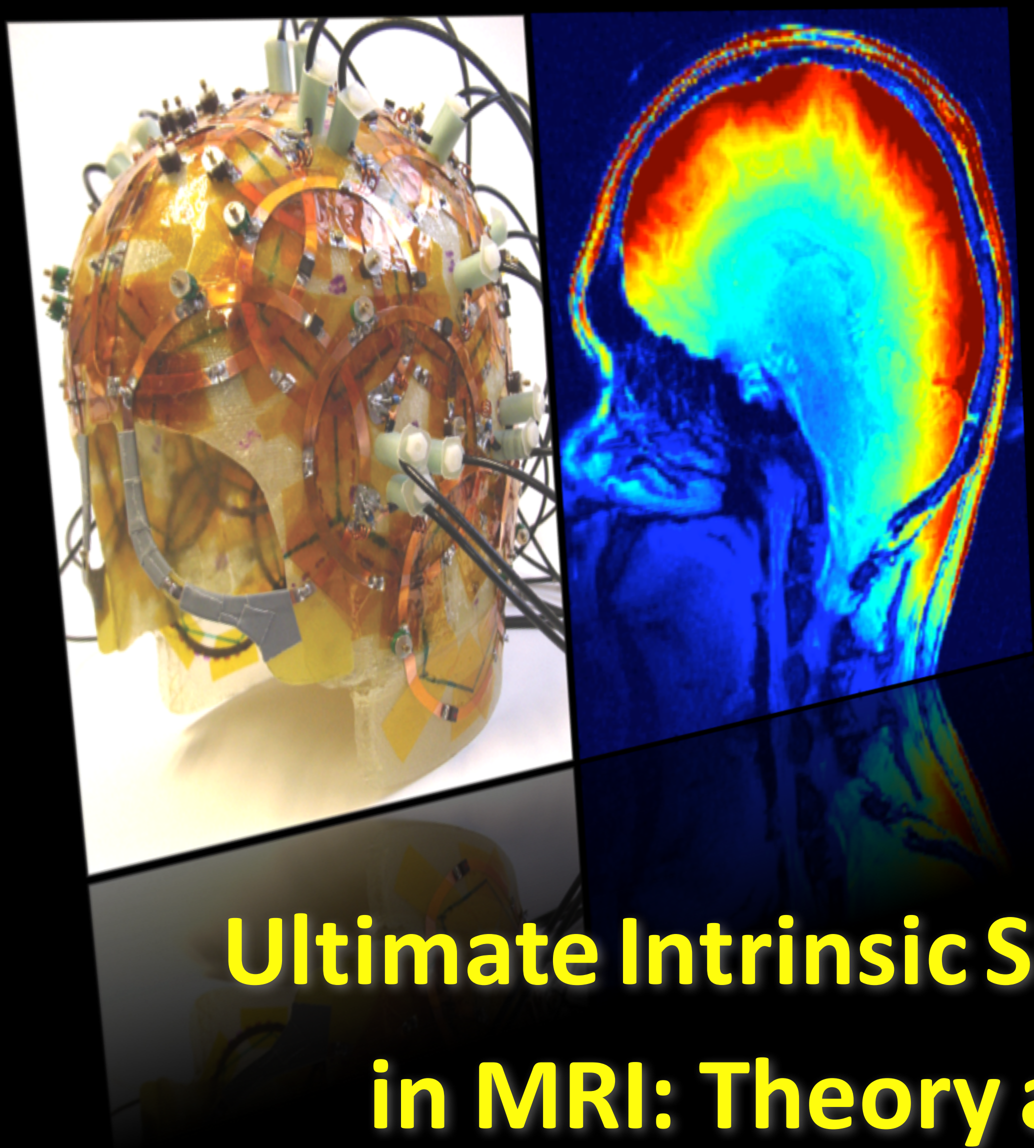


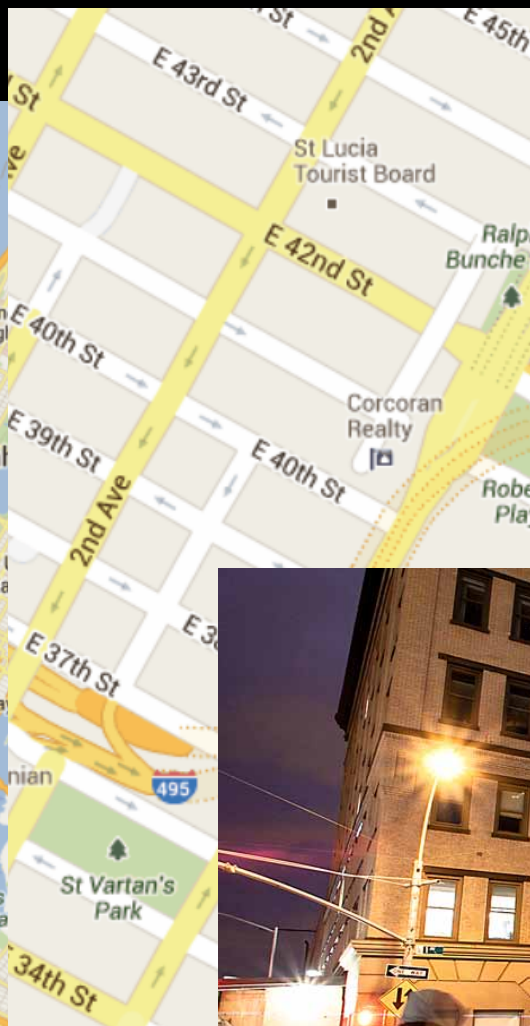

**Riccardo Lattanzi, Ph.D.**

Associate Professor of Radiology,
Electrical and Computer Engineering
New York University Langone Medical Center
New York, NY, USA


# Ultimate Intrinsic Signal-to-Noise Ratio in MRI: Theory and Applications


INFN – Pisa (Italy), September 22, 2016

# NYU Center for Biomedical Imaging

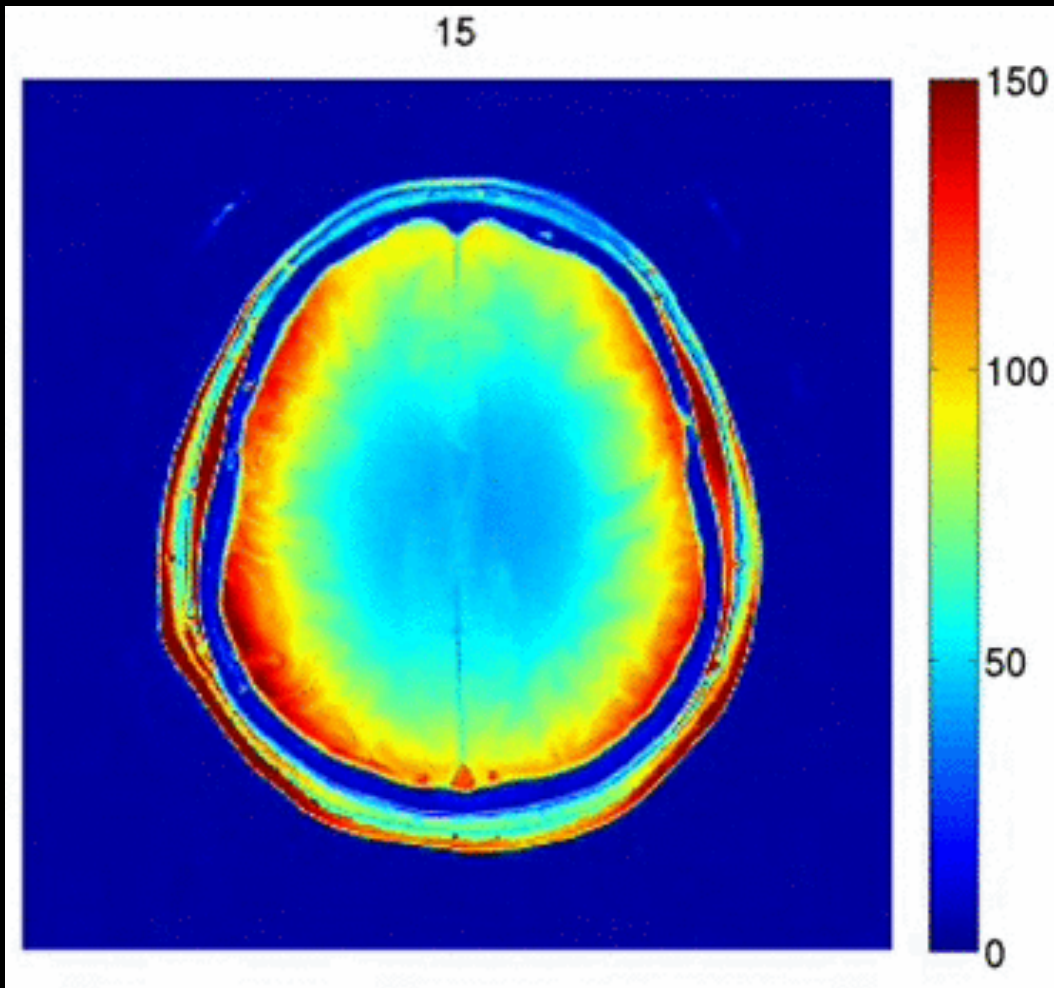




# Receive Arrays Are Critical in MRI

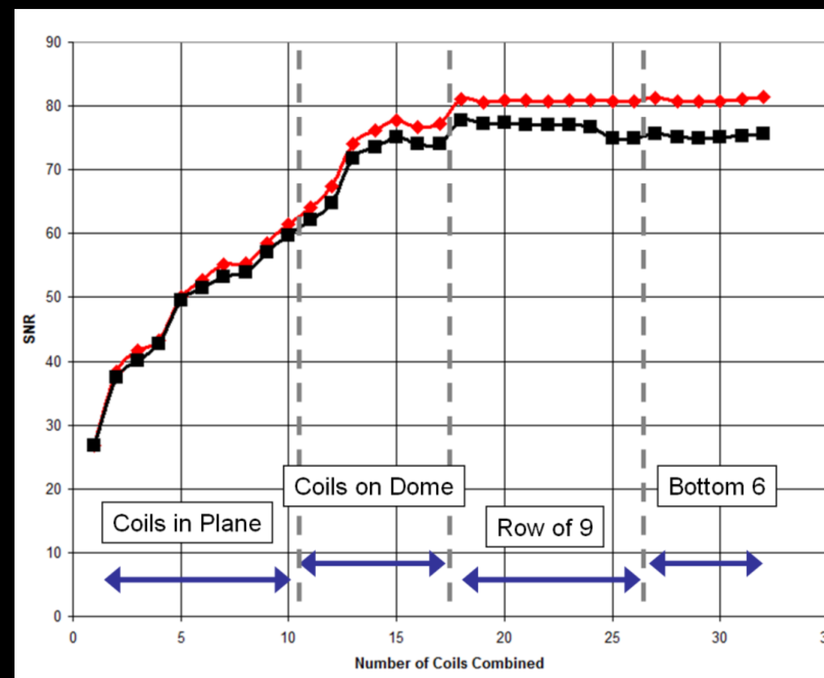
- Advantages
  - SNR
  - Speed (parallel MRI)
  - Volumetric coverage
  - Image quality
  - Simplicity
- Disadvantages
  - Cost
  - Complexity
  - Data load
  - …

➡ How many elements do we need?

# How to Increase Central SNR with an Array



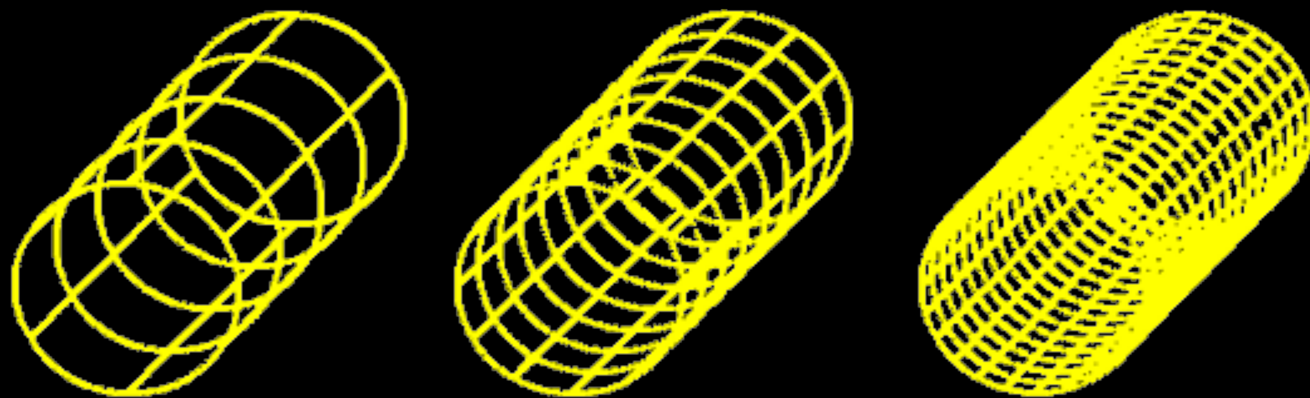

Central SNR vs. # of Active Elements

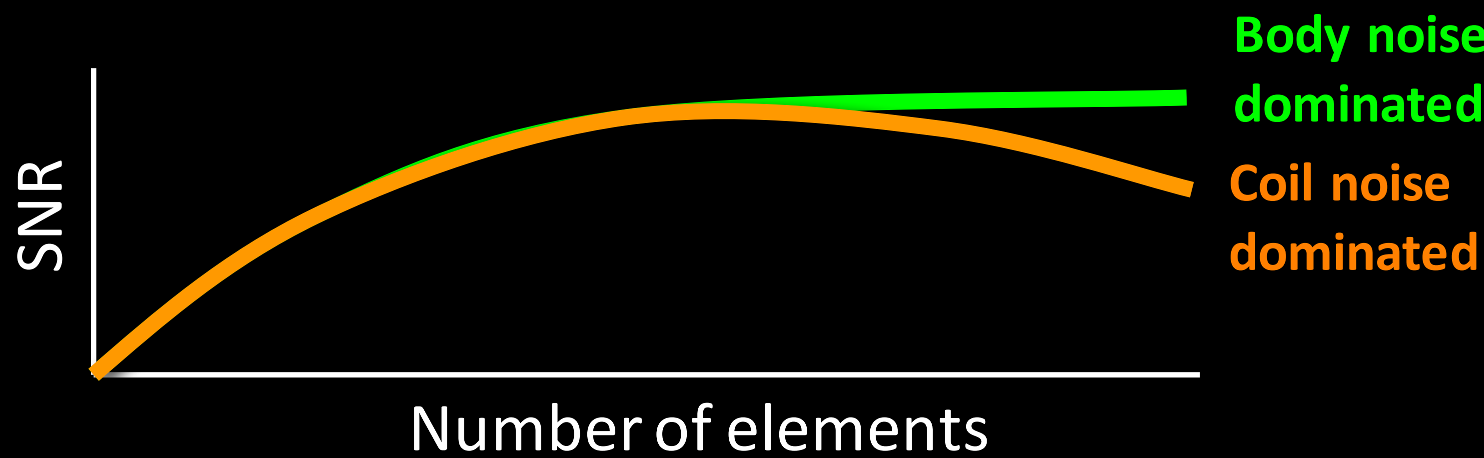



*Courtesy of Graham Wiggins (NYU)*

The best central SNR is achieved when the object is fully surrounded by coils

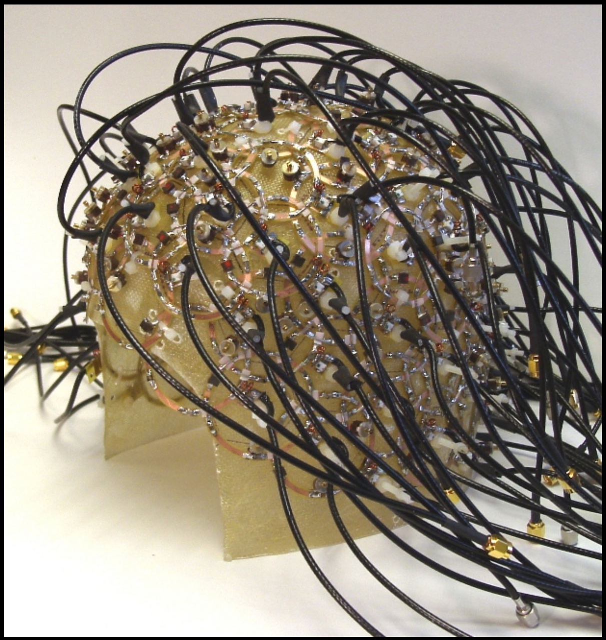
# SNR at Depth

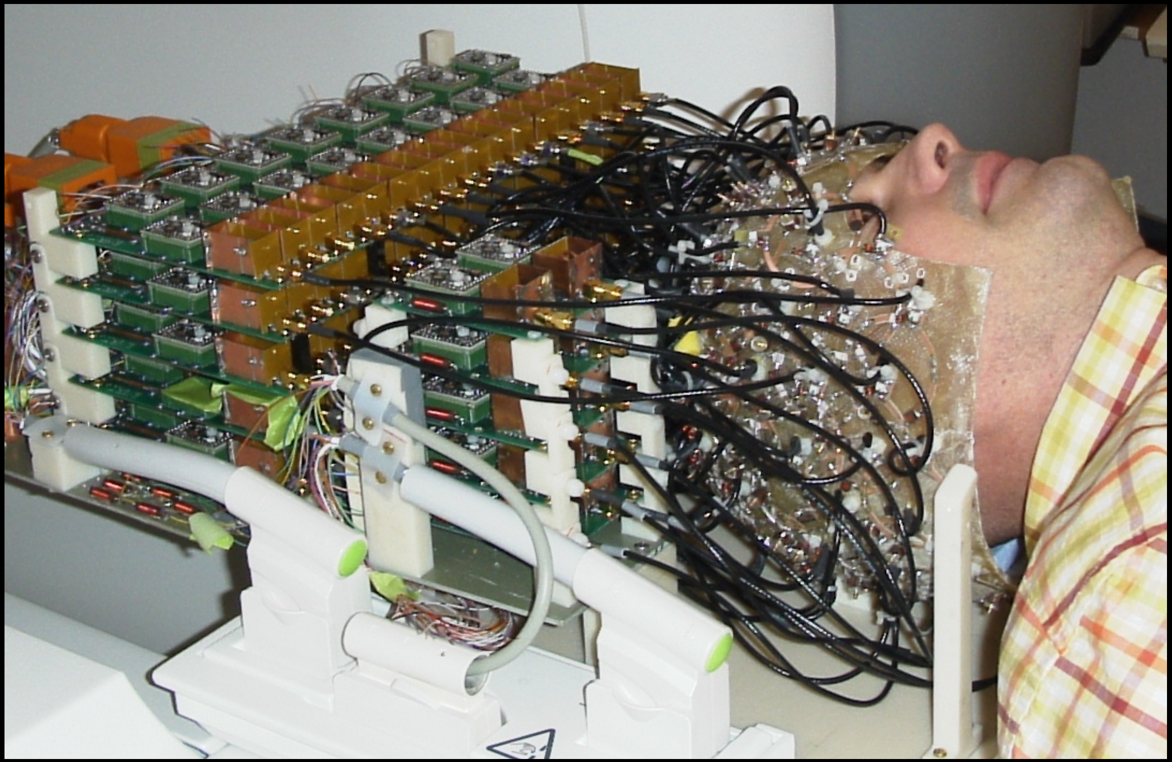





more coils are better up to a certain point !

# 96-Element Head Array





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*G.C. Wiggins et al., MRM 2009, vol. 62(3) p. 754*

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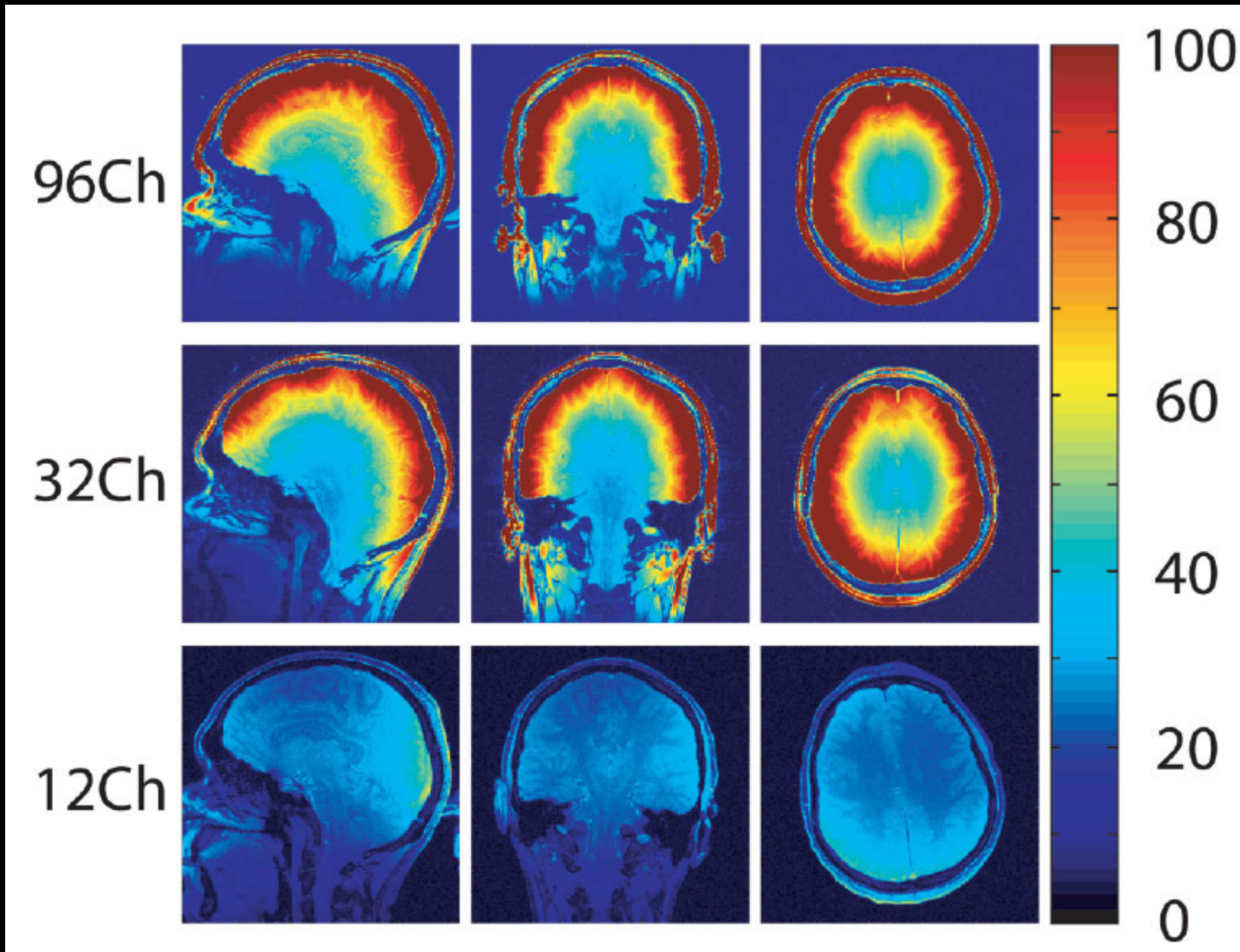
# Head Array SNR Comparisons




Courtesy of Graham Wiggins (NYU)

# Benefits for Parallel Imaging

- Max acceleration = # of detector coils
  - Need more coils to go faster!
- Intrinsic SNR loss
  - Need more coils for multi-dimensional acceleration and volumetric coverage!
- Noise amplifications (G factor)
  - Need more coils for improved encoding capabilities!

# G-Factor

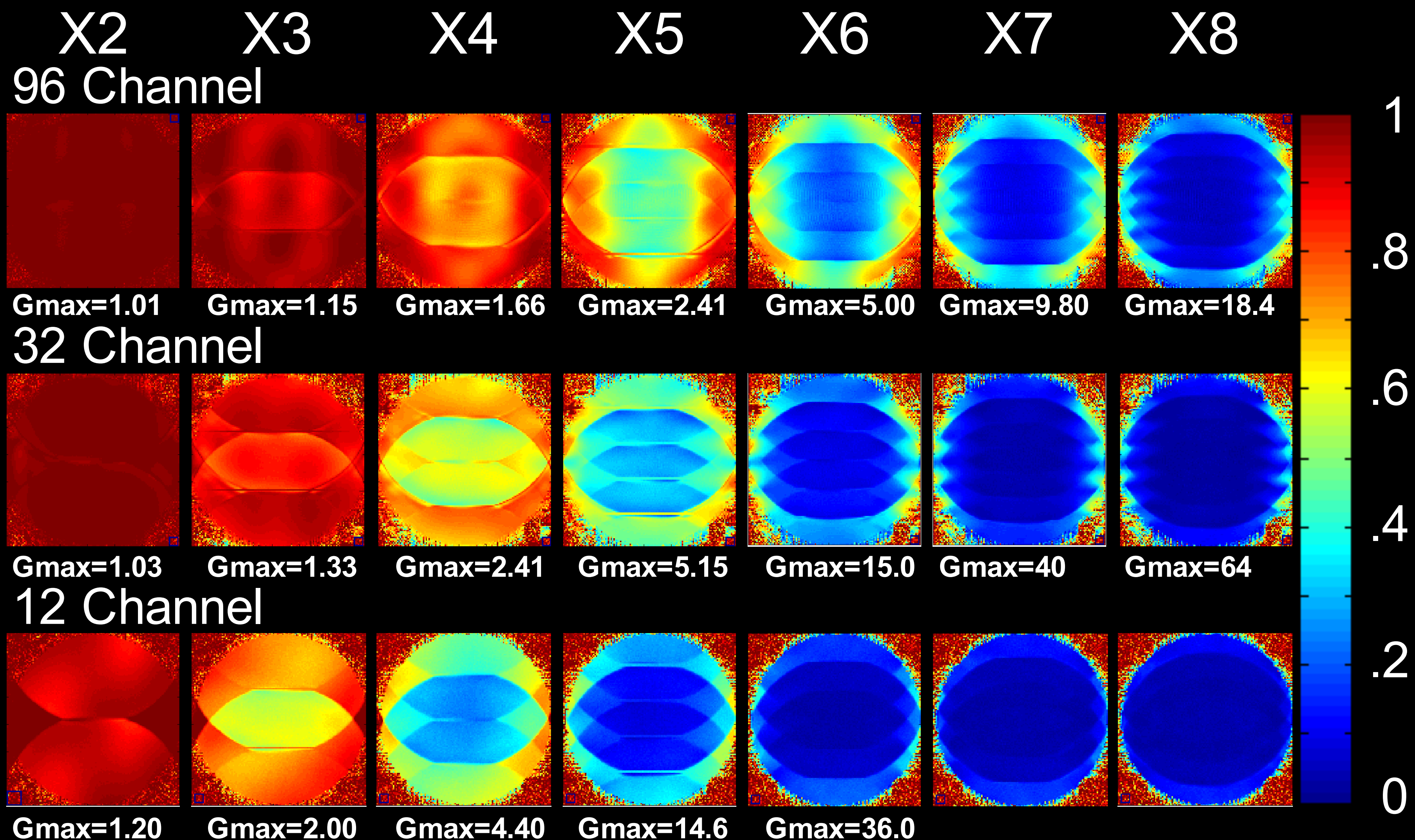
SNR for an accelerated image is defined as

$$SNR_{acc} = \frac{SNR_{full}}{\sqrt{R}\ g} \qquad g = \frac{SNR_{full}}{\sqrt{R}\ SNR_{acc}}$$

Where R = the reduction factor, and g (the "G-factor) is an additional SNR reduction connected with the number and geometrical configuration of the array elements.

We plot the "SNR Loss", or 1/g

# 1/G-Factor vs. Acceleration Rate

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*Courtesy of Graham Wiggins (NYU)*

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# Is SNR Alone Sufficient to Asses a Coil?

- Does it tell us how good is the design?
- Does it tell us how much room for improvement there is?
- Does it tell us if there is a better geometry?

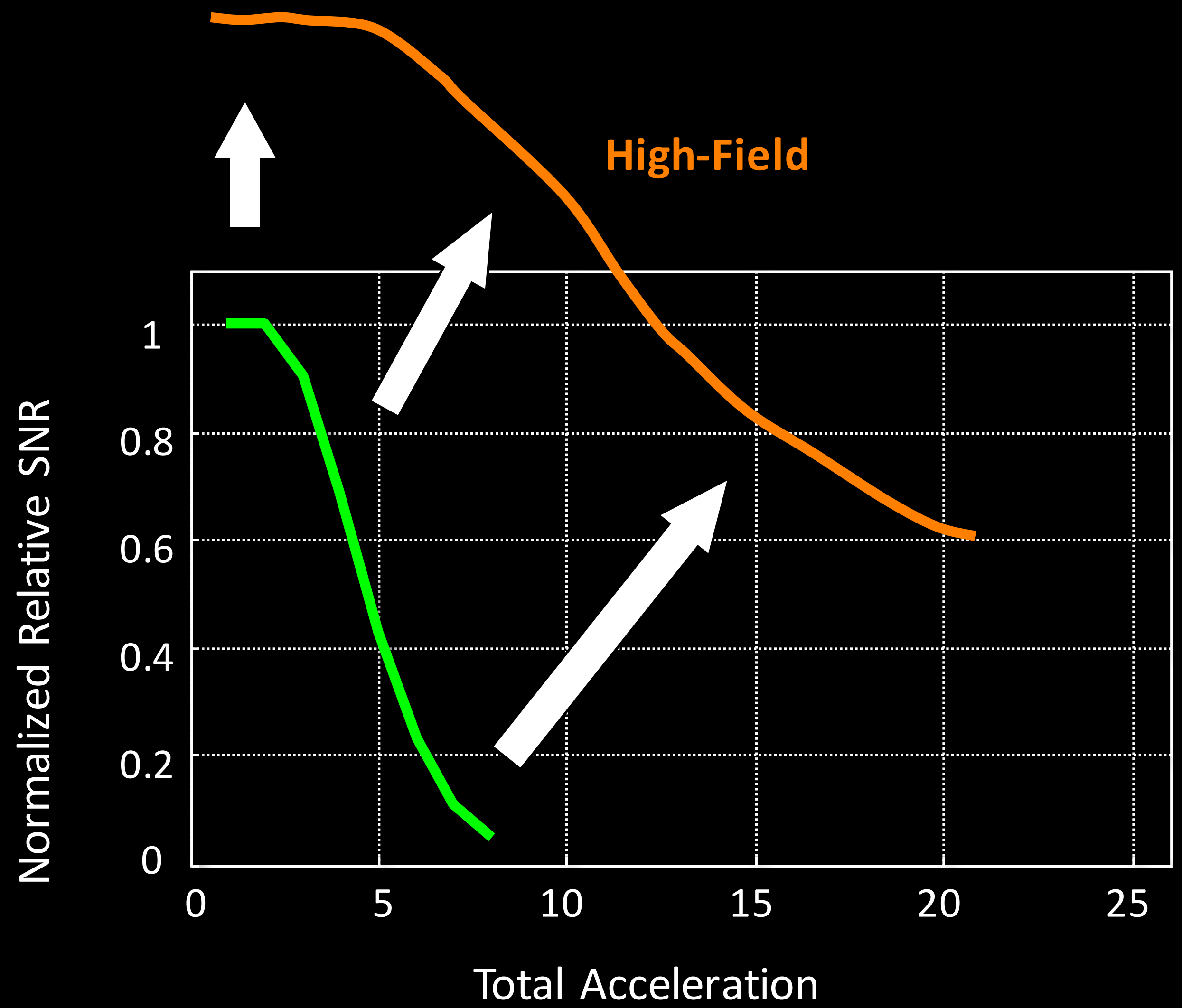
→ Need an absolute reference against which to evaluate the performance of any particular coil array

# Ultimate Intrinsic SNR

- Best possible SNR compatible with electrodynamics
- Computed using a complete set of carefully chosen distinct electromagnetic fields representing an infinite array
- Depends on object geometry and electrical properties
- Independent from coil design

# Is The Sky The Limit?

# Lessons from Ultimate Intrinsic SNR




- Increased baseline SNR
- Slower decline with acceleration

*M.A. Ohliger et al., MRM 2003, vol. 50(5) p. 1018*

# Optimal SNR in Parallel MRI

Intrinsic SNR

$$SNR(\mathbf{r}_n) \sim \frac{\omega_0 M_0 S^{net}(\mathbf{r}_n)}{\sqrt{4k_B T_S \int_V \sigma(\mathbf{r}) \left|\mathcal{E}^{net}(\mathbf{r})\right|^2 dV}}$$

Net coil sensitivity

$$S^{net}(\mathbf{r}_n) = \sum_l \beta_l \left(\mathcal{B}_{l,\mathrm{x}}(\mathbf{r}_n) - i\mathcal{B}_{l,\mathrm{y}}(\mathbf{r}_n)\right)$$

Net electric field

$$\mathcal{E}^{net}(\mathbf{r}_n) = \sum_l \beta_l \mathcal{E}_l(\mathbf{r}_n)$$

Cartesian SENSE reconstruction

minimize $\sum_{l,l'} \int_V \sigma(\mathbf{r}) \beta_l \mathcal{E}_l(\mathbf{r}) \cdot \beta_{l'}^* \mathcal{E}_{l'}^*(\mathbf{r}) dV = \sum_{l,l'} \beta_l \beta_{l'}^* \Psi_{l,l'}$

subject to $S^{net}(\mathbf{r}_{n'}) = \delta_{n,n'}$ $\quad$ *n, n′ = 1,…N*

Optimal weights

$$\tilde{\beta} = (\mathbf{S}^{\mathrm{H}} \mathbf{\Psi}^{-1} \mathbf{S})^{-1} \mathbf{S}^{\mathrm{H}} \mathbf{\Psi}^{-1}$$

# Calculation of Ultimate Intrinsic SNR

Complete set of surface current patterns

↓

Net Electric field

↓

Net Magnetic field

↓

Noise covariance matrix and coil sensitivities

↓

Ultimate Intrinsic SNR

current mode

$$\mathbf{J}^{net}(\mathbf{r}) = \sum_l w_l \mathrm{j}_l(\mathbf{r})$$

Dyadic Green's function

$$\mathcal{E}^{net}(\mathbf{r}) = i\omega\mu_o \int_S G(\mathbf{r},\mathbf{r}')\mathbf{J}^{net}(\mathbf{r})d\mathbf{r}' = \sum_l w_l \mathrm{e}_l(\mathbf{r})$$

$$\mathcal{B}^{net}(\mathbf{r}) = -\frac{1}{i\omega}\nabla\times\mathcal{E}^{net}(\mathbf{r}) = \sum_l w_l \mathrm{b}_l(\mathbf{r})$$

$$\Psi_{l,l'} = \int_V \sigma(\mathbf{r})\mathbf{e}_l(\mathbf{r})\cdot\mathbf{e}^*_{l'}(\mathbf{r})dv$$

$$S_l(\mathbf{r}_n) = (b_{l,x} - ib_{l,x})_{\mathbf{r}_n}$$

$$\boxed{SNR_{ult}(\mathbf{r}_n) \propto \frac{\omega_o B_o}{\sqrt{4k_B T_S (\mathbf{S}\Psi^{-1}\mathbf{S}^H)^{-1}_{n,n}}}}$$

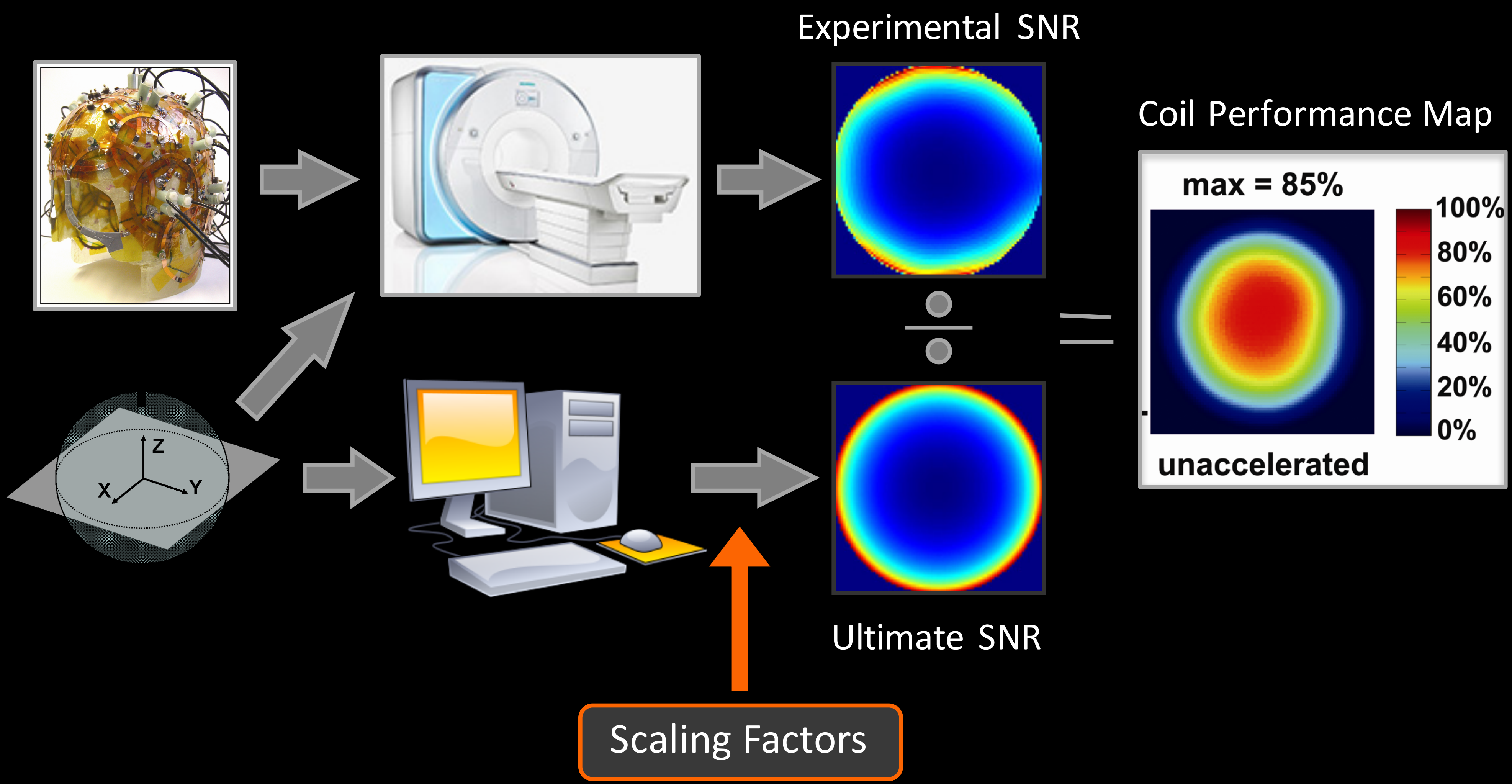
# How Good Are Our Designs?

# How Much Room For Improvement Is There?

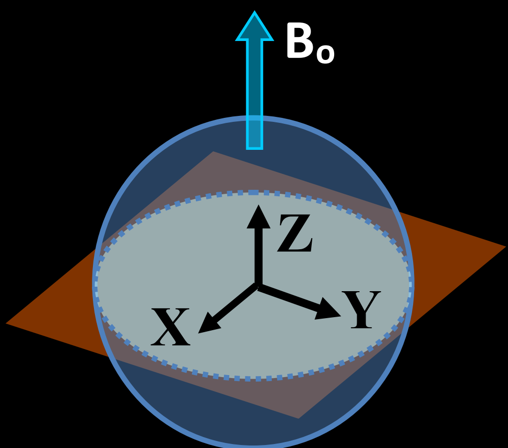
# Coil Performance Maps

- Ultimate intrinsic SNR is independent from any particular coil design, thus it can used as a reference against which the performance of any coil can be evaluated
- Coil performance maps show how closely a coil approaches the best possible SNR
  – Intuitive visual feedback of the quality of a coil
  – Absolute measure of coil performance

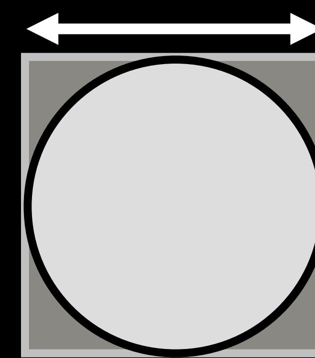
# Coil Performance Maps at a Glance

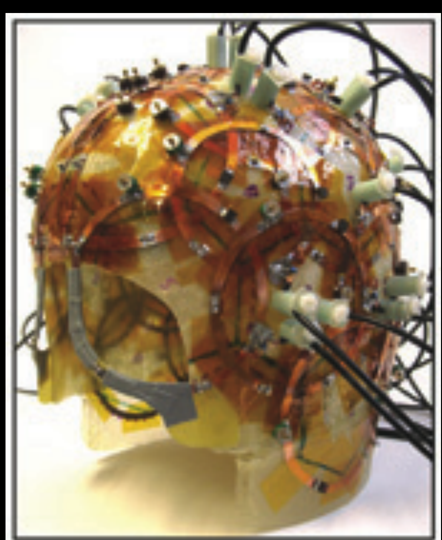


# Coil Performance Maps: Methods




- Braino uniform phantom
- $\sigma = 0.97 \quad \Omega^{-1}\,m^{-1}$
- $\varepsilon_r = 81.3$
- $\mu = \mu_0$



- Siemens TIM Trio 3T
- 32-element head coil
- Long TR
- Short TE

$$\frac{V_{voxel}\sqrt{N_{acq}NEX}\,\sin(\theta)}{F\sqrt{\Delta f}}$$

$F$ = noise factor (measured using "hot/cold resistor" method)

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*R. Lattanzi et al., NMR in Biomed 2010, vol. 23(2) p. 142*

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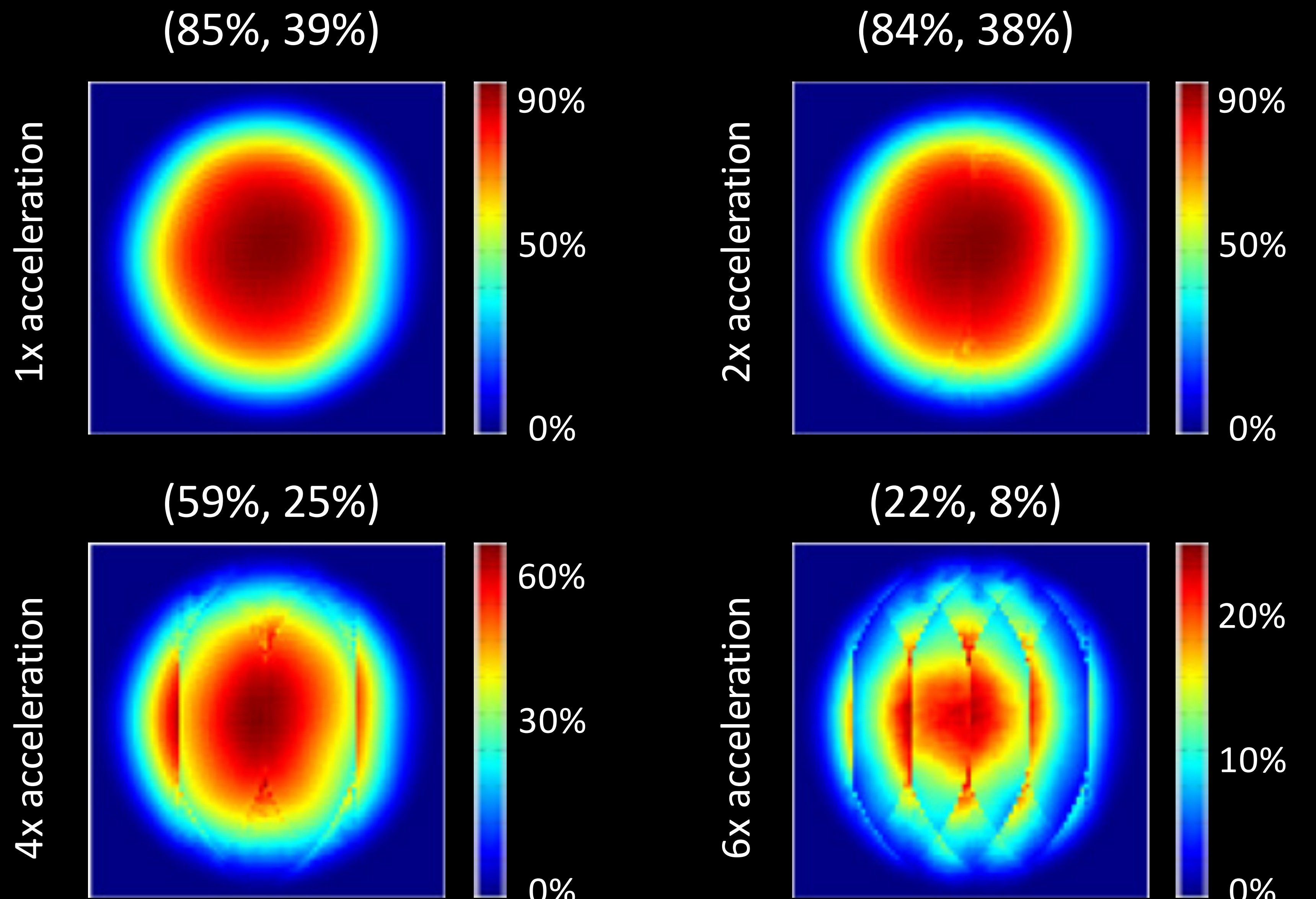
# Coil Performance Maps: Results

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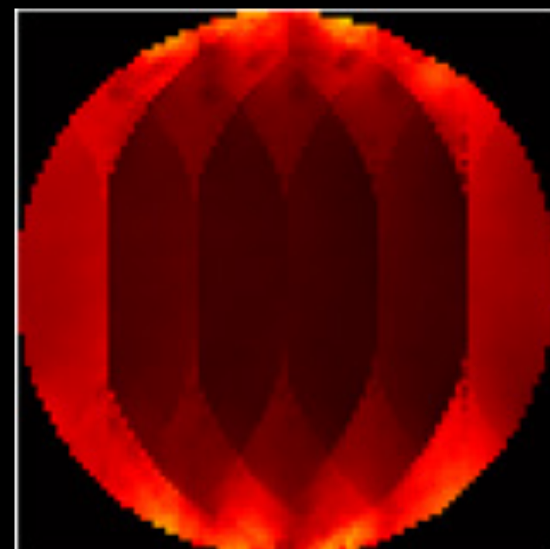
# g-Factor Performance Maps

$g^{\text{ultimate}} / g^{\text{array}}$

(100%, 98%) (89%, 62%) (66%, 19%)

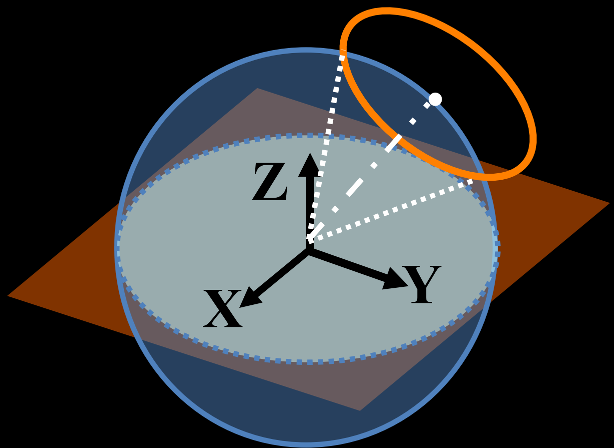


100%
80%
60%
40%
20%
0%

2x acceleration 4x acceleration 6x acceleration

# Can We Approach
# The Ultimate Intrinsic SNR?

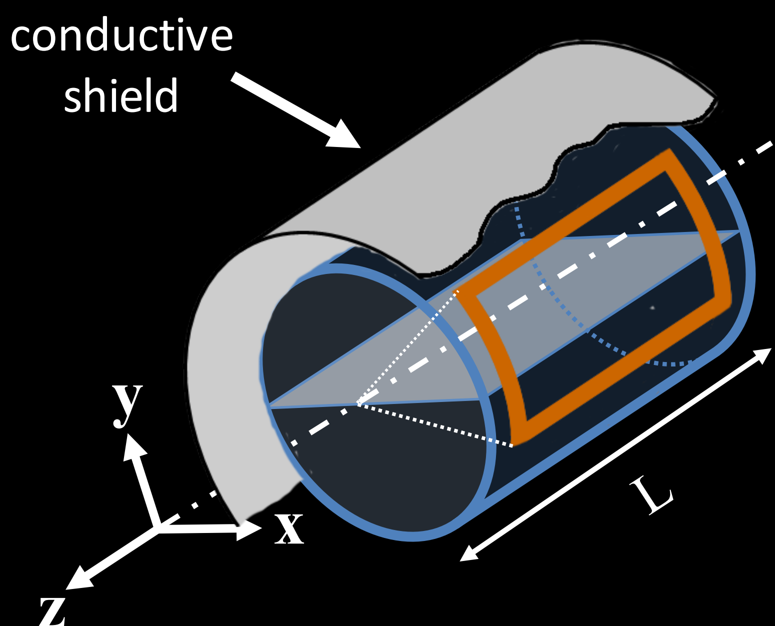
# Methods




- Radius = 10 cm
- Surface current distribution defined on a sphere with radius = 10.5 cm

$$\mathbf{J}^{net}(\theta,\varphi)=\sum_{l=0}^{l_{\max}}\sum_{m=-l}^{+l}\Big(\underbrace{W_{l,m}^{(M)}\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{divergence-free}}+\underbrace{W_{l,m}^{(E)}\hat{\mathbf{r}}\times\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{curl-free}}\Big)$$

*R. Lattanzi and D.K. Sodickson, MRM 2012, vol. 68(1) p. 286*




- Length = 40 cm, radius = 15 cm
- Surface current distribution defined on a cylinder with radius = 16 cm

$$\mathbf{J}^{net}(\varphi,z)=\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Big(\underbrace{W_{n}^{(M)}(m)\nabla\times e^{in\varphi}e^{imz}\hat{r}}_{\text{divergence-free}}+\underbrace{W_{n}^{(E)}(m)\nabla e^{in\varphi}e^{imz}}_{\text{curl-free}}\Big)dm$$

*W. Schnell et al., IEEE Trans Ant Prop 2000, vol. 48 p. 418*

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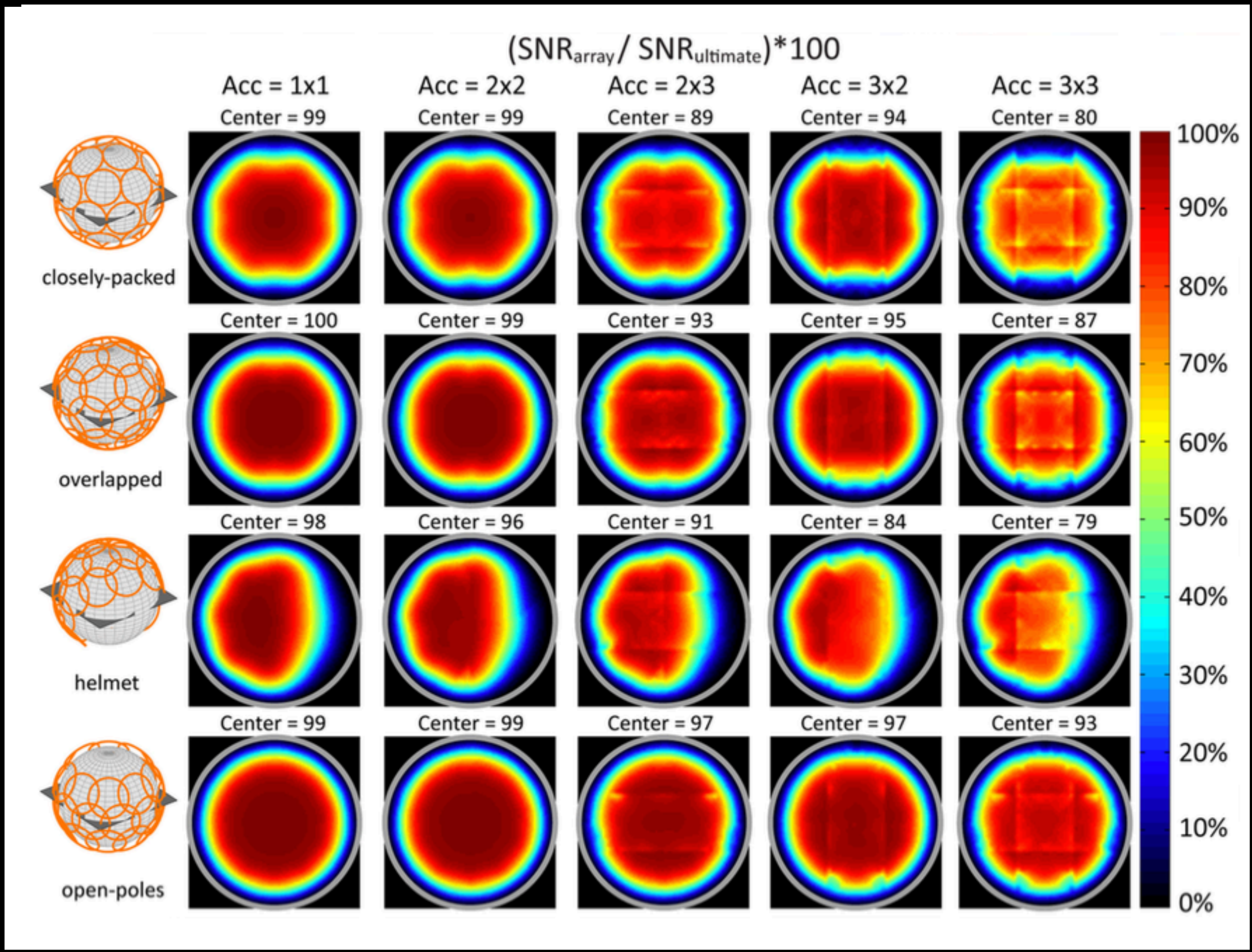
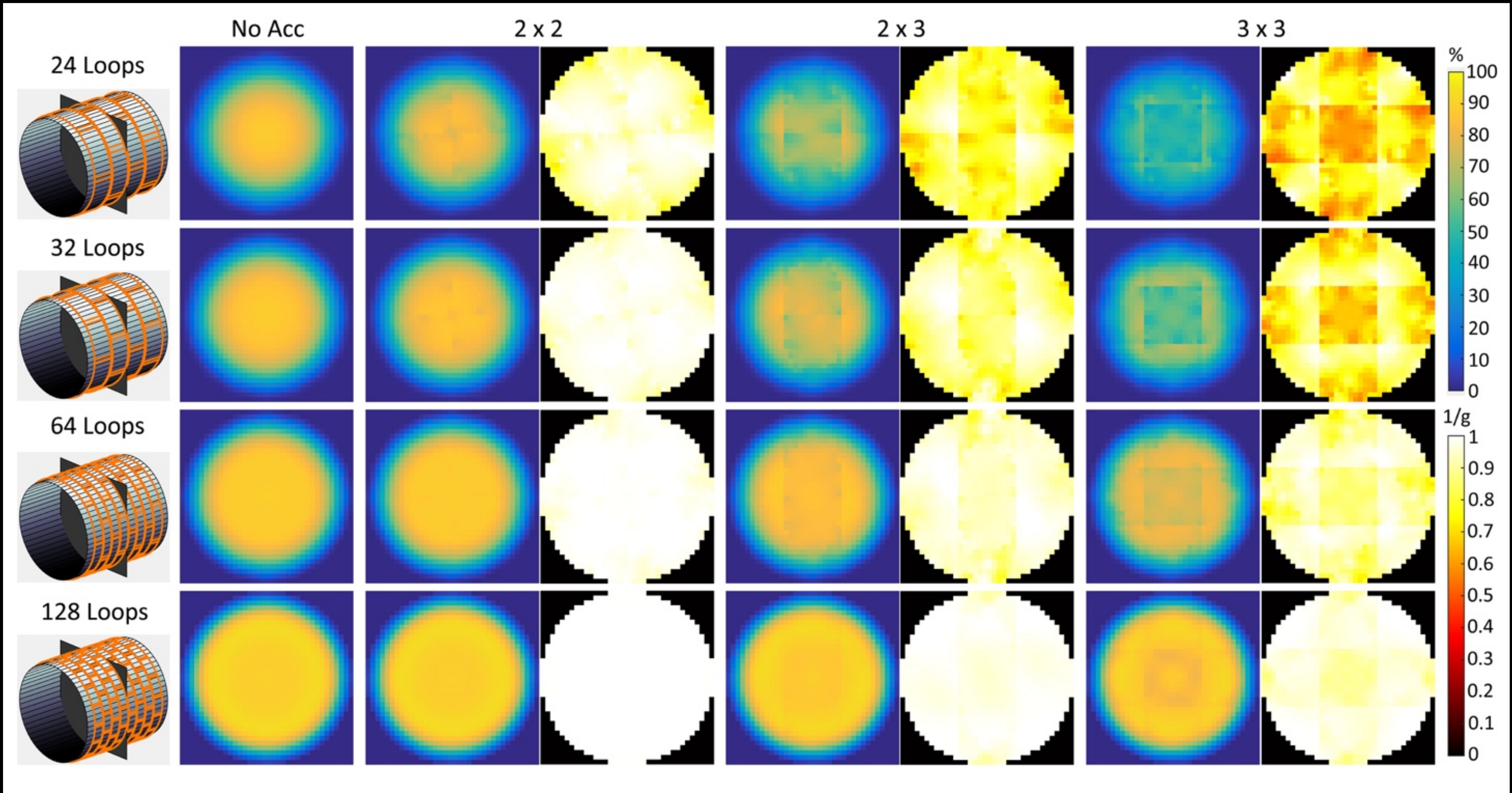
# Approaching Ultimate Intrinsic SNR with Finite Arrays at 3 T



# Approaching Ultimate Intrinsic SNR with Finite Arrays at 3 T

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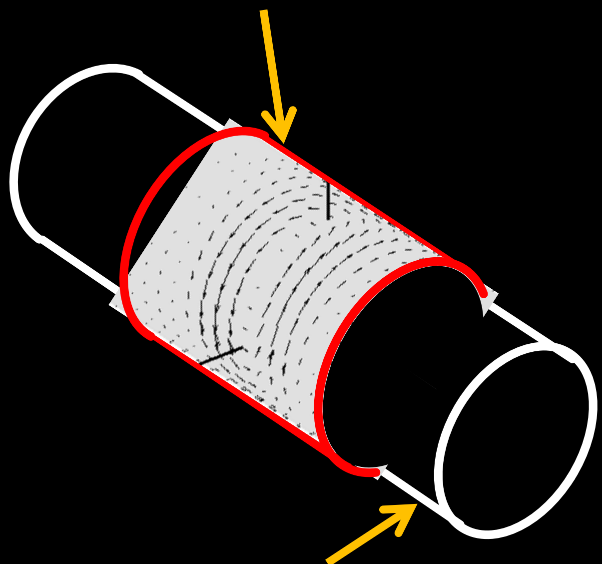
# How Do We Get Closer To The Ultimate Intrinsic SNR?

# Ideal Current Patterns

Current sheet
diameter = 32cm



Diameter = 30cm
$e_r$ = 44.7
s = 0.36
w = 63.9 MHz

- Given the complex weights for each mode in the optimum SNR combination, we can combine the modes to generate the ideal current pattern for maximum SNR at a given point in the sample

*R. Lattanzi and D.K. Sodickson, MRM 2012, vol. 68(1) p. 286*

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# Calculation of Ultimate Intrinsic SNR

Complete set of surface current patterns

Net Electric field

Net Magnetic field

Noise covariance matrix and coil sensitivities

Ultimate Intrinsic SNR

current mode

$$\mathbf{J}^{net}(\mathbf{r}) = \sum_l w_l \mathrm{j}_l(\mathbf{r})$$

Dyadic Green's function

$$\mathcal{E}^{net}(\mathbf{r}) = i\omega\mu_o \int_S G(\mathbf{r},\mathbf{r}')\mathbf{J}^{net}(\mathbf{r})d\mathbf{r}' = \sum_l w_l \mathrm{e}_l(\mathbf{r})$$

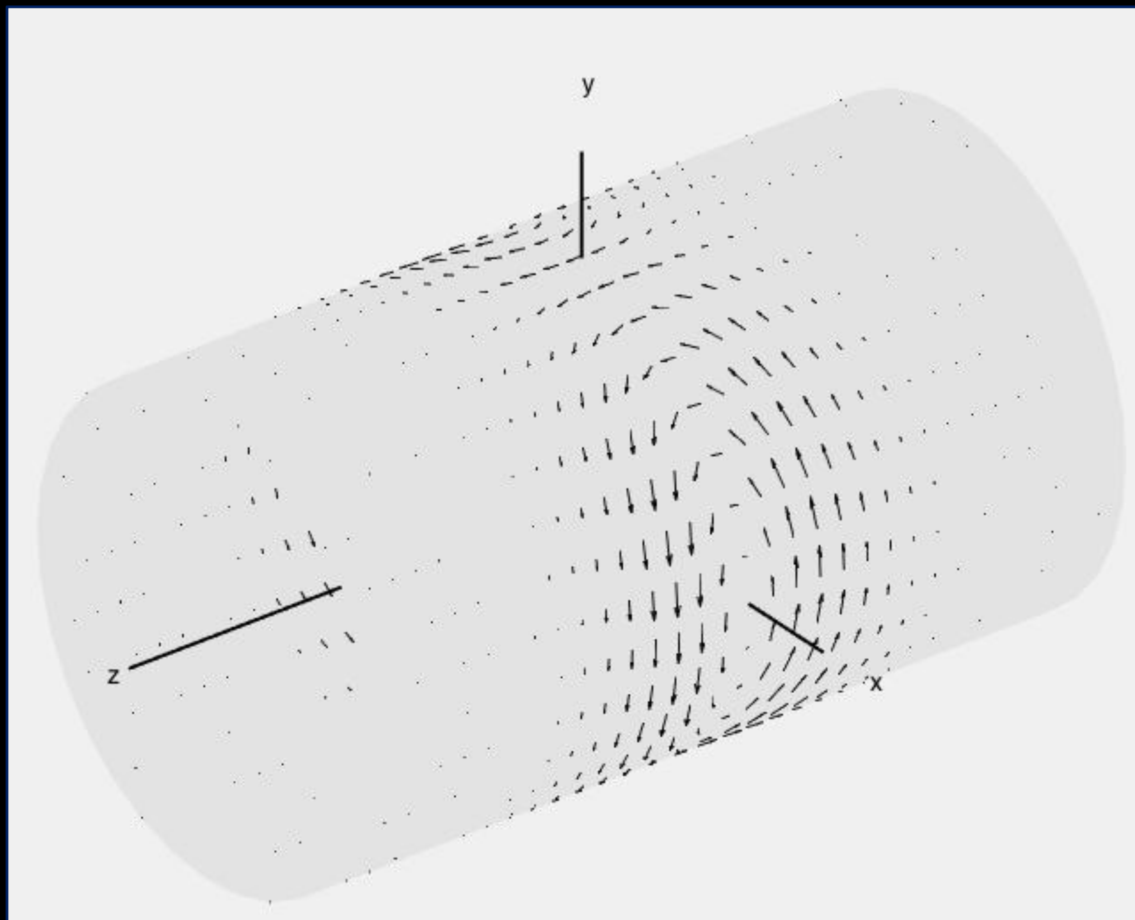
$$\mathcal{B}^{net}(\mathbf{r}) = -\frac{1}{i\omega}\nabla\times\mathcal{E}^{net}(\mathbf{r}) = \sum_l w_l \mathrm{b}_l(\mathbf{r})$$

$$\Psi_{l,l'} = \int_V \sigma(\mathbf{r})\mathbf{e}_l(\mathbf{r})\cdot\mathbf{e}^*_{l'}(\mathbf{r})dv$$

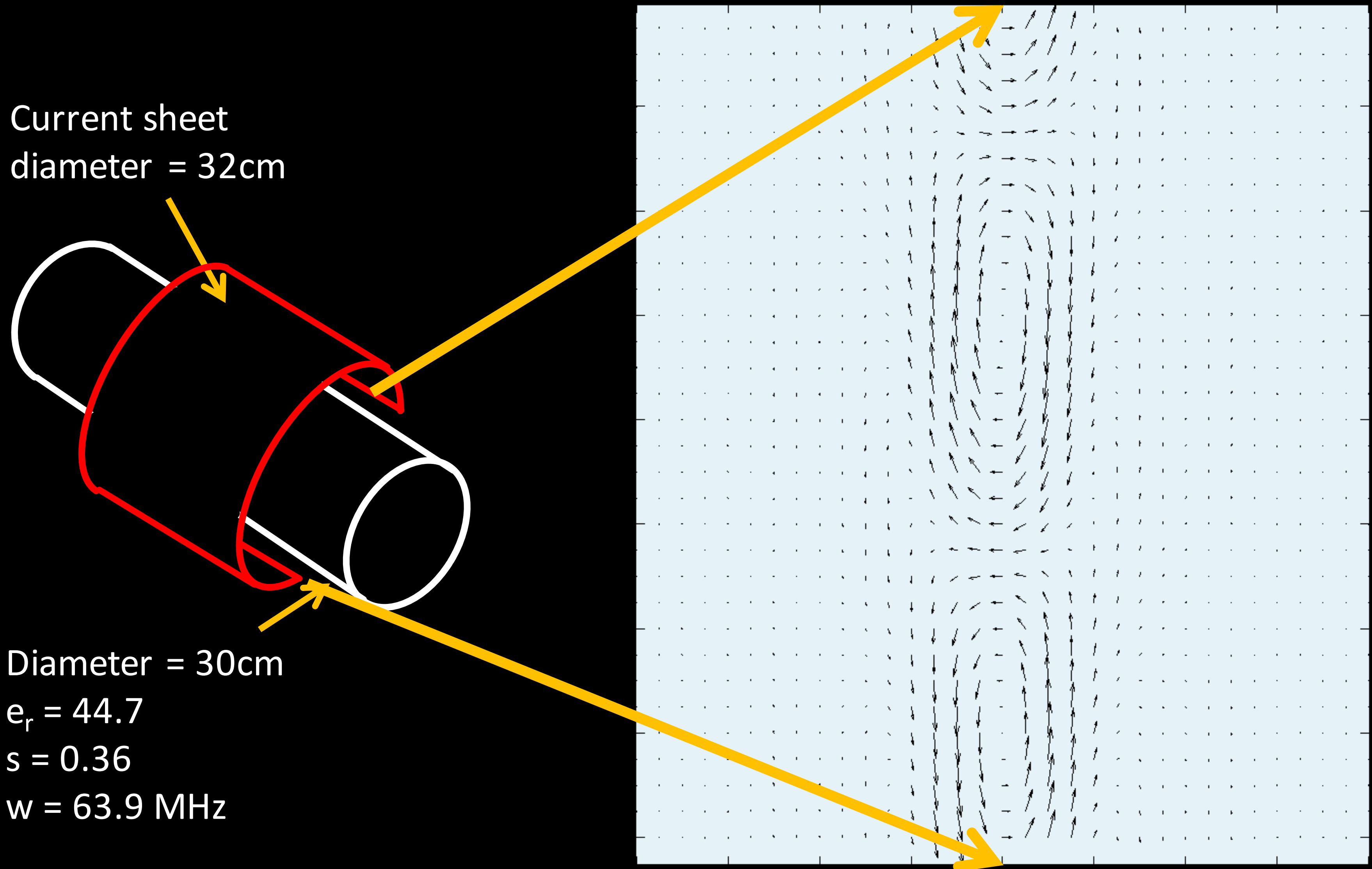
$$S_l(\mathbf{r}_n) = (b_{l,x} - ib_{l,x})_{\mathbf{r}_n}$$

$$\boxed{SNR_{ult}(\mathbf{r}_n) \propto \frac{\omega_o B_o}{\sqrt{4k_B T_S(\mathbf{S}\Psi^{-1}\mathbf{S}^H)^{-1}_{n,n}}}}$$

# Ideal Current Patterns for Central SNR at 0.2 T



# Ideal Current Patterns for Central SNR at 0.2 T

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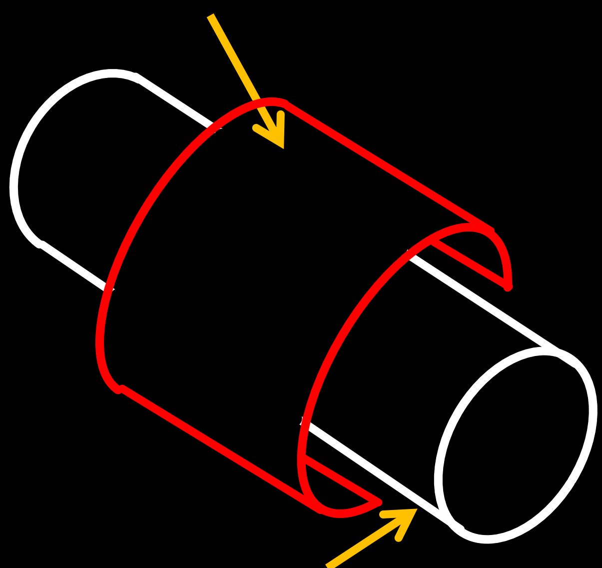
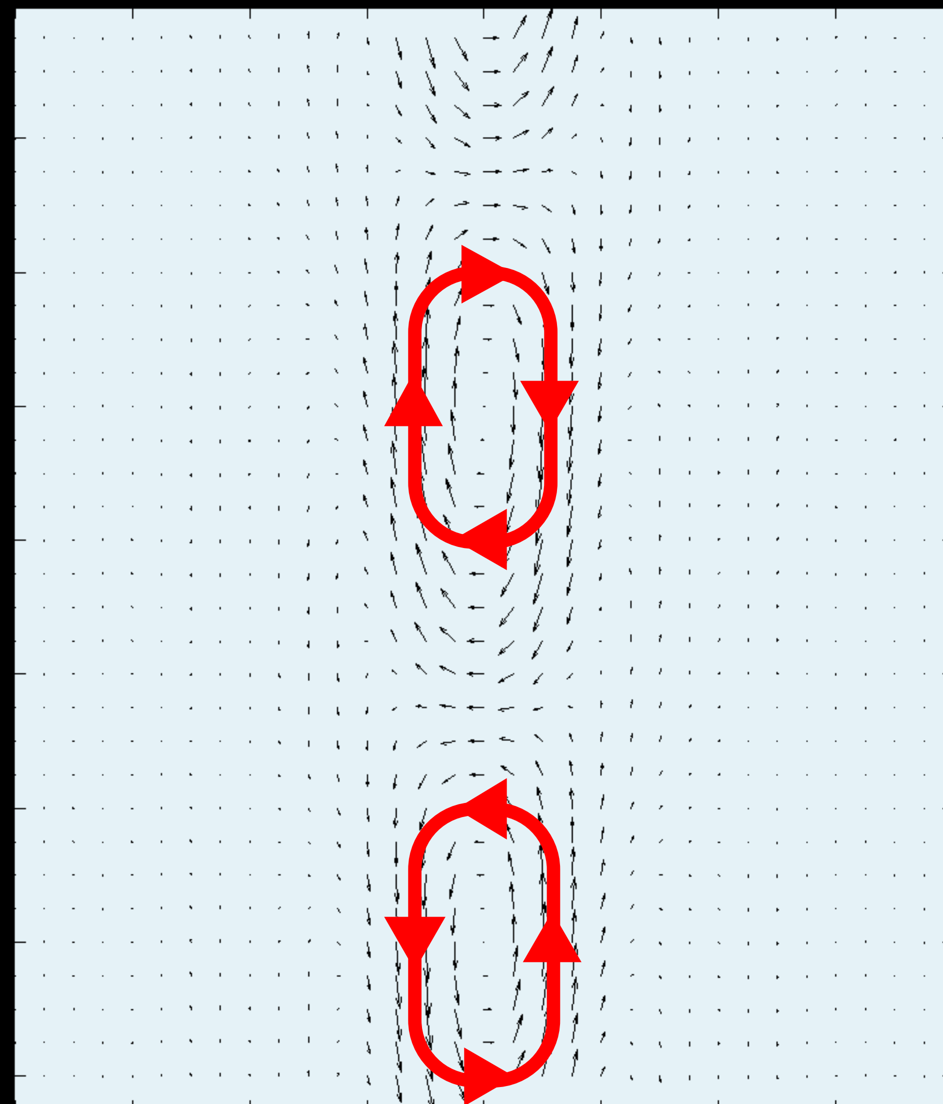
# Ideal Current Patterns for Central SNR at 0.2 T

Current sheet
diameter = 32cm

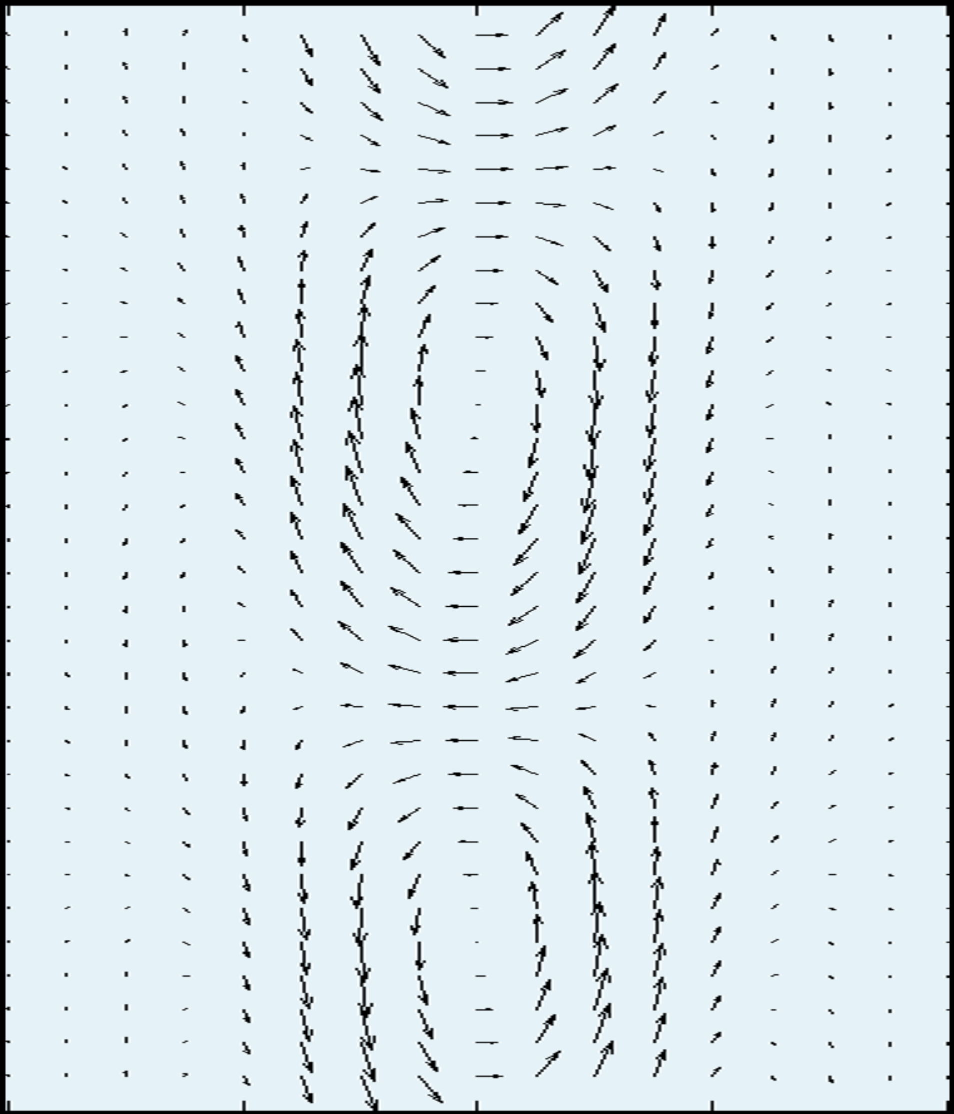


Diameter = 30cm
$e_r = 44.7$
s = 0.36
w = 63.9 MHz



# Ideal Current Patterns for Central SNR at 0.2 T

Expand the current plot along z



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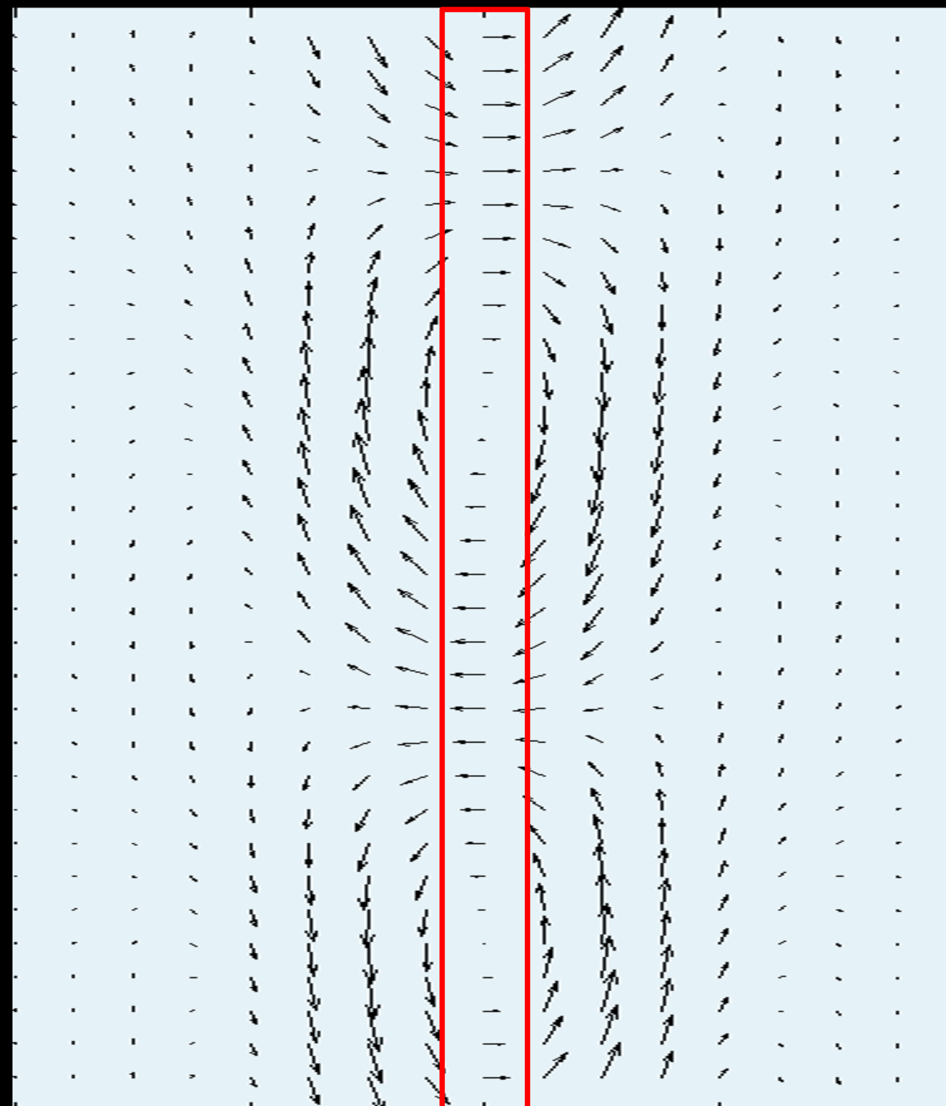
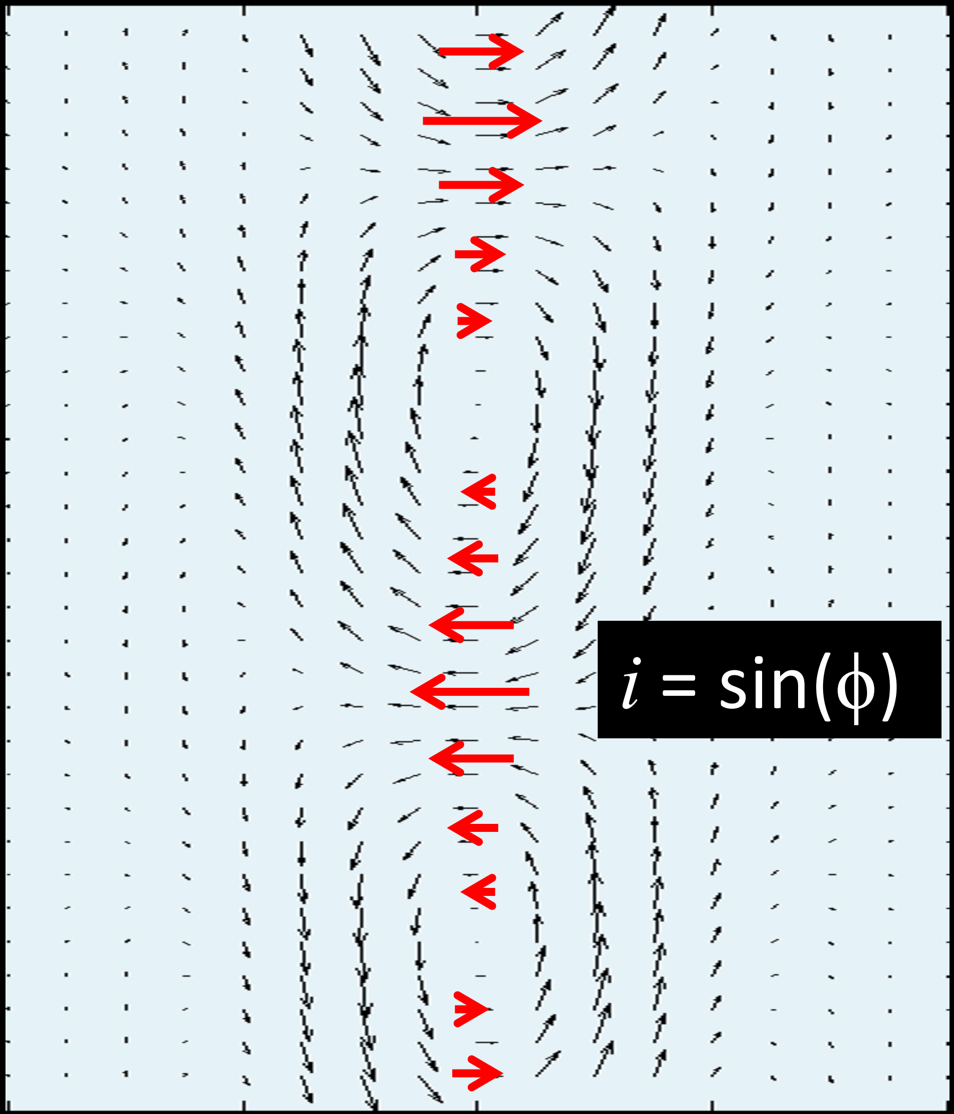
# Ideal Current Patterns for Central SNR at 0.2 T

Examine the current vectors at z = 0



# Ideal Current Patterns for Central SNR at 0.2 T

Current follows a sin(ϕ) function, just like the current in the rungs of a birdcage

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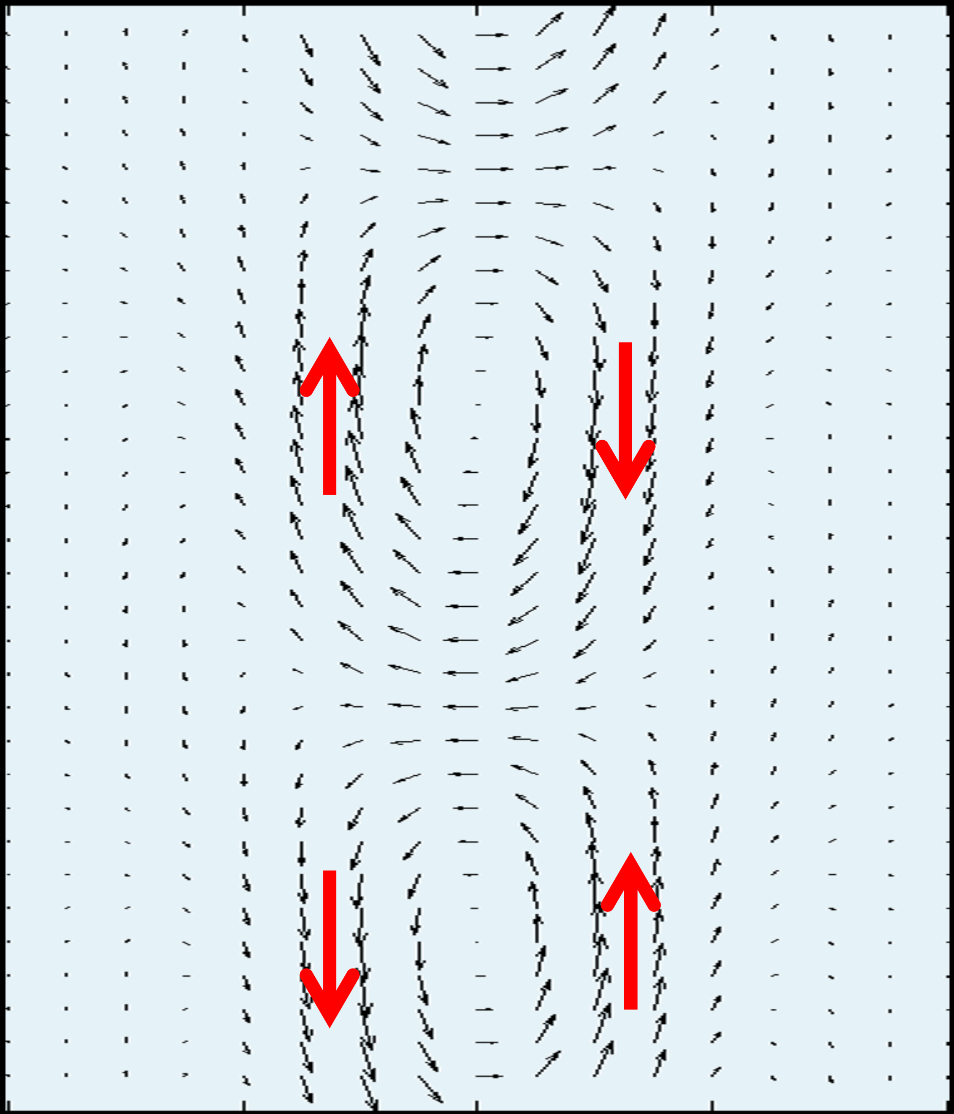
# Ideal Current Patterns for Central SNR at 0.2 T

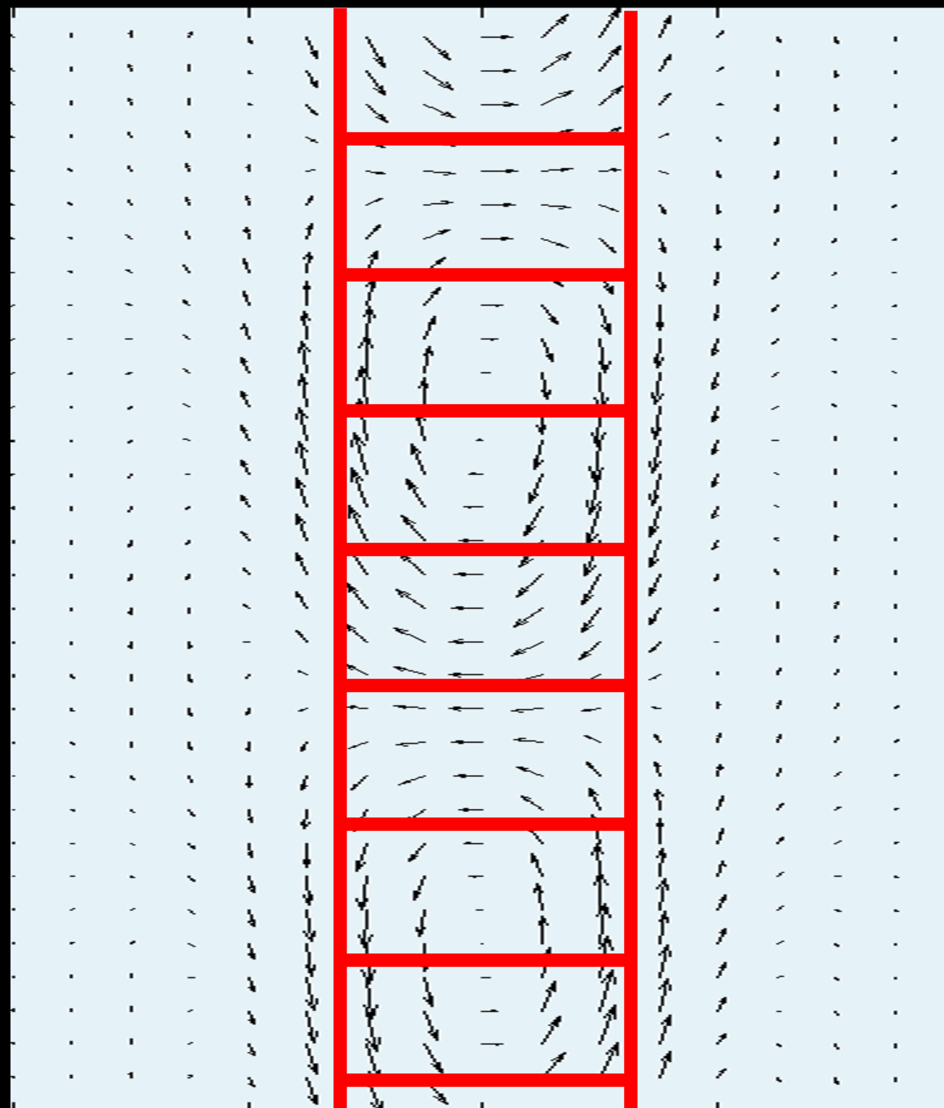
We also see strong azimuthal currents corresponding to end-ring currents in a birdcage



# Ideal Current Patterns for Central SNR at 0.2 T

For central SNR in a cylinder, a birdcage is close to optimal

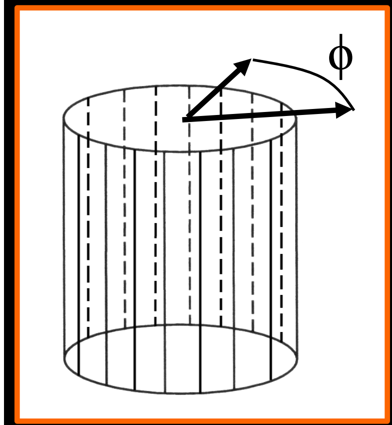
Ideal current patterns even point to optimum length to diameter ratio of ~1/2



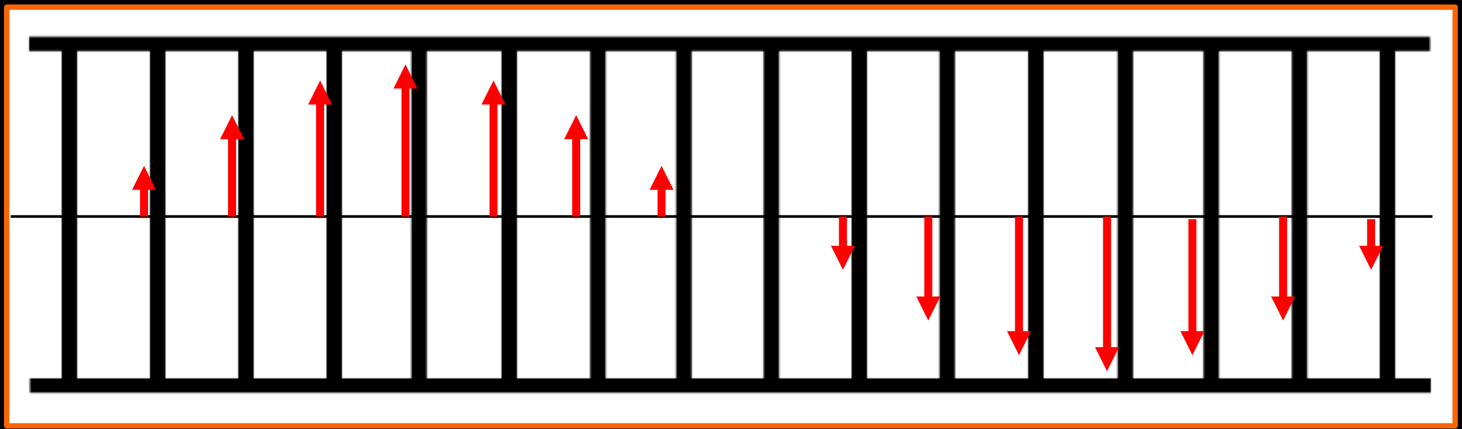
# Birdcage Coil

# Birdcage Modes

Uniform Mode: $I = I_0\sin(\phi)$




Unwrap



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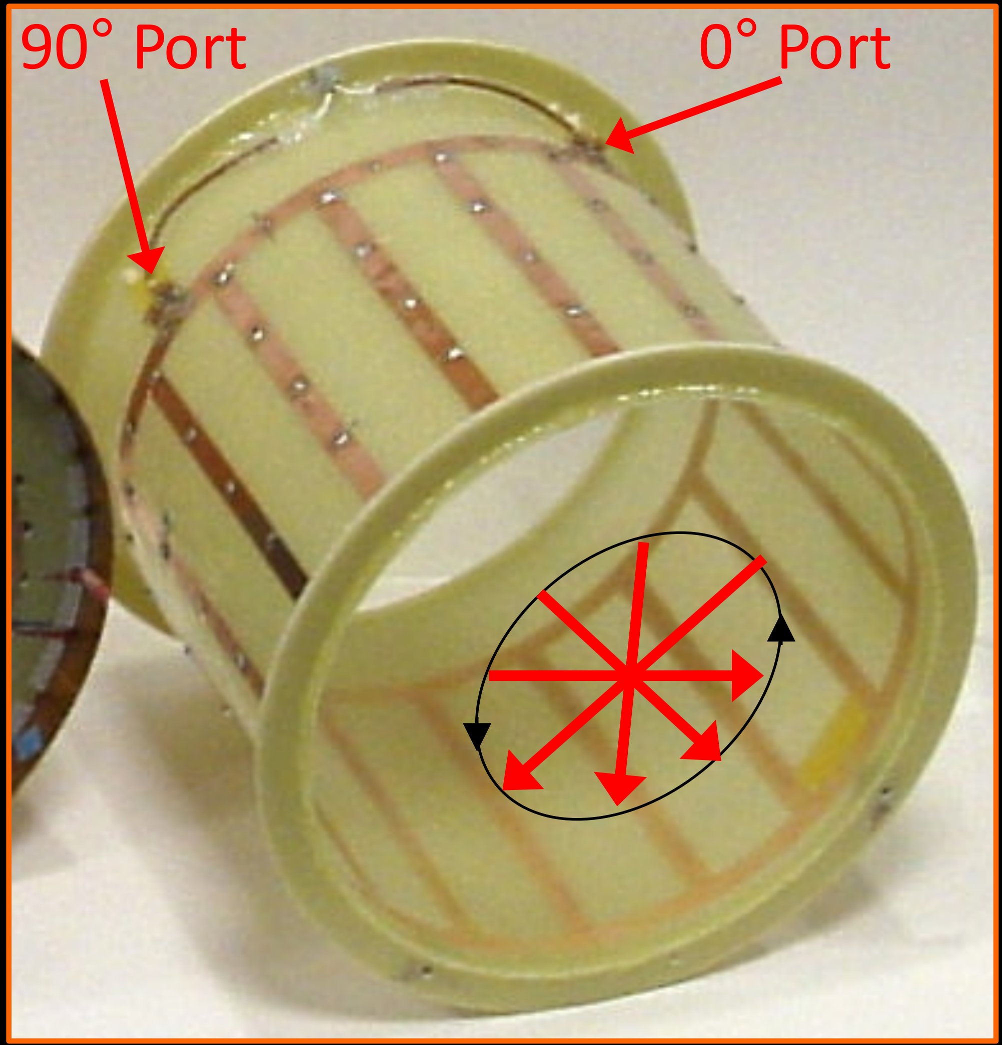
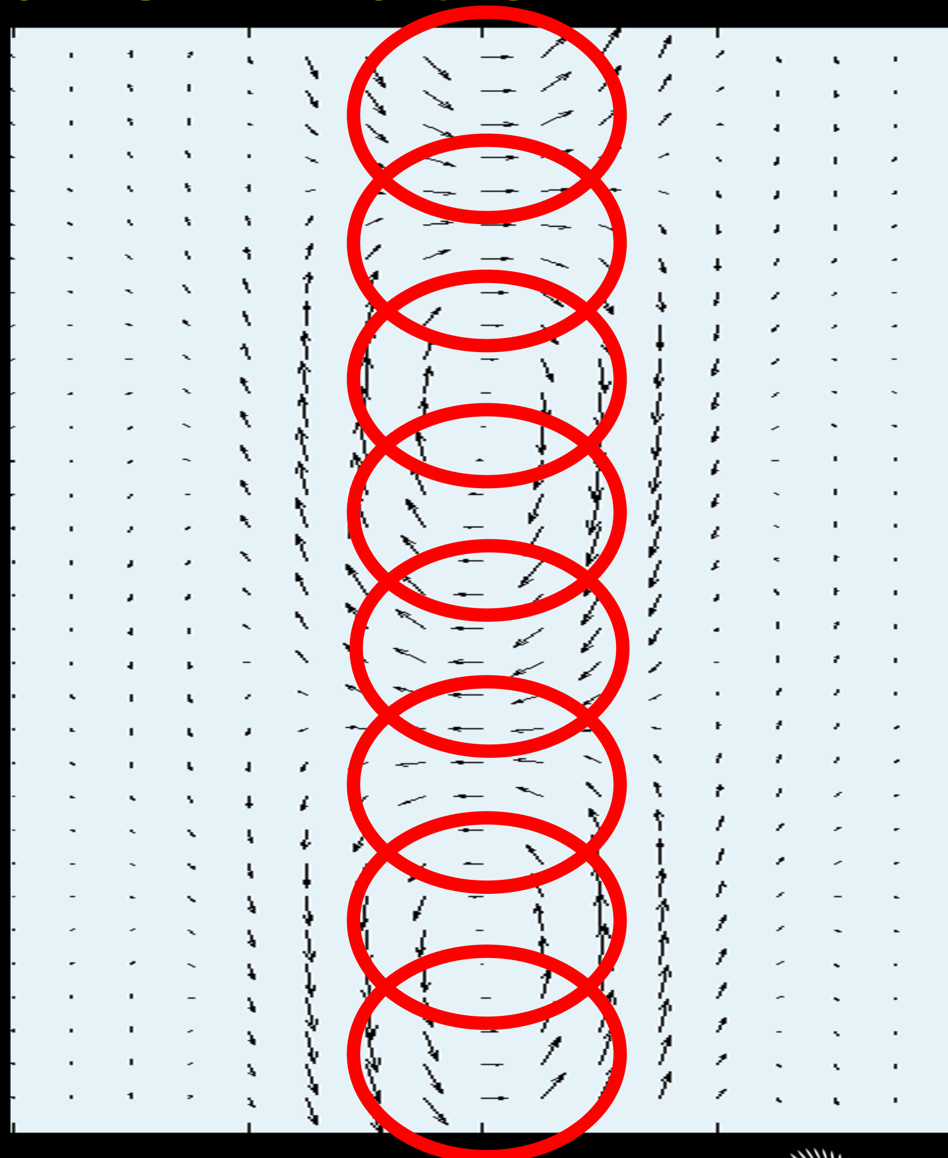
# Birdcage Coil: Quadrature Drive

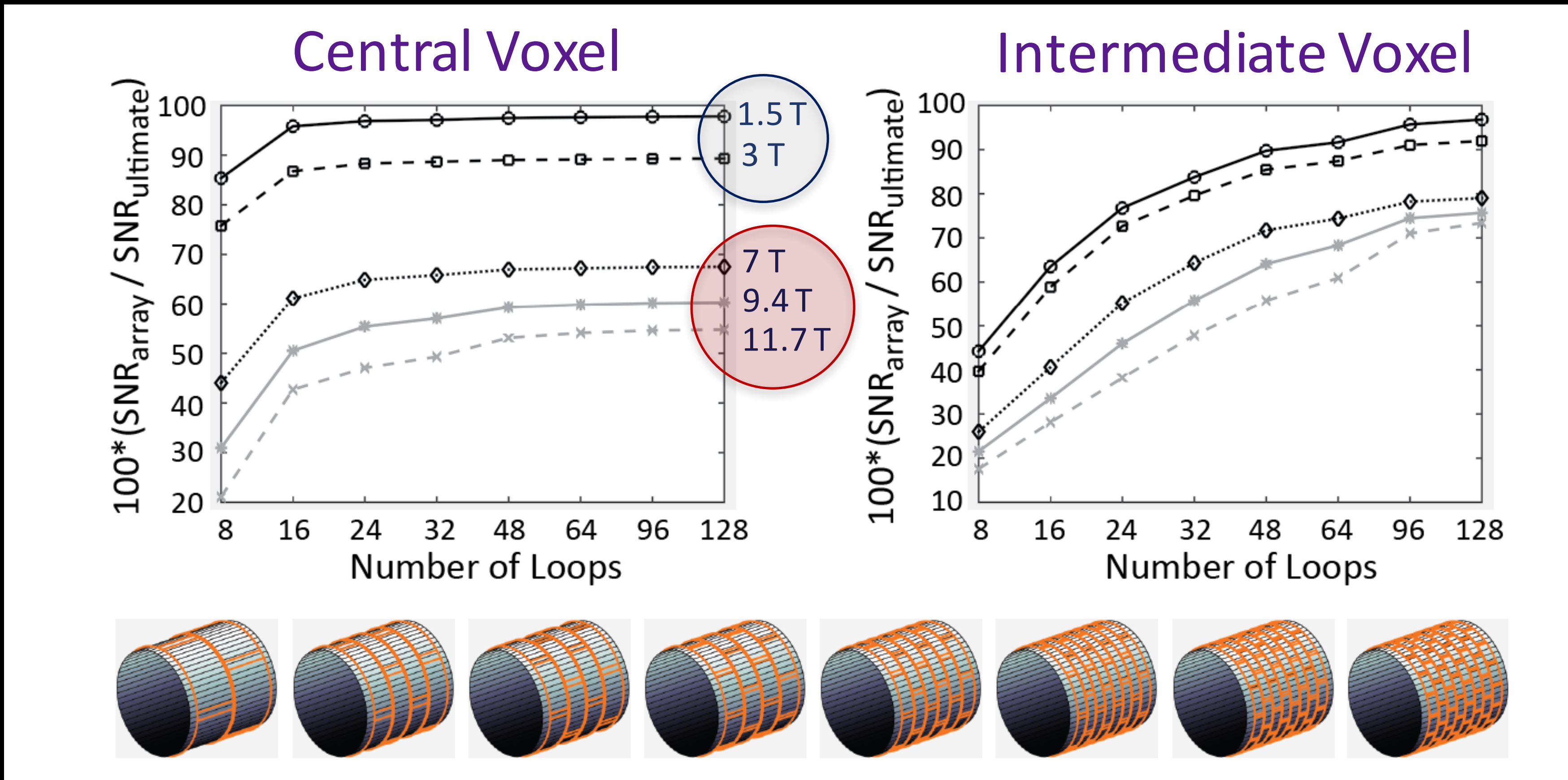


# Ideal Current Patterns for Central SNR at 0.2 T

An array of loops can also provide close to optimal SNR for the center of a cylinder



# Approaching Ultimate Intrinsic SNR with Loops




→ Loop arrays do not approach ultimate SNR at ultra-high field MRI !

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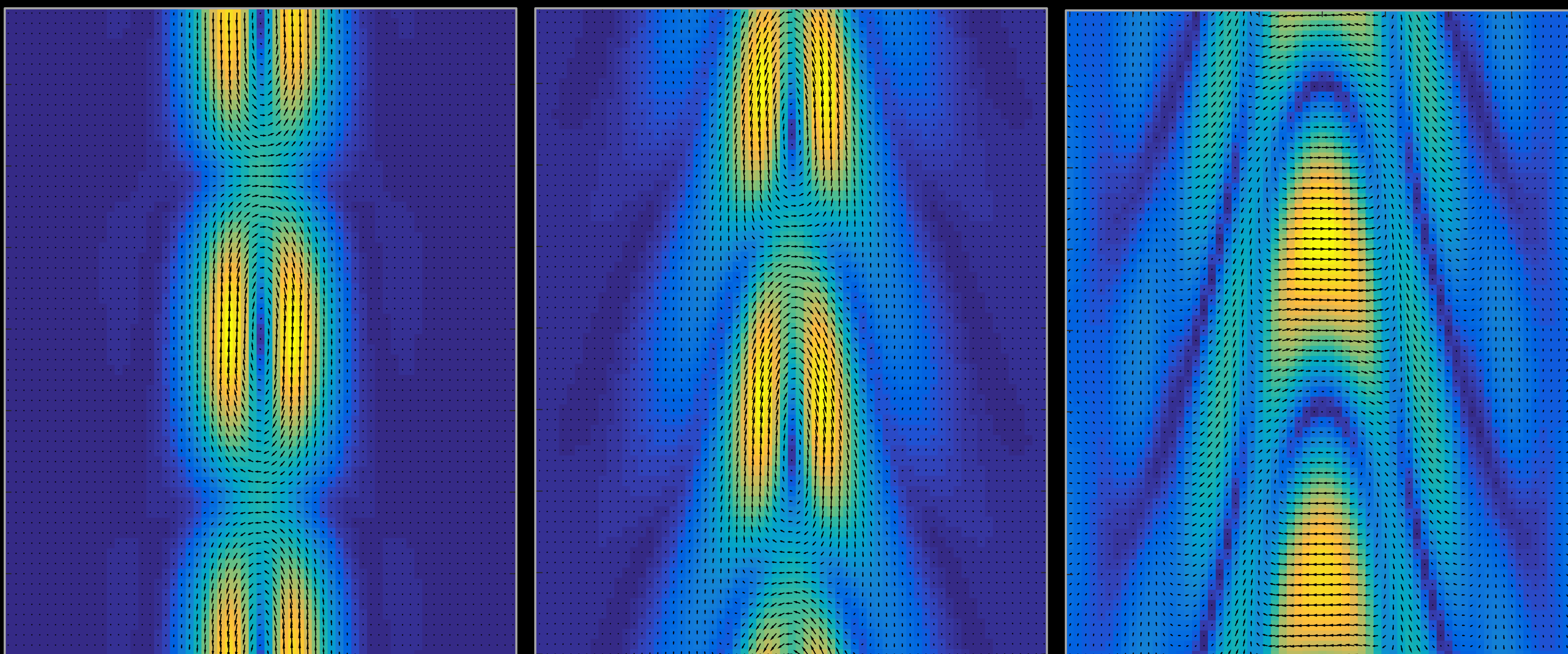
# What Happens to Ideal Current Patterns at High Field?

# Ideal Current Patterns vs. $B_0$

0.2 T | 1.5 T | 7 T



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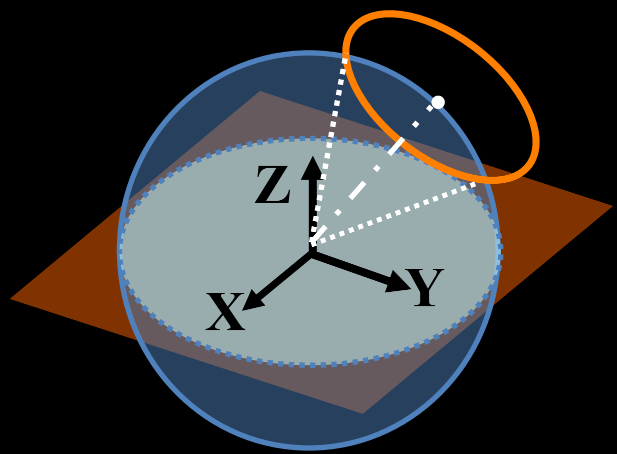
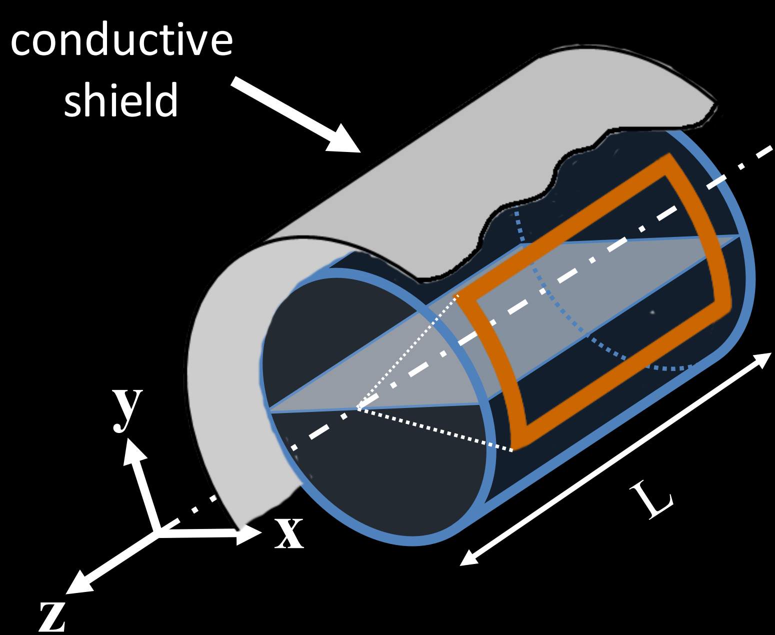
# Methods




- Radius = 10 cm
- Surface current distribution defined on a sphere with radius = 10.5 cm

$$\mathbf{J}^{net}(\theta,\varphi)=\sum_{l=0}^{l_{\max}}\sum_{m=-l}^{+l}\Big(\underbrace{W_{l,m}^{(M)}\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{divergence-free}}+\underbrace{W_{l,m}^{(E)}\hat{\mathbf{r}}\times\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{curl-free}}\Big)$$

*R. Lattanzi and D.K. Sodickson, MRM 2012, vol. 68(1) p. 286*

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*W. Schnell et al., IEEE Trans Ant Prop 2000, vol. 48 p. 418*

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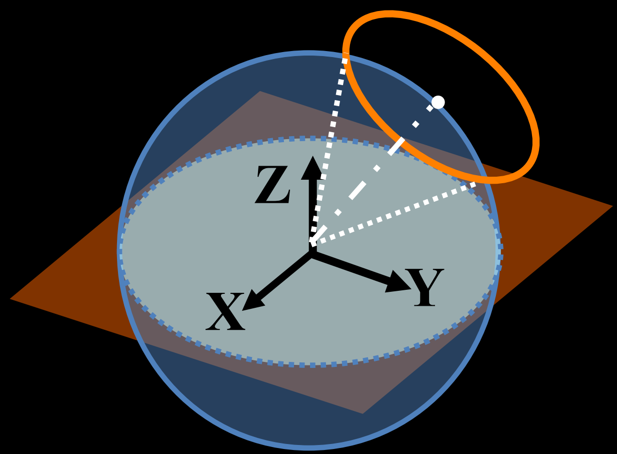
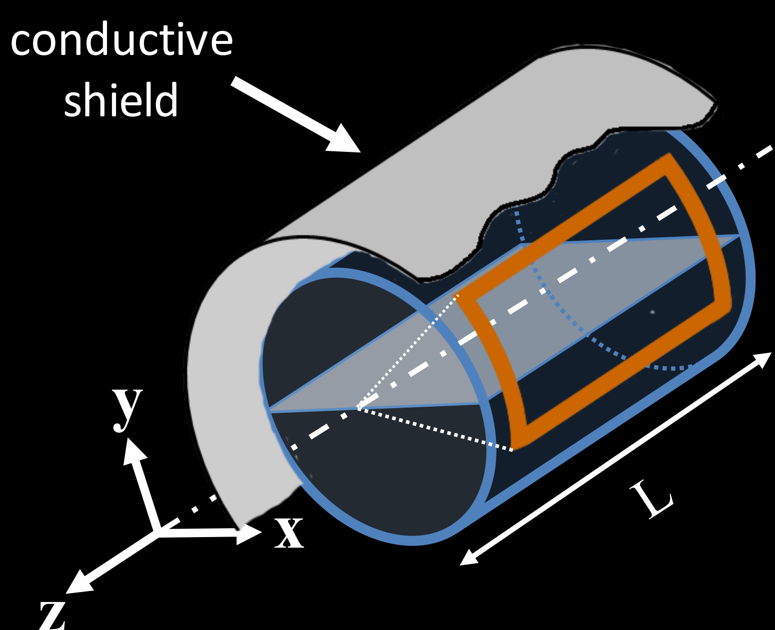
# Methods




- Radius = 10 cm
- Surface current distribution defined on a sphere with radius = 10.5 cm

$$\mathbf{J}^{net}(\theta,\varphi)=\sum_{l=0}^{l_{\max}}\sum_{m=-l}^{+l}\left(\boxed{\underbrace{W_{l,m}^{(M)}\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{divergence-free}}}+\underbrace{W_{l,m}^{(E)}\hat{\mathbf{r}}\times\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{curl-free}}\right)$$

*R. Lattanzi and D.K. Sodickson, MRM 2012, vol. 68(1) p. 286*

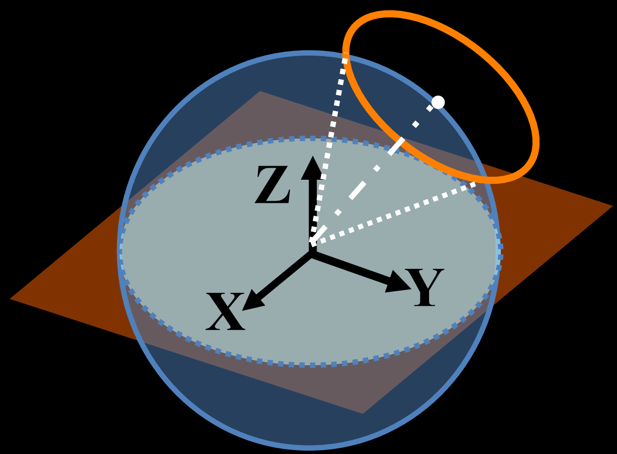



- Length = 40 cm, radius = 15 cm
- Surface current distribution defined on a cylinder with radius = 16 cm

$$\mathbf{J}^{net}(\varphi,z)=\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\boxed{\underbrace{W_{n}^{(M)}(m)\nabla\times e^{in\varphi}e^{imz}\hat{r}}_{\text{divergence-free}}}+\underbrace{W_{n}^{(E)}(m)\nabla e^{in\varphi}e^{imz}}_{\text{curl-free}}\right)dm$$

*W. Schnell et al., IEEE Trans Ant Prop 2000, vol. 48 p. 418*

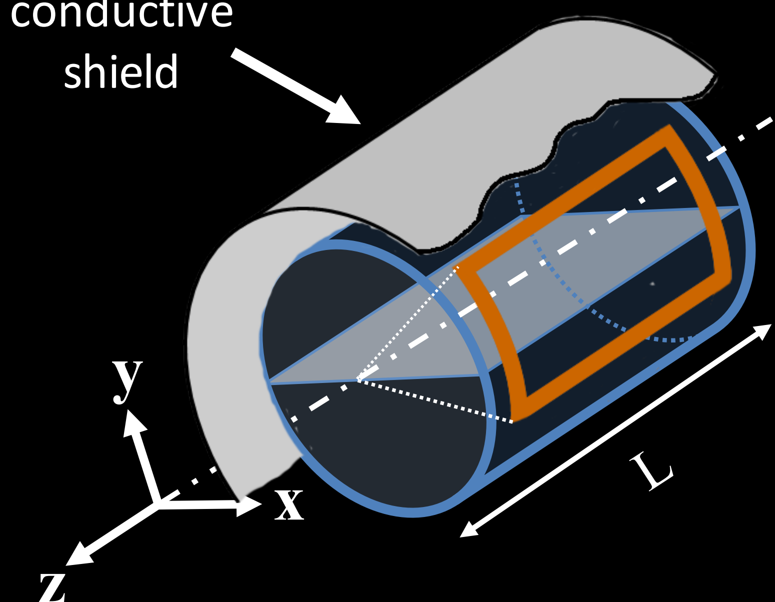
# Methods




- Radius = 10 cm
- Surface current distribution defined on a sphere with radius = 10.5 cm

$$\mathbf{J}^{net}(\theta,\varphi)=\sum_{l=0}^{l_{\max}}\sum_{m=-l}^{+l}\left(\underbrace{W_{l,m}^{(M)}\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{divergence-free}}+\boxed{\underbrace{W_{l,m}^{(E)}\hat{\mathbf{r}}\times\mathbf{X}_{l,m}(\theta,\varphi)}_{\text{curl-free}}}\right)$$

*R. Lattanzi and D.K. Sodickson, MRM 2012, vol. 68(1) p. 286*

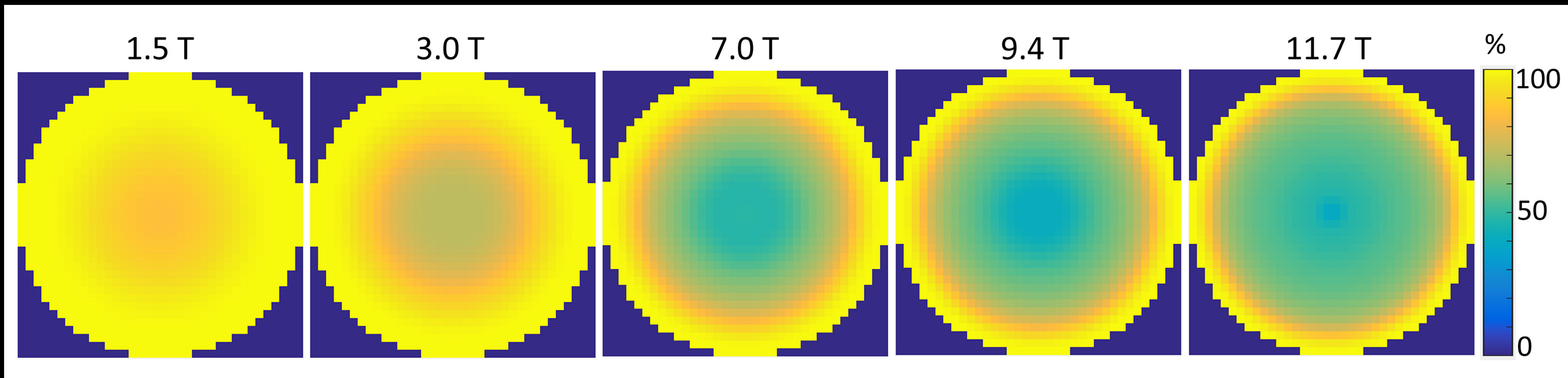



- Length = 40 cm, radius = 15 cm
- Surface current distribution defined on a cylinder with radius = 16 cm

$$\mathbf{J}^{net}(\varphi,z)=\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\underbrace{W_n^{(M)}(m)\nabla\times e^{in\varphi}e^{imz}\hat{r}}_{\text{divergence-free}}+\boxed{\underbrace{W_n^{(E)}(m)\nabla e^{in\varphi}e^{imz}}_{\text{curl-free}}}\right)dm$$

*W. Schnell et al., IEEE Trans Ant Prop 2000, vol. 48 p. 418*

# Divergence-Free Contribution to UISNR




- At low field divergence-free currents (e.g., loops) are sufficient to capture most of the UISNR
- At high field curl-free currents contribution increases
  - Loops are not enough to approach the UISNR

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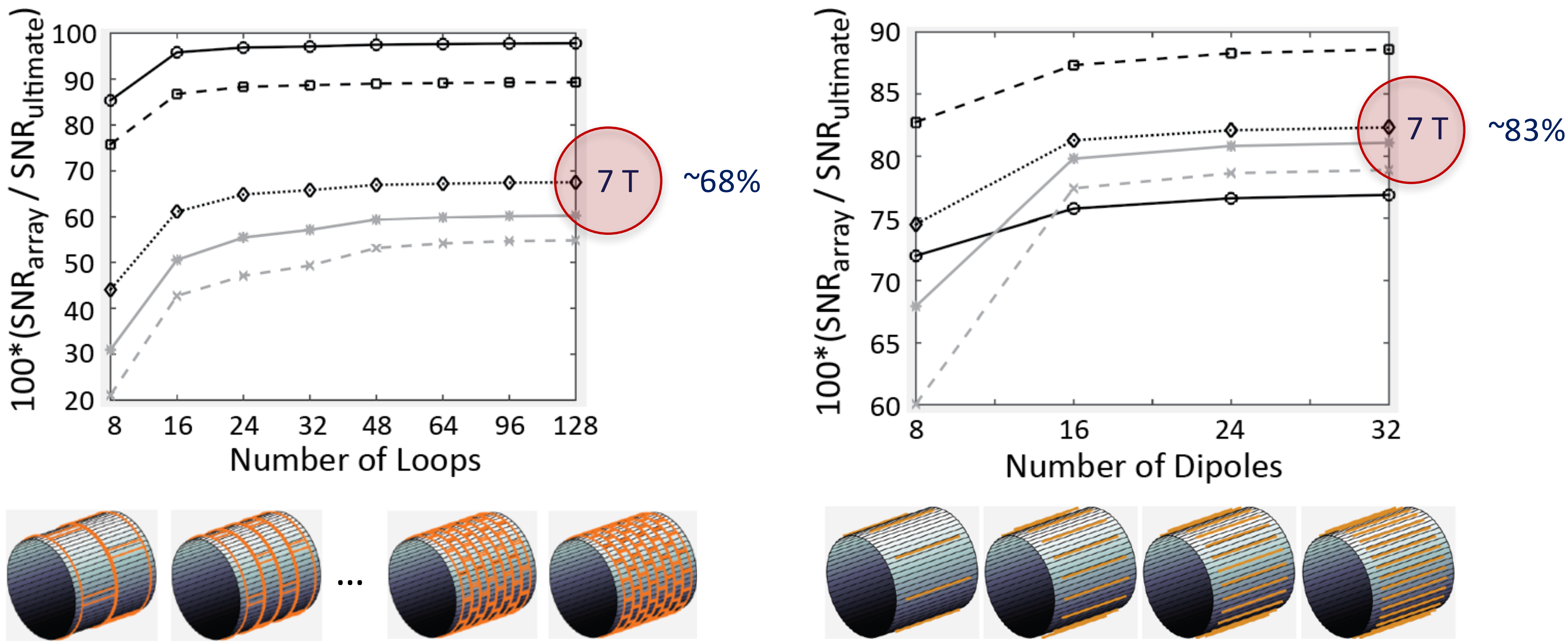
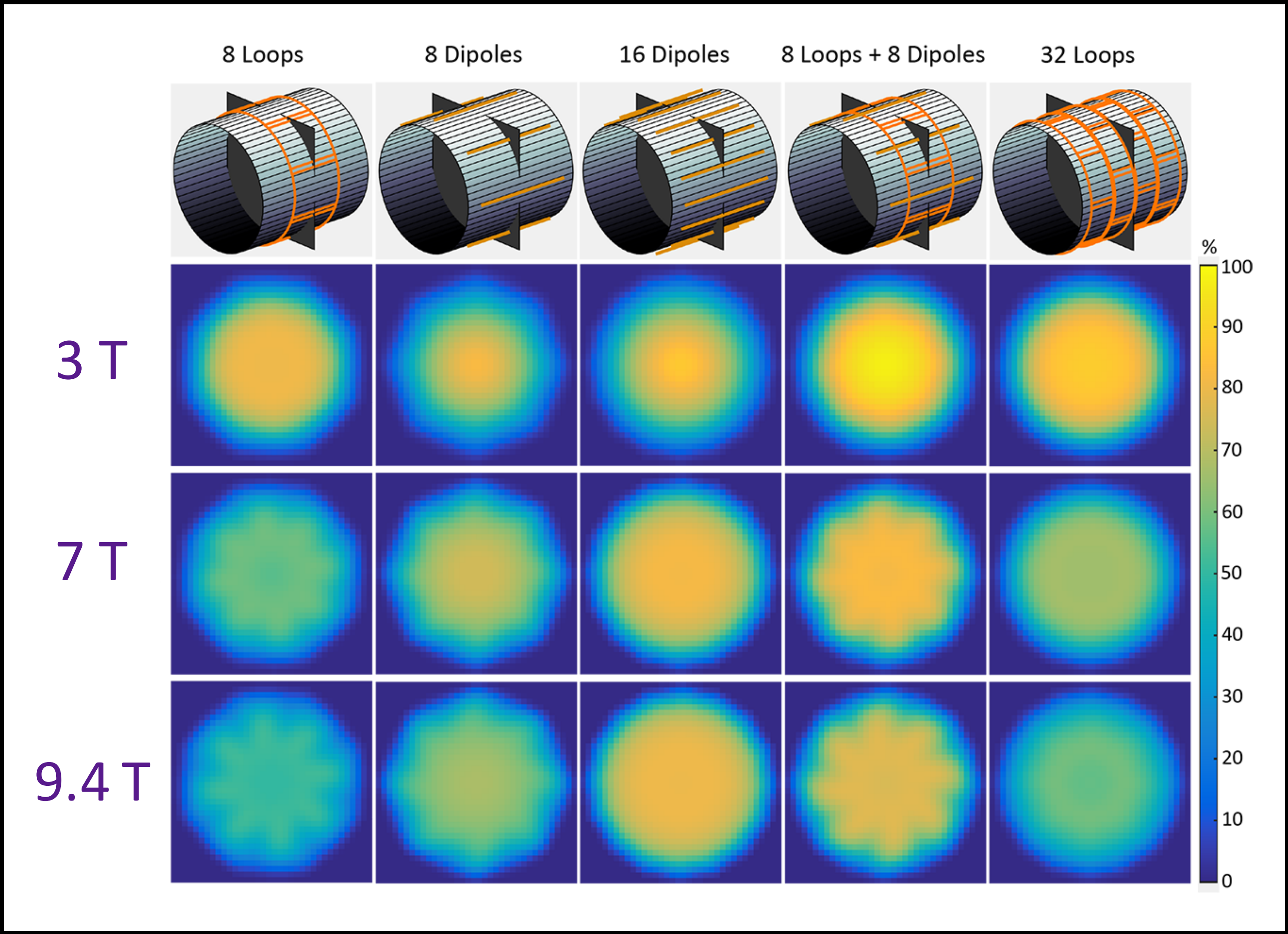
# Coil Performance: Loops vs. Dipoles

## Central Voxel



# Coil Performance: Loops vs. Dipoles

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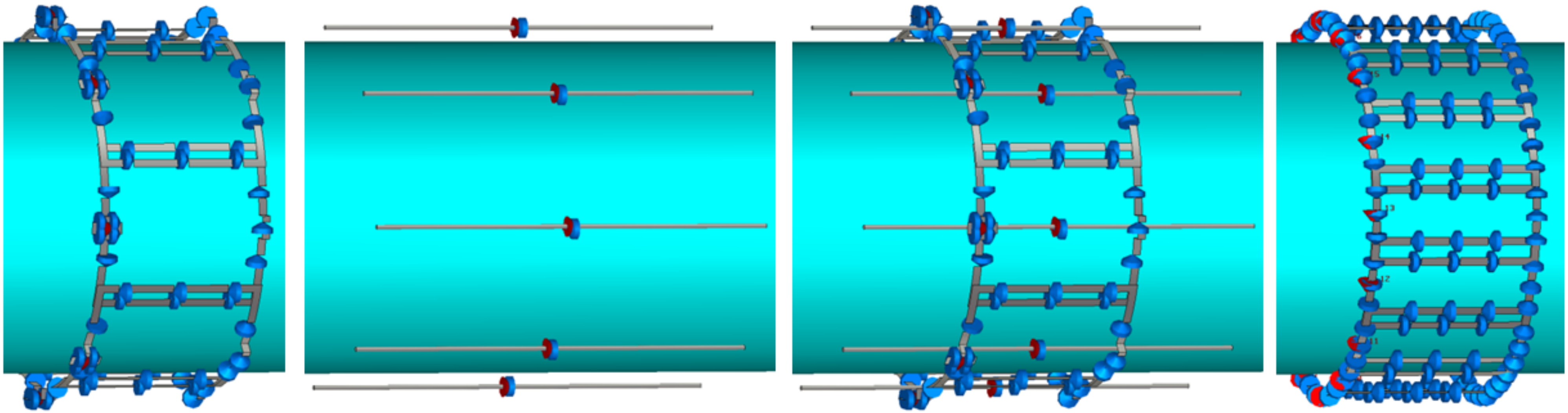
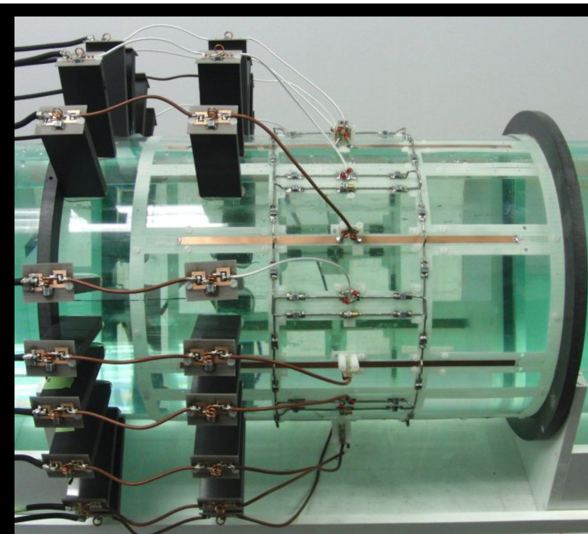
# Combined Dipole-Loop Coil

8 Loops | 8 Dipoles | 8 Loops + 8 Dipoles | 16 Loops



7 Tesla Coil
Phantom diameter = 29.5cm
Coil shell = 31.5cm
$e_r$ = 81.8, s = 0.604
(= Siemens phantom)



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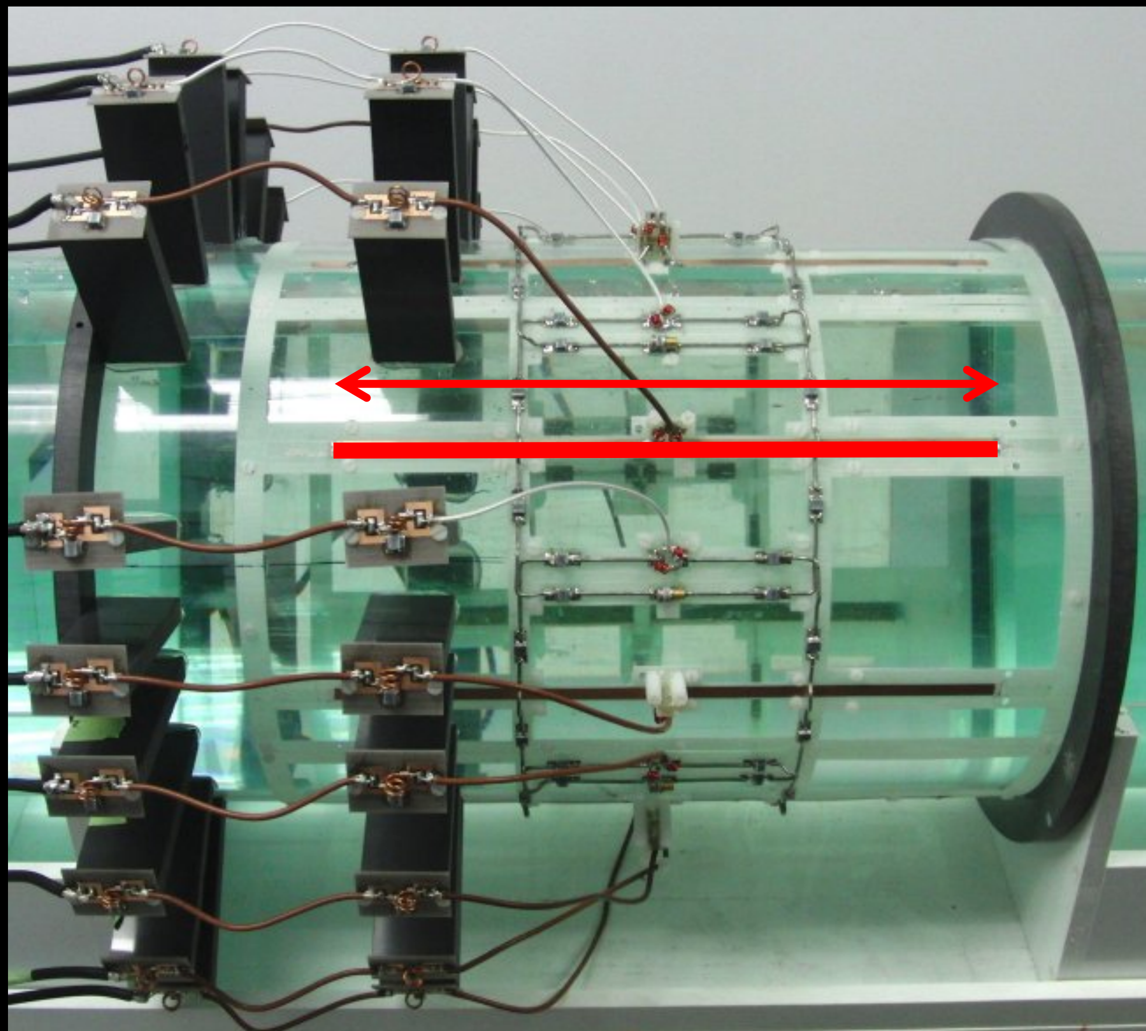
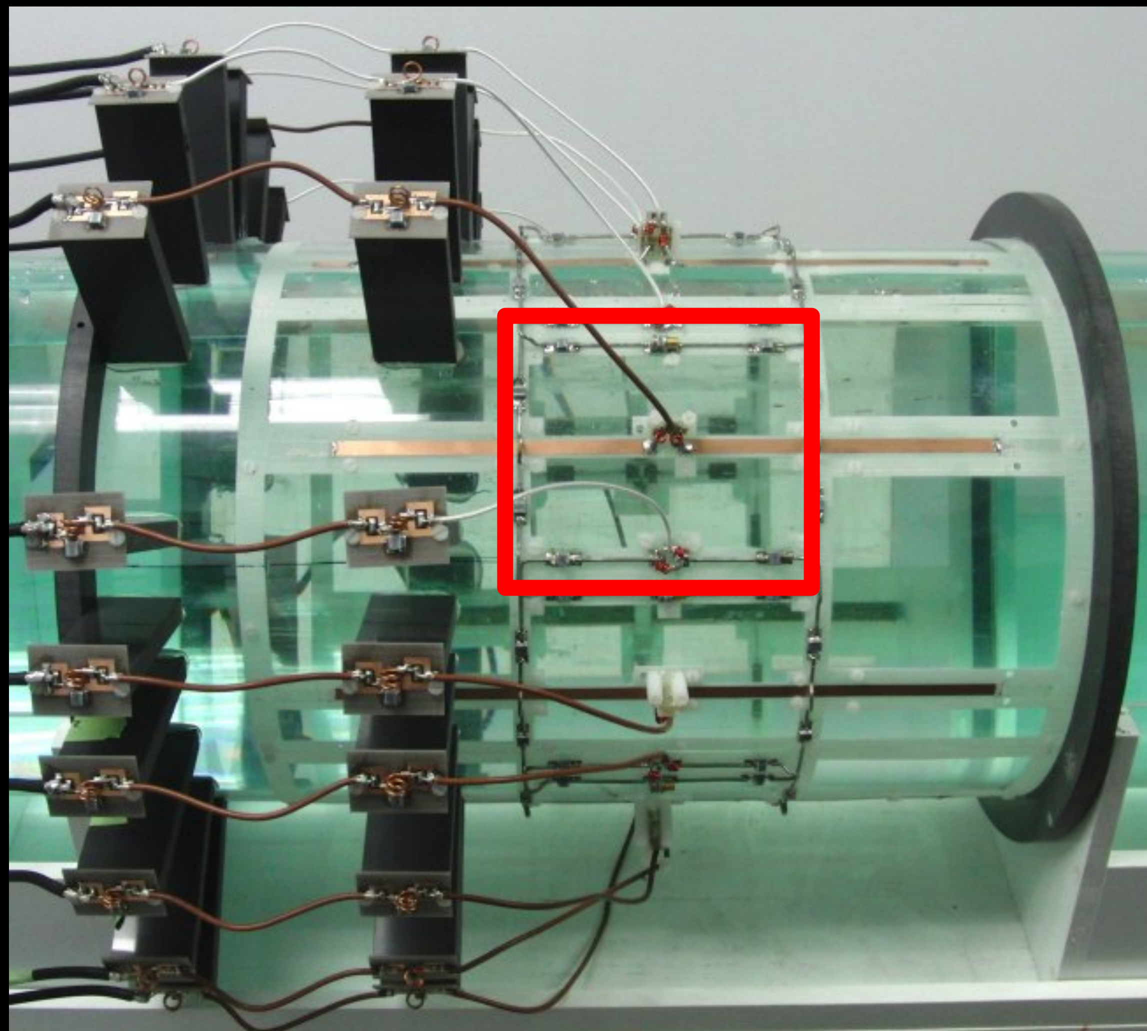
# Combined Dipole-Loop Coil



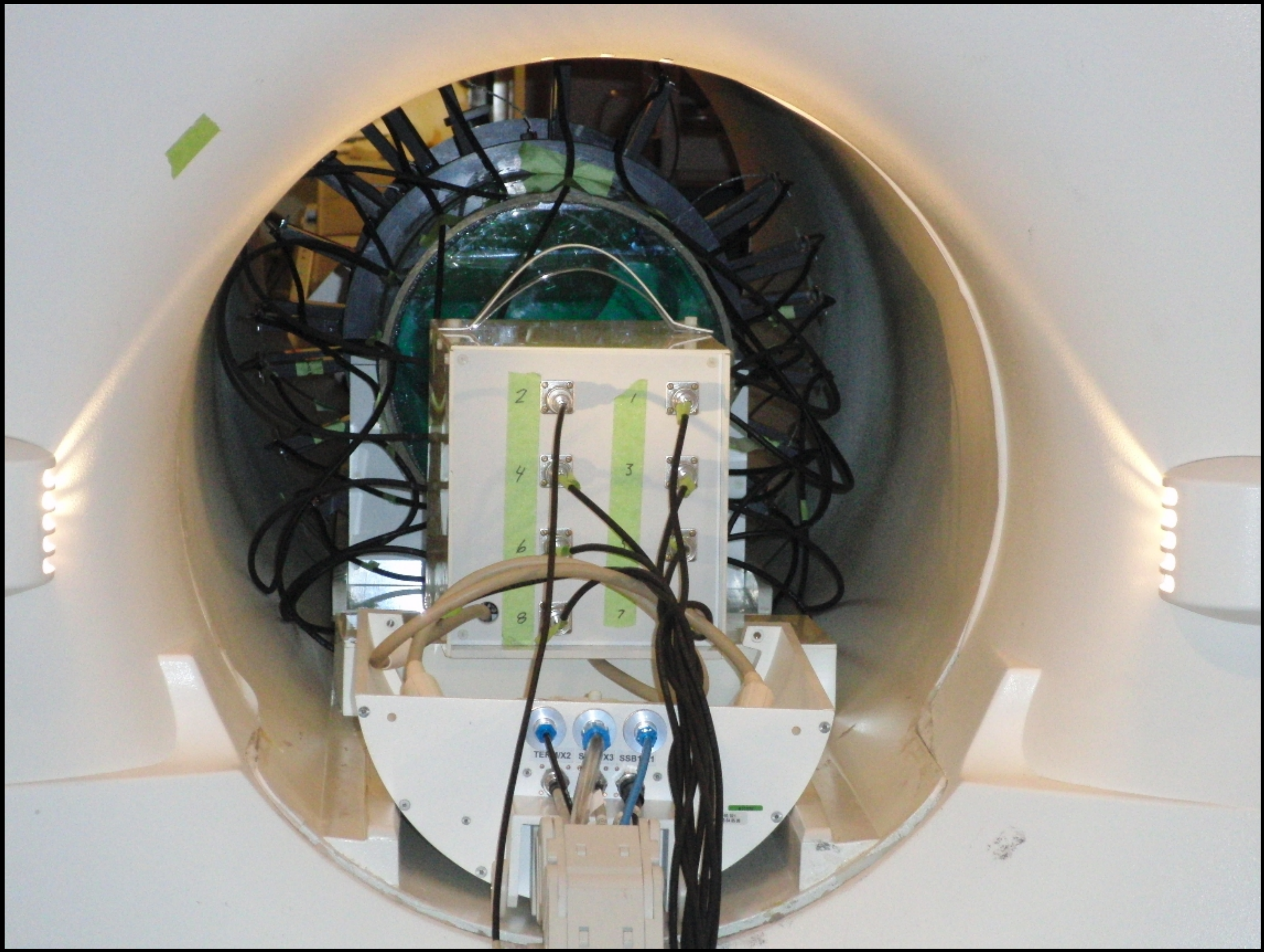
Dipole elements are self-resonant at 35.6 cm length

# Combined Dipole-Loop Coil



Overlapped Loop elements are 14.6 x 15 cm

# Scanner Measurements



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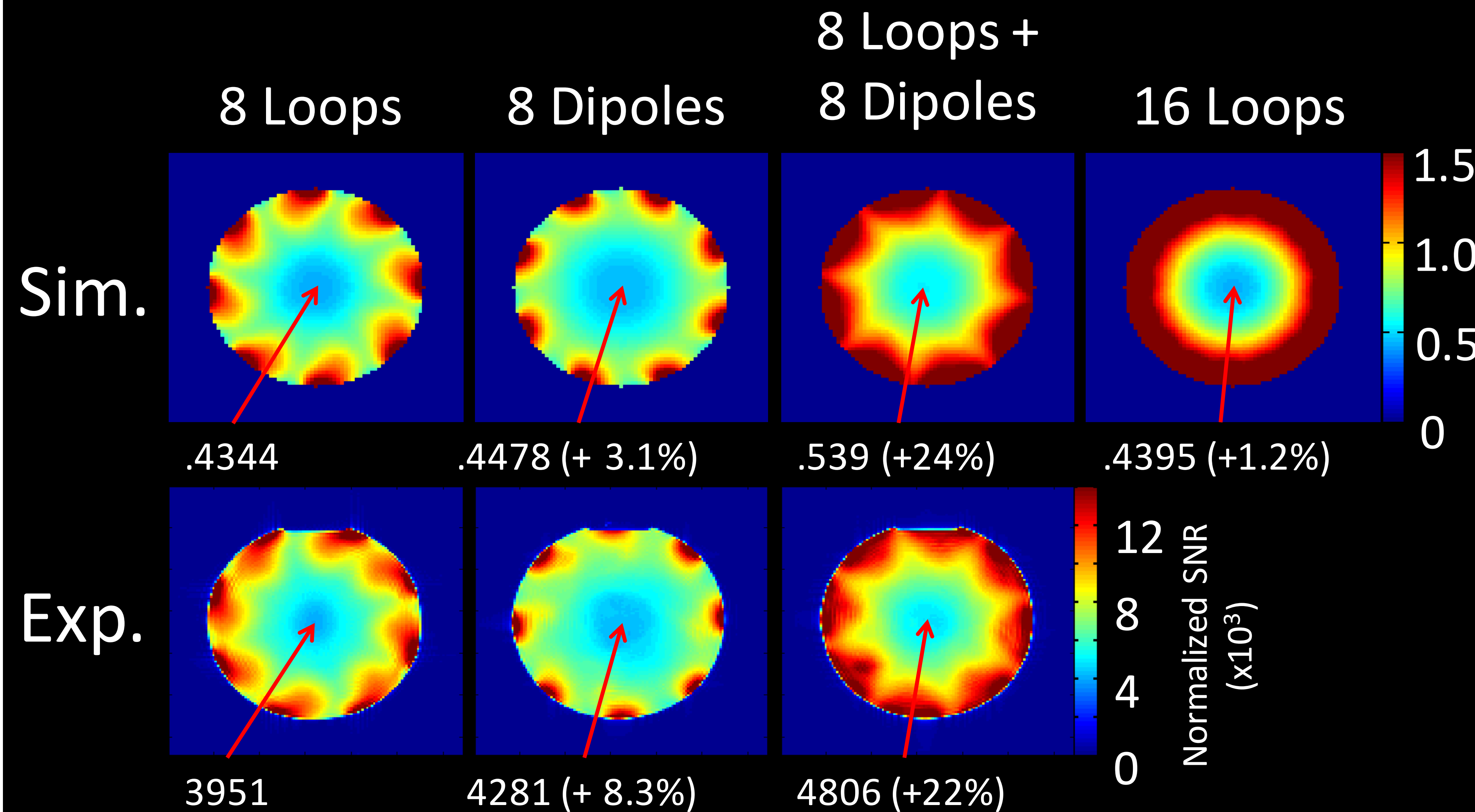
# SNR Maps

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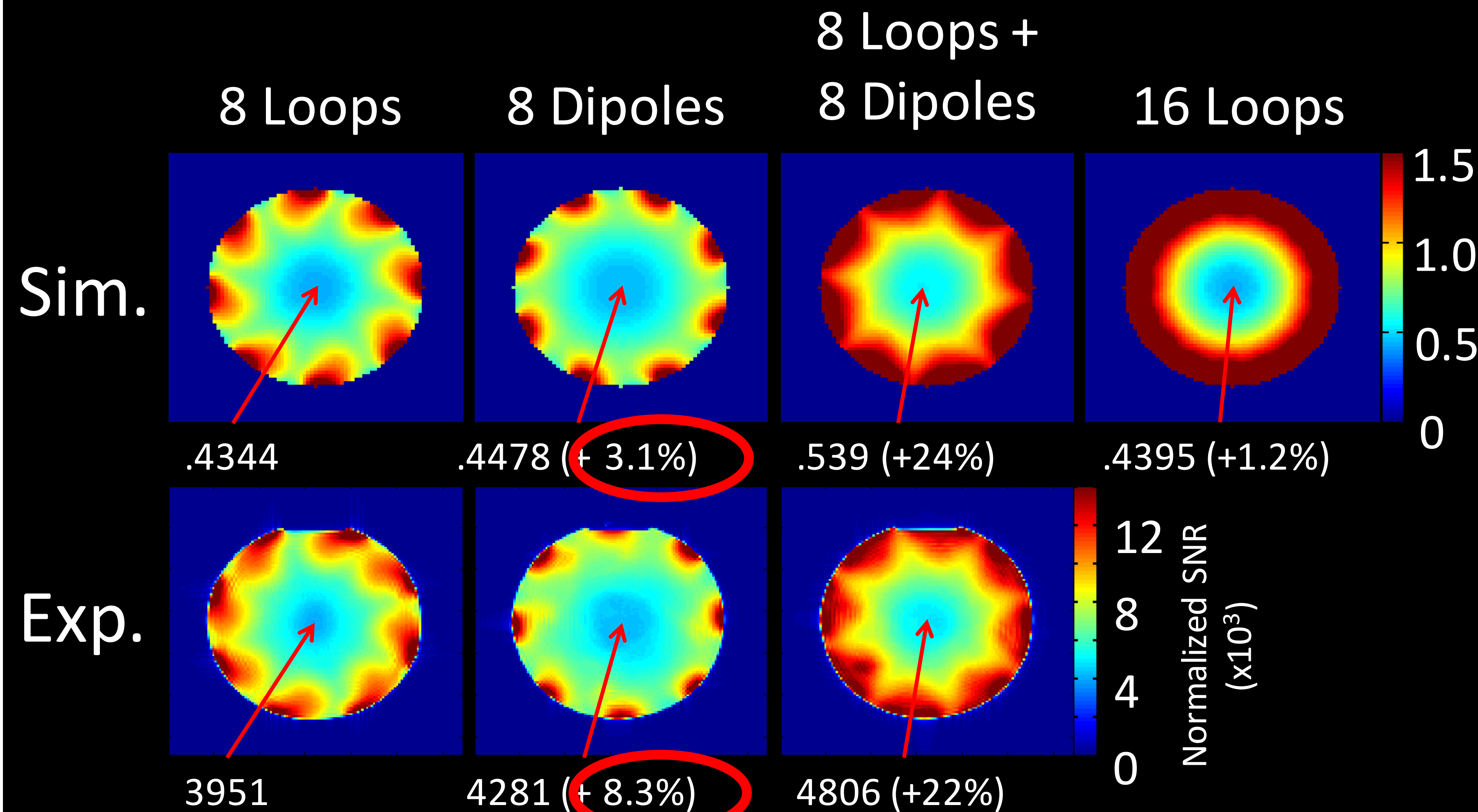
# SNR Maps

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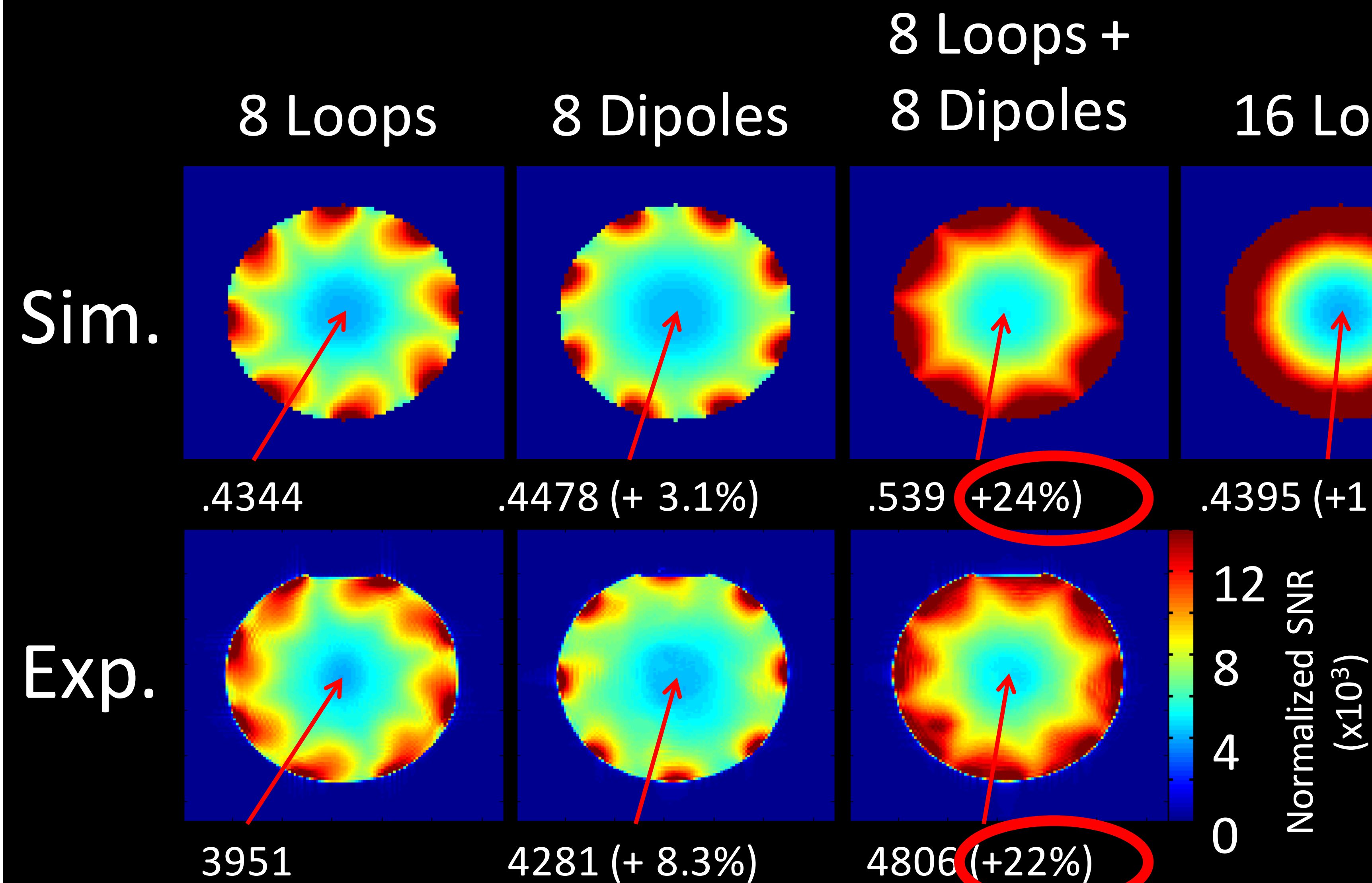
# SNR Maps

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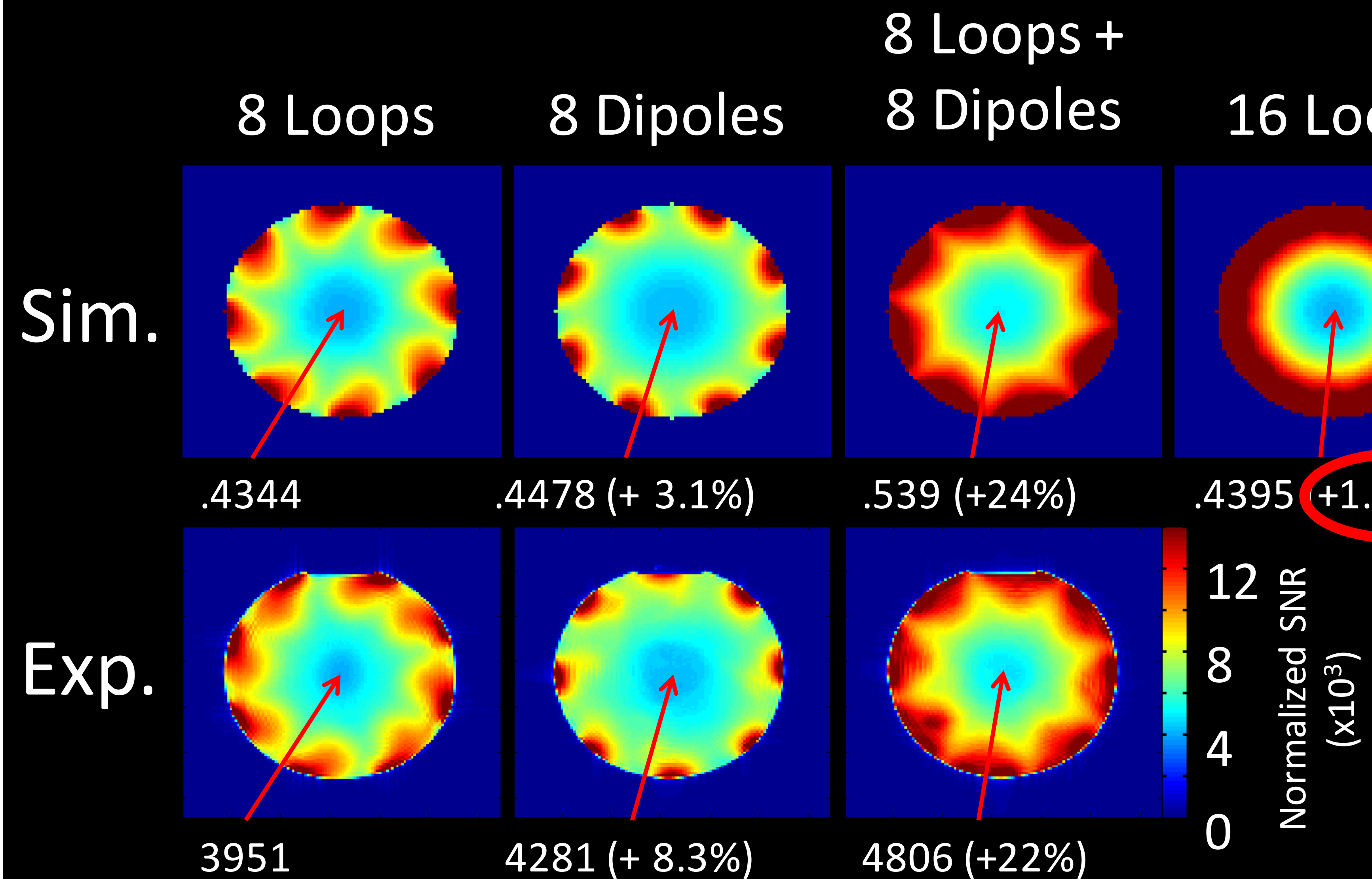
# SNR Maps

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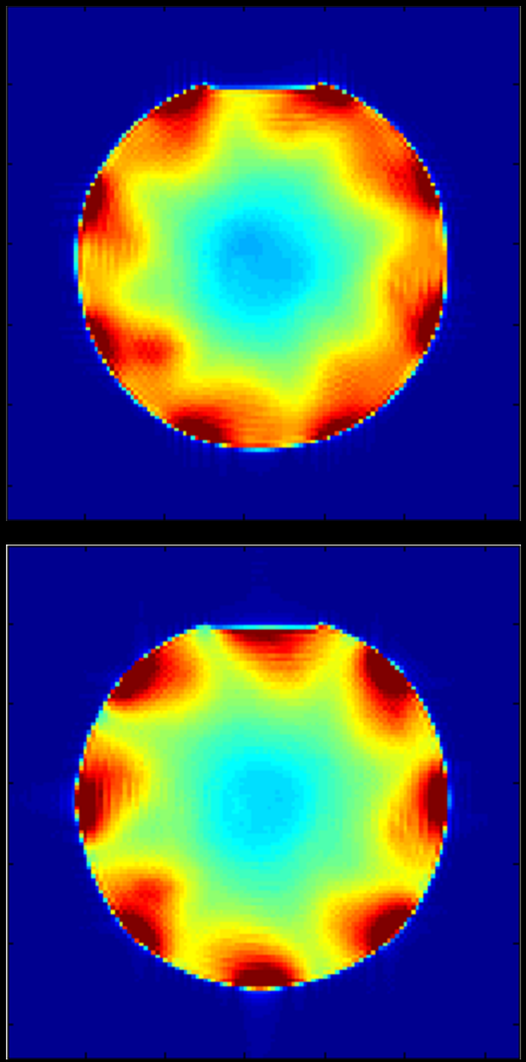
# SNR Normalized By $B_1$+

| | Tra | Sag | Cor |
|---|---|---|---|
| 8 loops | | | |
| 8 dipoles | | | |

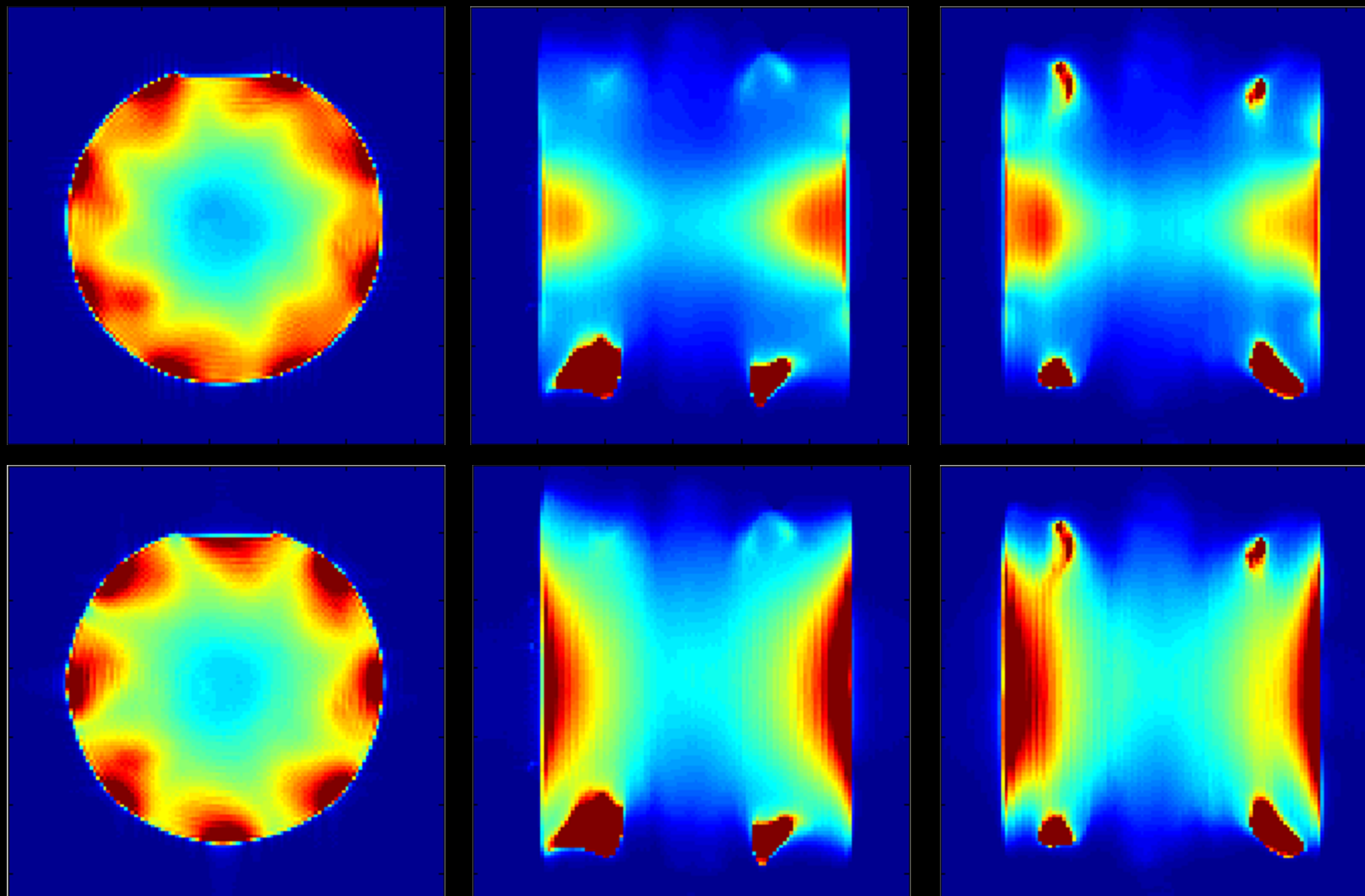


# Extra Bonus SNR With Dipoles!

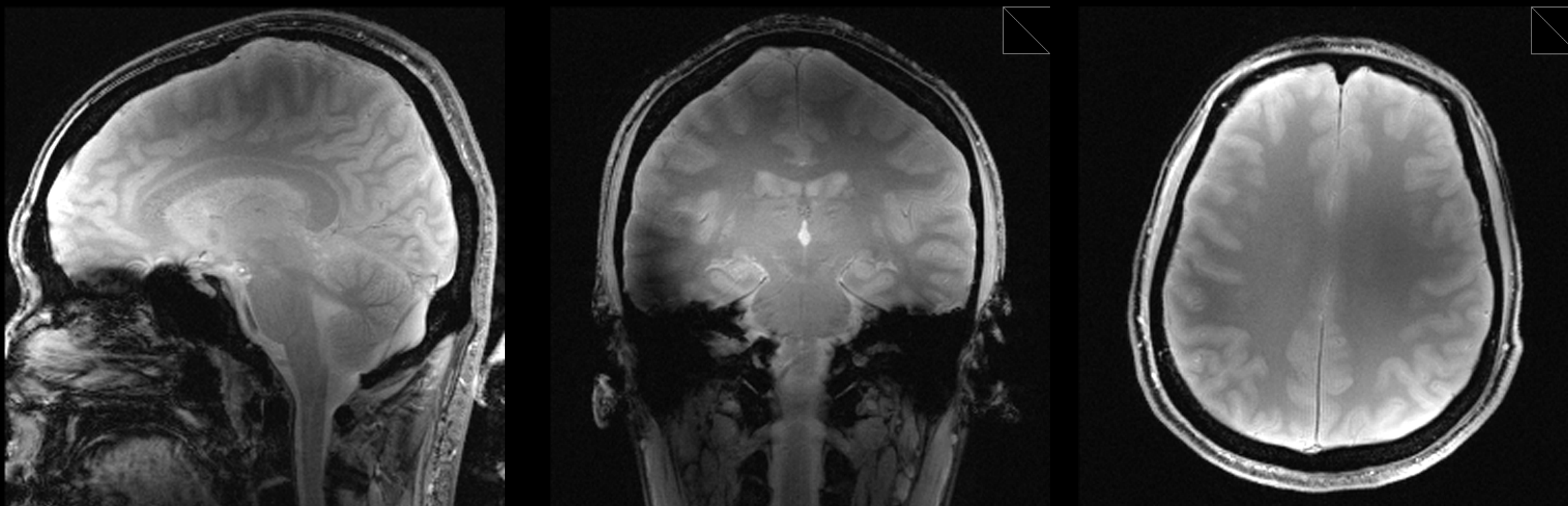
Tra Sag Cor

8 loops

8 dipoles



# GRE Images: 7 T Dipole Array



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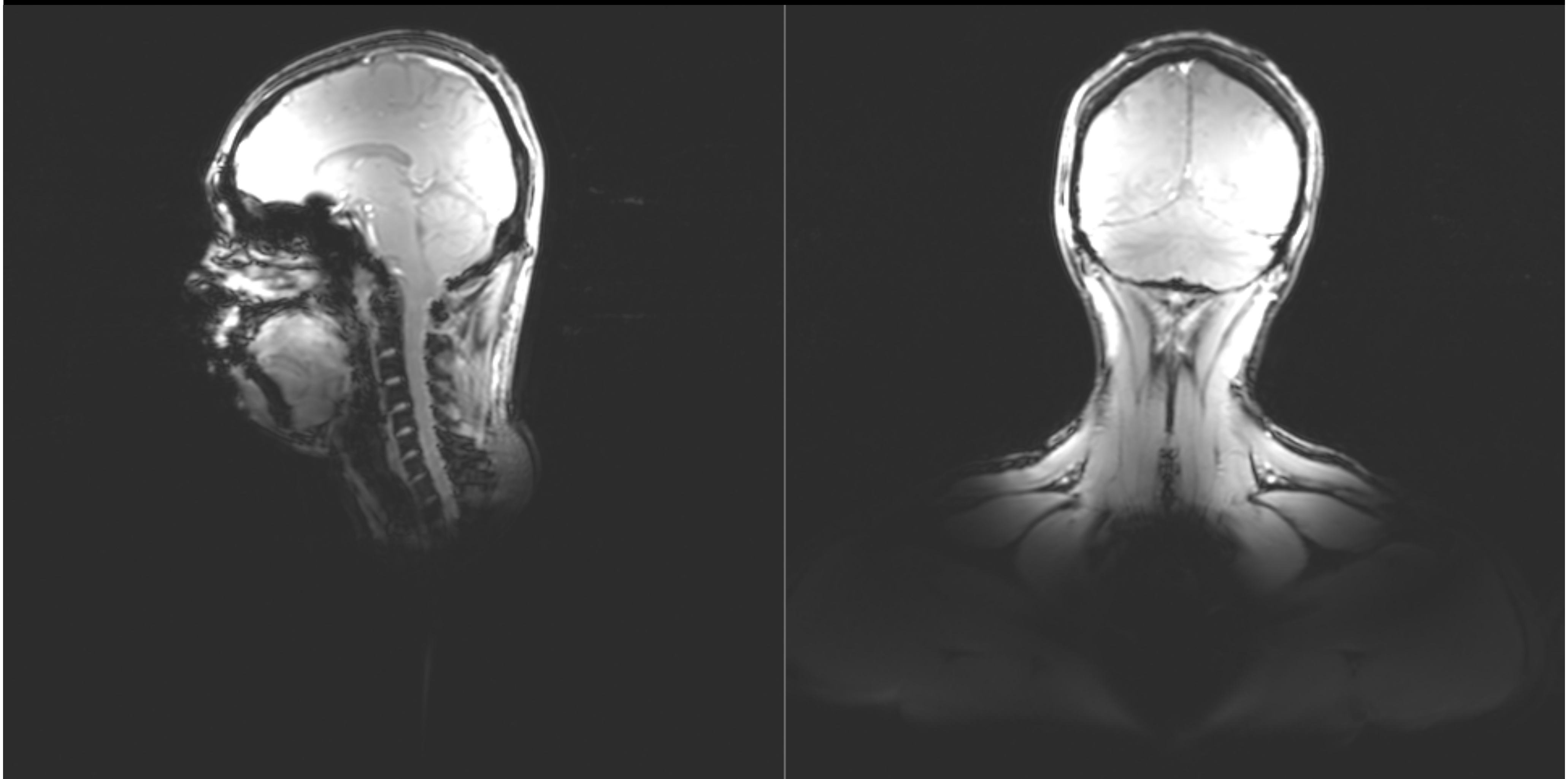
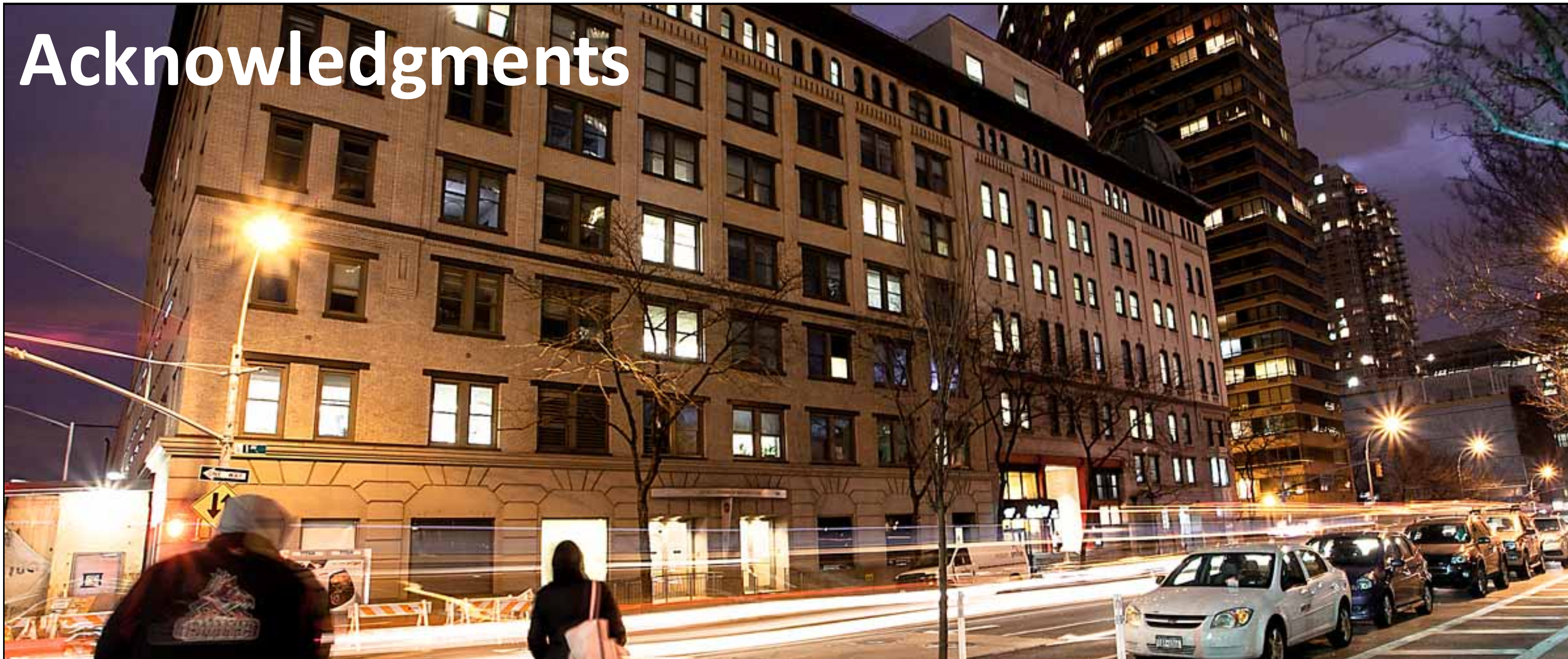
# GRE Images: 7 T Dipole Array



# Acknowledgments



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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Daniel Sodickson | Graham Wiggins | Ryan Brown | Cem Murat Deniz | Manushka Vaidya | Karthik Lakshmanan | Gillian Haemer | Gang Chen | Hong Hsi Lee |

## Thank You for Your Kind Attention!