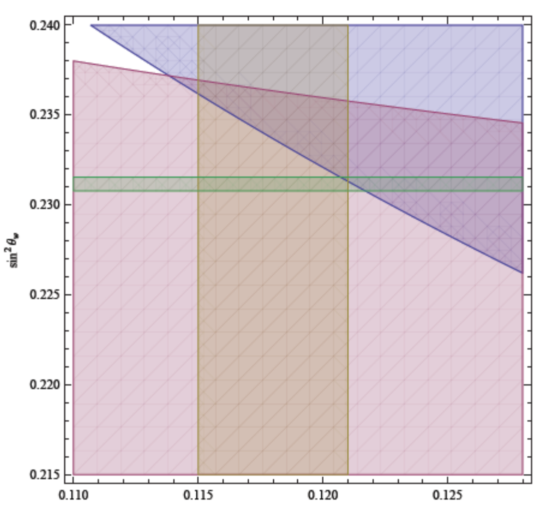

Fig. 6

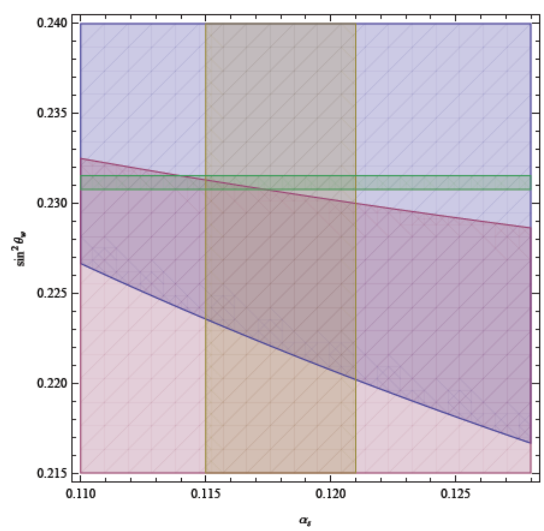

Fig. 7

Baryonic Asimmetry of the Universe

Up to now there is no evidence for a macroscopic antimatter, which would give rise to dramatic annihilation processes.

A positive value of the baryonic number

ΔB = B - $\bar{B}$

is required by the inflationary cosmological model.

Indeed nucleosynthesis depends from the relative abundance of baryons with respect to photons, which dissociate deuterium nuclei and disfavour the formation of helium.

The primordiale abundanzace of H, He, Li and Be imply for the ratio:

$\frac{\Delta B}{\gamma}$ about 6 10^{-10}.

The Leptogenesis Scenario

The property of the unified gauge theories of violaring the baryonic number evolved from the nightmere of proton stability into the intriguing property of being a necessary condition to produce the baryonic asymmetry .

In the framework pf the minimal SU(5) model one cannot produce a sufficient barionic asymmetry and at the electro-weak scale non perturbative contributions, the sphalerons, which violate B, but not B - L, would wash out any baryon asymmetry produced at a higher scale with B - L = 0, as the one produced in the minimale gauge SU(5) theory, where B - L is a global symmetry.

On the contrary a realistic SO(10) model implies the spontaneous breaking of its generator B - L

In particular one may consider a scenario, leptogenesis, where a leptonic asymmetry produced in the off-equilibrium decay of the lightest right-handed neutrino gives a leptonic asymmetry, which turns into an asymmerry in both B and L at the electroweak scale by the sphalerons.

To get the leptonic asymmetry needed to realize the leptogenesis scenario and avoid its washing out a compact spectrum for the eigenvalues of M_R around 10^{11} teV is welcome,

The Altarelli Meloni model

By adding a 45 scalar Higgs to the 210 and 126 for the symmetry breaking of SO(10) into SU(3) x SU(2) x U(1), they are able to comply with two the two relevant demands of dark matter and axion. This motivates the extension of the study performed several years ago (BCST) for the construction of an invariant renormalizable potential with absolute minimum in the directions of the VEV's proposed in the AM model.

The method for this construction is to consider positive quartic invariants, which vanish for symmetry in the desired directions.

This is possible for the 210, 126 and 45 with VEV respectively in the $SU(4) \times SU(2) \times SU(2)$, $SU(5)$ and $SU(4) \times SU(2) \times U(1)$ with common symmetry the gauge group of the standard model $SU(3) \times SU(2) \times U(1)$.

The only non-trivial quartic invariant built with a 45 is $\mathrm{Tr}(\Phi^4)$ and takes its maximum (minimum) value in the $SO(8) \times SO(2)$ ($SU(5)$) direction, but the terms quadratic in the 45 and in the 210 (or bilinear in 126 and its conjugate) may turn the absolute minimum in the $SU(4) \times SU(2) \times U(1)$ direction for the 45.

The conditions sufficient to get the minimum in the desired direction put constraints on the scalar spectrum and on the contributions of the scalar particles to the group evolution equations.

Conclusions

1)The SO(10) gauge unified theory with intermediate symmetry $SU(4) \times SU(2) \times SU(2)$ is well consistent with the present lower limits on proton lifetime and on the scale of $B-L$ spontaneous symmetry breaking in see-saw model for the left-handed neutrino mass matrix. A similar theory with an additional scalar in the adjoint (45) representation has been recently proposed by Altarelli and Meloni with an axion suitable to solve the strong CP problem and to account for the observed amount of Dark Matter.

2) The assumption that the determinant of the neutrino Dirac mass matrix, m_D, obeys
$Det(m_D)\ Det(m_d) = Det(m_l)\ Det(m_u)$
and the see-saw model for the neutrino mass matrix have as a consequence a lower limit for the scale of $B-L$ breaking, which increases with the decrease of the upper limit for the sum of the masses of the left-handed neutrinos.

Conclusions

3) The demand that the masses of the right-handed neutrinos are limited by the scale of the breaking of $B-L$, that the matrix V^L, which appears in the diagonalization of m_D, is similar to the CKM so that to be approximated with one acting only on the first two generation and that the highest eigenvalue of m_D be around 100 GeV, has important conseguences for the not yet measured osservables related to the mass matrix of left-handed neutrinos:
$7 \times 10^{-2} eV \leq \Sigma_i m_{Li} \leq 0.2 eV$
$m_{ee} \leq 6 \times 10^{-2}$ eV
$6 \times 10^{-3} eV \leq m_{\nu_e} \leq 6 \times 10^{-2}$ eV
and the lower limite 1.1×10^9 GeV for the scale of breaking of $B-L$.

4) The compact spectrum for right-handed neutrinos found in this framework favours the construction of a scenaria, where leptogenesis gives rise to the baryonic asymmetry in the universe.

5) the possibility of accounting for the dark matter and for the axions of the Altarelli Meloni model strongly increases the interest in non supersymmetric SO(10) unified theories.

The bibliography

1. AN SO(10) MODEL WITH 54 + 126 + 10 HIGGS. (with L. Cocco and C. Wetterich) Nucl.Phys.B243 (1984) 273

2. THE PRICE OF UNIFICATION SIGNATURES IN A SO(10) MODEL WITH LEFT-RIGHT SYMMETRY BROKEN AT THE HIGHEST SCALE. (with L. Cocco, A. Sciarrino and T. Tuzi) Nucl.Phys.B274 (1986) 559

3. SO(10) FROM SUPERSYMMETRIC E(6) (with G.Miele) Phys.Lett.B189 (1987) 115

4. AN SO(10) MODEL WITH $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times D$ INTERMEDIATE SYMMETRY (with L. Rosa) Z.Phys.C36 (1987) 425

5. A SO(10) MODEL WITH MAJORANA MASSES FOR THE NEUTRINO (R)'S AROUND $10^{11}\,GEV$. (with M.Abud,L.Rosa, A.Sciarrino) Z.Phys. C44 (1989) 589

6. AN UPPER LIMIT FOR THE PROTON LIFETIME IN SO(10). (with G. Miele, L. Rosa, P. Santorelli and T. Tuzi) Phys.Lett.B233 (1989) 178

7. BARYOGENESIS VIA B-L VIOLATION IN SO(10) UNIFIED MODELS. (with G.Mangano,A.Masiero and L.Rosa) Phys.Lett.B320 (1994) 313

8. PROTON DECAY AND NEUTRINO MASSES IN SO(10). (with F. Acampora,G.Amelino-Camelia,O.Pisanti,L.Rosa and T. Tuzi) Nuovo Cim.108A (1995) 375

9. BARYOGENESIS IN AN SO(10) GUT MODEL WITH PATI-SALAM INTERMEDIATE SYMMETRY (with O.Pisanti and L.Rosa) Mod.Phys.Lett.A13 (1998) 1539

10. NEUTRINO MASSES AND MIXING IN SO(10) (with M.Abud,D.Falcone,G.Ricciardi and F.Tramontano) Mod.Phys.Lett.A15 (2000) 15

11. SO(10) INSPIRED SEE-SAW MODEL (with M.Abud) Int.J.Mod.Phys. A16 (2001) 609

12. BARYOGENESIS VIA LEPTOGENESIS IN SO(10) MODELS (with D.Falcone and F.Tramontano) Phys.Lett. B524 (2002) 241

13. INTERMEDIATE SYMMETRIES IN THE SPONTANEOUS BREAKING OF SUPERSYMMETRIC SO(10) (with C.A.Savoy) Mod.Phys.Lett. A19 (2002) 1239

14. BOUNDS FOR THE MASS OF THE HEAVIEST RIGHT-HANDED NEUTRINO IN SO(10) THEORIES (with D. Falcone) Mod. Phys. Lett. A18 (2003) 1819

15. BOUNDS FOR NEUTRINOLESS DOUBLE BETA DECAY IN SO(10)INSPIRED SEESAW MODELS (with D. Falcone) Mod. Phys. Lett. A19 (2004) 2903

16. LEPTOGENESIS WITHIN A GENERALIZED QUARK-LEPTON SYMMETRY (with D.Falcone and L.Oliver) Phys. Rev. D77, (2008) 033002

17. BARYOGENESIS VIA LEPTOGENESIS FROM QUARK-LEPTON SYMMETRY AND A COMPACT HEAVY $N-R$ SPECTRUM (with D. Falcone and L. Oliver) Phys. Rev. D83 (2011) 093013

18. SEESAW MODEL IN SO(10) WITH AN UPPER LIMIT ON RIGHT-HANDED NEUTRINO MASSES (with M. Abud, D. Falcone and L. Oliver) Phys. Rev. D86, (2012) 033006

19. SQUEEZING OUT PREDICTIONSWITH LEPTOGENESIS FROM SO(1O) Phys. Rev. D86, (2012) 035012 (with D. Falcone, C.S. Fong, E. Nardi and G.Ricciardi)

20. NEUTRINO MASSES AND PROTON DECAY IN SO(10) (with M. Abud and D. Falcone) Phys. Rev. D86, (2012) 073014