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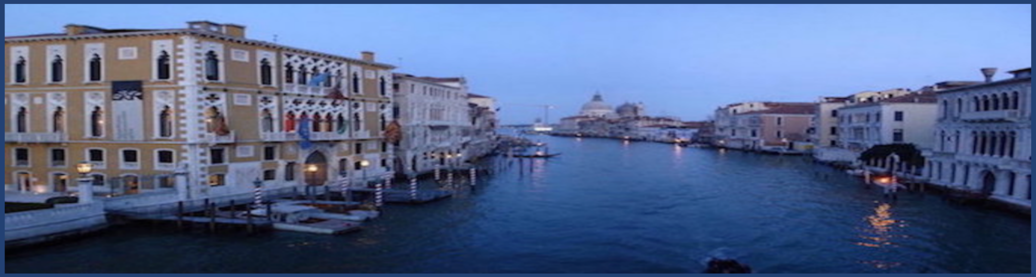
Neutrino Mass Hierarchy

(how to measure it)

- The present 3 standard neutrino *oscillation* framework
- Mass Ordering (MO), Experiments, Techniques (limited list)
- Issues on the Statistical Analysis
- A robust determination of MO at 95% C.L. ?
- Conclusions and Perspectives



XVII International Workshop on Neutrino Telescopes

13-17 March 2017 Venezia, Palazzo Franchetti - Istituto Veneto di Scienze, Lettere ed Arti

One slide on STATISTICS

What is largely accepted in particle physics is that:

- <u>Observations</u> are given by a **5 σ** measurement
- <u>Exclusions</u> are given at **95% C.L.** when systematics is well know, and at **3 σ** or more when detectors of new conception are used, or to reject previous results.

That is established by the last 30 years of experiences in our field.

Therefore, the still too large used 90% C.L. for exclusion limits remains a mystery to me.

Concerns about the use of 90% C.L.:

- It does not provide the "good feeling" (remember the case of CHOOZ)
- Choice for one-sided or two-sided is relevant (and the choice is not often obvious)
- Comparison of different measurements is not only a matter of the "median" values

Neutrino community should agree on a more robust statistical framework

What is the Mass Hierarchy determination?

is it an **exclusion** process or an **observation** one ?

Assume the presence of the signal and compute the compatibility of the data in the parameter space of the signal

(signal rejection)

Assume no signal and compute the compatibility of the data with the background

(background rejection)

The recent Neutrino History

1998

1rst Annus Mirabilis

Oscillations :
- Observation by *SuperK*
- Reactor exclusion by *Chooz*

2002

2nd A. M.

3 x 3 matrix :
- Solar by *SNO*
- Reactor by *KAMLAND*

2012

3rd A. M.

θ_{13}:
- Reactor by *Daya Bay*, *RENO*, *Double Chooz*, *T2K*…

2017-18 ?

4th A. M. ?

- Mass Hierachy by *NOvA/T2K* ?
- CPV by *T2K* ??

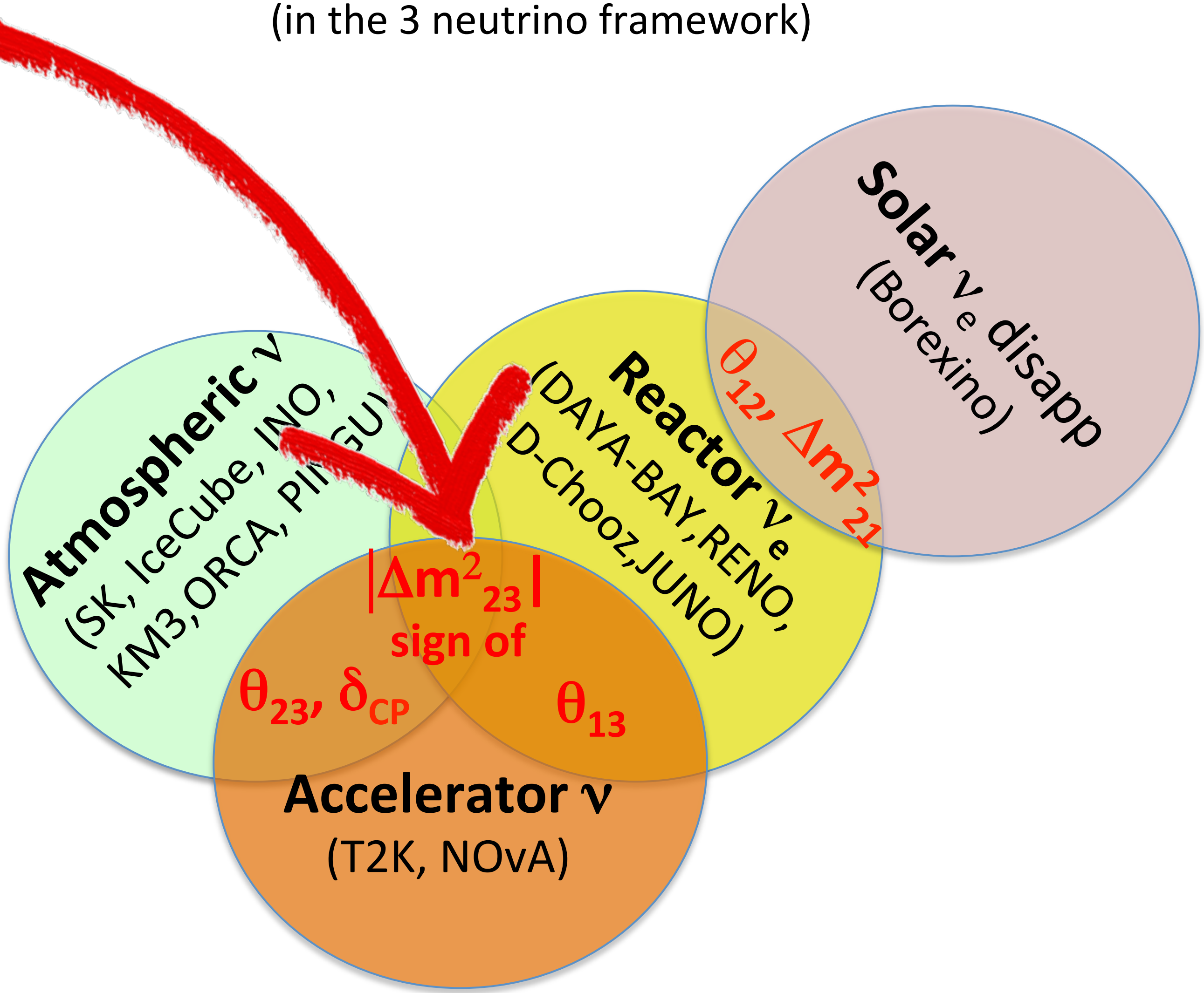
≥ 2020 ? —— δ_{CP} ?

– δ_{CP} by *T2K* ?

L. S., Rev. in Phys. 1 (2016) 90

Neutrino Oscillation Industry

(in the 3 neutrino framework)



From a Maxim Gonchar (DLNP) picture

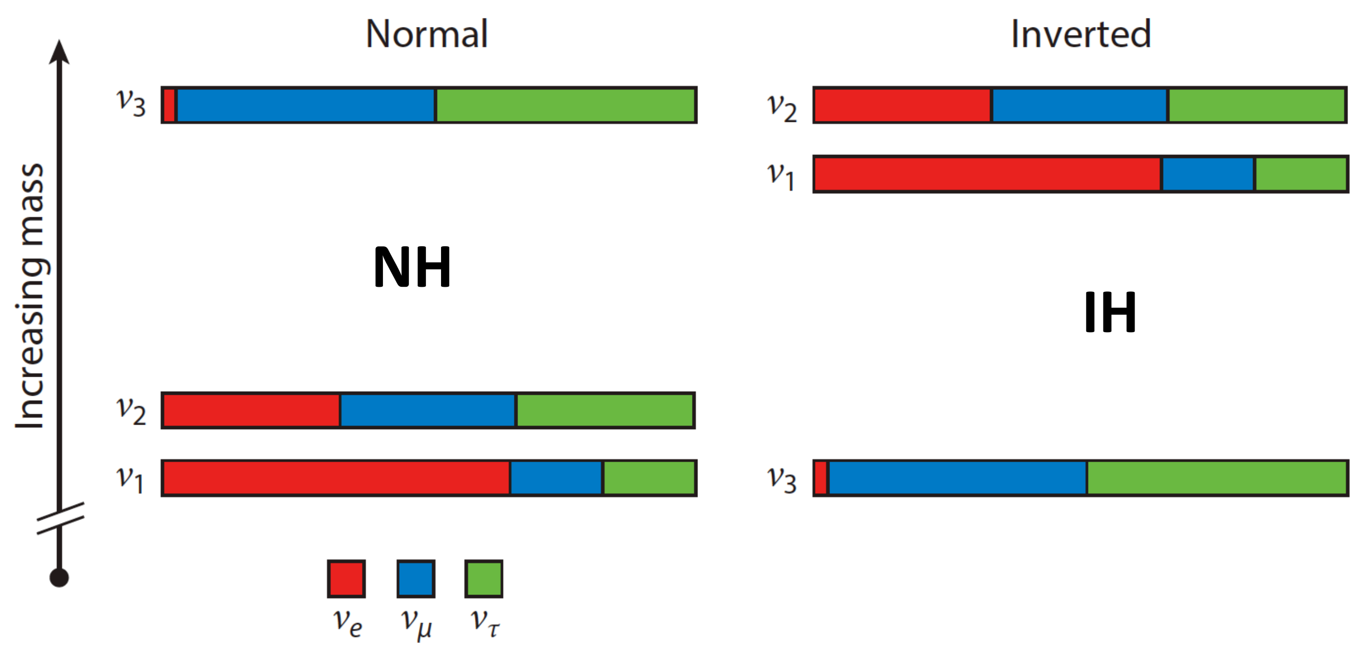
disclaimer: major actors only, not a full list…

The wonderful frame pinpointed for the 3 standard neutrinos
in less than 20 years,
beautifully adjusted by the θ_{13} measurement,
left out some relevant questions

The present scenario (cnt.)

Before really entering in the precision era, there are still 4 results to be obtained, at least at first order :

1) Leptonic CP violation (phase δ_{CP})
2) *Mass ordering (MO)*
3) (θ_{23} octet)
4) Presence or not of more (sterile ?) neutrinos states

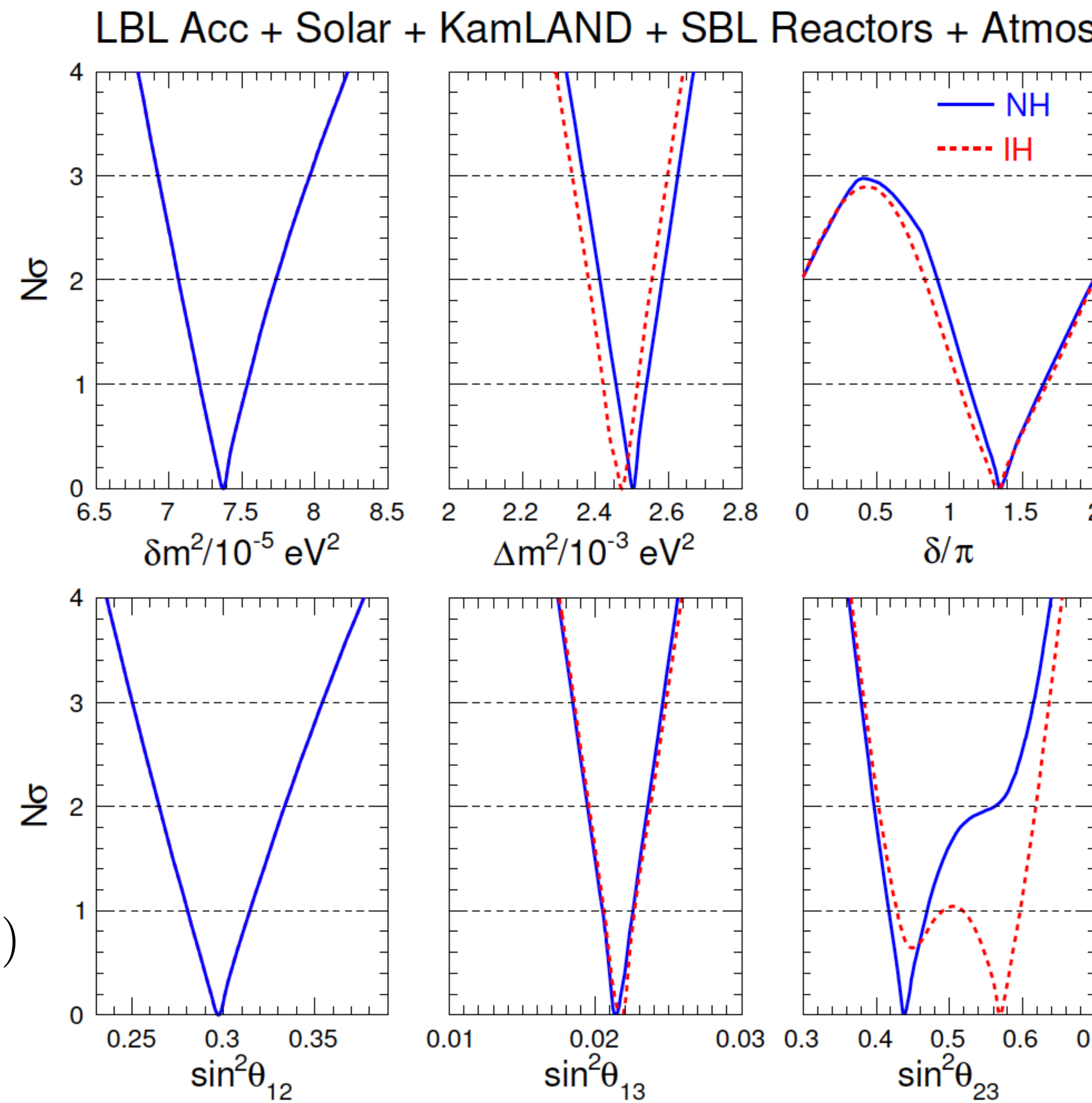


One or two states at higher mass ?

Current* knowledge from Global Fits

From few % to about 10%, but with no** knowledge on MO

$$\Delta\chi^2_{min} = \chi^2_{min}(IH) - \chi^2_{min}(NH)$$

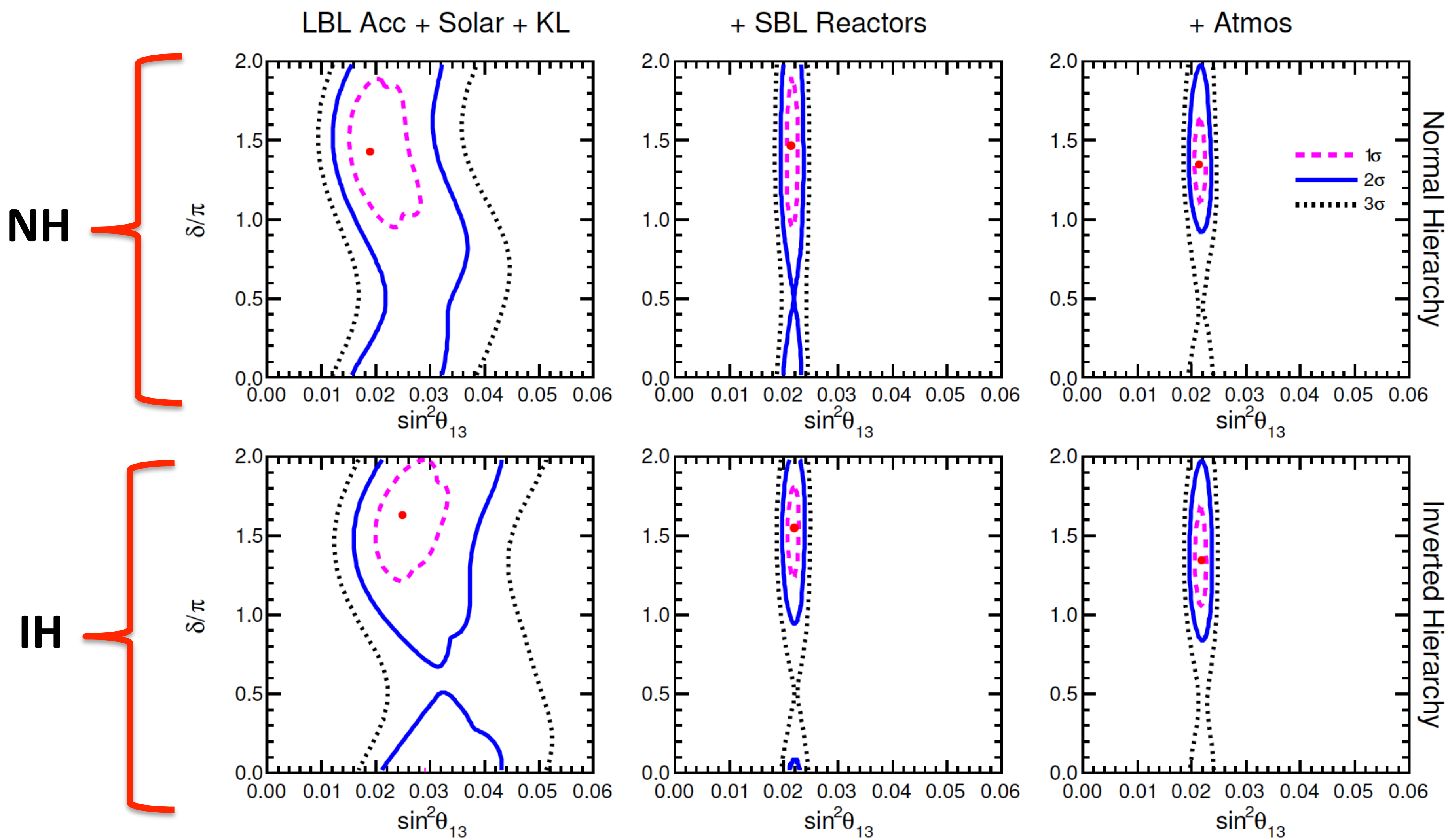


F. Capozzi et al., Nuclear Physics B 908 (2016) 218

*updated three days ago: arXiv:1703.04471

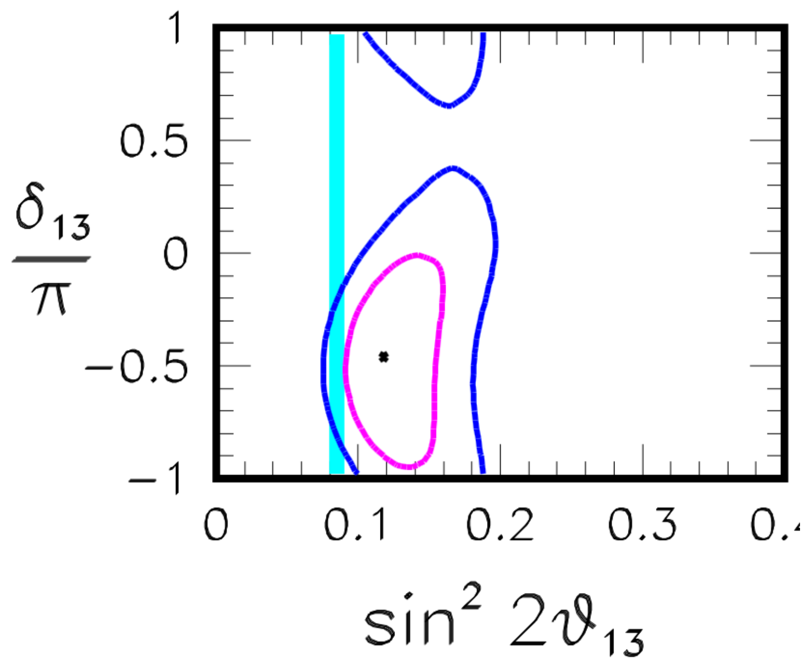
** $\Delta\chi^2 \approx 3 \div 4$ (dependent of θ_{23} mixing angle)

To better confirm it:

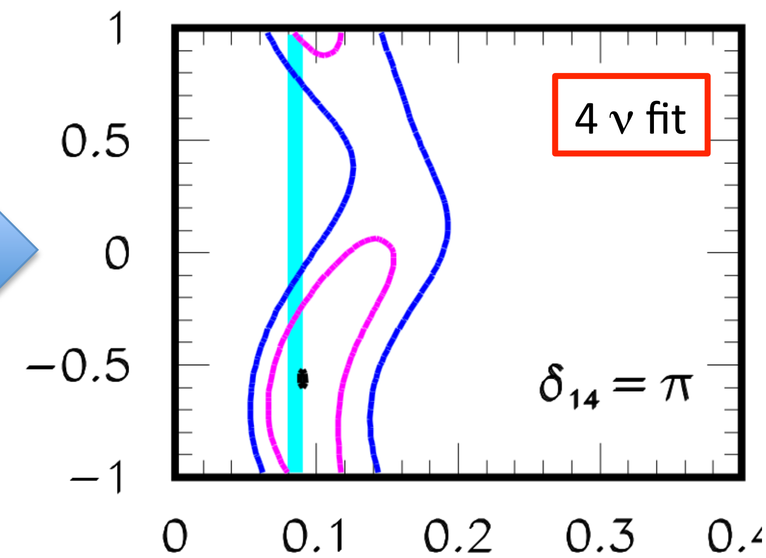


How steriles at 1 eV mass scale may wash out results on MH and δ_{CP}

3 ν fit after NOvA and T2K summer release



A.Palazzo: arXiv:1509.03148



(IH, θ_{23} marginalized, 68% e 90% C.L.)

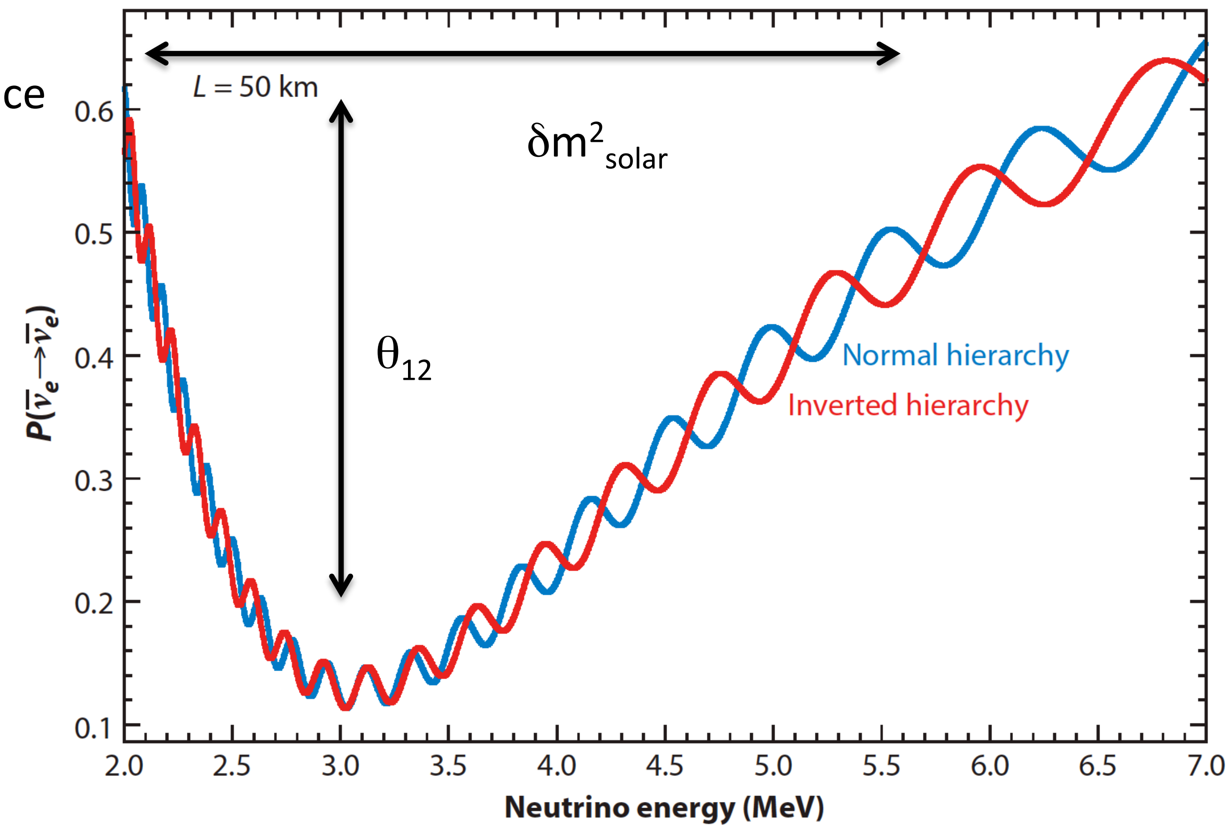
It is MANDATORY to close this SBL sterile story in the near future.

Let us have a closer look…

One possibility to access MO: interference of atmospheric and solar oscillations in reactors

(as originally pointed out by M.Piai&S.Petcov in 2002, PLB)

$\bar{\nu}_e$ disappearance



JUNO example

wiggles correspond to the subdominant atmospheric oscillation

F. An et al. (JUNO), J. Phys. G 43 (2016) 030401

See also JUNO's talk and poster

JUNO current effort to "**ascertain the true hierarchy**"

(is this really what we want ? A better approach would be ***to reject the wrong hierarchy*** *)*

Stat. discrim, power of **4σ** for:

- Baseline 53 km,
- Fiducial Volume 20 kt,
- Therm. React. Power 36 GW,
- Exposure Time 6 years,
- Energy resolution 3%

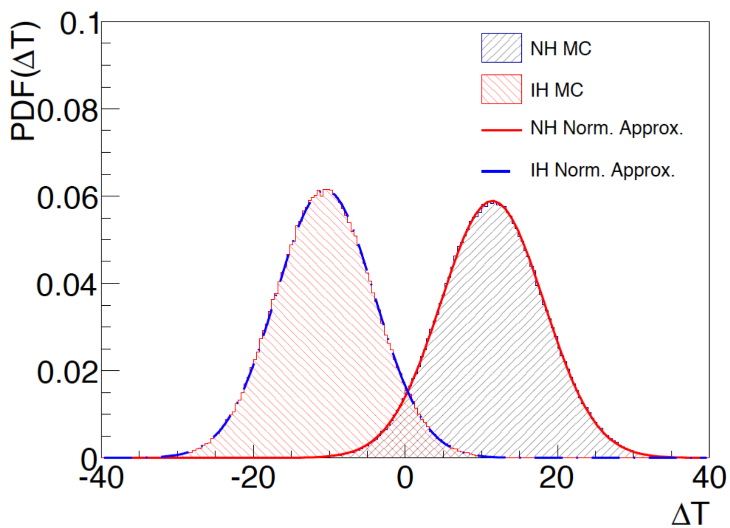
→ **3σ** → **4σ**

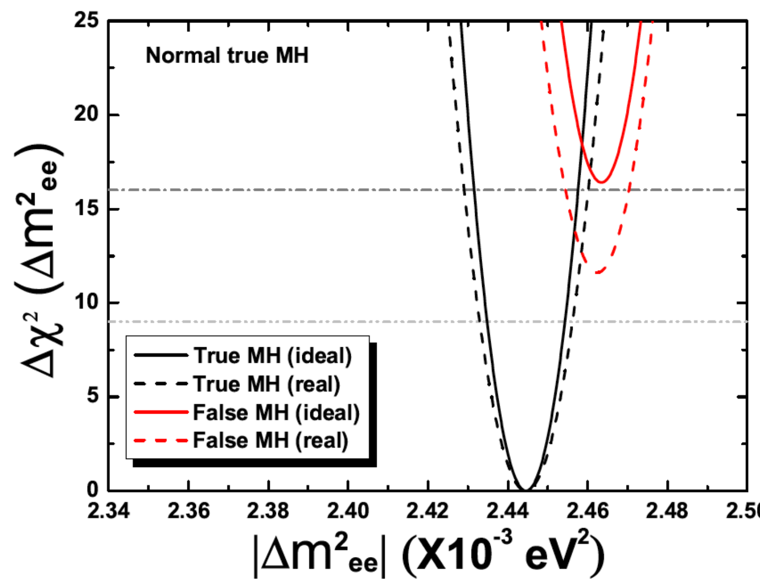
add syst. (position)

use Δm^2_{23}

$\Delta\chi^2=\chi^2_{min}(NH) - \chi^2_{min}(IH)$

Asimov data-set, prob. ratios etc.





$$\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}.$$

Another possibility: **Accelerators** for ν_μ Long Baseline with ν_e in Matter

$$P(\nu_\mu \to \nu_e) \simeq \left| \sqrt{P_{Atm}} e^{-i(\Delta_{32}+\delta)} + \sqrt{P_{Sol}} \right|^2$$
$$= P_{atm} + P_{sol} + 2\sqrt{P_{atm} P_{sol}} (\cos\Delta_{32}\cos\delta \mp \sin\Delta_{32}\sin\delta)$$

$$\sqrt{P_{atm}} = \sin\theta_{23}\sin 2\theta_{13} \frac{\sin(\Delta_{31} - aL)}{\Delta_{31} - aL} \Delta_{31}$$

$$\sqrt{P_{sol}} = \cos\theta_{23}\sin 2\theta_{12} \frac{\sin(aL)}{aL} \delta_{21}$$

$a = G_F N_e / \sqrt{2} \simeq 1/3500 \text{ km}$

$$aL = 0.08 \text{ for } L = 295 \text{ km}$$
$$aL = 0.23 \text{ for } L = 810 \text{ km}$$

Size of matter effect scales with size of θ_{13}, and sensitivity also depends on θ_{23} and true value of δ

But effect scales with Baseline and Energy:

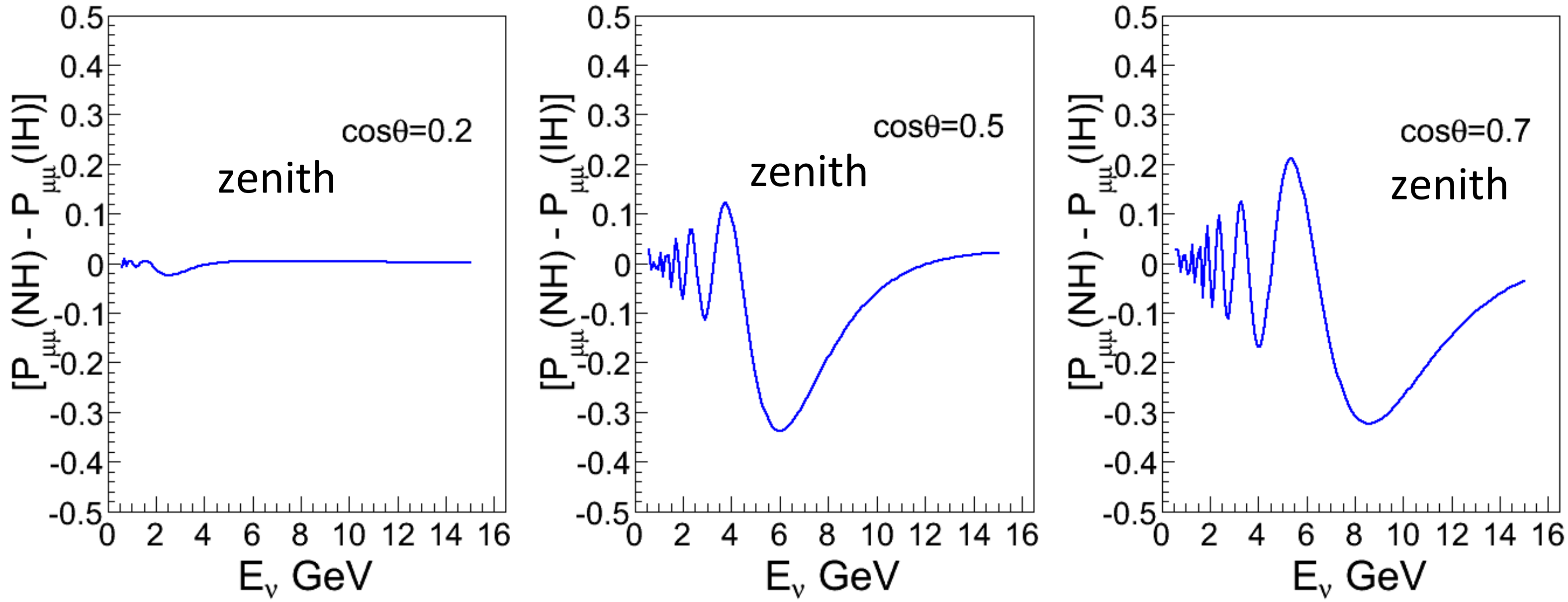
CP violating phase: neutrino and anti-neutrino oscillation probabilities at energies around 1 GeV and at relatively short baselines of L ≈ 300 km

⟺

Hierarchy: measurement at the point of maximal resonant conversion higher energies and baselines E ≈ 6 GeV and L ≈ 10^4 km.

Third possibility: high statistics for atmospheric ν_μ and anti-ν_μ

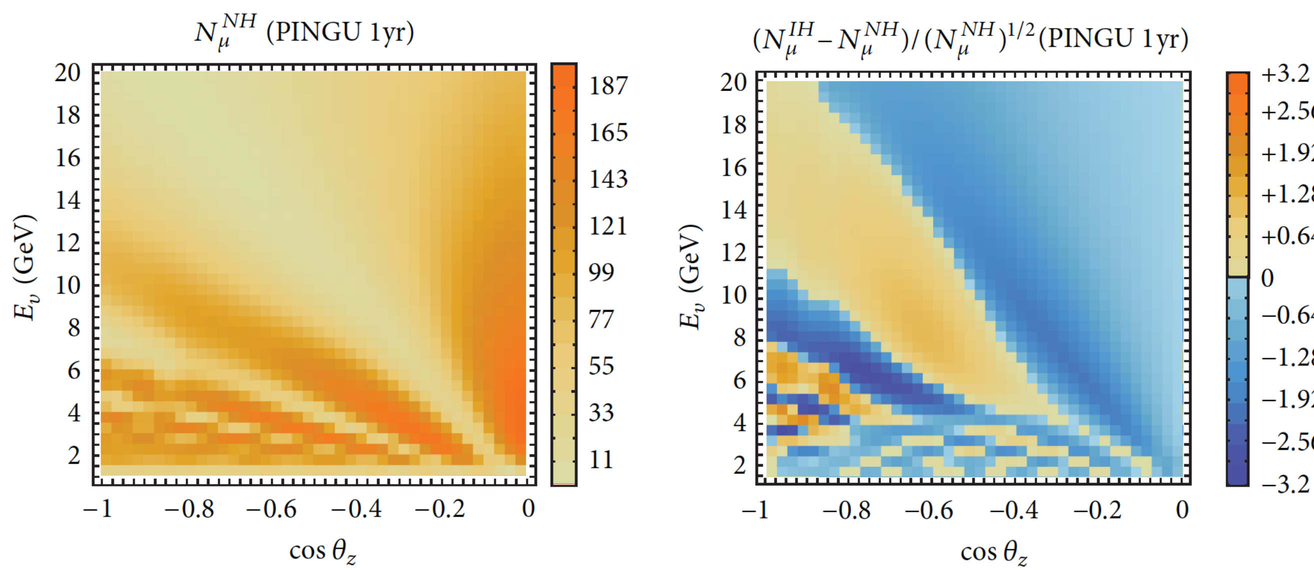
ICAL-INO



(opposite for anti-ν_μ)

S. Ahmed et al. (INO coll.), arXiv:1505.07380.

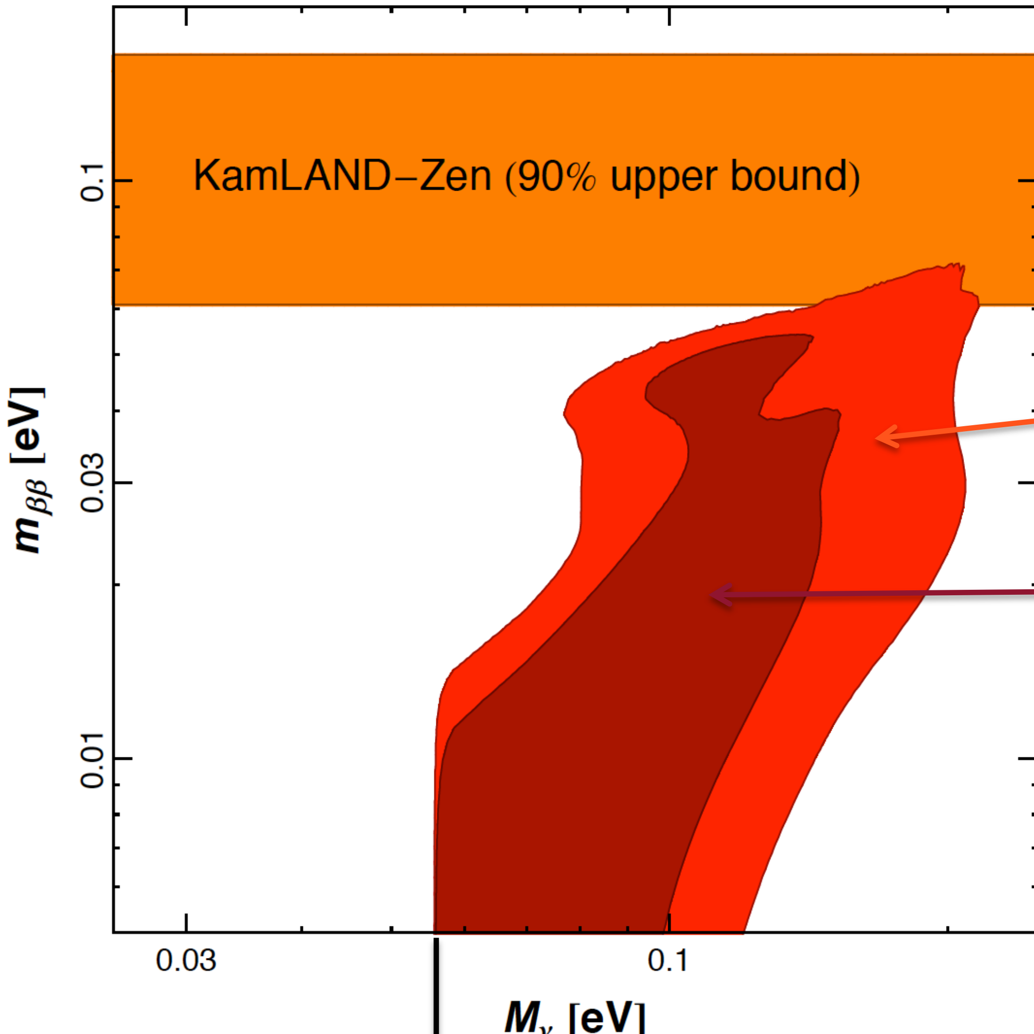
ORCA/PINGU



S. Adrián-Martinez et al. (KM3 coll.), Jour. Phys. G 43 , 084001 (2016), arXiv:1601.07459

Some prospects from Cosmology:

M. Gerbino et al., arXiv:1611.07847v1 [astro-ph.CO] 23 Nov. 2016



Bayesian analysis,
Posterior PDF(hyper-MO)

IH at 95% C.L.

NH at 95% C.L.

CMB+LS+BAO limit:

$$M_\upsilon \leq 0.21\, eV \ at\ 95\% C.L.$$

$$M_\upsilon = \sqrt{\Delta m_{31}^2} + \sqrt{\Delta m_{31}^2 + \Delta m_{21}^2} \approx 0.06\, eV$$ (NH limit)

(see also A. Meroni poster)

arXiv:1606.04691
Next generation experiments (EUCLID),
Posterior odds for normal versus inverted ordering of about 2:1 (low),
Cosmological + oscillation data bring the odds at 3:1
Or, expect to reject IH at > 95% with accuracy of 0.02 on the neutrino mass

However arXiv:1703.03425
claims odds at the level of 42:1 against IH ("strong" evidence in the Jeffrey scale)
with strong limit $M_{\upsilon} \leq 0.13\, eV\ at\ 95\% C.L.$
Choice of the prior…

(not sufficient experience in particle physics)

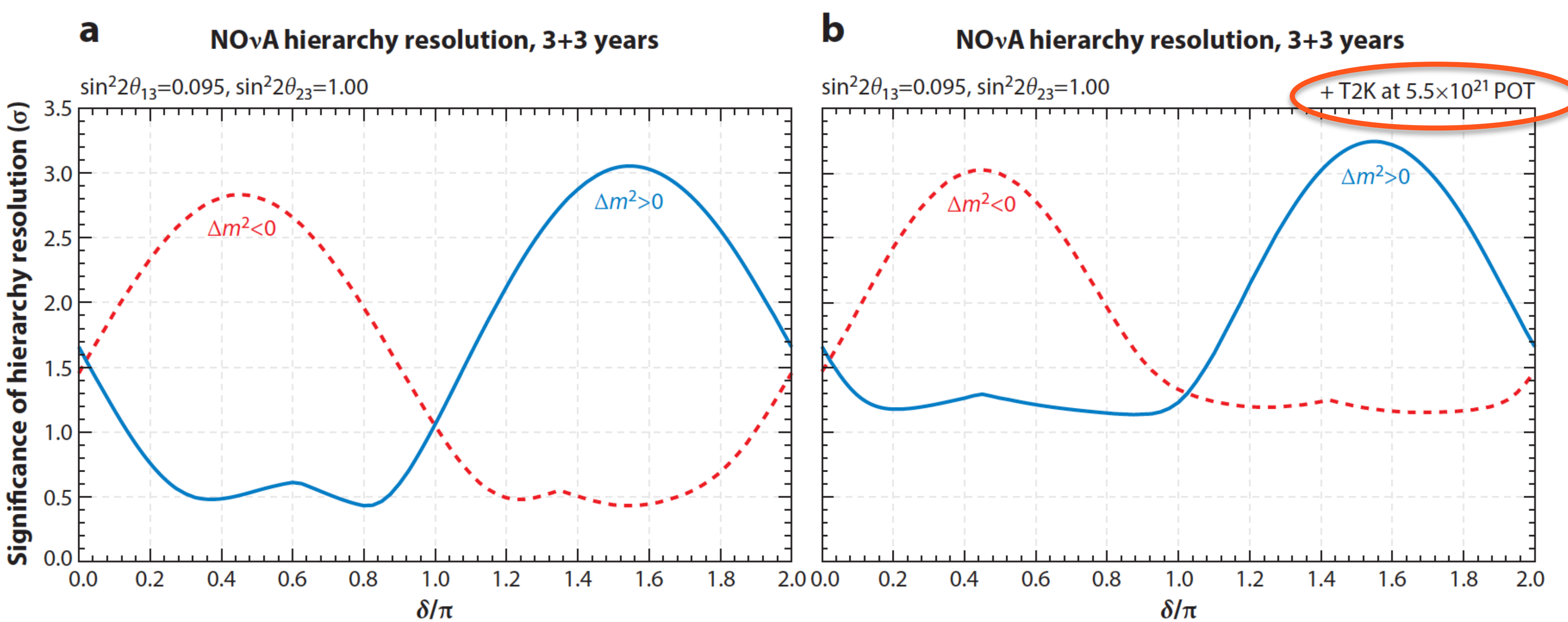
However arXiv:1703.04471
does <u>not</u> find a similar improvement from cosmological data

However arXiv:1703.04585
concludes <u>against</u> 1703.03425 claim due to prior assumptions

Statistical Issues: several studies (Frequentist and Bayesian, Asimov, full simulation…)

They are all based on χ^2 best fits, marginalized over the precisions of 3 ν parameters

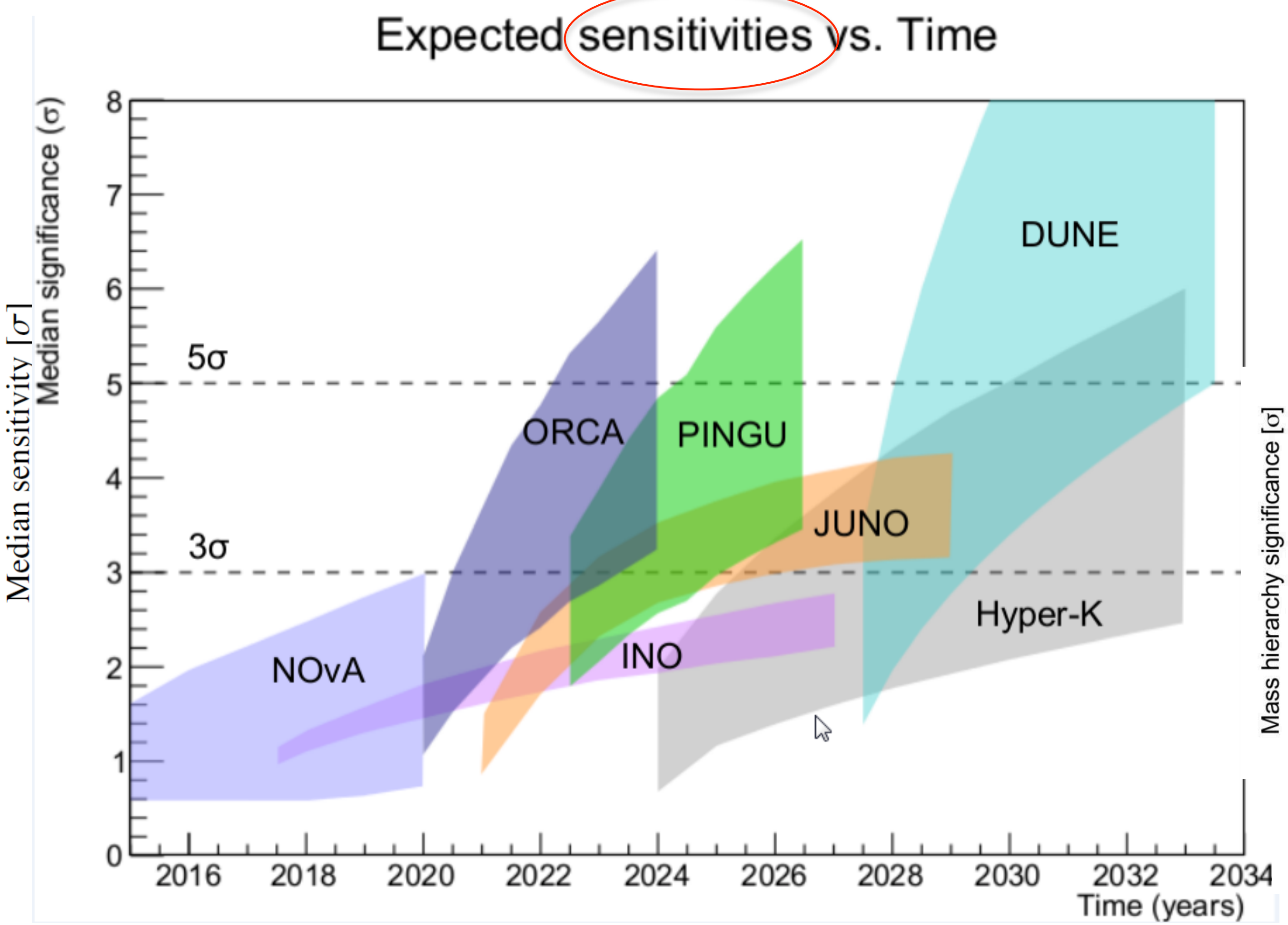
➔ Foreseen years and years of data taking and experiments

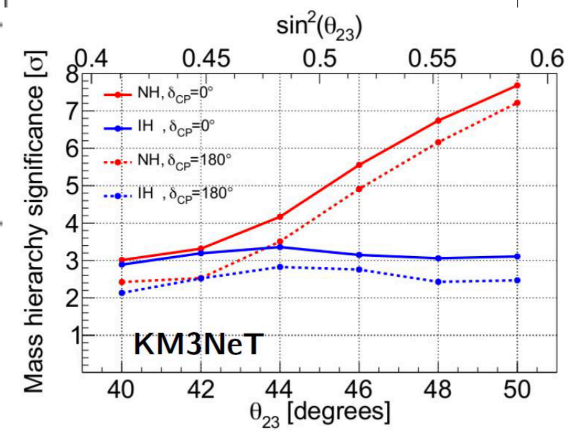


3.6×10^{21} pot

R.B. Patterson, Annu. Rev. Nucl. Part. Sci. 65 (2015) 177

UPDATE: A. Heijboer, talk at NOW2016, Otranto (Italy), 4-11 September 2016





median sensitivities

Many different ways to define "sensitivity"

CAVEATS:

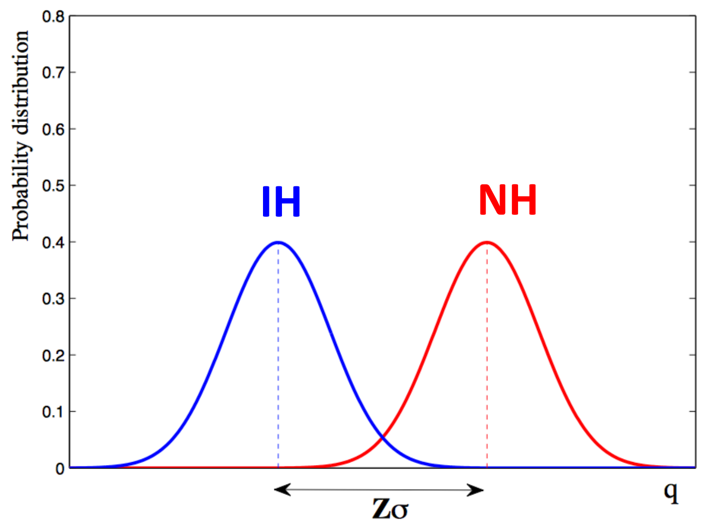
- NOvA: luckiness ?
- PINGU/ORCA: funded ? systematics ?
- INO: really ?
- JUNO: technical challenge on energy resolution ?
- DUNE/HY: ok they will got it, in 10 years from now **IF exploited**

A second slide on STATISTICS

Not all the estimators correspond to the same significance
(Frechet-Cramer-Rao theorem and limit)

For example, the arithmetic mean and the median own two different variances:
Mean ➔ $\sigma/\sqrt{n}$ *(this is also the Cramer-Rao limit)*
Median ➔ $\sqrt{(\pi/2)} \times \sigma/\sqrt{n}$
i.e. the median has an error 25% larger than the mean.

That matters when doing test of hypotheses, determination of NH against IH, etc.

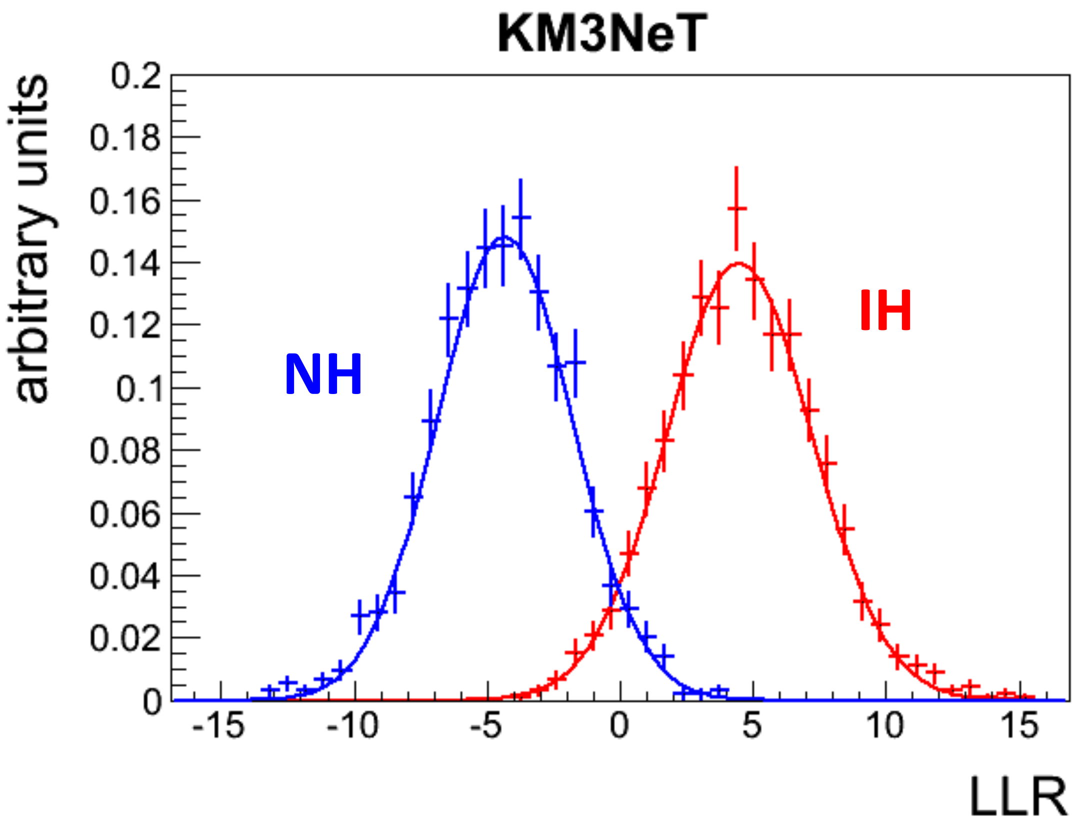


My rule of thumb:
only above 5 σ the scientific result
of an experiment will NOT depend
of the used statistical analysis

An experiment with an estimation of 4 σ but forced
to use medians (e.g. due to systematics, outliers etc.) will get 3.2 σ

How is the sensitivity computed for ORCA ?

(see KM3NeT, J. Phys. G 43 (2016) 084001)



Suppose NH true, IH sensitivity is computed (from Asimov sample or pseudo-experiments) as distance to the <u>median</u> of NH, to quote n-σ against IH,
LLR are Gaussians, take usual Z distance…

 **CRITICAL…** *Full distributions of IH and NH should be taken into account, then sensitivity is generally small*

A third slide on STATISTICS

Within the chosen hypothesis (either NH or IH) everything looks fine:
good Gaussian properties, χ^2 distributions, use of LLR etc.

Problems come when the $\Delta\chi^2$ quantity is used.

Nobody really knows how to properly handle $\Delta\chi^2$ to quote the sensitivity,
(and therefore also the measurement)
because $\Delta\chi^2$ is a rather bad estimator:

- not following the Wilk's theorem;
- NH/IH are disjoint and NOT nested hypotheses;
- marginalization or minimization ?
- frequentist or bayesian computation ?

and, additionally (personal thought) it does not correctly approach
the observation/rejection issue*,
it deals instead with the fuzzy concept of "separation of two hypotheses"
(type I and type II errors)

* *When dealing with $\Delta\chi^2$ the solution is given by the best χ^2. Thus you are trying to define the observation through the right hypothesis itself, and NOT trying to reject the wrong hypothesis.*

Come back to JUNO: very recent idea from Bilenky, Capozzi and Petcov, arXiv:1701.06328

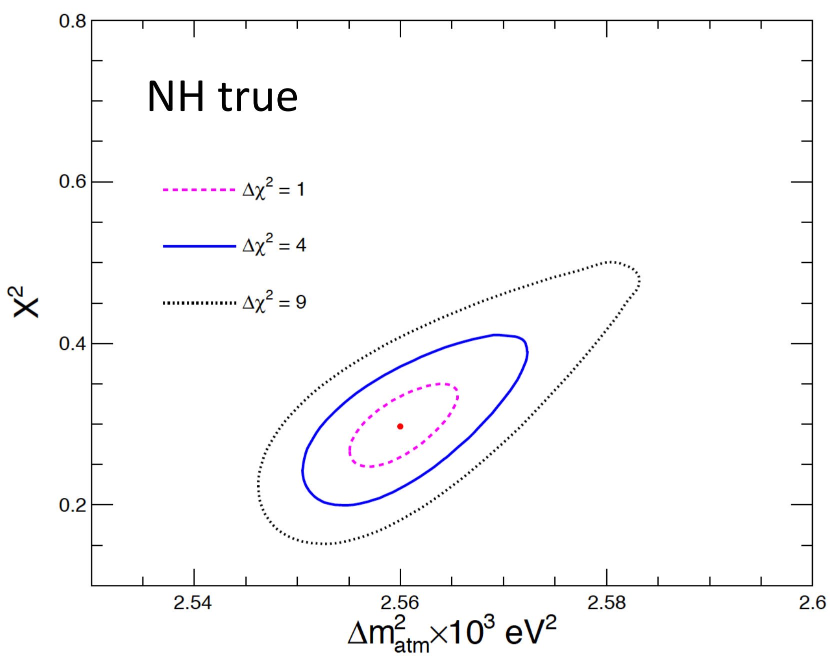
Writing the survival probability

complementary method,
no increasing in sensitivity,
it cures the "nested" problem

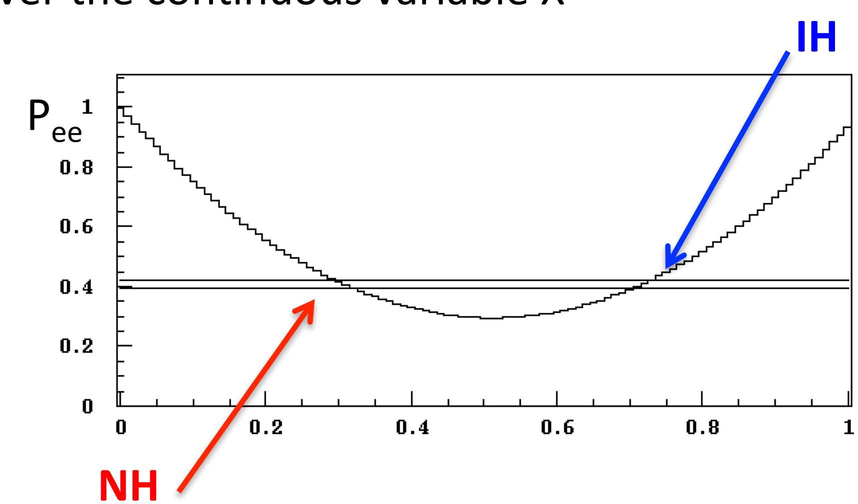
IO

$$
\begin{aligned}
P^{NO}(\bar{\nu}_e \to \bar{\nu}_e) \\
= \; & 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\rm atm} L}{2\,E_\nu}\right) \\
- \; & \frac{1}{2}\cos^4\theta_{13}\,\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m^2_{\odot} L}{2\,E_\nu}\right) \\
+ \; & \frac{1}{2}\sin^2 2\theta_{13}\,\sin^2\theta_{12}\left(\cos\left(\frac{\Delta m^2_{\rm atm} L}{2\,E_\nu} - \frac{\Delta m^2_{\odot} L}{2\,E_\nu}\right) - \cos\frac{\Delta m^2_{\rm atm} L}{2\,E_\nu}\right)
\end{aligned}
$$

$\cos^2\theta_{12}$



Define **X=$\sin^2\theta_{12}$** and perform minimization over the continuous variable X

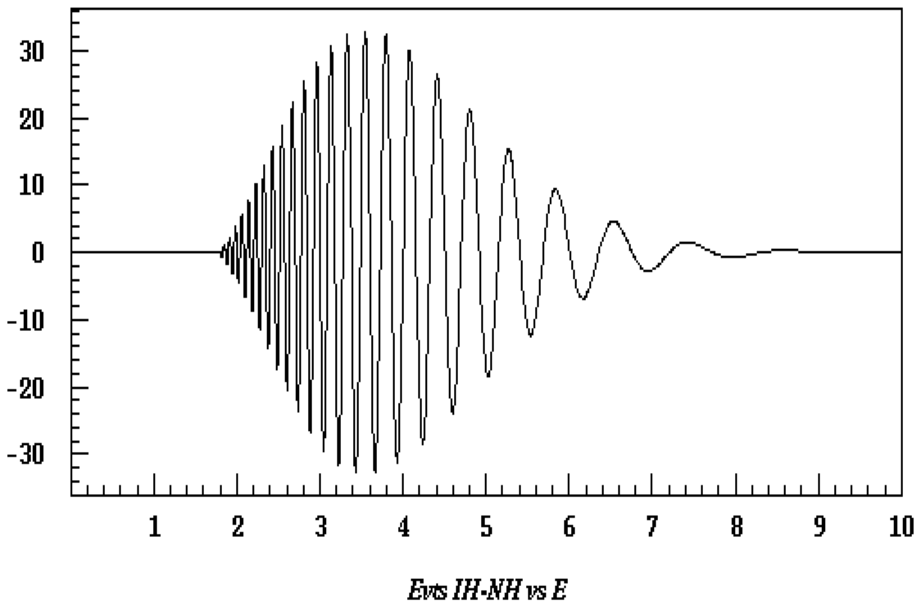


(see S. Dusini, F. Sawy, LS poster)

A simple consideration:

why to use in JUNO non-informative points ?
This is irrelevant within the $\Delta\chi^2$ method,
but it is not for data with systematics,
and for possible other estimators.

Amount of difference in events for 180 GW•year



New estimator: F

$$F = \sum_j \Delta_j^+ + \sum_j \Delta_j^-$$

$$\Delta^+ = n_{obs} - n^{IH}_i \text{ when } n_{obs} > n^{IH}_i \text{ in } I^+$$

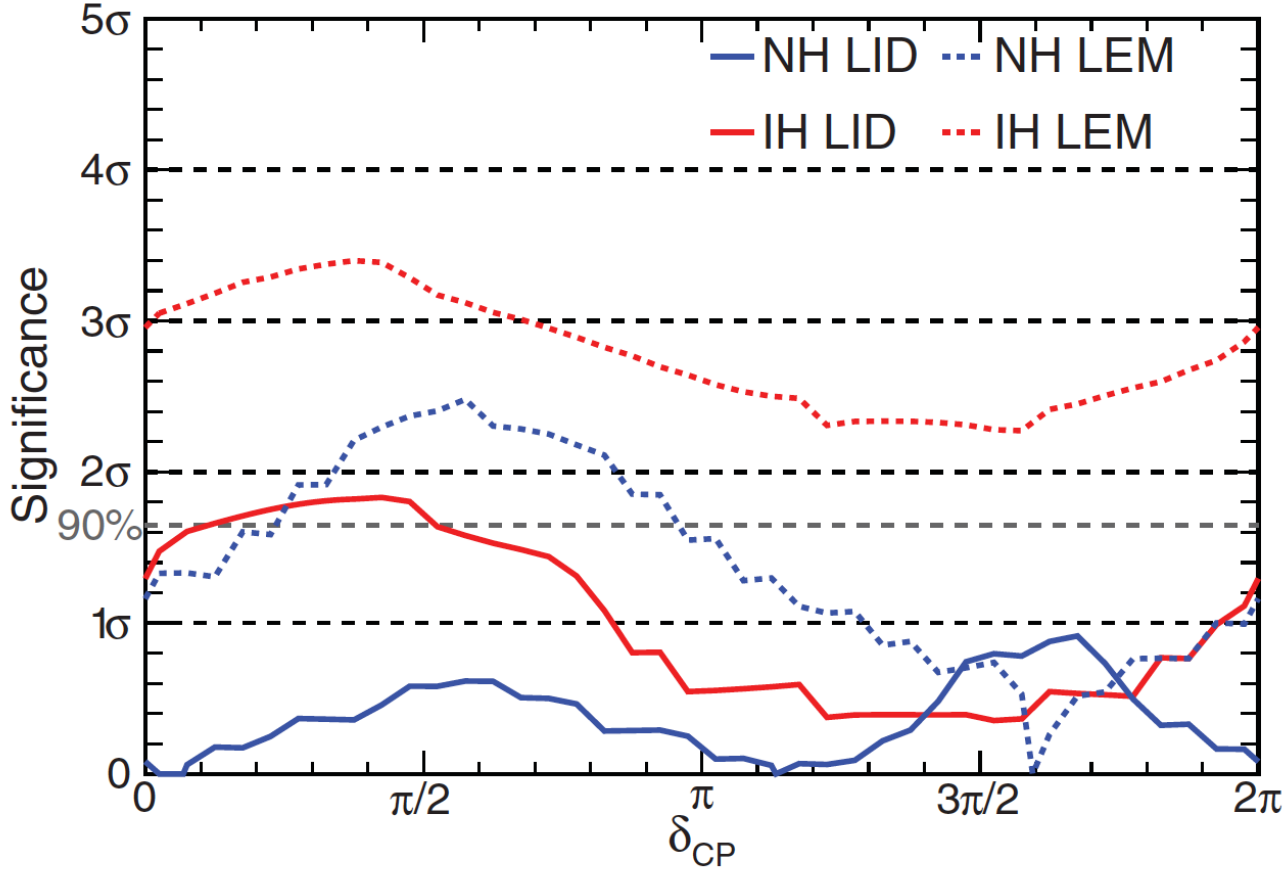
$$\Delta^- = n^{IH}_i - n_{obs} \text{ when } n_{obs} < n^{IH}_i \text{ in } I^-$$

$$I^+ \text{ intervals when } n^{IH}_i < n^{NH}_i$$

$$I^- \text{ intervals when } n^{IH}_i > n^{NH}_i$$

➔ *a possible complementary analysis*

Meanwhile data are released: NOvA 2015, PRL, 116, 151806 (2016)

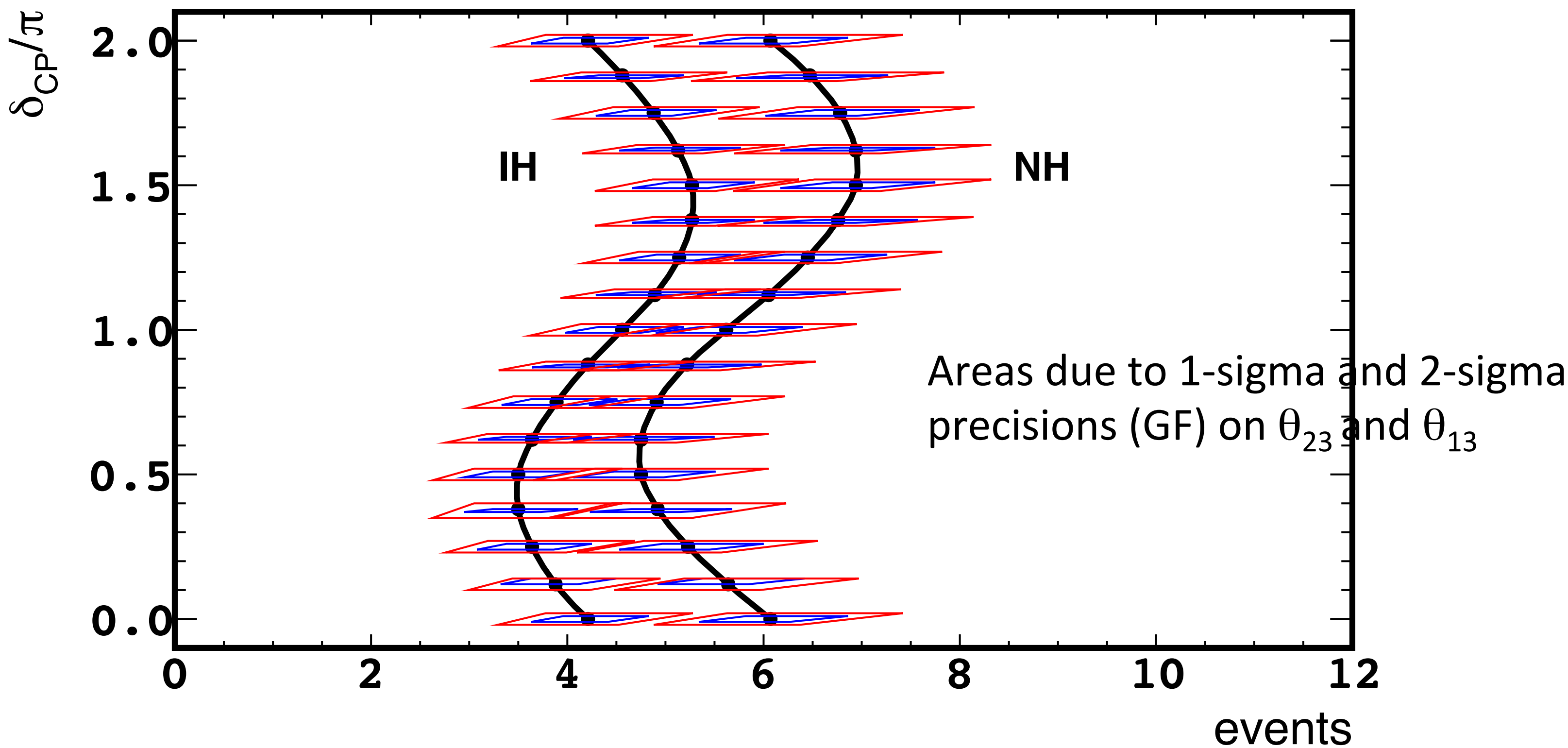


2.74×10^{20} pot

(1/13 luminosity)

LID: 6 events
LEM: 11 events

NOvA 2015 luminosity + Global Fits + GLoBES simulation*



(signal+bck)

Several possible configurations with $\chi^2 \leq 1$

AAA: the standard 3ν oscillation framework is assumed !

➔ new method *(it means new estimator)*

L. S., S. Dusini and M. Tenti, Phy.Rev. D, 95, 053002 (2017), arXiv:1606.09454v3

(see also poster from Matteo Tenti)

1 Take the Poisson distributions, $\boldsymbol{f_{IH}}$ and $\boldsymbol{f_{NH}}$ and the new estimators, $\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$, as function of a threshold $\boldsymbol{n}$, nb. of events, and of δ_{CP}:

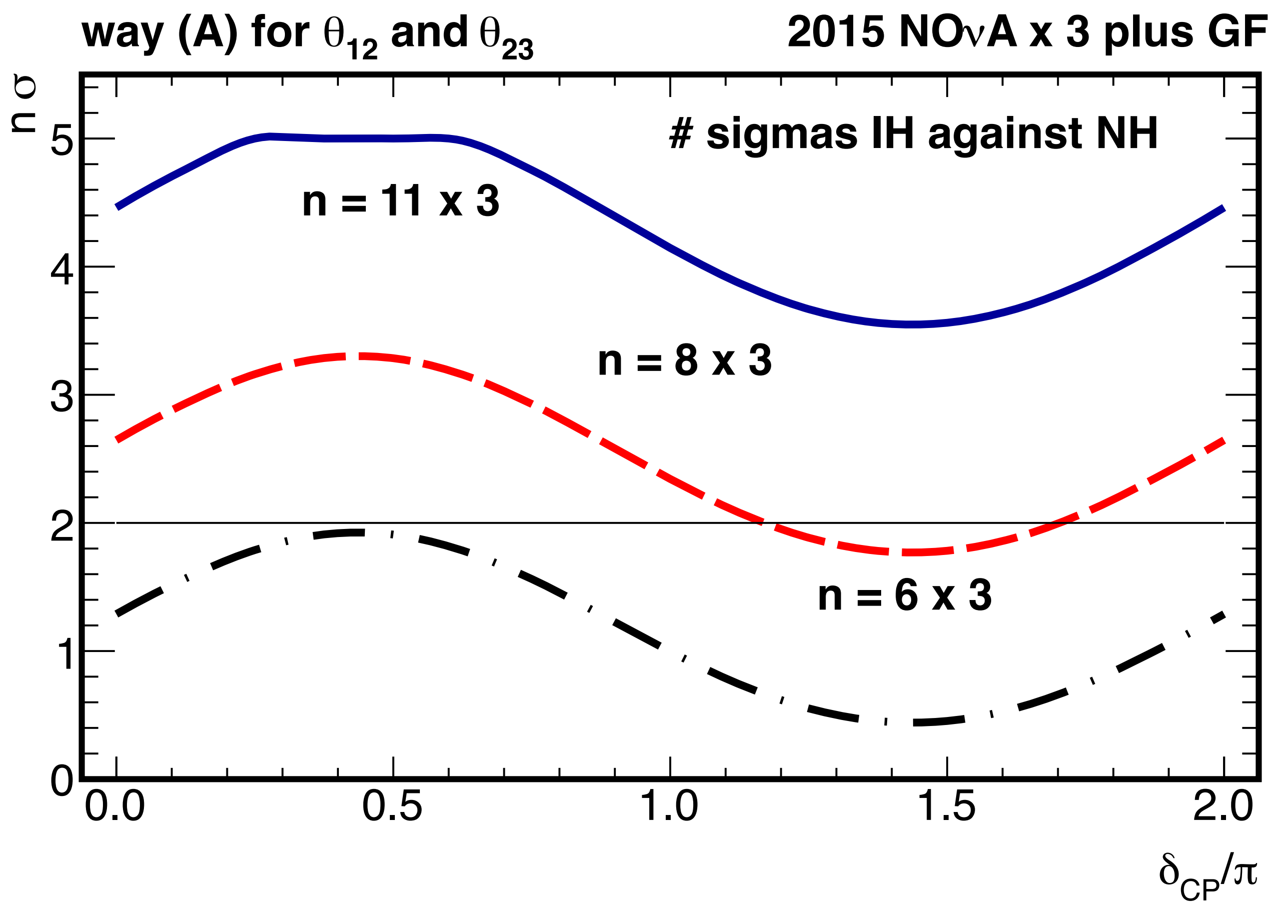
$$q_{IH}(n,\delta_{CP}) = \frac{\sum_{n_i^{IH}\geq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}{\sum_{n_i^{NH}\geq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}$$

$$q_{NH}(n,\delta_{CP}) = \frac{\sum_{n_i^{NH}\leq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}{\sum_{n_i^{IH}\leq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}$$

$\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$ are discrete random variables in [0,1]

2 Compute probability mass functions $\boldsymbol{P_{IH}}$ and $\boldsymbol{P_{NH}}$ of $\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$
Assume one Hierarchy, compute its ***p***-value for the corresponding estimator
Compute n-sigma with the <u>one-sided option</u>

Suppose to have a factor 3 more of 2015 NOvA luminosity*:

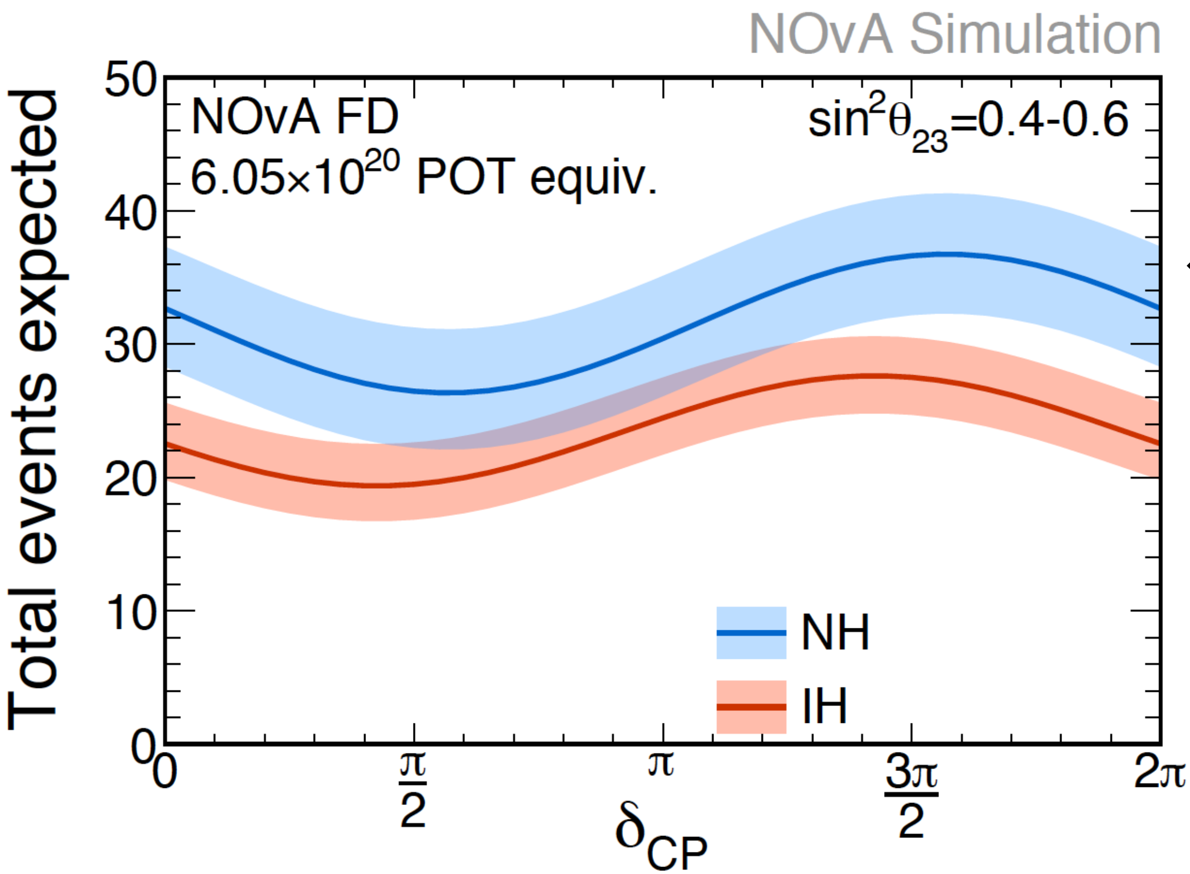


In case 24 events be observed the IH is excluded at 95% C.L. in almost all δ_{CP} the range !

(* with the same analysis conditions)

NOvA released 2016 analysis with 6.02*10^{20} p.o.t.

(factor 2.21 more luminosity than 2015)



← 33 events observed

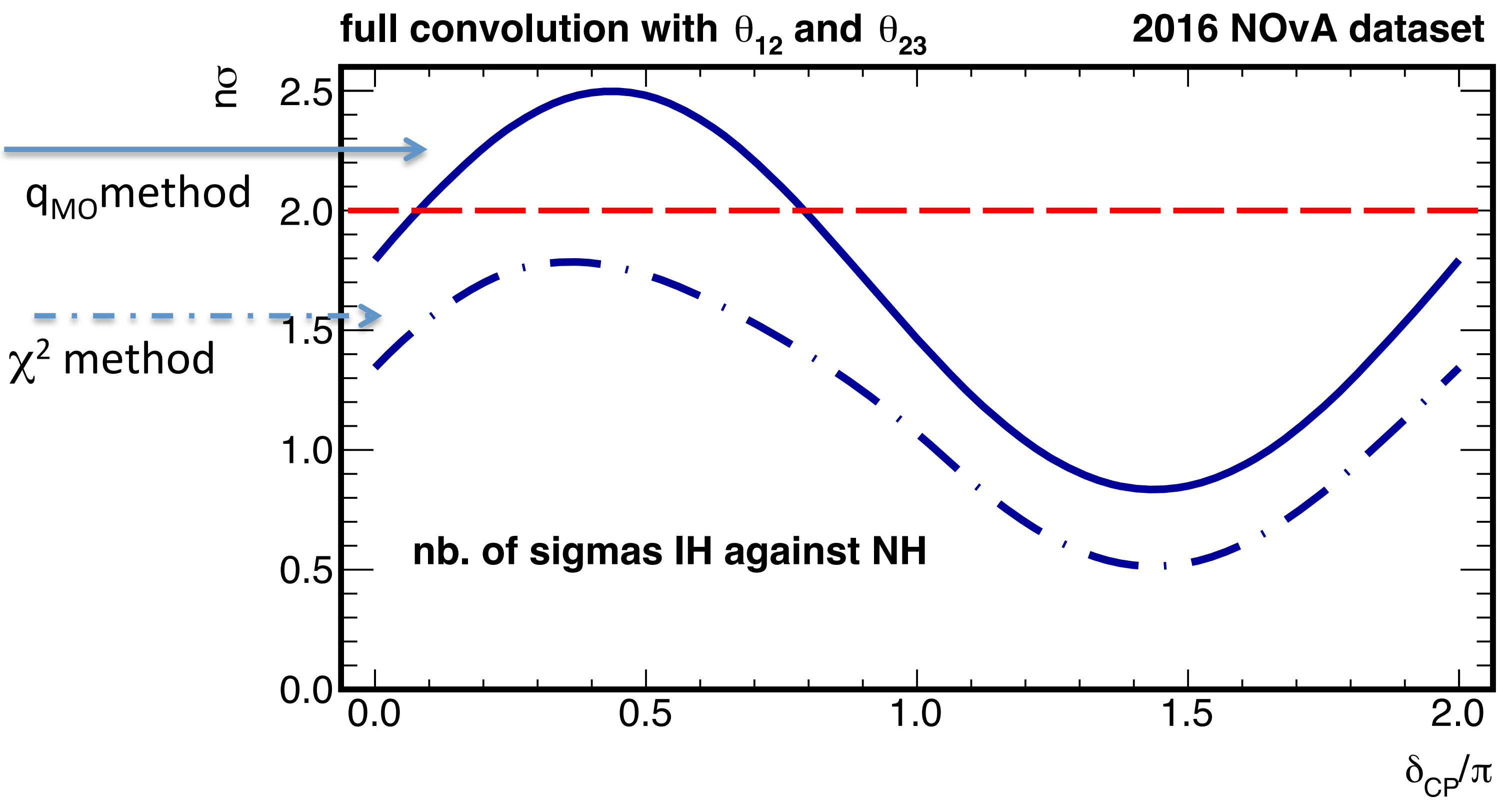
NOvA decided to change rather deeply its analysis:

2.5 more efficient for the signal

against

an increase of a factor 4.5 for the background

Our result for the updated NOvA results (Neutrino2016)



IH is excluded at 95% C.L. in δ_{CP} the range the 0.10 π – 0.77 π, robustly.

Note: this result is NOT equivalent to the NOvA statement, mainly because there is no fit on θ_{23}

A final slide on Statistics and Mass Hierarchy

MH determination is it a discovery or a exclusion analysis ?

For a discovery sensitivity is much easier to estimate and quote.
Then the right process is to "reject the wrong hierarchy".
A 5 σ result would be needed.
But that process for MH can be also seen as an exclusion one.
Then a 95% C.L. quotation could be sufficient (*) !

* If
- properly computed with a good estimator and billion (trillion) of simulated events
- to properly evaluate its Probability Density Function and
- to further properly extract its significance

Conclusions and perspectives

- The MASS ORDERING is one of the 4 missing parameters to complete the 3ν oscillation framework and start the precision era
- Hierarchy is highly dependent of CP angle, in most of the experiments

- Statistical analysis IS an issue since we are dealing with ”sensitivity” below 5 σ
- A new statistical method has been introduced: arXiv:1606.09454v3 (robust, all MC simulated, without statistical concerns)
- With this method NOvA may quite soon set a comfortable exclusion level for IH (or NH). With 2016 result, IH is excluded in 0.10 - 0.77π with the new method
- We plan to extend our technique to other data, like JUNO and PINGU-like, and add T2K data.

- It is time to look forward and evaluate the consequences of MO knowledge on the future programs !

  ➔ T2K should have easier life for δ_{CP},

  ➔ JUNO will become relevant for checking the consistency of the 3ν framework (measurement of MO without δ_{CP} dependence)

  ➔ HK/DUNE may be forced to re-analysis their goals (δ_{CP} accuracy)

Finally…

When dealing with EXCLUSION LIMITS
reality is usually* more severe than
Monte-Carlo/statisticians expectations.

When dealing with OBSERVATIONS
physicists are usually much better than
Monte-Carlo/statisticians expectations.

* Not always true: first release of Higgs exclusion at 2008 ICHEP (Philadelphia) from CDF

BACKUP

The present scenario

From masses to flavours:

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{e\mu 3}|\nu_3\rangle$$

$$|\nu_\tau\rangle = U_{\tau 1}|\nu_1\rangle + U_{\tau 2}|\nu_2\rangle + U_{\tau 3}|\nu_3\rangle$$

U is the 3 × 3 Neutrino Mixing Matrix, mixing given by 3 angles, θ_{23}, θ_{12}, θ_{13} and one phase δ (CPV for $\delta \neq 0,\pi$).

Oscillations have amplitudes driven by the mixing angles and frequencies by
$\delta m^2_{solar} = \Delta m^2_{21}$
$\Delta m^2_{atm} = |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$
(and L, E experimental parameters)

E.g. if only two flavours are taken, at leading order, neglecting interference terms, and taking some approximations:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

APPEARANCE

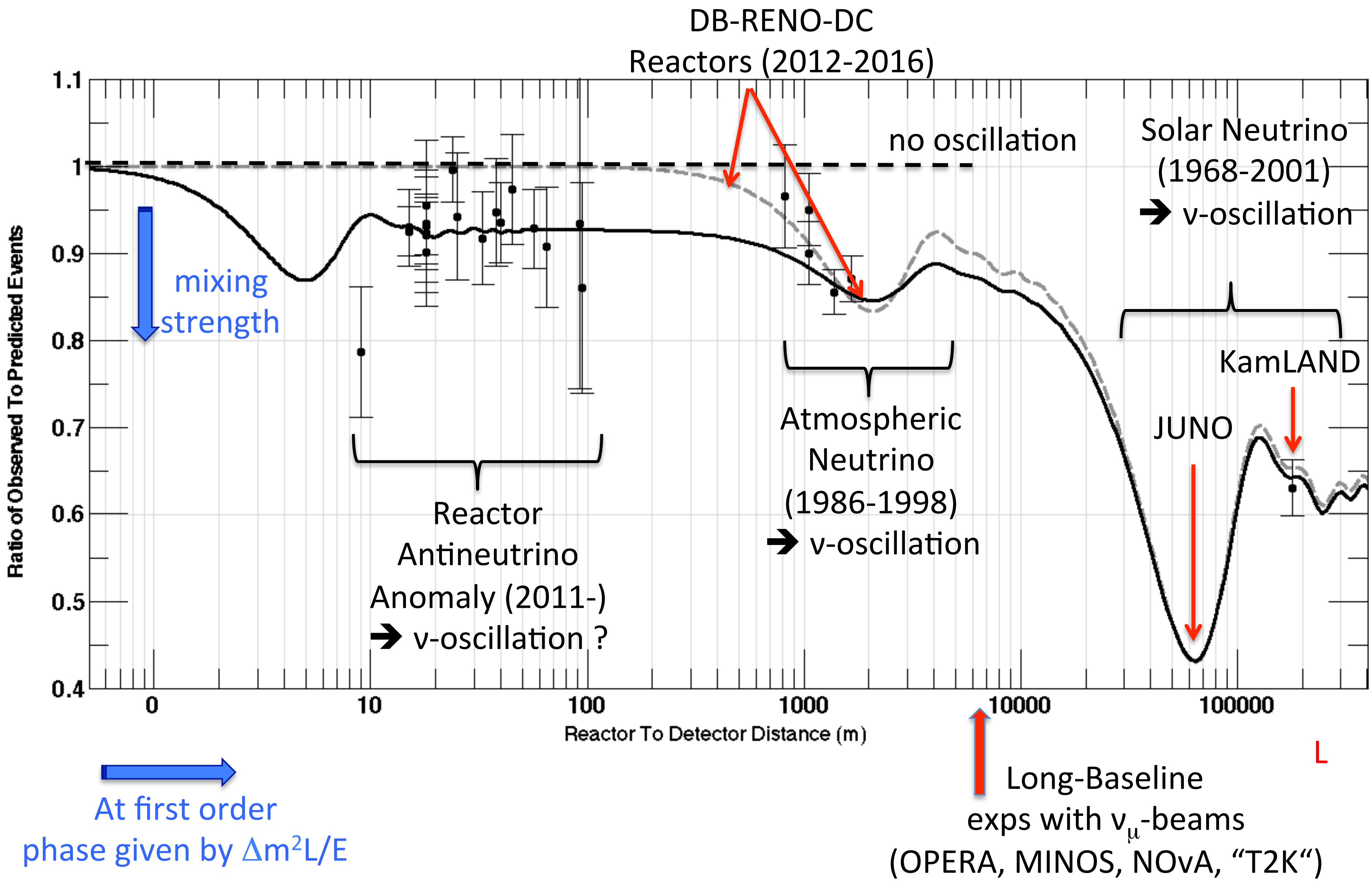
$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

DISAPPEARANCE

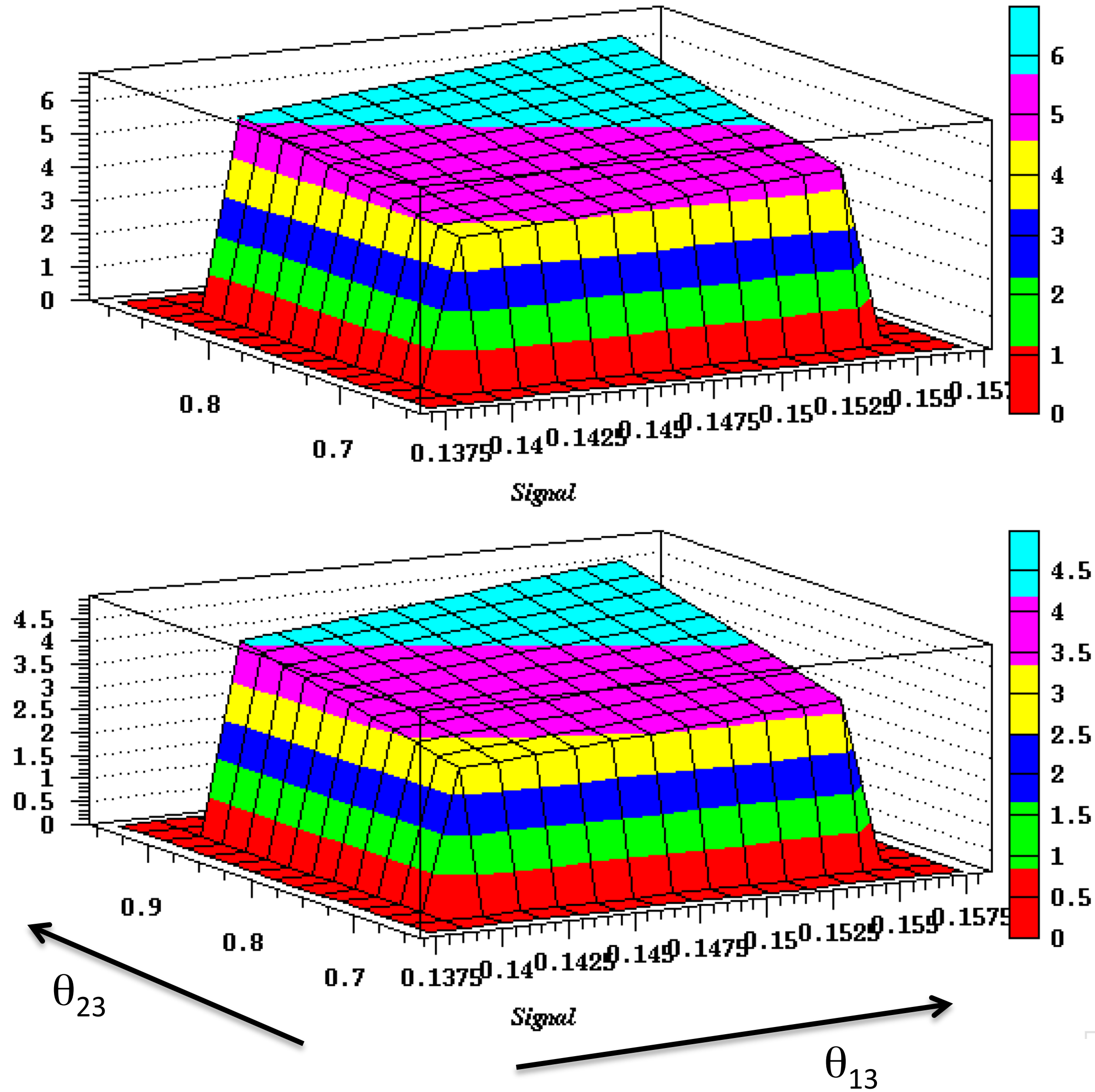
amplitude

frequency

The oscillation picture in terms of **distance L** from the (*reactor*) source



Variation on the number of events due to θ_{23} and θ_{13} precisions (➔not really degeneracy)

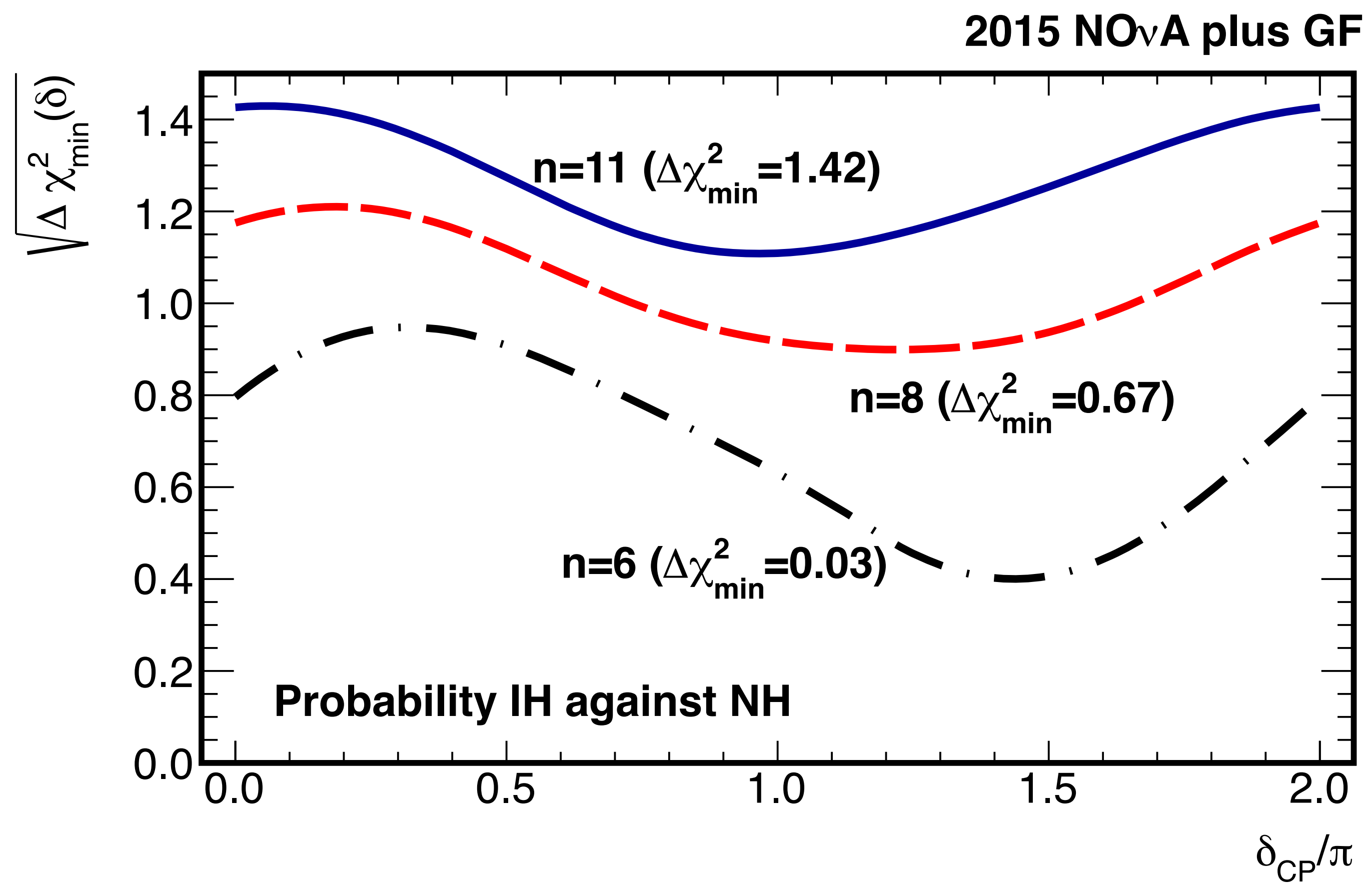


NH

IH

(NOvA 2015 luminosity + Global Fits + GLoBES simulation)

Usual procedure with χ^2 (find minimum χ^2 over best fits for IH and NH)



No strong gain when luminosity increases, even for the many concerns about the χ^2 statistics (Wilck not valid, no statistical and systematic fluctuations, no Gaussian)

➔ new method

1 Take the Poisson distributions, $\boldsymbol{f_{IH}}$ and $\boldsymbol{f_{NH}}$ and the new estimators, $\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$, as function of a threshold $\boldsymbol{n}$, nb. of events, and of δ_{CP}:

$$q_{IH}(n,\delta_{CP}) = \frac{\sum_{n_i^{IH}\geq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}{\sum_{n_i^{NH}\geq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}$$

$$q_{NH}(n,\delta_{CP}) = \frac{\sum_{n_i^{NH}\leq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}{\sum_{n_i^{IH}\leq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}$$

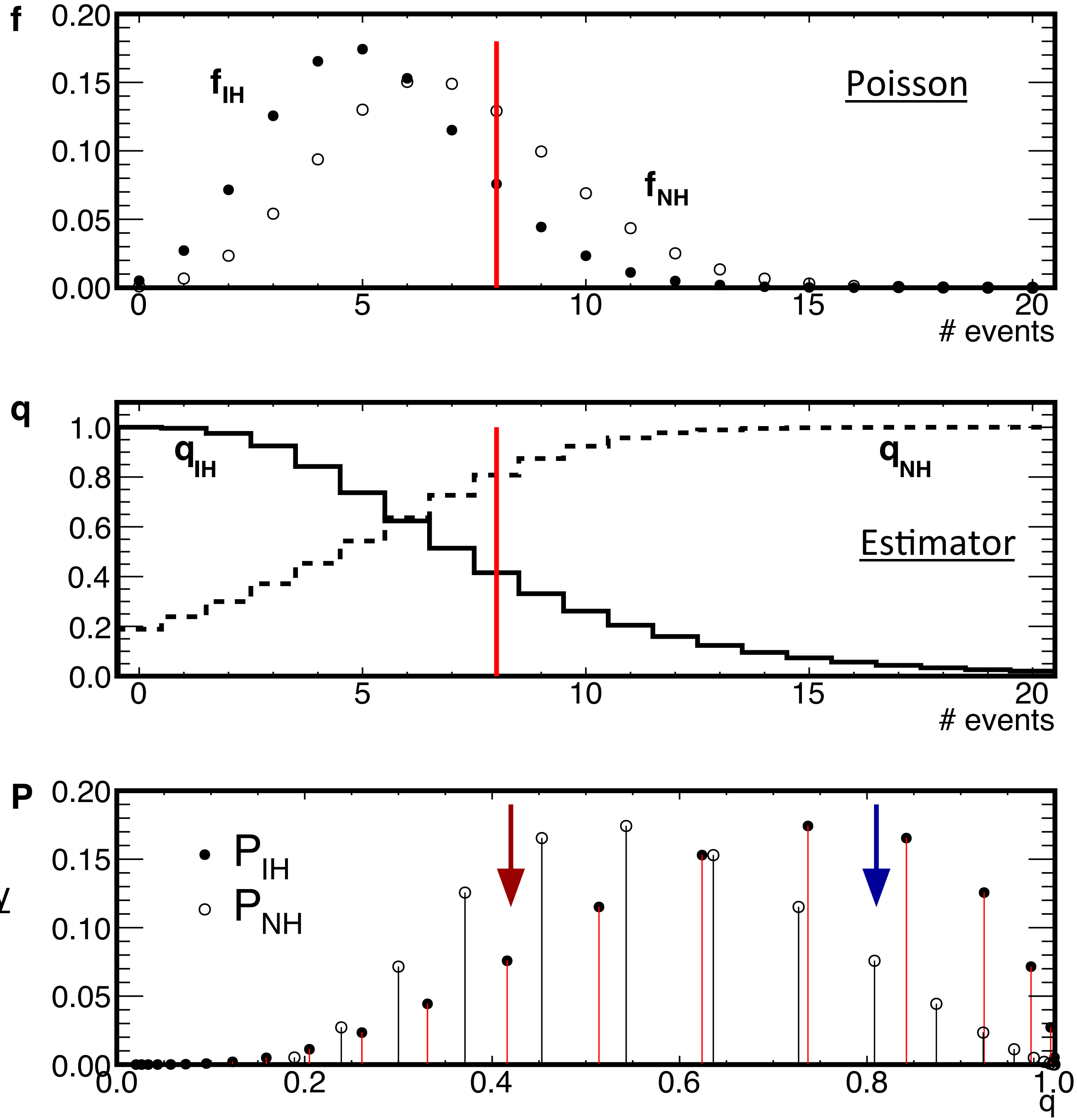
$\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$ are discrete random variables in [0,1]

2 Compute probability mass functions $\boldsymbol{P_{IH}}$ and $\boldsymbol{P_{NH}}$ of $\boldsymbol{q_{IH}}$ and $\boldsymbol{q_{NH}}$, based on

- $\boldsymbol{f_{IH}}$ (for test of IH against NH)
- $\boldsymbol{f_{NH}}$ (for test of NH against IH)

Example for
$\delta_{CP} = 3/2\pi$ and
2.74×10^{20} p.o.t. ,
n = 8

Probability

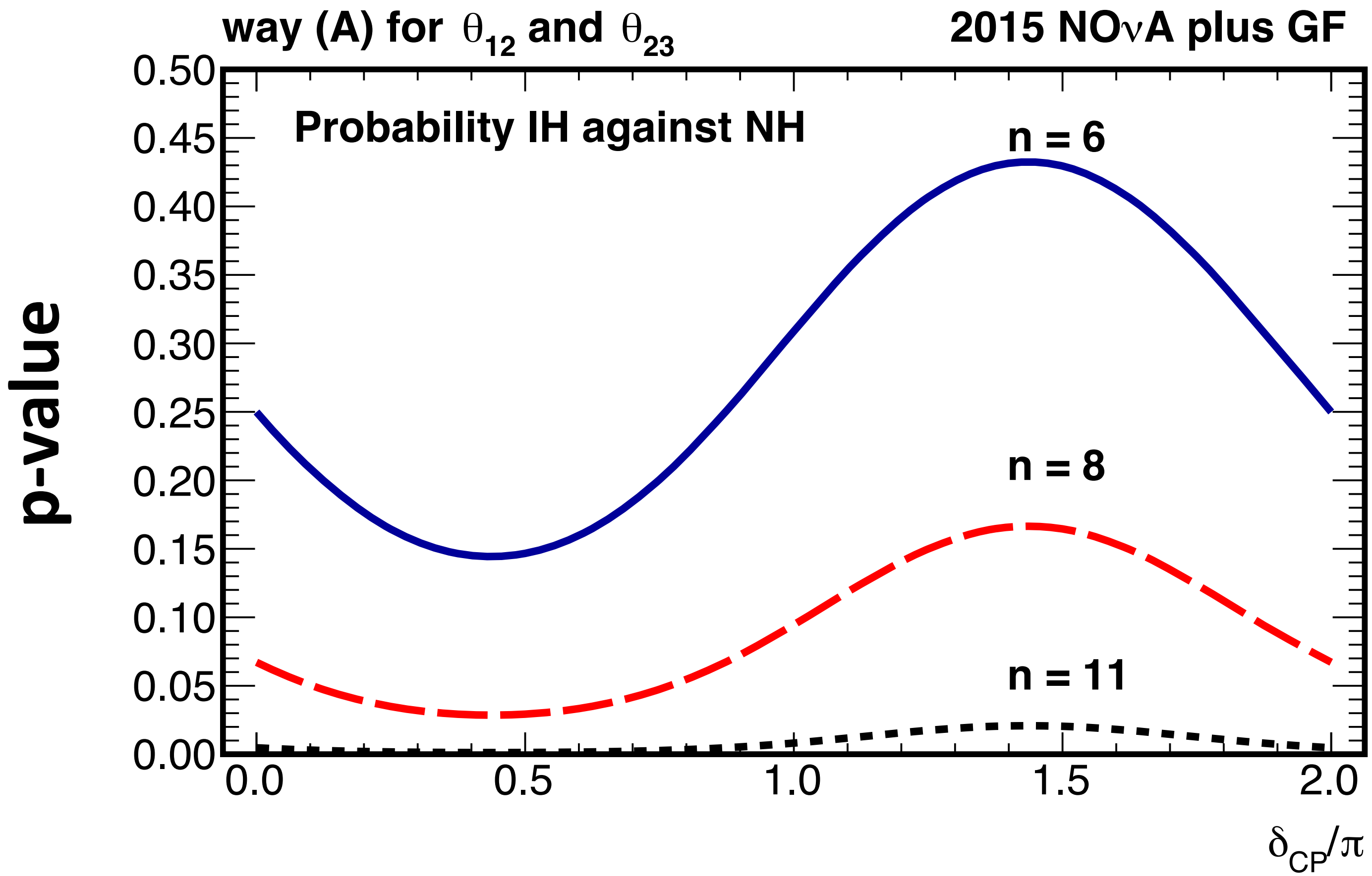


4 Assume one Hierarchy, compute its ***p***-value for the corresponding estimator

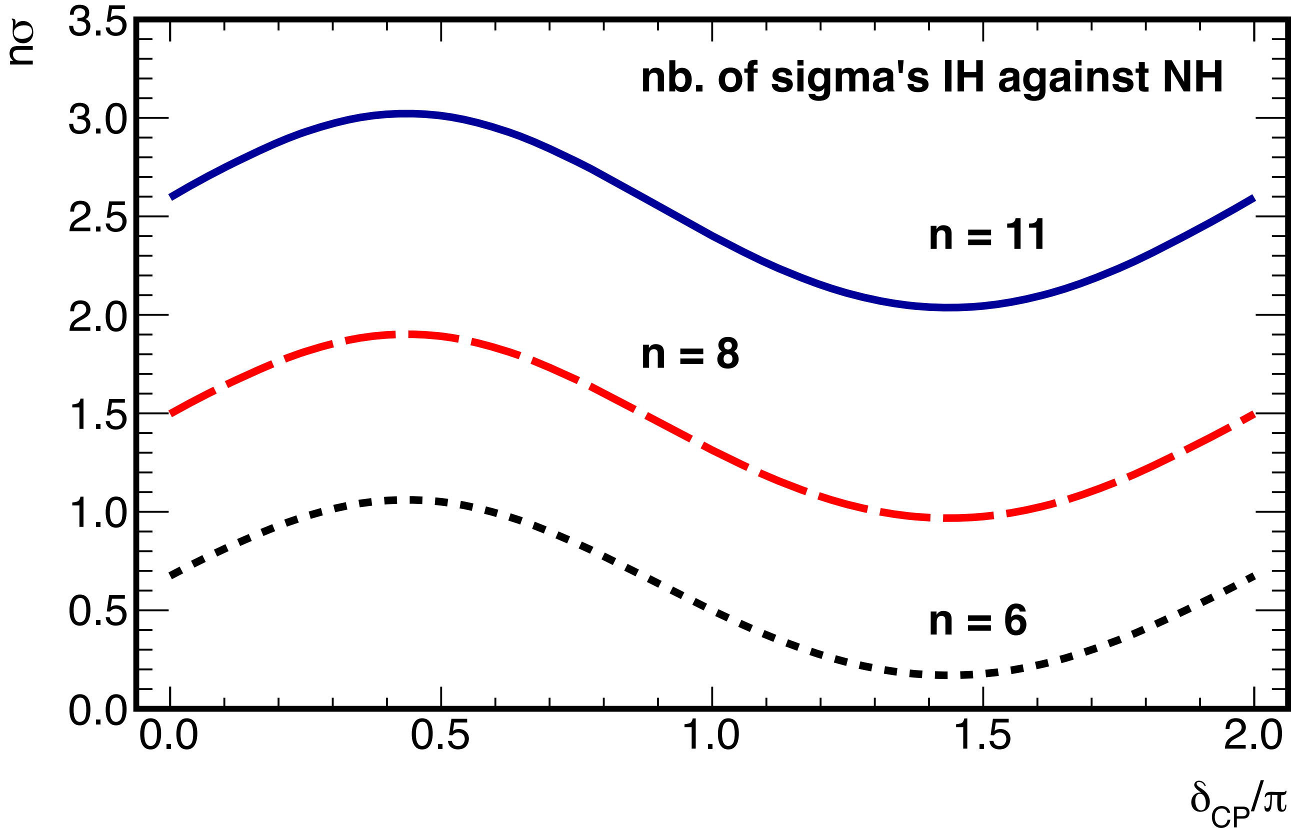
$$p_{IH}(n_D, \delta_{CP}) = \sum_{q'_{IH} \leq q_{IH}(n_D)} P_{IH}(q'_{IH} | \delta_{CP})$$

$$p_{NH}(n_D, \delta_{CP}) = \sum_{q'_{NH} \leq q_{NH}(n_D)} P_{NH}(q'_{NH} | \delta_{CP})$$

5 Compute n-sigma with the <u>one-sided option</u>
(***p***=0.5 ➔ 0 sigma)



Treatment of θ_{23} and θ_{13} uncertainties as priors



- **gain in average 0.5 σ over the χ^2 method**
- **it increases with marginality of Poisson tail…**

WHY we gain ?

We look at the intrinsic statistical fluctuation on the signal.

No fluctuation, i.e. just around the average,
➔small gain

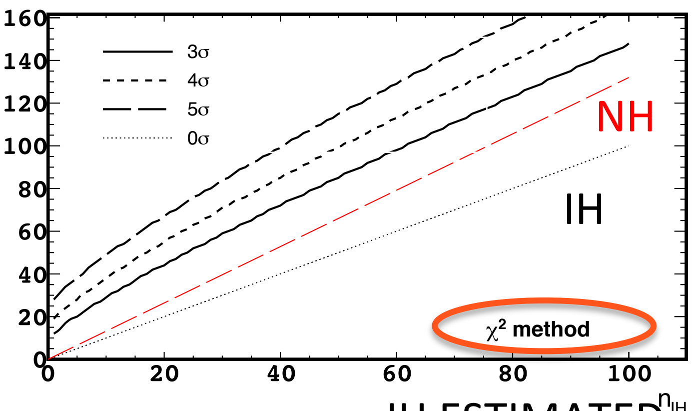
Fluctuation ➔ large gain

and choose an appropriate test statistic.

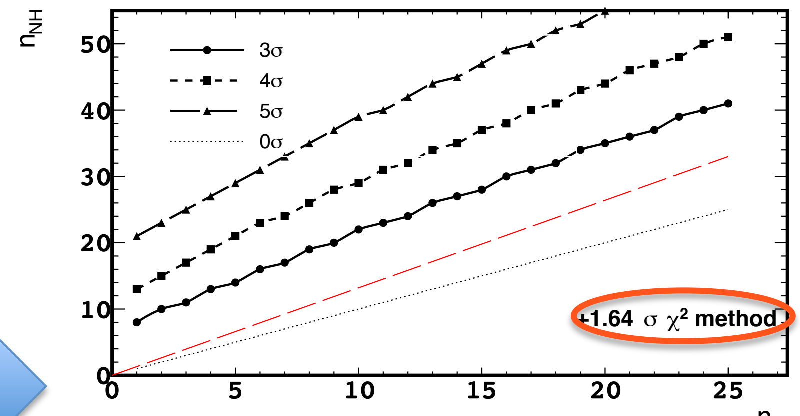
Performances of the χ^2 and the q_{MO} method by increasing the sample and the fluctuation

Isolines for 1,2,3 σ distinction (δ_{CP}, = 1.5 π, 2015 NOvA conditions)

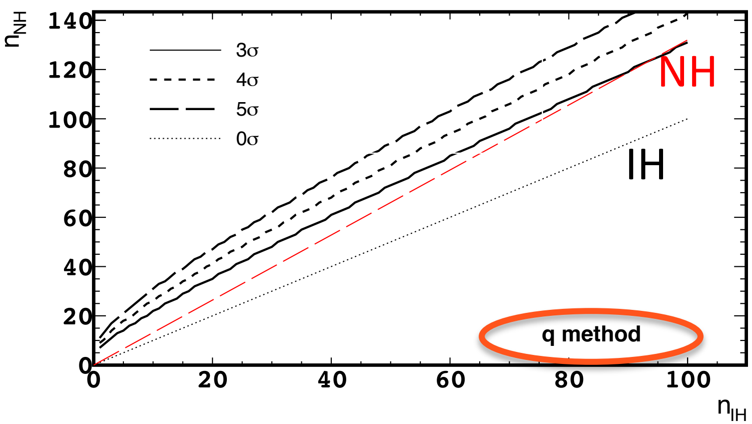
(NH) OBSERVED

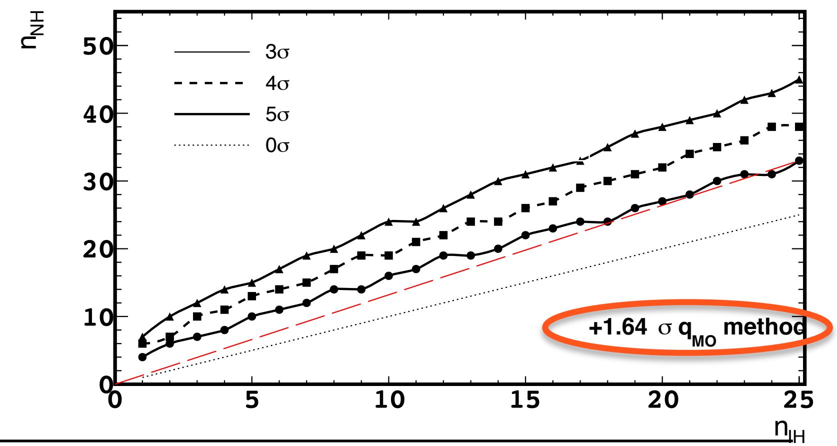


IH ESTIMATED



Median - NO fluctuation



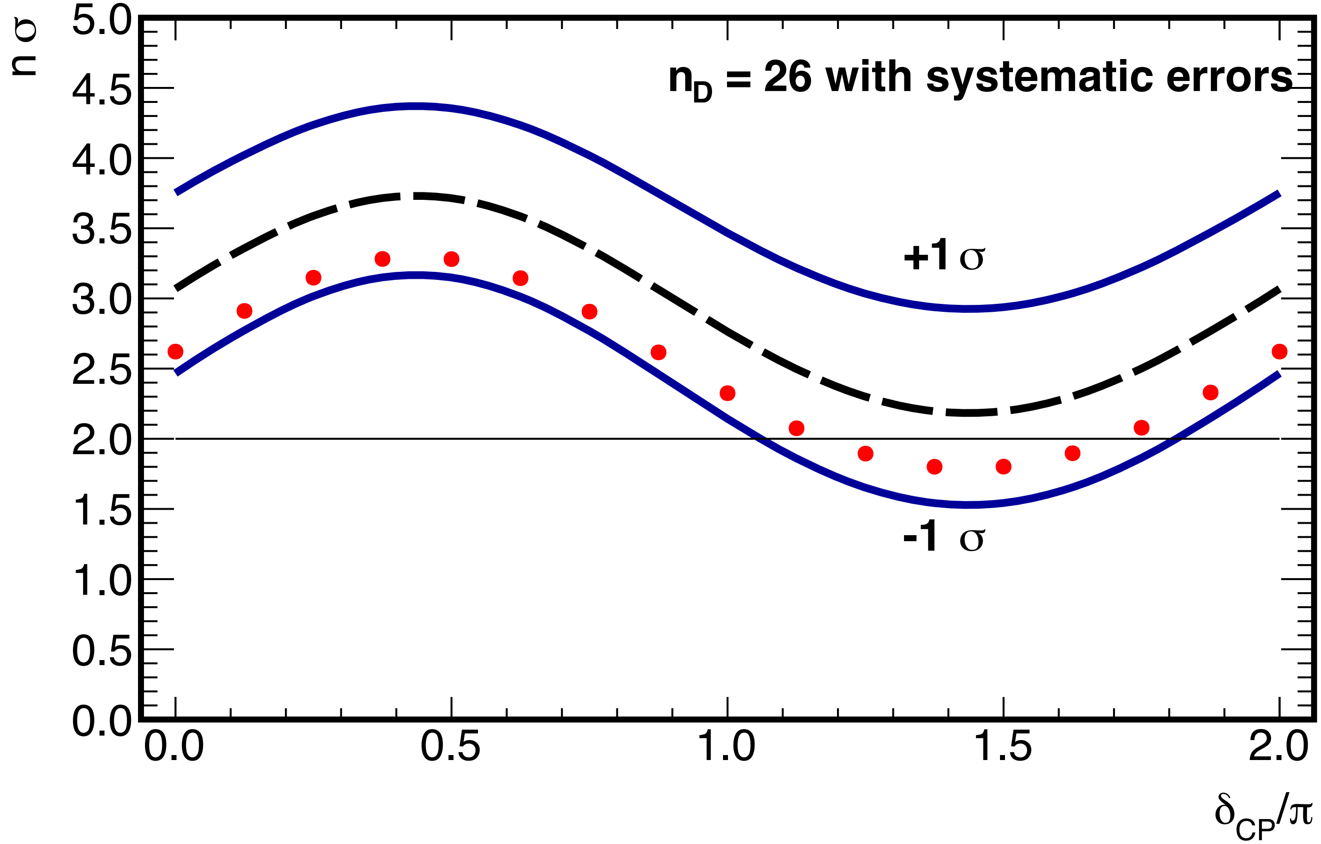


1.64 σ fluctuation

Gaining Factors

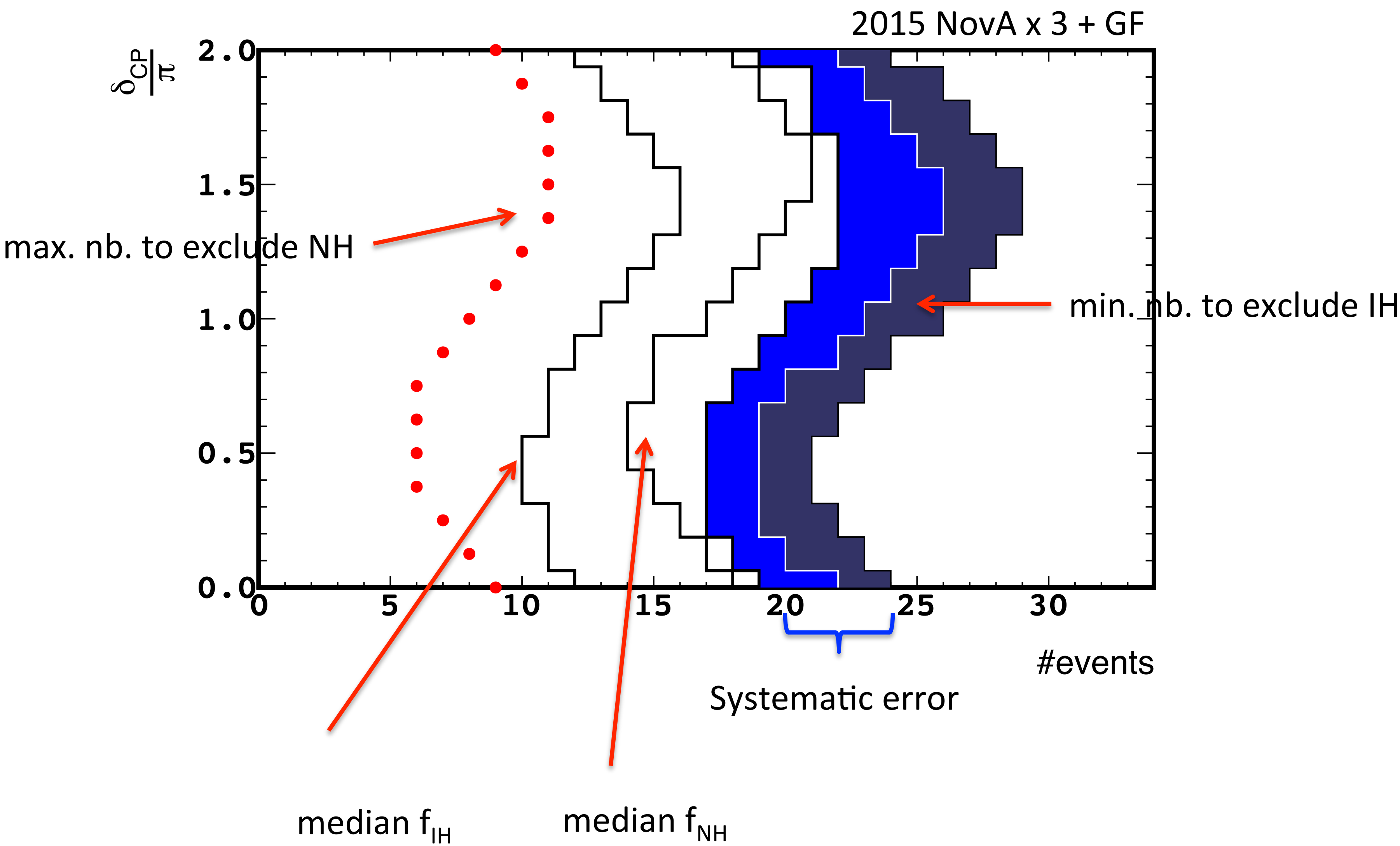
median	average on δ	spread
100% prob	2.27	+0.18-0-12
32% prob.	2.75	+0.51-0.19
10% prob.	3.78	+0.79-0.39

Include 2015 NOvA systematic error (take 17.6% for signal and 11% for background)



- **For the CPV range around 0.4 π – 0.6 π IH is excluded at 3 sigma even including 1 sigma systematic error**
- **Red dots corresponds to a prior treatment**

What is the minimum (maximum) number of events to be observed by NOvA, with a factor 3 in luminosity, to exclude IH (NH) ?



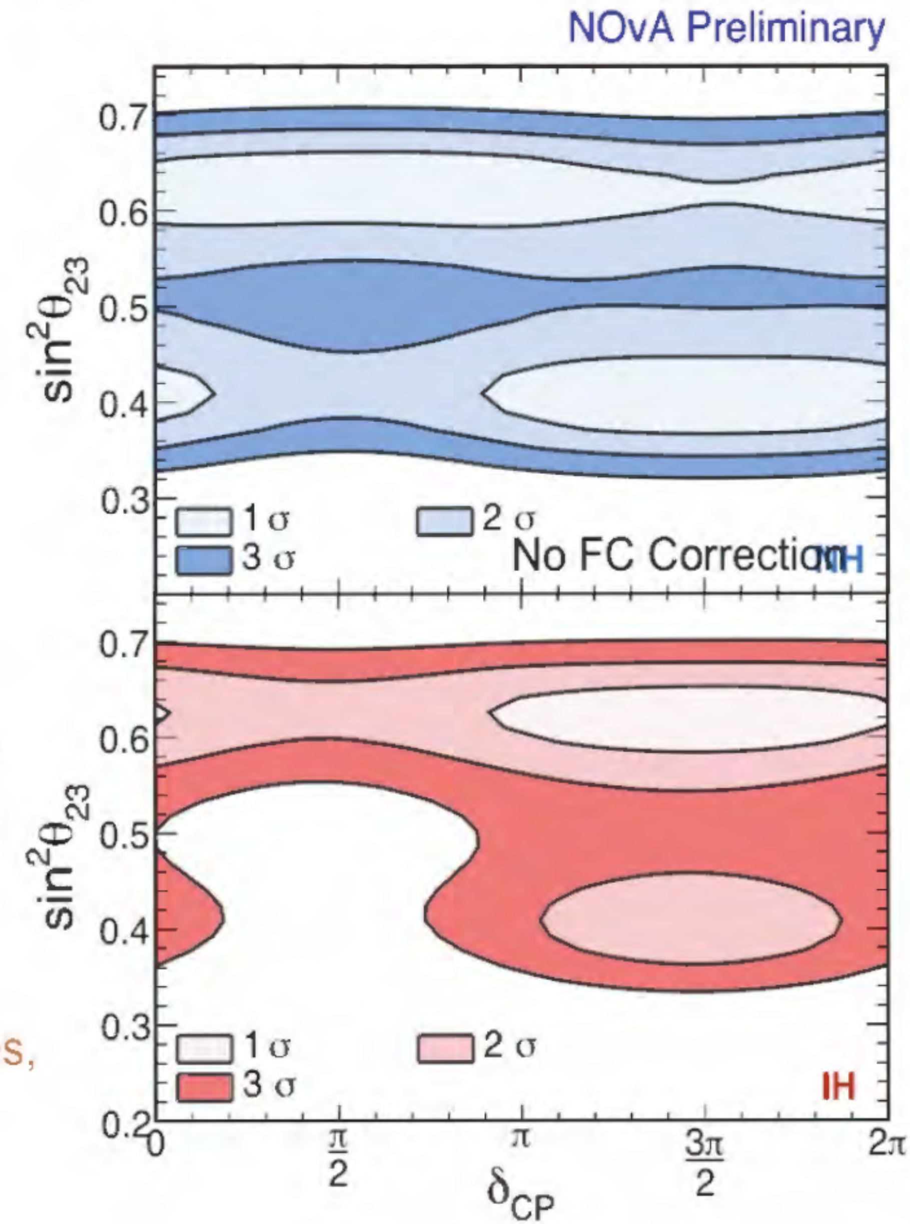
NOvA statements on IH/NH

Contours

- Fit for hierarchy, $\boldsymbol{\delta}_{CP}$, $\sin^2\theta_{23}$
  - Constrain Δm^2 and $\sin^2\theta_{23}$ with NOvA disappearance results
  - Not a full joint fit, systematics and other oscillation parameters not correlated
- Global best fit Normal Hierarchy

$$\delta_{CP} = 1.49\pi$$
$$\sin^2(\theta_{23}) = 0.40$$

  - best fit IH-NH, $\Delta\chi^2$=0.47
  - both octants & hierarchies allowed at 1σ
  - 3σ exclusion in IH, lower octant around $\boldsymbol{\delta}_{CP}=\pi/2$



Antineutrino data will help resolve degeneracies, particularly for non-maximal mixing

Planned for Spring 2017