Theory and Phenomenology of Leptonic CP Violation

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The Pontecorvo Prize for 2016 (24/02/2017): Prof. Yifang Wang (Daya Bay), Prof. Soo-Bong Kim (RENO), Prof. K. Nishikawa (T2K)

"For their outstanding contributions to the study of the neutrino oscillation phenomenon and to the measurement of the Theta13 mixing angle in the Daya Bay, RENO and T2K experiments."

The relatively large value of the "reactor" angle $\theta_{13} \cong 0.15$ measured in the Daya Bay, RENO and Double Chooz experiments, indications for which were obtained first in the T2K experiment, opened up the possibility to search for CP violation effects in neutrino oscillations.

Determining the status of CP symmetry in the lepton sector is one of the principal goals of the program of research in neutrino physics.

Information on leptonic Dirac CP violation is currently provided by the T2K and NOνA neutrino oscillatoion experiments (input - the data on θ_{13}); global analyses of the neutrino oscillation data; in the future it is expected to be provided principally by the planned DUNE and T2HK (or T2HKK) experiments.

Of fundamental importance are also

- the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics);

- determination of the type of spectrum neutrino masses possess, or neutrino mass ordering (T2K + NOνA; JUNO; PINGU, ORCA; T2HKK, DUNE);

- determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology).

The program of research extends beyond 2030.

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.37 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.53\ (2.43) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437\ (0.569)$, NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0214\ (0.0218)$, Capozzi et al. NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.

- $\Delta m^2_\odot \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47$ (2.42) $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 2.6\%$, $1\sigma(\sin^2\theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2\theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m^2_{21})$: $(6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.259 - 0.359)$;
($3\sigma(\Delta m^2_{21})$: $(6.93 - 7.97) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.250 - 0.354)$;)
- $3\sigma(|\Delta m^2_{31(23)}|)$: $2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV^2;
($2.40(2.30) - 2.66(2.57) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23})$: $0.374(0.380) - 0.628(0.641)$;
($3\sigma(\sin^2\theta_{23})$: $0.379(0.383) - 0.616(0.637)$)
- $3\sigma(\sin^2\theta_{13})$: $0.0176(0.0178) - 0.0296(0.0298)$
($3\sigma(\sin^2\theta_{13})$: $0.0185(0.0186) - 0.0246(0.0248)$.)

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)
(F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.)

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

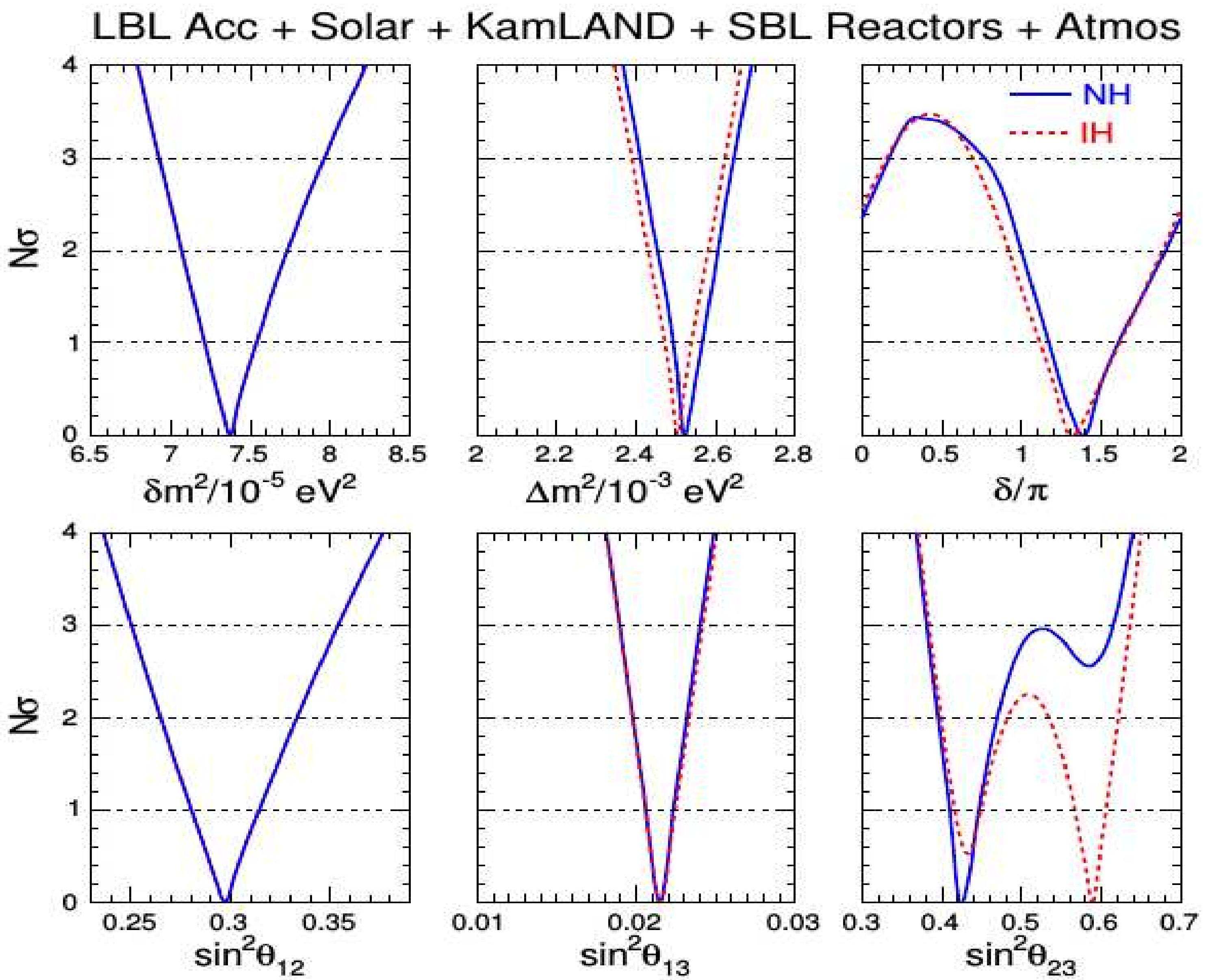
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left| < m > \right|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

$$\delta \cong 3\pi/2?$$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\}$$
$$= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$



F. Capozzi, E. Lisi *et al.*, Proc. of ν2016 Int. Conf.

The Quest for Nature's Message

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

Understanding the Pattern of Neutrino Mixing: Symmetry Approach.

Examples of Predictions and Correlations.

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}(1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13})$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545$ **(small uncert.)**.

- $\sin^2\theta_{12} \cong \frac{1}{3}(1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.340$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Understanding the Pattern of Neutrino Mixing: Predictions for the CPV Phase δ.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$,
$\theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 \mp 0.10$.
- $U_{\rm PMNS}$ due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi,\omega) U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)\,,$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix} ; \quad \mu - \tau \text{ symmetry}: \quad \theta^{\nu}_{23} = \mp\pi/4\,;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} ; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad \theta^{\nu}_{12} = \pi/6\,.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$**;** $a/b = a + b/a$**,** $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

• $U_{\rm TBM}$: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected .

• $U_{\rm BM}$: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

$U_{\rm TBM(BM)}$: Groups A_4, T' (S_4), ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• $U_{\rm GRA}$: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• $U_{\rm LC}$: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

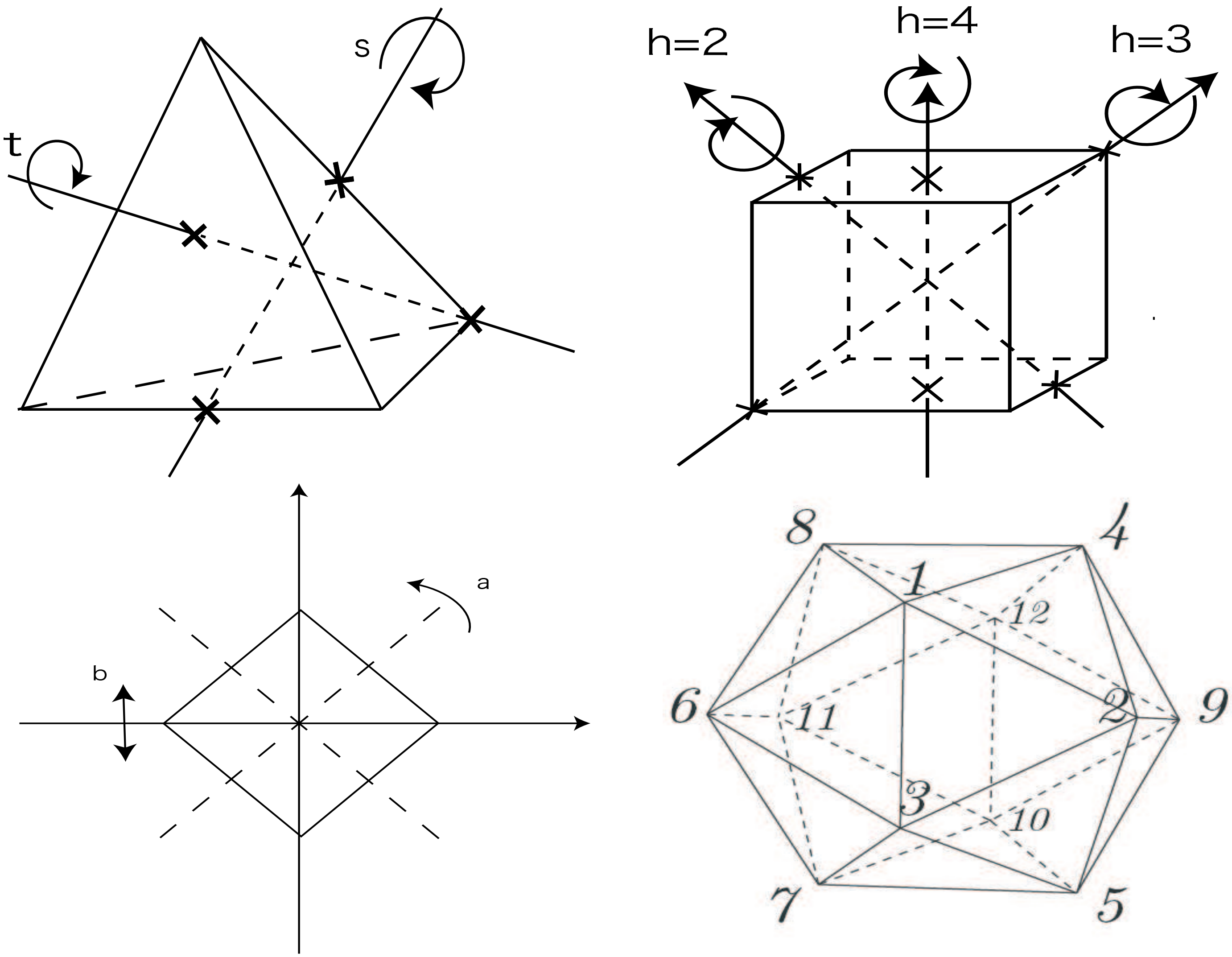
S.T.P., 1982

• $U_{\rm LC}$: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• U_{GRB}: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• U_{HG}: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta^\nu_{13} = 0$, $\theta^\nu_{23} = \mp\pi/4$.
They differ by the value of θ^ν_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^\nu_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



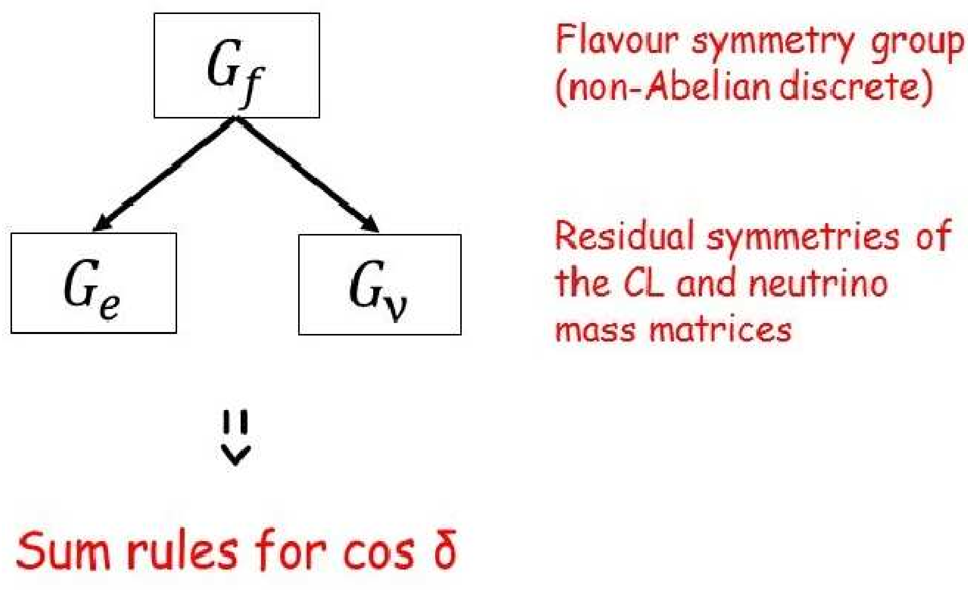
Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	$S,\ T,\ U$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{2},\ \mathbf{3},\ \mathbf{3'}$
A_4	12	$S,\ T$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{1''},\ \mathbf{3}$
T'	24	$S,\ T,\ R$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{1''},\ \mathbf{2},\ \mathbf{2'},\ \mathbf{2''},\ \mathbf{3}$
A_5	60	$S,\ T$	$\mathbf{1},\ \mathbf{3},\ \mathbf{3'},\ \mathbf{4},\ \mathbf{5}$
D_{10}	20	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
D_{12}	24	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4,\ \mathbf{2}_5$

Number of elements, generators and irreducible representations of some discrete groups.

How does it Work.



ν_j, **Majorana mass term,** $m_j \neq m_k$, $j \neq k = 1,2,3$: $G_\nu = Z_2 \times Z_2$, Z_2

$G_e = Z_2$; Z_n, $n > 2$; $Z_n \times Z_m$, $n, m \geq 2$

ν_j, **Majorana mass term,** $m_j \neq m_k$, $j \neq k = 1,2,3$: $G_\nu = Z_2 \times Z_2$, Z_2
(max. $G_\nu = Z_2 \times Z_2 \times Z_2$ **+** G_ν **subgroup of** $SU(3)$**: max.** $G_\nu = Z_2 \times Z_2$**)**

$G_e = Z_2$; Z_n, $n > 2$; $Z_n \times Z_m$, $n, m \geq 2$
(max. $G_e = U(1) \times U(1) \times U(1)$ **+** G_e **subgroup of** $SU(3)$**: max.** $G_e = U(1) \times U(1)$**)**

In models with $G_\nu = Z_2 \times Z_2$:

U_ν - determined up to re-phasing on the right and permutations of columns; the latter can be fixed within a specific model.

In models with $G_\nu = Z_2$:

U_ν - two free parameters, one angle and a phase, as long as the neutrino Majorana mass term does not have additional "accidental" symmetries, e.g., the $\mu - \tau$ symmetry; otherwise, determined up to re-phasing on the right and permutations of columns.

- TBM form of $\tilde{U}_\nu$:

from $G_f = A_4$, $G_\nu = Z_2$ (S generator of A_4 is unbroken) $+ \mu - \tau$ accidental symmetry.

G. Altarelli, F. Feruglio, arXiv:1002.0211; see also, e.g., I. Girardi et al., arXiv:1509.02502

- TBM form of $\tilde{U}_\nu$:

from $G_f = T'$, $G_\nu = Z_2$ ($+$ TST^2 element of T' - unbroken) $+$ $l_L(x)$, $\nu_{lL}(x)$, $l = e, \mu, \tau$ - triplets of T'.

- BM form of $\tilde{U}_\nu$:

from $G_f = S_4$, $G_\nu = Z_2 + \mu - \tau$ accidental symmetry.

G. Altarelli et al., arXiv:0903.1940; see also, e.g., I. Girardi et al., arXiv:1509.02502

M_e **- charged lepton mass matrix (L-R convention).**

U_e: $U_e^\dagger M_e M_e^\dagger U_e = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$.

G_e **- residual symmetry group of** $M_e M_e^\dagger$**:**

$$\rho(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger,$$

g_e: **an element of** G_e;
ρ: **the unitary representation of** G_f **acting on** $l_L(x)$;
$\rho(g_e)$: **action of** G_e **on** $l_L(x)$, $l = e, \mu, \tau$.

$\rho(g_e)$ **and** $M_e M_e^\dagger$ **commute: both are diagonalised by** U_e.

M_ν - **neutrino Majorana mass matrix (R-L convention).**
U_ν: $U_\nu^T M_\nu U_\nu = \mathrm{diag}(m_1, m_2, m_3)$.

G_ν - **residual symmetry group of** M_ν:

$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$,

g_ν: **an element of** G_ν;
ρ: **the unitary representation of** G_ν **acting on** $\nu_L(x)$;
$\rho(g_\nu)$: **action of** G_ν **on** $\nu_L(x)$, $l = e, \mu, \tau$.

$\rho(g_\nu)$ **and** $M_\nu^\dagger M_\nu$ **commute: both are diagonalised by** U_ν.

None of the symmetries leading to U_{TBM}, U_{BM} or other approximate forms of U_{PMNS} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In the cases of U_ν given by U_{TBM}, U_{BM}, etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\text{TBM,BM}}$, etc. and on the form of U_{lep}, one obtaines diiffer- ent experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spec- trum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the ob- served patterns of neutrino mixing and Δm^2_{ij}.

For arbitrary fixed θ^{ν}_{12} and any θ_{23}
("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right].$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\,\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

Predictions for δ

Assume:

• $U_{PMNS} = U_{\rm lep}^{\dagger}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi, \omega) U_{\rm TBM,BM,GR,HG} \bar{P}(\xi_1, \xi_2)$,

• $U_{\rm lep}^{\dagger}$ - minimal, such that

i) $\sin\theta_{13} \cong 0.16$; BM: $\sin^2\theta_{12} \cong 0.31$;

ii) $\sin^2\theta_{23}$ can deviate significantly (by more than $\sin^2\theta_{13}$) from 0.5 (b.f.v. = 0.40-0.45 or 0.55-0.60) .

The "minimal" = simplest case $(SU(5) \times T', \ldots)$

$U_{\text{lep}} \cong O^{\ell}_{12}(\theta^{\ell}_{12})$; now $Q = \text{diag}(e^{i\varphi}, 1, 1)$;

$\sin^{\ell}\theta_{13}$, $\sin^{\ell}\theta_{23}$ - negligibly small $(SU(5) \times T', \ldots)$.

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta^{\ell}_{12}, \varphi$ and θ^{ν}_{12}.

$\theta_{23} = \theta_{23}(\theta_{13}), \quad \delta = \delta(\theta_{12}, \theta_{13}, \theta^{\nu}_{12})(!)$

The exact sum rule will be given later.

$\sin^2\theta_{23} = \frac{1-2\sin^2\theta_{13}}{2(1-\sin^2\theta_{13})} \cong 0.5(1 - \sin^2\theta_{13})$.

Problem for $U_{\text{TBM,BM,GRA(B),HG}}$ if $\sin^2\theta_{23} \cong 0.44-0.45$:

Larger correction to $\sin^2\theta^{\nu}_{23} = 0.5$ might be needed.

Next-to-Minimal case: $U_{\rm lep} \cong U_{\rm lep}(\theta^{\ell}_{12}, \theta^{\ell}_{23})$,
$Q = {\rm diag}(1, e^{-i\psi}, e^{-i\omega})$.

"Standard" Ordering:
$U_{\rm lep}(\theta^{\ell}_{12}, \theta^{\ell}_{23}) = O^T_{23}(\theta^{\ell}_{23}) O^T_{12}(\theta^{\ell}_{12})$ (GUTs typically);
in many theories - a consequence of $m^2_e \ll m^2_{\mu} \ll m^2_{\tau}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

S.T.P., arXiv:1405.6006

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^{\circ}$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^{\circ}$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^{\circ}$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^{\circ}$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ one can distiguish between different symmetry forms of $\tilde{U}_\nu$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340; P. Coloma *et al.*, arXiv:1203.5651; R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984.

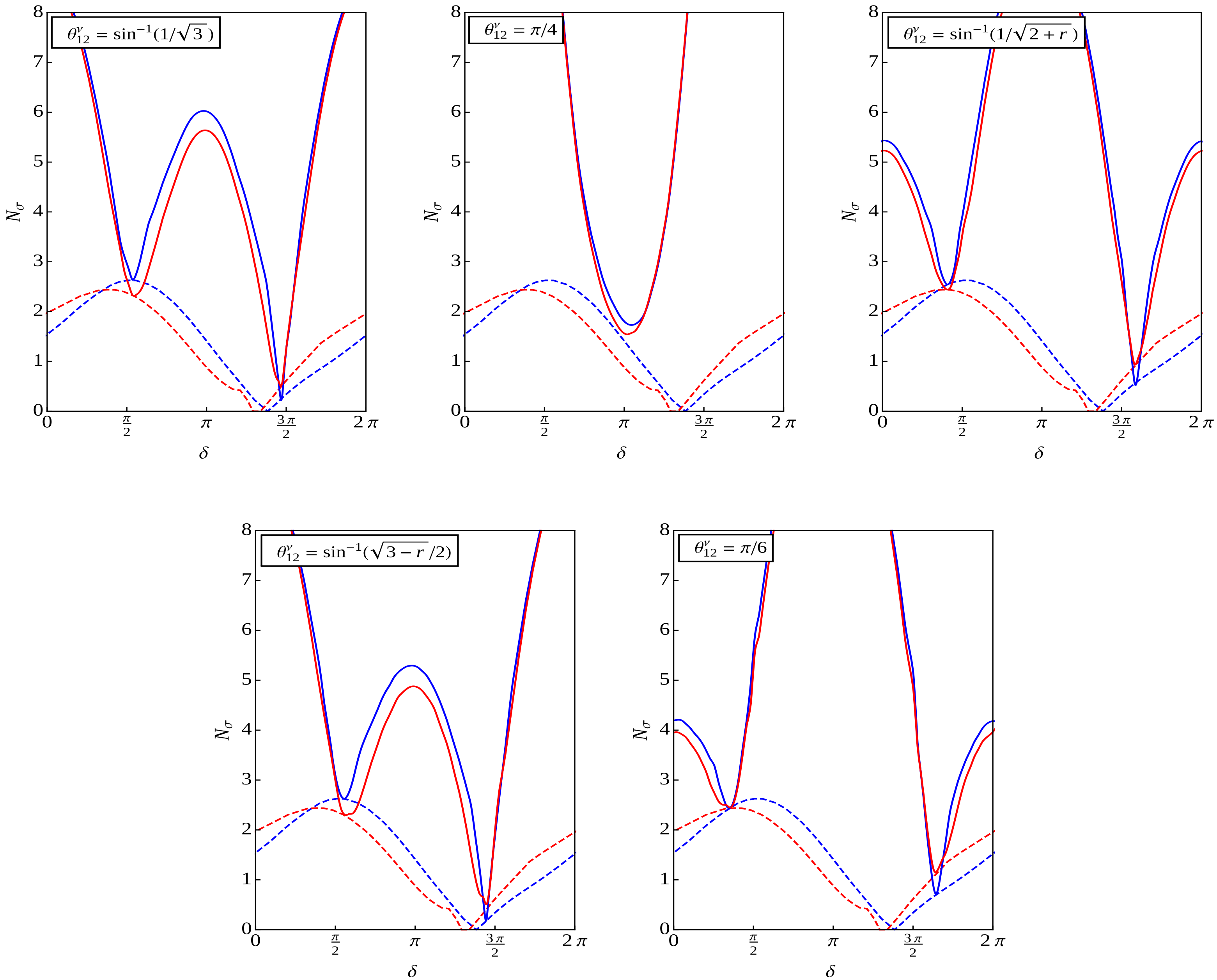
Statistical analysis, likelihood method; input "data": $\sin^2\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{12}$, δ from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

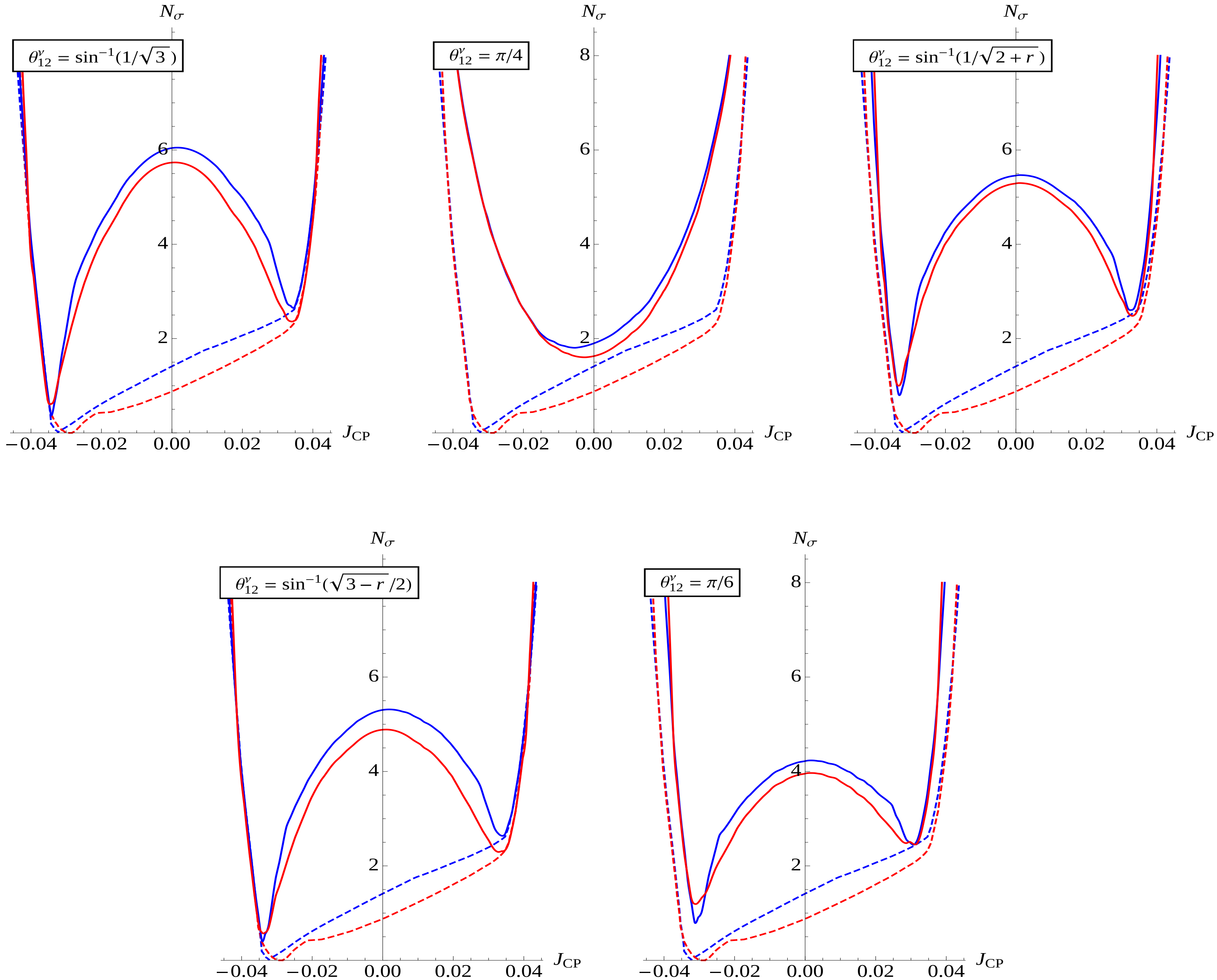
nσ confidence level interval of values of $\cos\delta$:

$$L(\cos\delta) \geq L(\chi^2_{\rm min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J = 0$ **excluded at** 5σ**,** 4σ**,** 4σ**,** 3σ **confidence level.**

At 3σ**:** $0.020 \leq |J_{\rm CP}| \leq 0.039$**.**

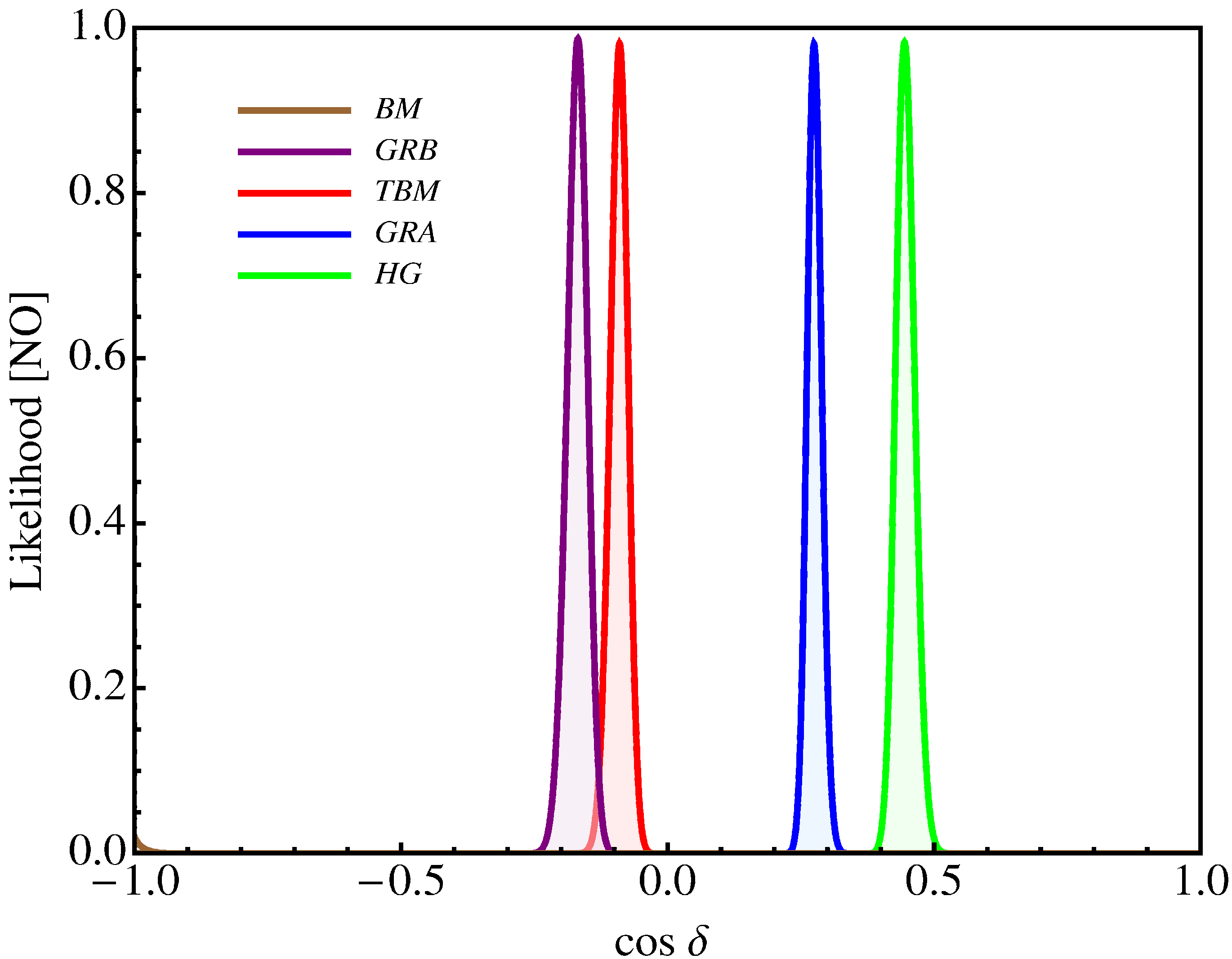
BM (LC), b.f.v.: $J_{\rm CP} = 0$**;**
at 3σ**:** -0.026 $(-0.025) \leq J_{\rm CP} \leq 0.021$ (0.023) **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

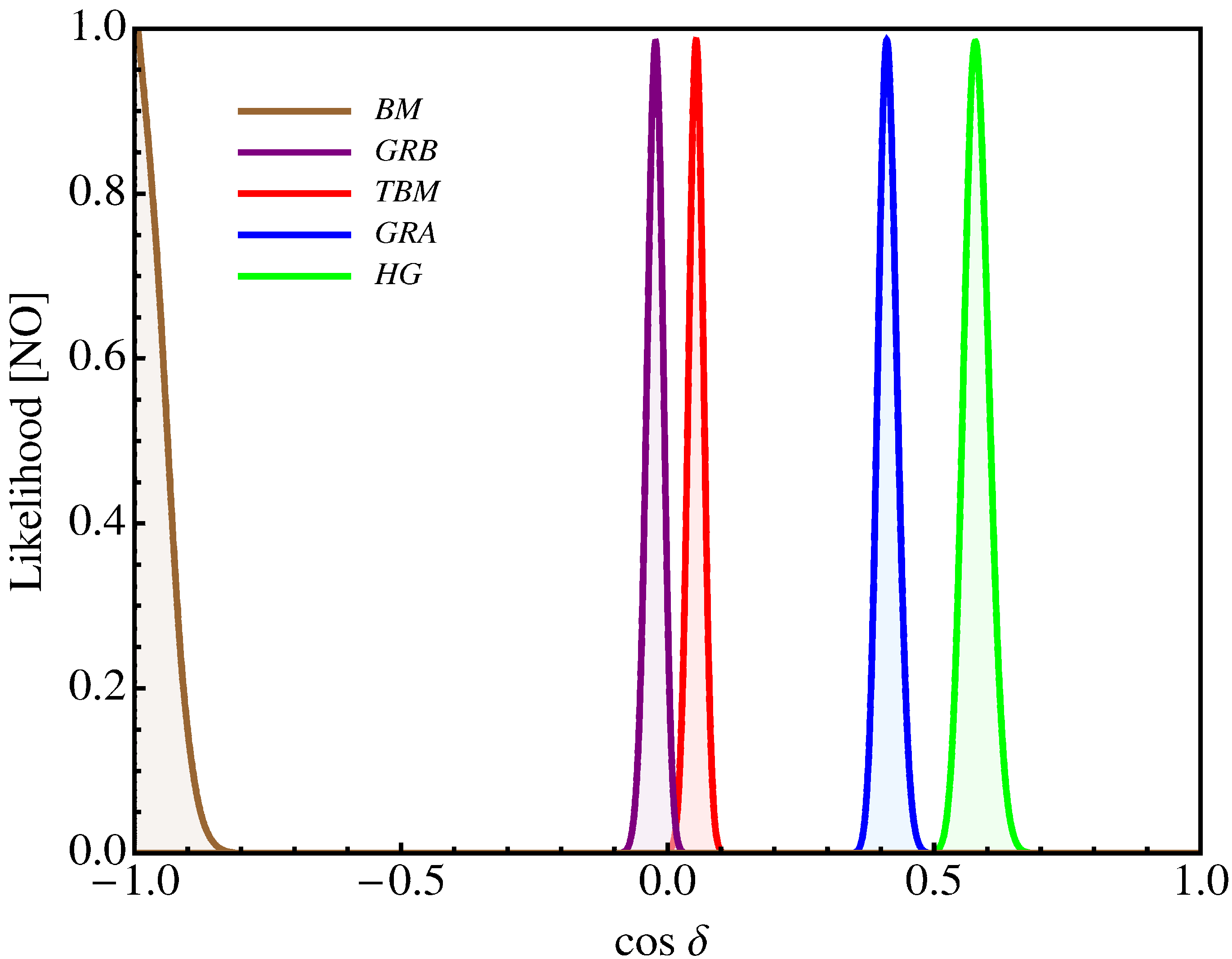
$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 5\%$ **(T2K, NOνA combined).**



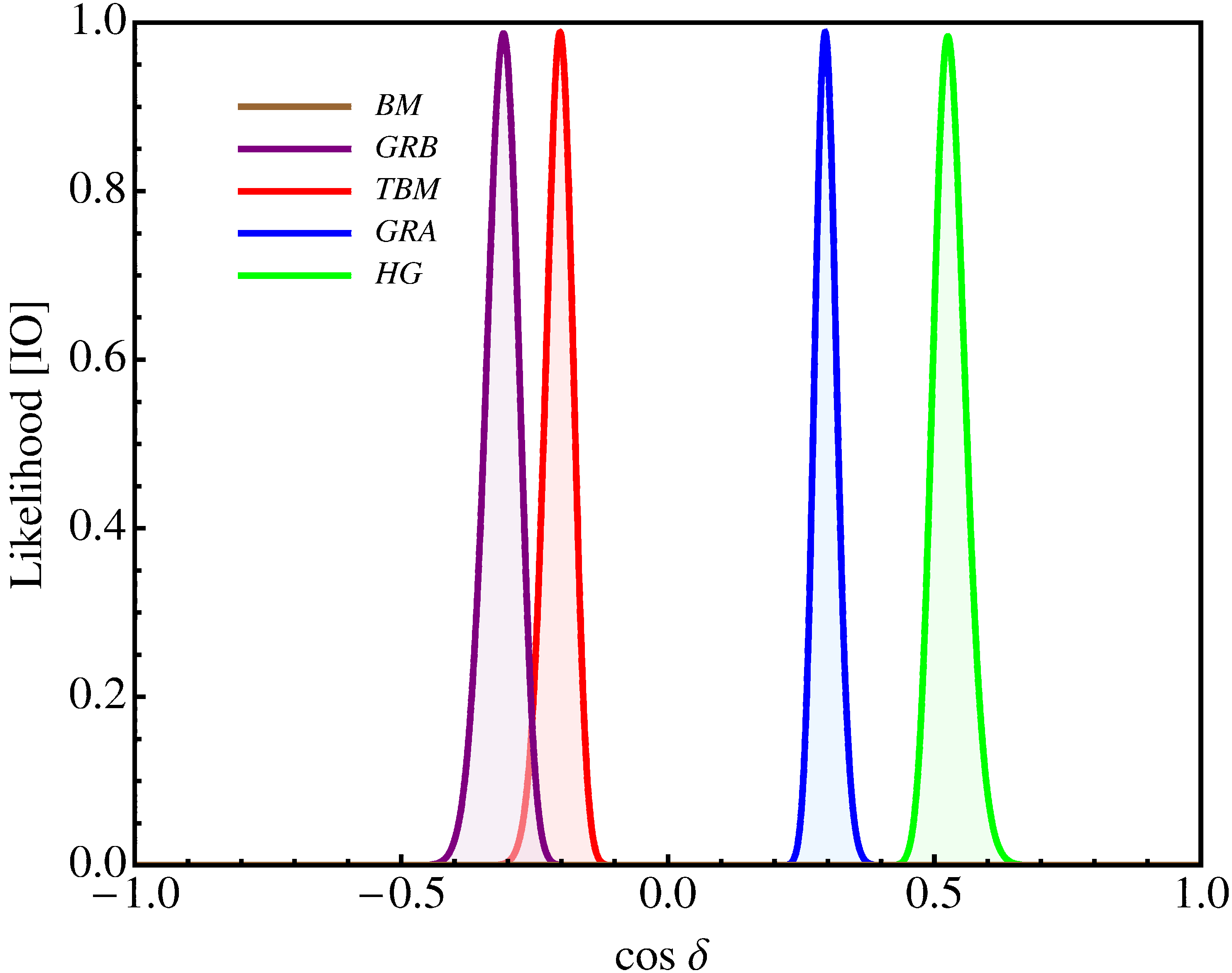
I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2\theta_{ij}$ (Capozzi et al., 2014) + the prospective precision used.



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

The same, but for $\sin^2\theta_{12} = 0.33$ **(the BM prediction dependence on** $\sin^2\theta_{12}$**).**



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2\theta_{23} = 0.557$ **(b.f.v.: C. Gonzales-Garcia et al., 2014, IO case).**

For, e.g., $|\cos\delta| < 0.93$ **(76% of values of δ), and** $\Delta(\cos\delta) = 0.10\,(0.08)$:
$\Delta\delta\Delta(\cos\delta)/\sqrt{1-0.93^2} = 16^\circ\,(12^\circ)$.
Planned to be reached, e.g., in T2HK.
Thus, a measurement of $\cos\delta$ **in the quoted range will allow to distinguish between the TBM/GRB, BM (LC) and GRA/HG forms at** $\sim 3\sigma$ **C.L., if** $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ **and** $\sin^2\theta_{23}$ **are measured with the prospective precisions.**

Distinguishing between GRA and HG forms at 3σ **C.L. requires** $1\sigma(\cos\delta) \cong 0.03$ **(if b.f.v. of** $\cos\delta$ **at one of the two maxima).**

TBM and GRB cannot be distinguished at 3σ **C.L. with the prospective uncertainties on** $\sin^{\theta}_{ij}$;
for zero uncertainties on $\sin^{\theta}_{ij}$ **(infinite precision), can be distinguished if** $1\sigma(\cos\delta) \cong 0.03$.

I. Girardi, S.T.P., A. Titov, arXiv:1504

In I. Giradi et al., arXiv:1410.8056, we have investigated also the dependence of the predictions for $\cos\delta$ **on** $\sin^2\theta_{13}$ **and** $\sin^2\theta_{12}$ **when the latter are varied in their respective** 3σ **allowed intervals in the cases of** $\theta^{\ell}_{23} \neq 0$ **and** $\theta^{\ell}_{23} \cong 0$**. In the latter case:**

$$\sin^2\theta_{23} = \frac{1-2\sin^2\theta_{13}}{2(1-\sin^2\theta_{13})} \cong 0.5(1-\sin^2\theta_{13}).$$

The predictions for $\cos\delta$ **are very similar in the two cases are very similar.**

Sum rule for $\cos\delta$**:** $U = U_e^{\dagger}\, U_\nu = (\tilde{U}_e)^{\dagger}\, \Psi \tilde{U}_\nu\, Q_0$

Results shown for: $\tilde{U}_\nu = U_{\text{TBM,BM,LC,GRM,HGM}}$ **and**
$\tilde{U}_e = R_{12}^{-1}(\theta^e_{12})$;
$\tilde{U}_e = R_{23}^{-1}(\theta^e_{23}) R_{12}^{-1}(\theta^e_{12})$ **(figures);**
$\tilde{U}_e = R_{13}^{-1}(\theta^e_{13})$;
$\tilde{U}_e = R_{23}^{-1}(\theta^e_{23})\, R_{13}^{-1}(\theta^e_{13})$.

The same method can be used for obtaining predictions for leptonic Dirac and Majorana CP violation in a large number of cases, see S.T.P., arXiv:1405.6006; I. Girardi *et al.*, arXiv:1410.8056, arXiv:1504.00658, arXiv:1509.02502, and arXiv:1605.04172.

Sum rule for $\cos\delta$**:** $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$

In I. Girardi *et al.*, **rXiv:1504.00658, results for**
$\tilde{U}_\nu = R_{23}(\theta^\nu_{23} = -\pi/4)\, R_{13}(\theta^\nu_{13})\, R_{12}(\theta^\nu_{12})$, $\theta^\nu_{13} \neq 0$,

i) $[\theta^\nu_{13}, \theta^\nu_{12}] = [\pi/20, -\pi/4]$, $[\pi/10, -\pi/4]$, $[\sin^{-1}(1/3), -\pi/4]$, $[\pi/20, \sin^{-1}(1/\sqrt{2+r})]$, $[\pi/20, \pi/6]$,
ii) $[\theta^\nu_{13}, \theta^\nu_{12}] = [\pi/20, \sin^{-1}(1/\sqrt{3})]$, $[\pi/20, \pi/4]$, $[\pi/10, \pi/4]$, $[\sin^{-1}(1/3), \pi/4]$, $[\pi/20, \sin^{-1}(\sqrt{3-r}/2)]$,
and
$\tilde{U}_e = R^{-1}_{12}(\theta^e_{12}),\ R^{-1}_{23}(\theta^e_{23}) R^{-1}_{12}(\theta^e_{12});$
$\tilde{U}_e = R^{-1}_{13}(\theta^e_{13}),\ R^{-1}_{23}(\theta^e_{23})\, R^{-1}_{13}(\theta^e_{13});$
$\tilde{U}_e = R^{-1}_{13}(\theta^e_{13})\, R^{-1}_{12}(\theta^e_{12}).$

In I. Girardi *et al.*, arXiv:1509.02502, sum rule for $\cos\delta$ were derived when $U_e^\dagger$ and/or U_ν of $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, are partially (or fully) determined by residual discrete symmetries of the lepton flavour symmetry groups $G_f = S_4$, A_4, T' and A_5.

The following cases of residual symmetries G_e and G_ν were analysed:

1. $G_e = Z_2$ and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$;

2. $G_e = Z_n$, $n > 2$ or $G_e = Z_n \times Z_m$, $n, m \geq 2$ and $G_\nu = Z_2$;

3. $G_e = Z_2$ and $G_\nu = Z_2$;

4. G_e is fully broken and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$;

5. $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and G_ν is fully broken.

In the case of $G_e = Z_2$ ($G_\nu = Z_2$), U_e (U_ν) is determined up to a $U(2)$ transformation in the degenerate subspace.
When the residual symmetry is large enough, namely, $G_e = Z_n$, $n > 2$ or $G_e = Z_n \times Z_m$, $n, m \geq 2$ and $G_\nu = Z_2 \times Z_2$ ($G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$) for Majorana (Dirac) neutrinos, U_e and U_ν are fixed (up to diagonal phase matrices on the right, which are either unphysical for Dirac neutrinos, or contribute to the Majorana phases otherwise, and permutations of columns) by the residual symmetries of the charged lepton and neutrino mass matrices.
In the case when the discrete symmetry G_f is fully broken in one of the two sectors, the corresponding mixing matrix U_e or U_ν is unconstrained and contains in general three angles and six phases.

Predicting the Majorana Phases in $U_{\rm PMNS}$

$$U = O_{12}(\theta^{\ell}_{12}) O_{23}(\theta^{\ell}_{23}) {\rm diag}(1, e^{-i\psi}, e^{-i\omega}) O_{23}(\theta^{\nu}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P}\,,$$

$$\bar{P} = {\rm diag}(1, e^{i\xi_{21}}, e^{i\xi_{31}})\,.$$

Can be shown to be equivalent to:

$$U = O_{12}(\theta^{\ell}_{12}) {\rm diag}(1, e^{i\phi}, 1) O_{23}(\hat{\theta}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P}(\xi_{21}, \xi_{31} + \beta)$$

$$\sin^2\hat{\theta}_{23} \;=\; \frac{1}{2}\left(1 - 2\sin\theta^\ell_{23}\cos\theta^\ell_{23}\cos(\omega - \psi)\right)\,,$$

$$\phi \;=\; \arg\left(e^{-i\psi}\cos\theta^\ell_{23} + e^{-i\omega}\sin\theta^\ell_{23}\right)\,,$$

$$\gamma \;=\; \arg\left(-\,e^{-i\psi}\cos\theta^\ell_{23} + \,e^{-i\omega}\sin\theta^\ell_{23}\right)\,,$$

$$\bar{P} \;=\; \mathrm{diag}(1, e^{i\xi_{21}}, e^{i(\xi_{31}+\beta)})\,,\;\; \beta = \gamma - \phi\,,$$

$$U \;=\; O_{12}(\theta^\ell_{12})\mathrm{diag}(1, e^{i\phi}, 1)O_{23}(\hat{\theta}_{23})O_{12}(\theta^\nu_{12})\bar{P}\,.$$

$$\frac{\alpha_{21}}{2} = \beta_{e2} - \beta_{e1} + \frac{\xi_{21}}{2}\,,$$
$$\frac{\alpha_{31}}{2} = \beta_{e2} + \beta + \frac{\xi_{31}}{2}\,.$$

$$\beta_{e2} = \text{arg}(U_{\tau 1}) = \text{arg}\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\delta}\right)\,,$$

$$\beta_{e1} - \pi = \text{arg}(U_{\tau 2}\, e^{-\frac{\alpha_{21}}{2}}) = \text{arg}\left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\delta}\right)\,.$$

Using the b.f.v. of $\sin^2\theta_{ij}$,

for TBM, GRA, GRB, HG forms: $\beta_{e1} \cong \pm(6^\circ - 7^\circ)$, $\beta_{e2} \cong \pm(12^\circ - 13^\circ)$, $\beta_{e2} - \beta_{e1} \cong \pm(18.7^\circ - 19.6^\circ)$;

BM (LC) form: $\beta_{e1} \cong \pm 1.87^\circ$, $\beta_{e2} \cong \pm 2.6^\circ$, $\beta_{e2} - \beta_{e1} \cong \pm 4.35^\circ$.

S.T.P., arXiv:1405.6006

The simplest case: $\theta^{\ell}_{23} = 0$, $\beta = 0$.

Generalised CP invariance: $\xi_{21} = 0$ or π, $\xi_{31} = 0$ or π.

I. Girardi, S.T.P., A. Titov, arXiv:1605.04172

The predictions obtained for $\cos\delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, "An $SU(5) \times A_5$ Golden Ratio Flavour Model", arXiv:1410.2095;

I. Girardi *at al.*, "Generalised Geometrical CP Violation in a T' Lepton Flavour Model", arXiv:1312.1966, JHEP 1402 (2014) 050.

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- ν_j - Majorana particles.
- Diagonalisation of M_ν: $U_{\text{TBM}}\Phi$, $\Phi = \text{diag}(1, 1, 1(i))$
- U_{TBM} "corrected" by

$$U_{\text{lep}}^{\dagger} Q = R_{12}(\theta_{12}^{\ell})\, R_{23}(\theta_{23}^{\ell}) Q\,, \quad Q = \text{diag}(1, e^{i\phi}, 1)$$

T' model of lepton flavour: U_{TBM} , $\delta \cong 3\pi/2$ or $\pi/2$.

• T': double covering of A_4 (tetrahedral symmetry group).

• T': **1**, $1'$, $1''$; **2**, $2'$, $2''$; **3**.

• T' model: $\psi_{eL}(x)$,$\psi_{\mu L}(x)$,$\psi_{\tau L}(x)$ - triplet of T';
$e_R(x)$,$\mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T';
$\nu_{eR}(x)$,$\nu_{\mu R}(x)$,$\nu_{\tau R}(x)$ - a triplet of T';
the Higgs doublets $H_u(x)$, $H_d(x)$ - singlets of T'.

• The discrete symmetries of the model are $T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2:

$$\frac{1}{m_1} - \frac{2}{m_2} = \frac{1}{m_3}, \quad \text{NO}$$

$$\text{NO, A}: \ (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ eV},$$

$$\text{NO, B}: \ (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ eV},$$

$$\text{IO}: \ (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3} \text{ eV},$$

$$\text{NO A}: \quad \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2} \text{ eV},$$

$$\text{NO B}: \quad \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2} \text{ eV},$$

$$\text{IO}: \quad \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2} \text{ eV},$$

- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

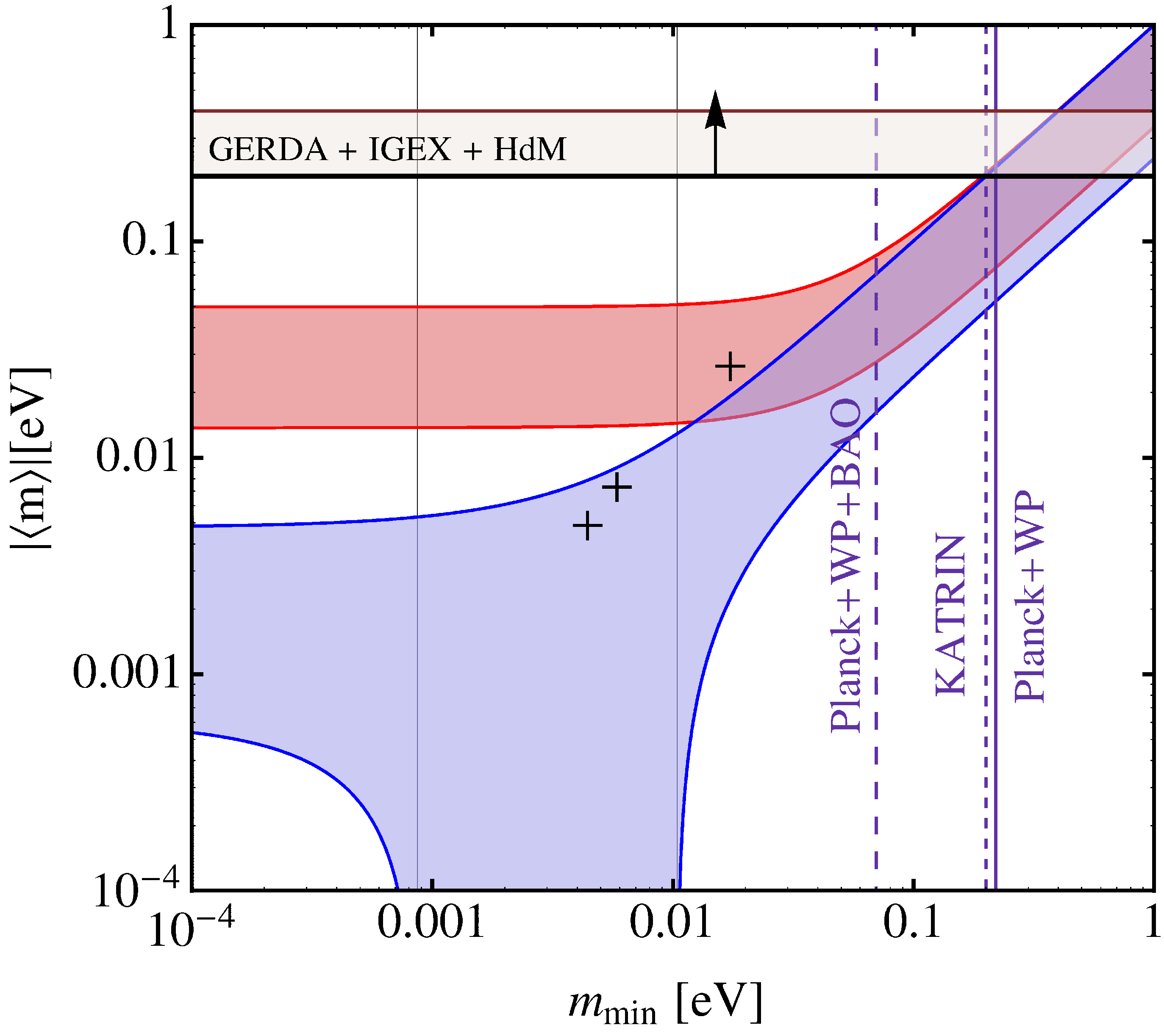
Given the values of $\theta_{12}, \theta_{23}, \theta_{13}$, $\delta, \alpha_{21}, \alpha_{31}$ are predicted:

$\delta \cong 3\pi/2$ (266°) (or $\pi/2$ (94°));

NO A: $\alpha_{21} \cong +47.0^\circ$ (or -47.0°) $(+2\pi)$,

$\alpha_{31} \cong -23.8^\circ$ (or $+23.8^\circ$) $(+2\pi)$.

The model is falsyfiable.


1
0.1
0.01
0.001
10^{-4}
10^{-4}
0.001
0.01
0.1
1
$|\langle m \rangle|$[eV]
$m_{\rm min}$ [eV]
GERDA + IGEX + HdM
Planck+WP+BAO
KATRIN
Planck+WP

LEPTOGENESIS

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

• Explains the smallness of $\nu-$masses.

• Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.

• In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

• The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

In GUTs, $M_{1,2,3} < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_{1,2,3} = (10^{11}, 10^{12}, 10^{13})$ GeV, $m_D \sim 1$ GeV.

TeV Scale (Resonant) Leptogenesis:

$M_{1,2,3} \sim (10^2 - 10^3)$ GeV (requires fine-tuning (severe)); observation of N_j at LHC - problematic (low production rates); observable LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- - e^-$ conversion.

Can the CP violation necessary for the generation of the observed value of the Baryon Asymmetry of the Universe (BAU) be provided exclusively by the Dirac and/or Majorana CPV phases in the neutrino PMNS matrix?

Demonstrated in (incomplete list):

S. Pascoli *et al.*, **hep-ph/0609125 and hep-ph/0611338.**
E. Molinaro *et al.*, **arXiv:0808.3534.**
A. Meroni *et al.*, **arXiv:1203.4435.**
C. Hagedorn *et al.*, **arXiv:0908.0240.**
J. Gehrlein *et al.*, **arXiv:1502.00110 and arXiv:1508.07930.**
J. Zhang, Sh. Zhou, arXiv:1505.04858 (FGY 2002 model).
P. Chen *et al.*, **arXiv:1602.03873.**
C. Hegdorn, E. Molinaro, arXiv:1602.04206.
P. Hernandez *et al.*, **arXiv:1606.06719 and 1611.05000.**
M. Drewes *et al.*, **arXiv:1609.09069.**
G. Bambhaniya *et al.*, **arXiv:1611.03827.**

The Seesaw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) \;=\; \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,,$$

$$\mathcal{L}_{\text{CC}} \;=\; -\,\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,$$

$$\mathcal{L}_{\text{Y}}(x) \;=\; \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,$$

$$\mathcal{L}_{\text{M}}^{\text{N}}(x) \;=\; -\,\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.$$

ψ_{lL} - LH doublet, $\psi_{lL}^{\text{T}} = (\nu_{lL}\ l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\ v = 174\ \text{GeV},\ v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\ \lambda^T\,D_N^{-1}\,\lambda = U_{\text{PMNS}}^{*}\ D_\nu\ U_{\text{PMNS}}^{\dagger}\ .$$

$$m_\nu \simeq v^2 \; \lambda^T \, D_N^{-1} \, \lambda = U^*_{\rm PMNS} \, D_\nu \, U^\dagger_{\rm PMNS} \, ,$$

$$\lambda \equiv Y_\nu$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\; R\; \sqrt{D_\nu}\; (U_{\rm PMNS})^\dagger / v_u$, all at M_R;

R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

$D_N \equiv {\rm diag}(M_1, M_2, M_3)$, $D_\nu \equiv {\rm diag}(m_1, m_2, m_3)$.

Theories, Models:

- R - CP conserving ($SU(5) \times T'$, A. Meroni *et al.*, arxiv:1203.4435; S_4, P. Cheng *et al.*, arXiv:1602.03873; C. Hagedorn, E. Molinaro, arXiv:1602.04206).
- CPV parameters in R determined by the CPV phases in U (e.g., class of A_4 theories).
- Texture zeros in Y_ν: CPV parameters in R determined by the CPV phases in U (Frampton, Glashow Yanagida (FGY), 2002: $N_{1,2}$, two texture zeros in Y_ν; LG in FGY model: J. Zhang, Sh. Zhou, arXiv:1505.04858).

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j,\;\; C(\overline{N}_k)^T = N_k,\;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U^{\dagger}_{\rm CP} &= \eta^{NCP}_j\, \gamma_0\, N_j(x')\,, \quad \eta^{NCP}_j = i\rho^N_j = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U^{\dagger}_{\rm CP} &= \eta^{\nu CP}_k\, \gamma_0\, \nu_k(x')\,, \quad \eta^{\nu CP}_k = i\rho^{\nu}_k = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda^*_{jl} = \lambda_{jl}\, (\eta^{NCP}_j)^*\, \eta^l\, \eta^{H*}\,, \quad j = 1,2,3,\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i,\;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\begin{aligned} \lambda^*_{jl} &= \lambda_{jl}\, \rho^N_j\,,\;\; \rho^N_j = \pm 1\,, \\ U^*_{lj} &= U_{lj}\, \rho^{\nu}_j\,,\;\; \rho^{\nu}_j = \pm 1\,, \\ R^*_{jk} &= R_{jk}\, \rho^N_j\, \rho^{\nu}_k, \quad j,k = 1,2,3,\; l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,,\;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^{\nu}_k)^2\, (\rho^{\nu}_m)^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at "high" E due to the interplay of R and U: $\mathrm{Im}(P_{123\tau}) \neq 0$ (!)

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \qquad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B **number non-conservation.**
- **Violation of C and CP symmetries.**
- **Deviation from thermal equilibrium.**

Leptogenesis

• **The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.**

• **When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.**

• **If $\Gamma(N_1 \Phi^- \ell^+) \neq \Gamma(N_1 \Phi^+ \ell^-)$, a lepton asymmetry will be generated.**

• **Wash-out processes, like $\Phi^+ + \ell^- N_1$, $\ell^- + \Phi^+ \Phi^- + \ell^+$, etc. tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final resullt is a net (non-zero) lepton asymmetry.**

• **This lepton asymmetry is then converted into a baryon asymmetry by $(B + L)$ violating but $(B - L)$ conserving sphaleron processes which exist within the SM (at $T \gtrsim M_{\rm EWSB}$).**

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B:

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

2. solve the Boltzmann (or similar) equation to account for the wash-out of the asymmetry:

$$Y_{\mathsf{L}} = \kappa\, \varepsilon$$

where $\kappa = \kappa(\widetilde{m})$ is the "efficiency factor", $\widetilde{m}$ is the "the wash-out mass parameter" - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_{\mathsf{B}} = -\frac{c_s}{g_*}\kappa\varepsilon, \quad c_s \cong 1/3, \quad g_* = 215/2$$

Baryon number violation in the SM

Instanton and Sphaleron processes

SU(2) instantons lead to (leading order) to effective 12 fermion $(B+L)$ **nonconserving, but** $(B-L)$ **conserving, interactions:**

$O(B+L) = \Pi_i q_{Li} q_{Li} q_{Li} l_{Li}$

These would induce $\Delta B = \Delta L = 3$ **processes:**

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \rightarrow \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at T = 0 the probability of such processes is $\Gamma/V \sim e^{-4\pi/\alpha} \sim 10^{-165}$**.**

't Hooft, 1976

At finite T, the transitions proceed via thermal fluctuations (over the barrier) with an unsuppressed probability (due to sphaleron (static) configurations - saddle "points" of the field energy of the $SU(2)$ gauge - Higgs field system):

$$\Gamma/V \sim \alpha^4 T^4.$$

Kuzmin, Rubakov, Shaposhnikov, 1985;
Arnold et al., 1987 and 1997.

Sphaleron processes are efficient (in the case of interest) at

$$T_{\rm EW} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Can generate $B \neq 0$, $L \neq 0$ at $T_{\rm EW} < T (< 10^{12}$ GeV) from $(B-L)_0 \neq 0$ (with $(B-L) = const.$).

Harvey, Turner, 1990

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma\!: \sim 6.1 \times 10^{-10})$$

$$Y_B \cong -3 \times 10^{-3} \;\; \varepsilon \, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

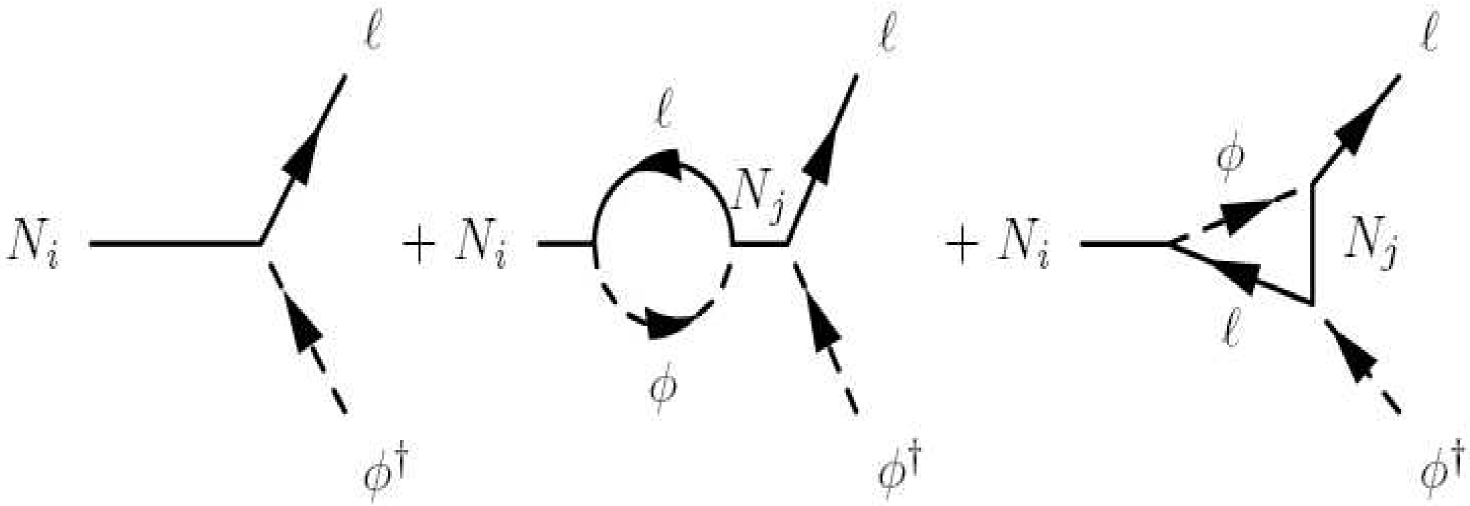
$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- N_1$, $\ell^- + \Phi^+ \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004


N_i
ℓ
$\phi^\dagger$
$+ N_i$
ℓ
ϕ
N_j
ℓ
$\phi^\dagger$
$+ N_i$
ϕ
ℓ
N_j
ℓ
$\phi^\dagger$

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{lj} U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j\,|R_{1j}|^2}\,, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\,v^2}{M_1} \; = \; \left|\sum_k R_{1k}\, m_k^{1/2}\, U^*_{lk}\right|^2\,, \quad l = e, \mu, \tau\,.$$

The "one-flavor" approximation - $\mathbf{Y}_{\mathbf{e},\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l\ H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 \;=\; \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k\,|R_{1k}|^2}\,,$$

$$\widetilde{m_1} \;=\; \sum_l \widetilde{m_l} = \sum_k m_k\,|R_{1k}|^2\,.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \to (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \to N_1$;

$\tau_L^- + \Phi^0 \tau_R^-$, $\tau_L^- + \tau_L^+ N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 \ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2 , \quad l = e, \mu, \tau \, .$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*} \left(\epsilon_2 \, \eta \left(\frac{417}{589} \widetilde{m_2} \right) + \epsilon_\tau \, \eta \left(\frac{390}{589} \widetilde{m_\tau} \right) \right) ,$$

$$\eta \left(\widetilde{m_l} \right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3} \, \text{eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \, \text{eV}}{\widetilde{m_l}} \right)^{-1.16} \right)^{-1} .$$

$Y_{\mathcal{B}} = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$\varepsilon_{1\tau} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}$$

$$= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j) \, R_{1j} R_{1k} \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \quad R_{1j} R_{1k} = \pm |R_{1j} R_{1k}|,$$

$$= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j) \, |R_{1j} R_{1k}| \, \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \quad R_{1j} R_{1k} = \pm i |R_{1j} R_{1k}|$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, $\mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589} \widetilde{m_\tau}\right) - \eta\left(\frac{417}{589} \widetilde{m_2}\right) \right)$$

$m_1 \ll m_2 \ll m_3$, $M_1 \ll M_{2,3}$; $R_{12}R_{13}$ − real; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}}|R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi$, $\delta = 0$: $\mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0$, CPV due to the interplay of R and U.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}| \cong 0.86$, $|R_{13}|^2 = 1 - |R_{12}|^2$, $|R_{13}| \cong 0.51$ - maximise $|Y_B|$:

$$|Y_B| \ \cong 2.1 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.15}\right) \left(\frac{M_1}{10^9\ \text{GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \cong 0.15\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \cong 0.15\,; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

Realised in a theory based on the S_4 symmetry: P. Cheng *et al.*, arXiv:1602.03873.

The requirement $\sin\theta_{13} \gtrsim 0.09$ (0.11) - compatible with the Daya Bay, RENO, Double Chooz results: $\sin\theta_{13} \cong 0.15$.

$|\sin\theta_{13}\sin\delta| \gtrsim 0.11$ implies $|\sin\delta| \gtrsim 0.7$ - compatible with $\delta \cong 3\pi/2$.

$\sin\theta_{13} \cong 0.15$ and $\delta \cong 3\pi/2$ imply relatively large (observable) CPV effects in neutrino oscillations: $J_{\rm CP} \cong -3.5 \times 10^{-2}$.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2.2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}}\right) \left(\frac{M_1}{10^9 \text{ GeV}}\right) \frac{|\sin(\alpha_{32}/2)|}{\sin \pi/4}. \tag{2}$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m^2_{\odot}/|\Delta_{32}| \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 8.1 \times 10^{-12}\, |s_{13} \sin\delta| \left(\frac{M_1}{10^9\ \text{GeV}}\right).$$

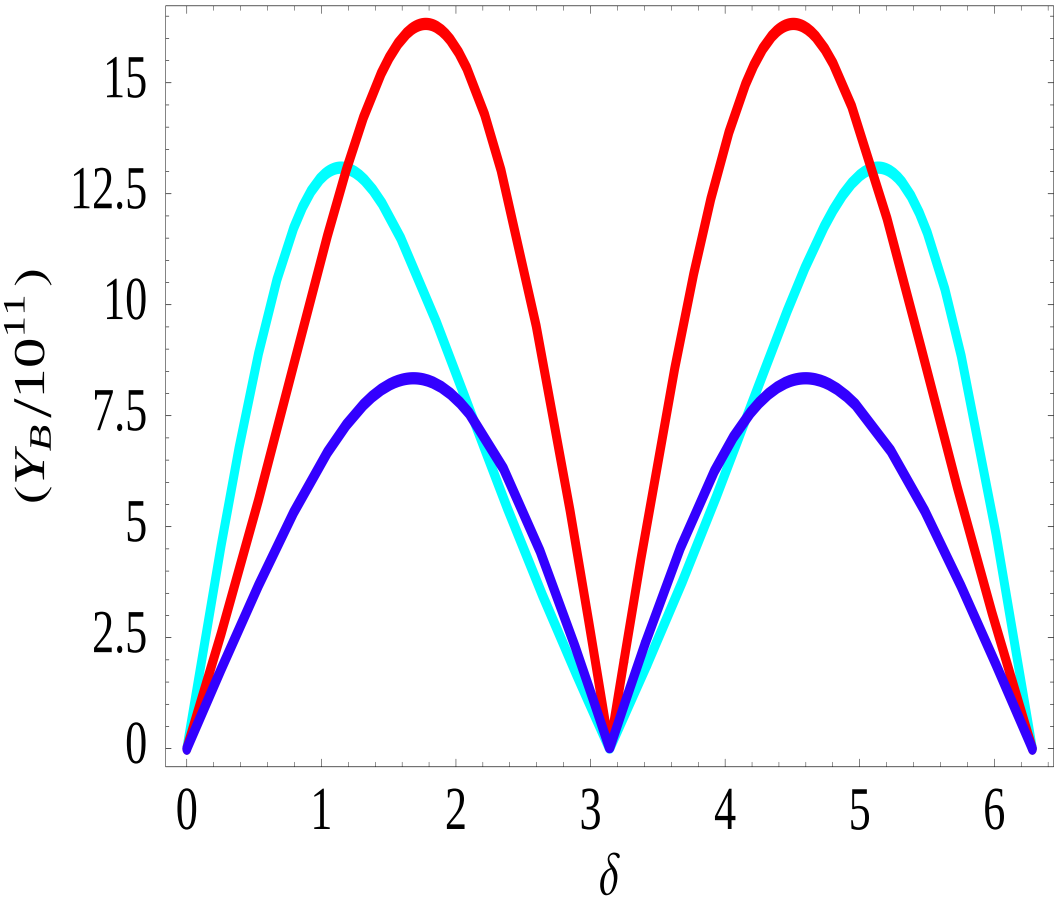
$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \cong 0.15\,.$$

The lower limit corresponds to

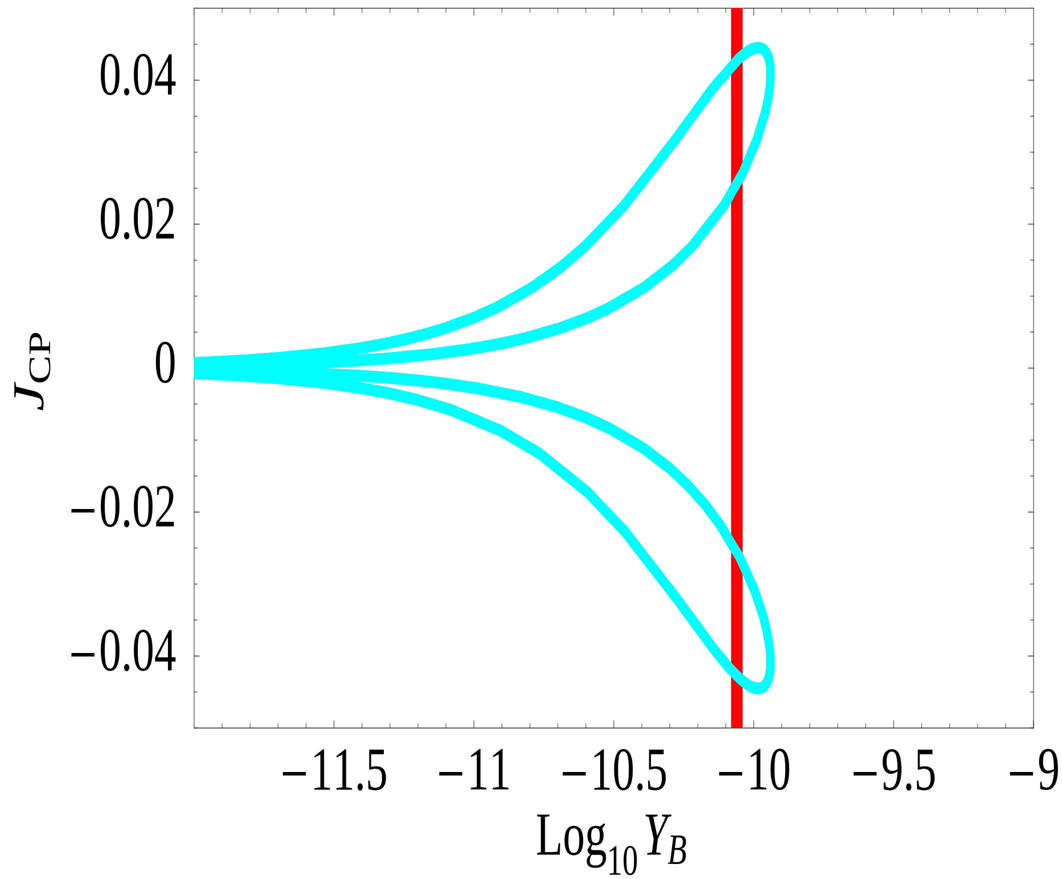
$$|J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

Realised in a theory based on the S_4 symmetry: P. Cheng *et al.*, arXiv:1602.03873.



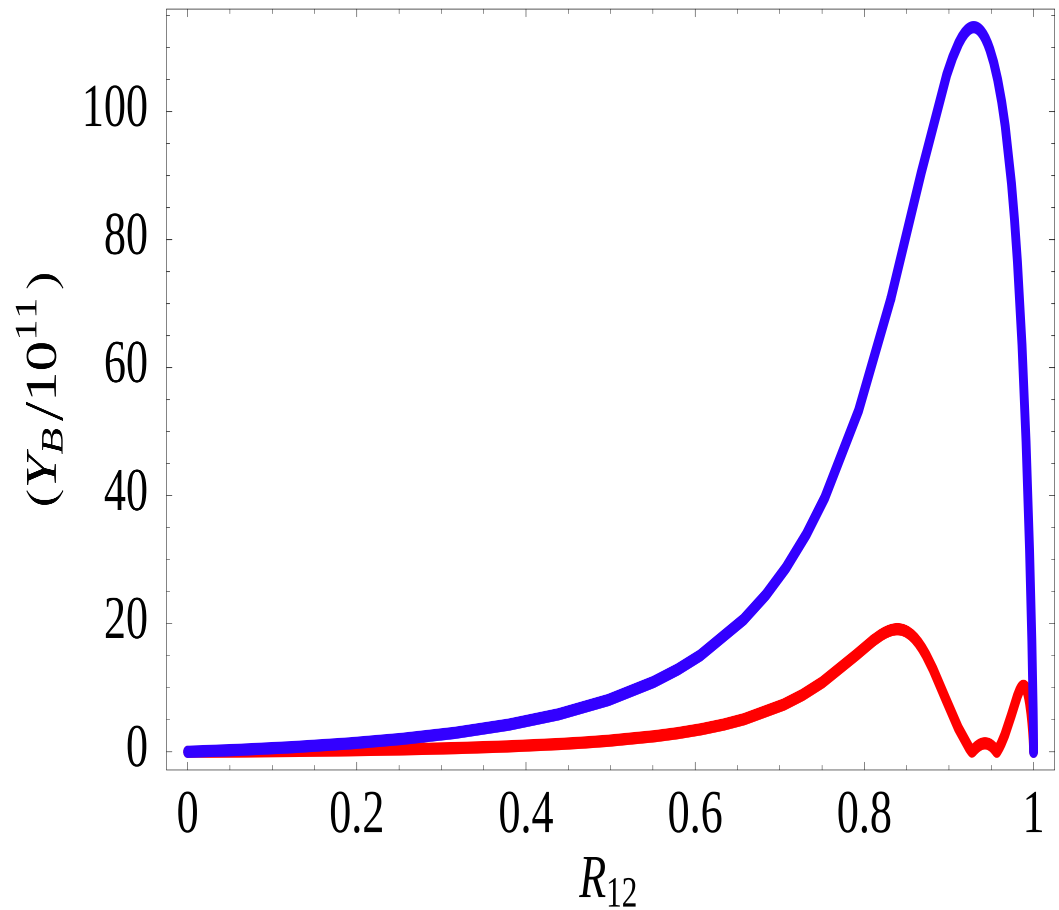
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ $(\kappa' = +1)$, $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ $(\kappa' = -1)$, $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

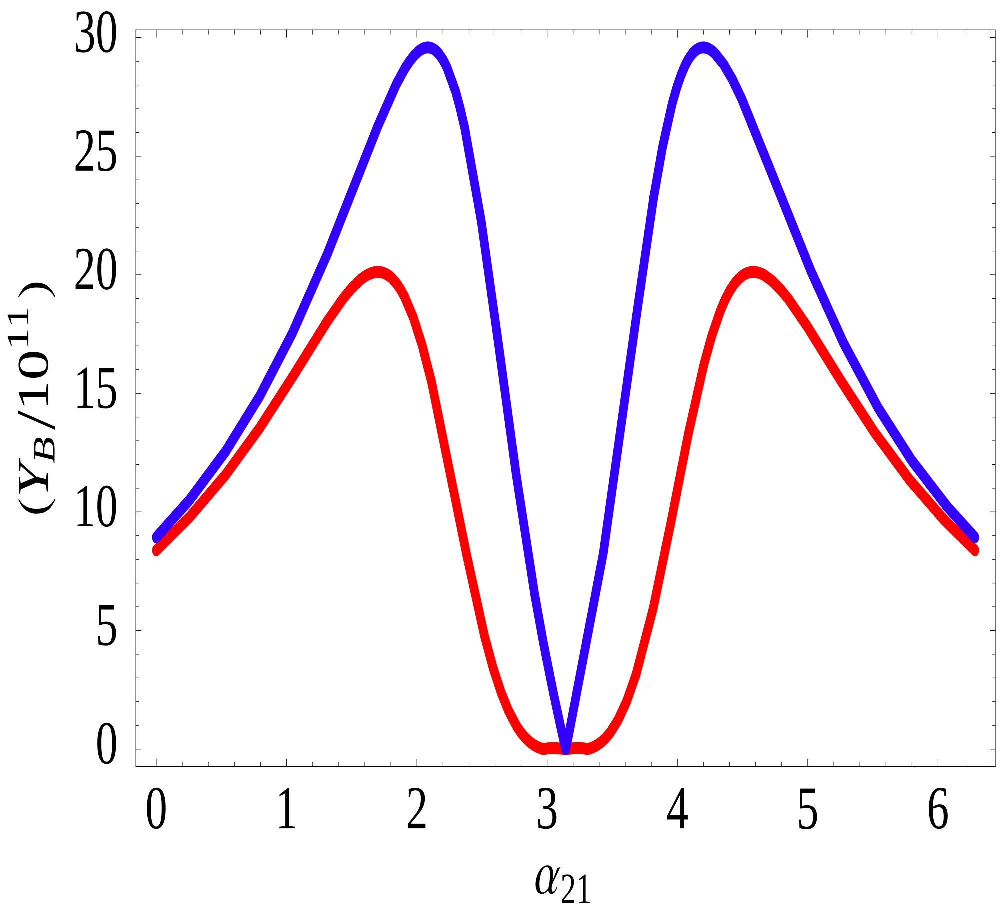


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0$ (2π);
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$\,(R_{12}R_{13}) = +1$ (-1) ($\beta_{23} = 0$ (π), $\kappa' = +1$);
The red region denotes the 2σ allowed range of $Y_{\rm B}$.

S. Pascoli, S.T.P., A. Riotto, 2006.

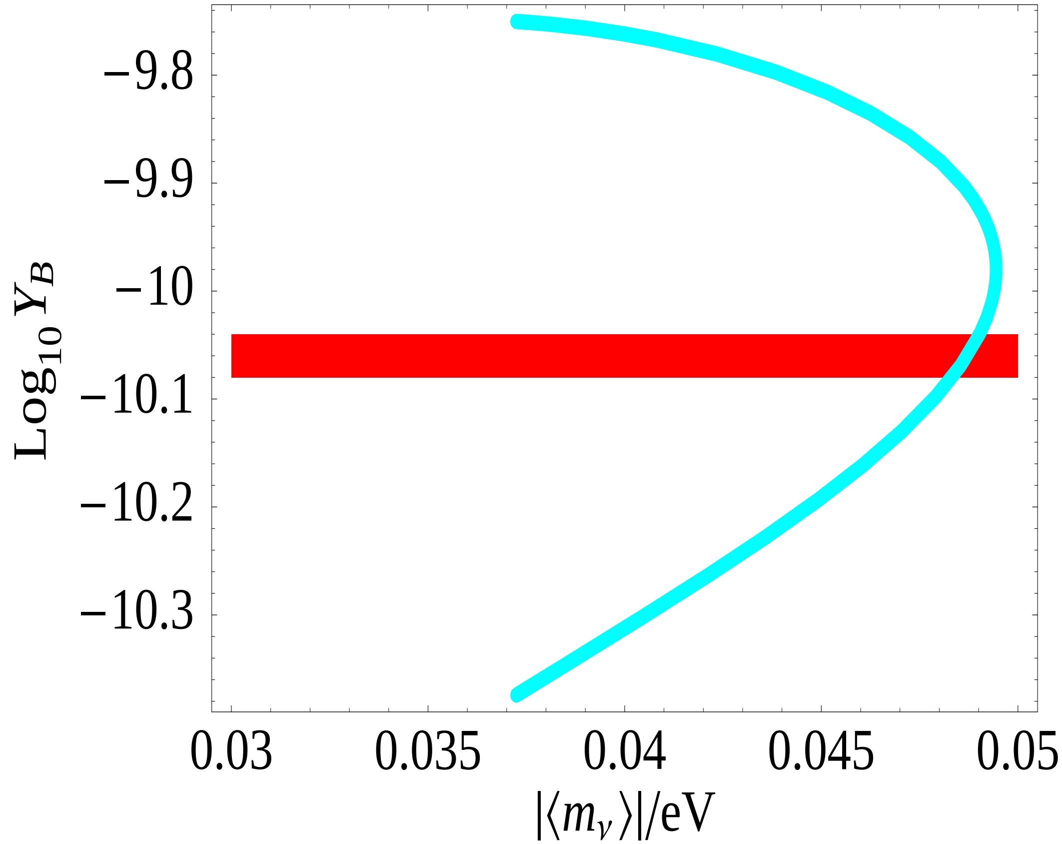


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign$\,(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m_{\odot}^2$, $\sin^2\theta_{12}$, Δm_{31}^2, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

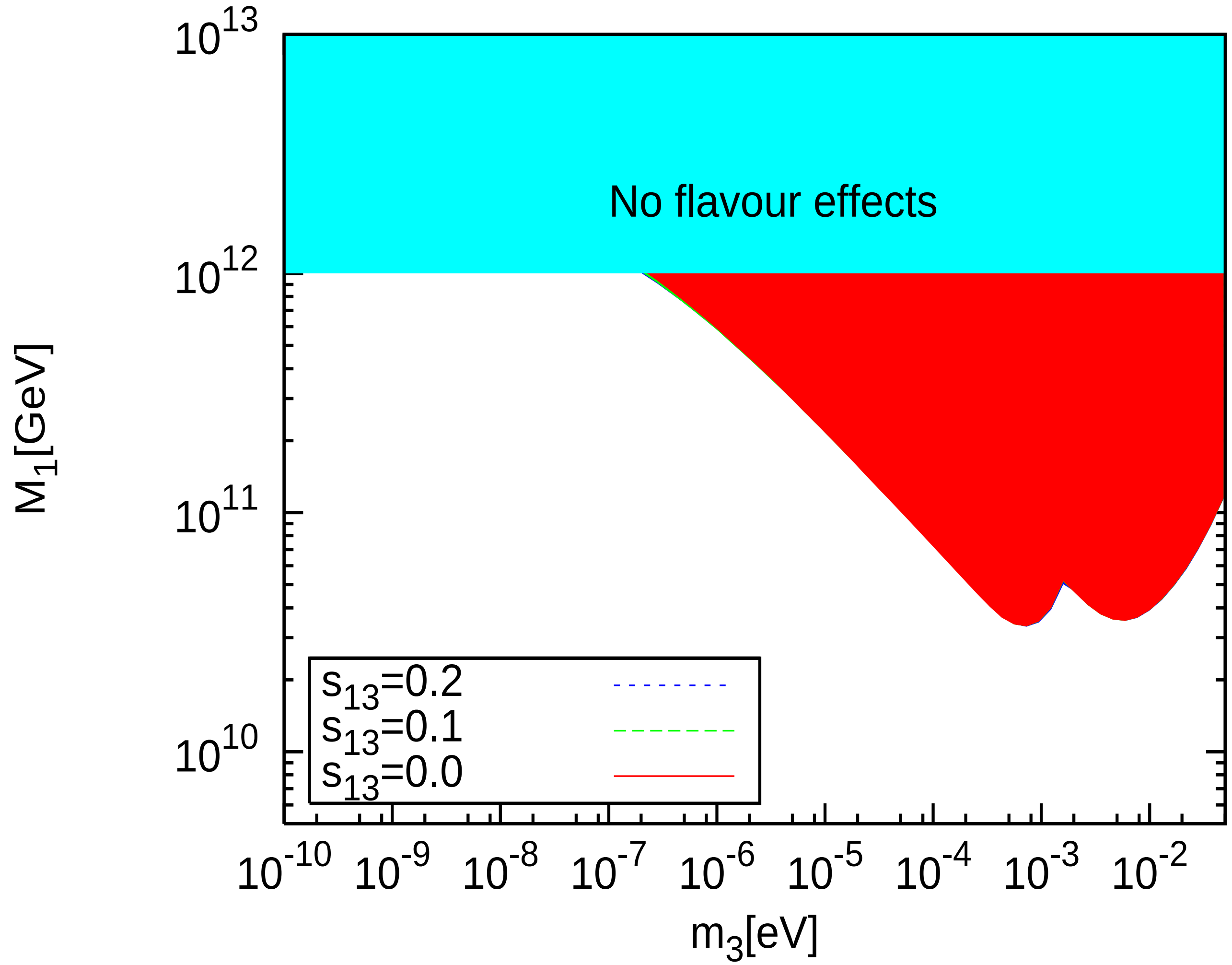
$$|\sin\theta_{13} \sin\delta| \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 <$ ($\ll$) $m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0;$

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.11\,; \qquad |J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02\,; \qquad |J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

LOW SCALE (TeV,...) LEPTOGENESIS

The CP violation necessary for the generation of the observed value of the Baryon Asymmetry of the Universe (BAU) can be provided exclusively by the Dirac and/or Majorana CPV phases in the neutrino PMNS matrix also in the low scale (TeV, GeV,...) leptogenesis.

P. Hernandez *et al.*, **arXiv:1606.06719 and 1611.05000.**
M. Drewes *et al.*, **arXiv:1609.09069.**
G. Bambhaniya *et al.*, **arXiv:1611.03827.**

Conclusions.

- The observed pattern of neutrino mixing can be due to a new fundamental (approximate) symmetry of particle interactions leading to an approximate symmetry form of the PMNS matrix. We have considered the following symmetry forms: TBM, BM (LC), GRA, GRB and HG. Each of these forms can be obtained from a specific discrete flavour symmetry.
- For all the forms considered $\theta^{\nu}_{13} = 0$ and $\theta^{\nu}_{23} = -\pi/4$. The forms differ by the value of θ^{ν}_{12}. Values of the neutrino mixing angles θ_{ij} compatible with the observations are obtained with the help of subleading (perturbative) corrections generated by U_{lep} coming from the diagonalisation of the charged lepton mass matrix.
- The most important testable consequence of this approach to understanding the pattern of of neutrino mixing is the correlation between the value of $\cos\delta$ and the values of the neutrino mixing angles: $\delta = \delta(\theta_{12}, \theta_{13}, \theta_{23}; \theta^{\nu}_{12})$. The correlation depends on the underlying approximate symmetry form of the U_{PMNS}.
- The precise knowledge of the value of $\sin^2\theta_{23}$, in particular, is crucial for testing the predictions obtained following the approach discussed by us and for discriminating between various cases possible within this approach.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Conclusions (contd.)

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

Dirac and Majorana CPV may have the same source.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

The see-saw mechanism provides a link between the ν-mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

These results underline further the importance of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.

Supporting Slides

Leptonic CP Violation

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} \ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} \ , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21}, \ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger, S. Pakvasa et al.,1980.

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'}) \ , \quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l), \ l \neq l'$$

3ν—mixing:

$$A^{(l,l')}_{\text{CP}} \equiv P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'}) \ , \quad l \neq l' = e, \mu, \tau$$

$$A^{(l,l')}_{\text{T}} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l), \ l \neq l'$$

$$A^{(e,\mu)}_{\text{T(CP)}} = A^{(\mu,\tau)}_{\text{T(CP)}} = -A^{(e,\tau)}_{\text{T(CP)}}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa *et al.*, 1980

3-Neutrino Oscillations in Vacuum

$$P(\nu_l \to \nu_{l'}) = \sum_j |U_{l'j}|^2 \, |U_{lj}|^2$$

$$+ 2 \sum_{j>k} |U_{l'j} \, U^*_{lj} \, U_{lk} \, U^*_{l'k}| \, \cos(\frac{\Delta m^2_{jk}}{2p} L - \phi_{l'l;jk}) \, , \;\; l, l' = e, \mu, \tau \, ,$$

$$P(\bar{\nu}_l \to \bar{\nu}_{l'}) = \sum_j |U_{l'j}|^2 \, |U_{lj}|^2$$

$$+ 2 \sum_{j>k} |U_{l'j} \, U^*_{lj} \, U_{lk} \, U^*_{l'k}| \, \cos(\frac{\Delta m^2_{jk}}{2p} L + \phi_{l'l;jk}) \, , \;\; l, l' = e, \mu, \tau \, ,$$

$$\phi_{l'l;jk} = \arg\left(U_{l'j} \, U^*_{lj} \, U_{lk} \, U^*_{l'k}\right) \, .$$

- Spatial localisation condition

ΔL - dimensions of the ν- source (and/or detector):

$2\pi \Delta L / L^{v}_{jk} \lesssim 1.$

- Time localisation condition

ΔE - detector's energy resolution:

$2\pi (L/L^{v}_{jk})(\Delta E/E) \lesssim 1.$

If $2\pi \Delta L / L^{v}_{jk} \gg 1$, and/or $2\pi (L/L^{v}_{jk})(\Delta E/E) \gg 1$,

$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2 \, |U_{lj}|^2$

In vacuum:

$$A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\}$$

$$= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin\left(\frac{\Delta m^2_{21}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E} L\right)$$

P.I. Krastev, S.T.P., 1988

$$\sin\left(\frac{\Delta m^2_{21}}{2E} L\right) \cong 0: \quad F^{vac}_{osc} \cong 0$$

In matter: **Matter effects violate**

$$\mathrm{CP}: \quad \mathbf{P}(\nu_l \to \nu_{l'}) \neq \mathbf{P}(\bar{\nu}_l \to \bar{\nu}_{l'})$$

$$\mathrm{CPT}: \quad \mathbf{P}(\nu_l \to \nu_{l'}) \neq \mathbf{P}(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

Can conserve the T-invariance **(Earth)**

P. Langacker et al., 1987

$$\mathbf{P}(\nu_l \to \nu_{l'}) = \mathbf{P}(\nu_{l'} \to \nu_l), \; l \neq l'$$

In matter with constant density (e.g., Earth mantle): $A_T^{(e,\mu)} = J_{CP}^{mat} F_{osc}^{mat}$,

$$J_{CP}^{mat} = \frac{1}{8} \sin 2\theta_{12}^m \sin 2\theta_{13}^m \cos \theta_{13}^m \sin 2\theta_{23}^m \sin \delta^m$$

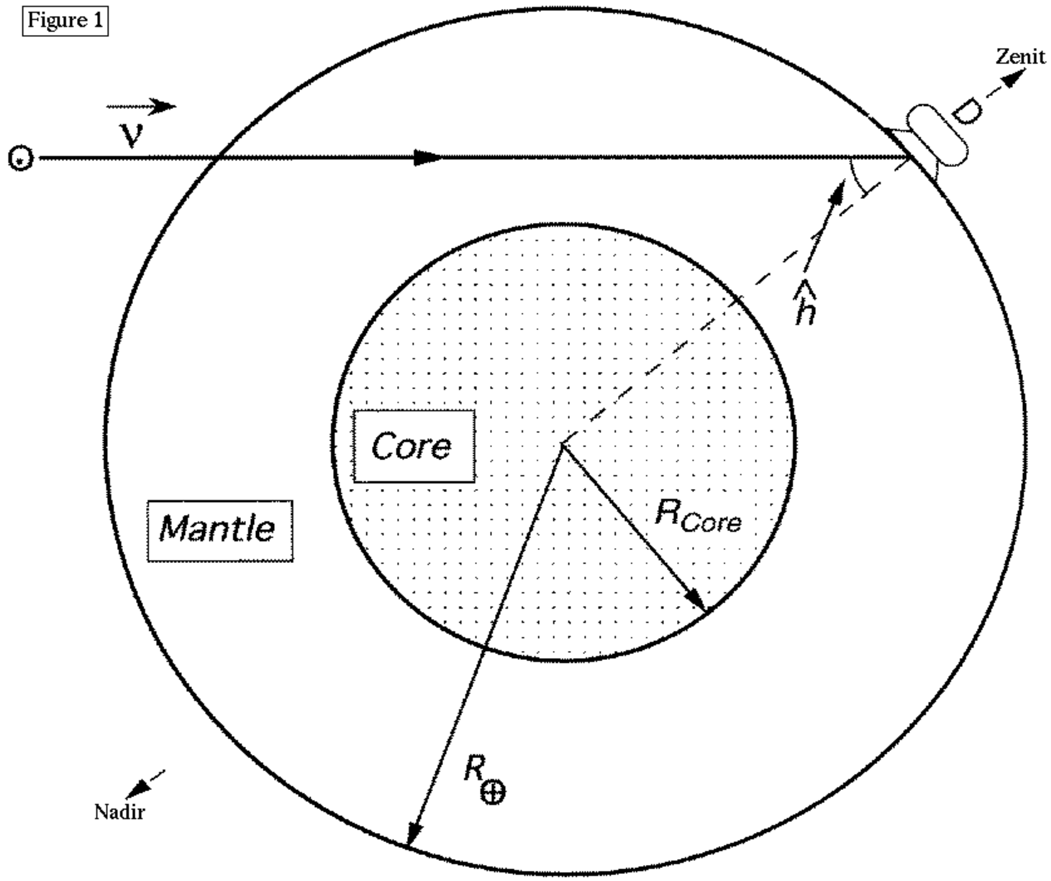
$$J_{CP}^{mat} = J_{CP}^{vac} \, \mathbf{R}_{CP}$$

R_{CP} **does not depend on** θ_{23} **and** δ**:** $\sin 2\theta_{23}^m \sin \delta^m = \sin 2\theta_{23} \sin \delta$

$\sin 2\theta_{12} \cong 0.92$, $\sin 2\theta_{13} \cong 0.3$: $|R_{CP}| \lesssim 3.6$

P.I. Krastev, S.T.P., 1988

The Earth



Earth: $R_{core} = 3446$ km, $R_{mant} = 2885$ km

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

The Earth

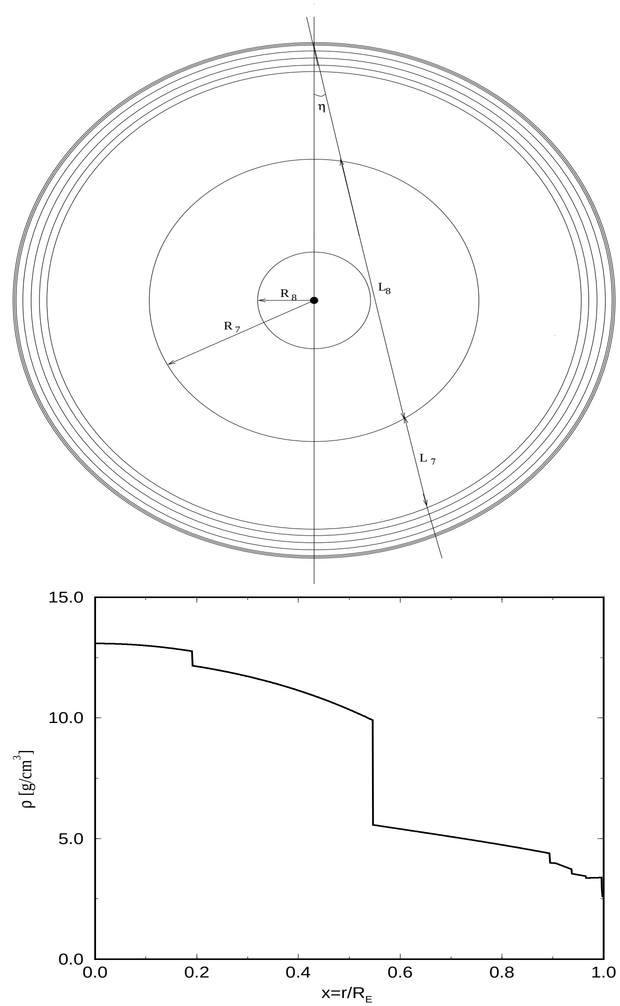
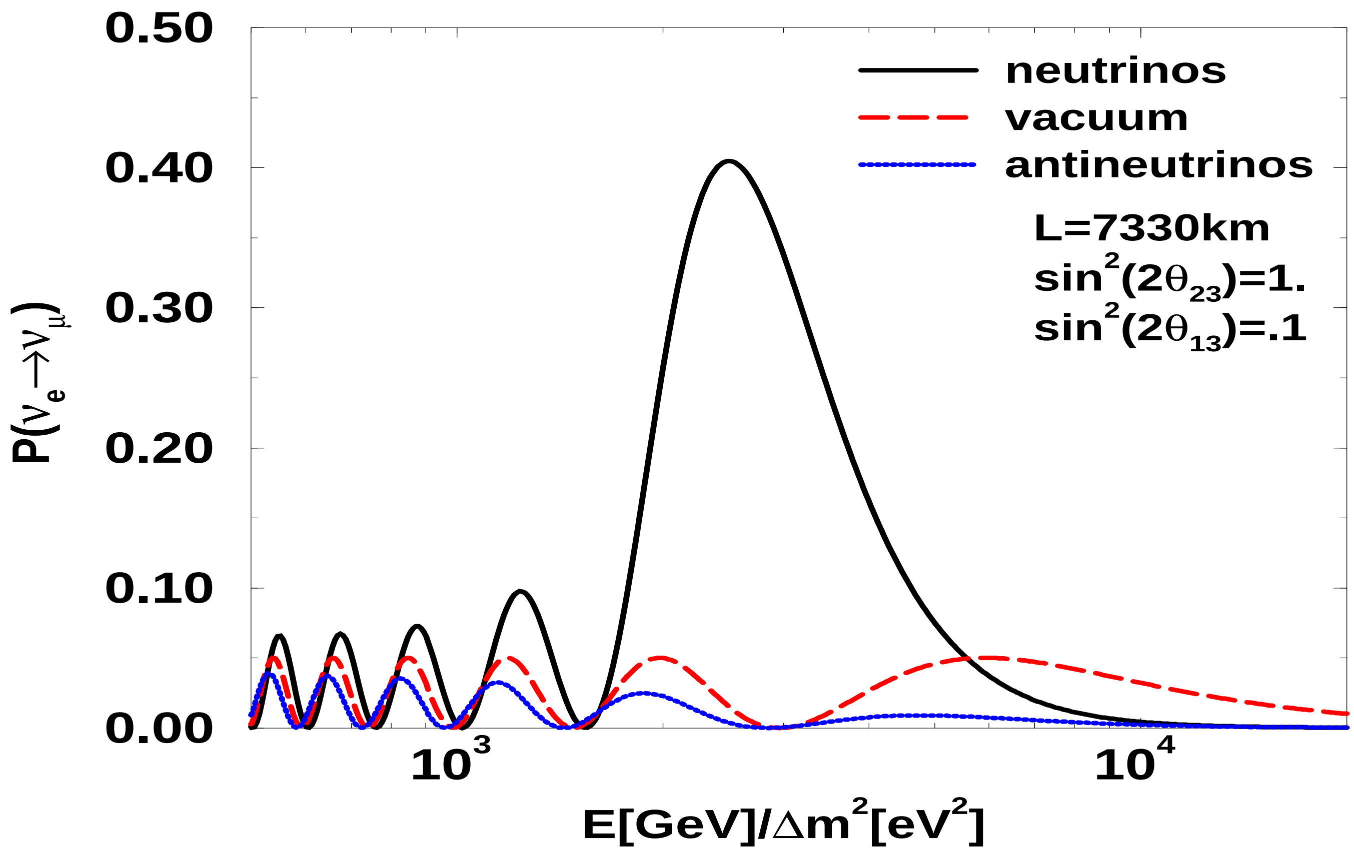


FIG. 1. Density profile of the Earth.

$R_c = 3446$ km, $R_m = 2885$ km; $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ GeV; $P^{3\nu} = \sin^2\theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;
$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250/0.32$ km; $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

Earth mantle: up to 2nd order in $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \cong \frac{1}{30}$ and $\sin^2\theta_{13} \cong 0.0214$:

$$P^{3\nu\ man}_m(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_{\mathsf{F}}N^{man}_e\frac{2E}{\Delta m^2_{31}} \cong \frac{N^{man}_e}{N^{res}_e}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: δ, $A \to (-\delta)$, $(-A)$

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\} \;.$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\,\{U_{e1}U^*_{e3}\}\,, \quad S_2 = \text{Im}\,\{U_{e2}U^*_{e3}\} \quad \text{(not unique);} \quad \text{or}$$
$$S'_1 = \text{Im}\,\{U_{\tau 1}U^*_{\tau 2}\}\,, \quad S'_2 = \text{Im}\,\{U_{\tau 2}U^*_{\tau 3}\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

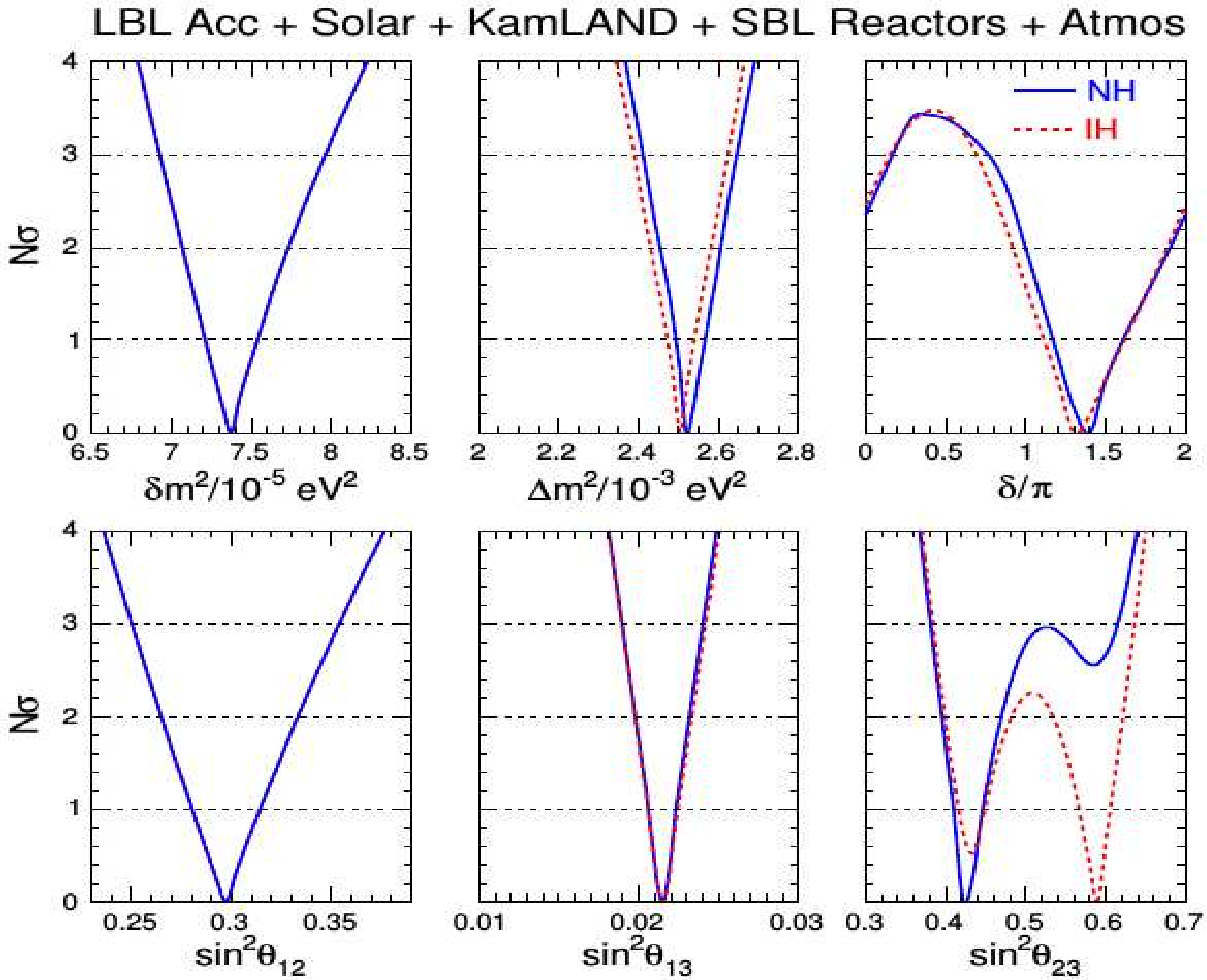
CP-violation: both $\text{Im}\,\{U_{e1}U^*_{e3}\} \neq 0$ and $\text{Re}\,\{U_{e1}U^*_{e3}\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

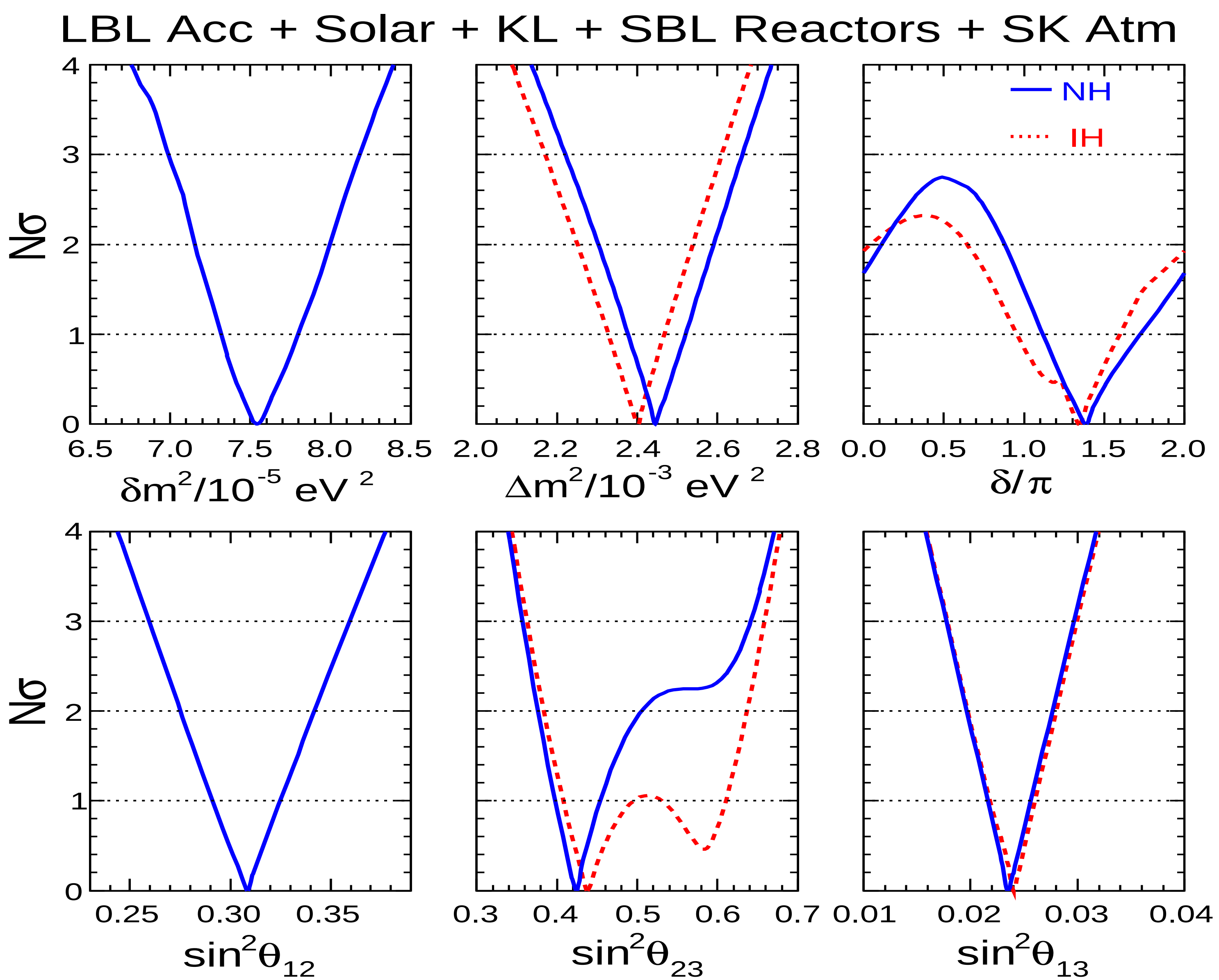
In general, J_{CP}, S_1 and S_2 are independent.

$$\delta \cong 3\pi/2?$$

$$\begin{aligned} J_{CP} &= \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta \end{aligned}$$

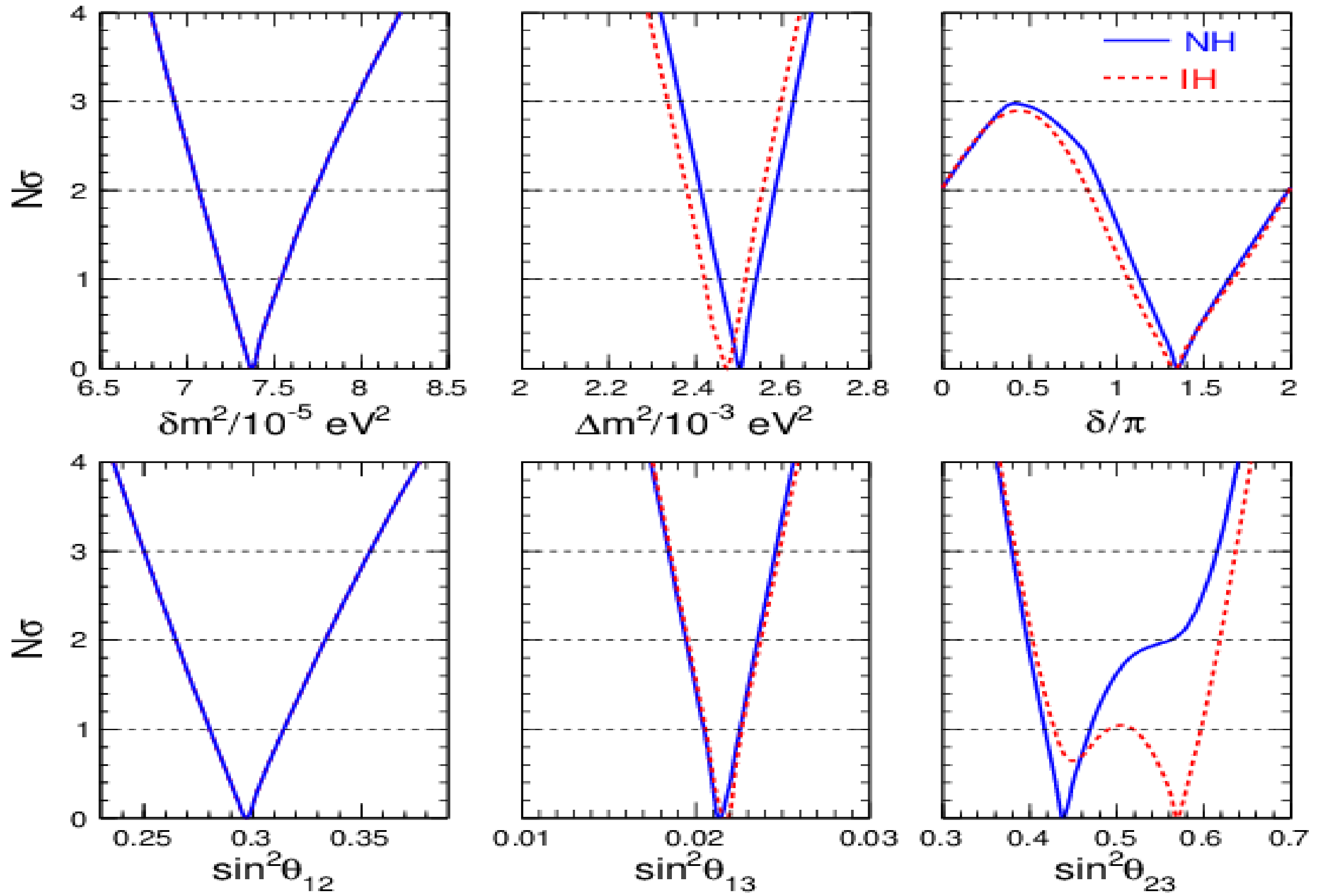


F. Capozzi, E. Lisi *et al.*, Proc. of ν2016 Int. Conf.



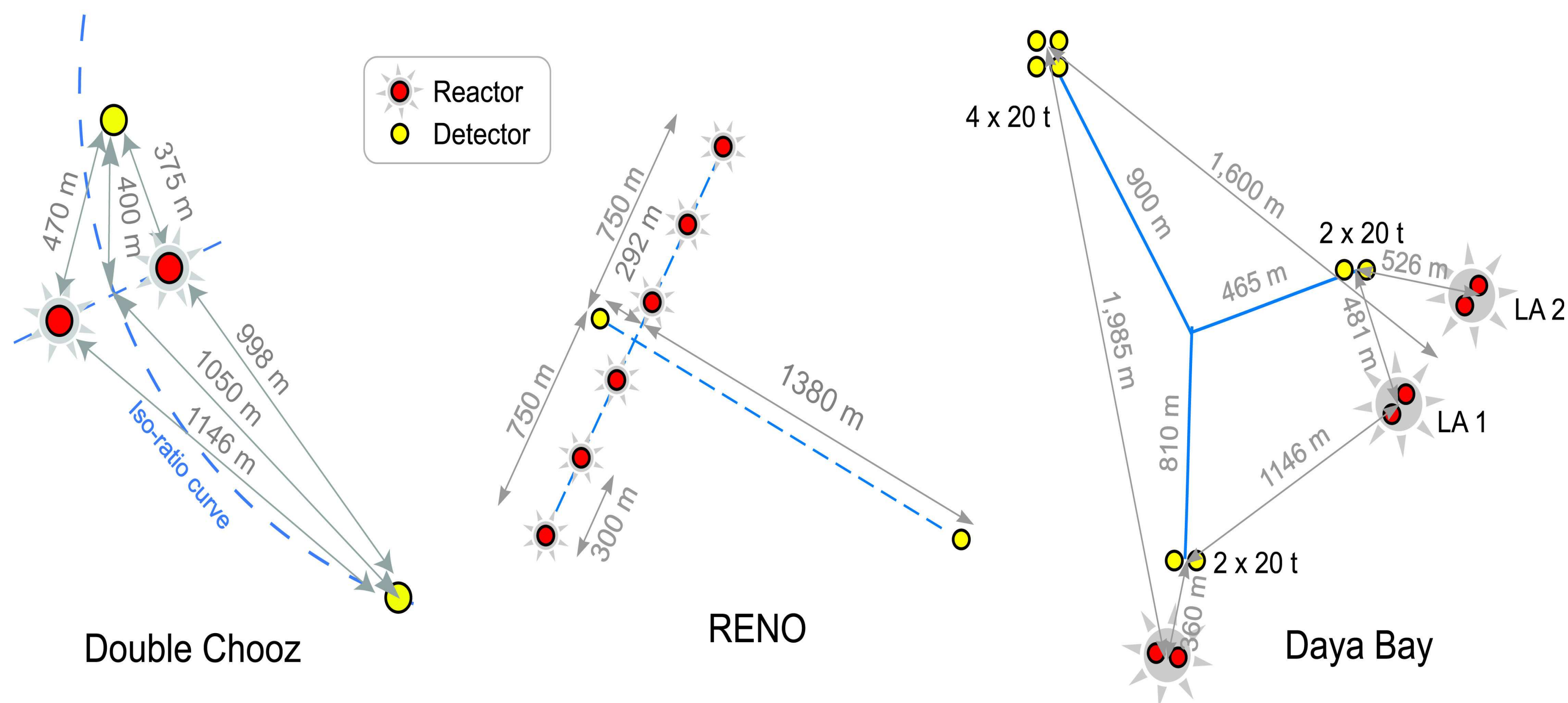
F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



F. Capozzi, E. Lisi *et al.*, arXiv:1601.01777v1


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]


Ontario
NOvA
Soudan
Minnesota
Wisconsin
735 kilometers
Iowa
Fermilab
Illinois

LBL Oscillation Experiments NOνA, T2K (Detector=SuperKamiokande (SK)

T2K: Tokai - Kamioka; off-axis ν, $\bar{\nu}_\mu$ beams, $E \cong 0.6$ GeV, $L \cong 295$ km, SK (50 kt water Cherenkov).

NOνA: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

Reactor Neutrino Experiments on θ_{13}:

$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m^2_{31(32)}; \theta_{12}, \Delta m^2_{21}) \cong$
$1 - \sin^2 2\theta_{13} \sin^2(\frac{\Delta m^2_{31(32)}}{4E} L)$, no dependence on θ_{23}, δ.

LBL Oscillation Experiments T2K (Detector=SuperKamiokande (SK), NOνA:

$P^{3\nu}_m(\nu_\mu \rightarrow \nu_e) = P^{3\nu}_m(\theta_{13}, \Delta m^2_{31(32)}, \theta_{12}, \Delta m^2_{21}, \theta_{23}, \delta)$.

T2K, NOνA: up to 2nd order in $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \cong \frac{1}{30}$ and $\sin^2\theta_{13} \cong 0.0214$:

$$P^{3\nu\ man}_m(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

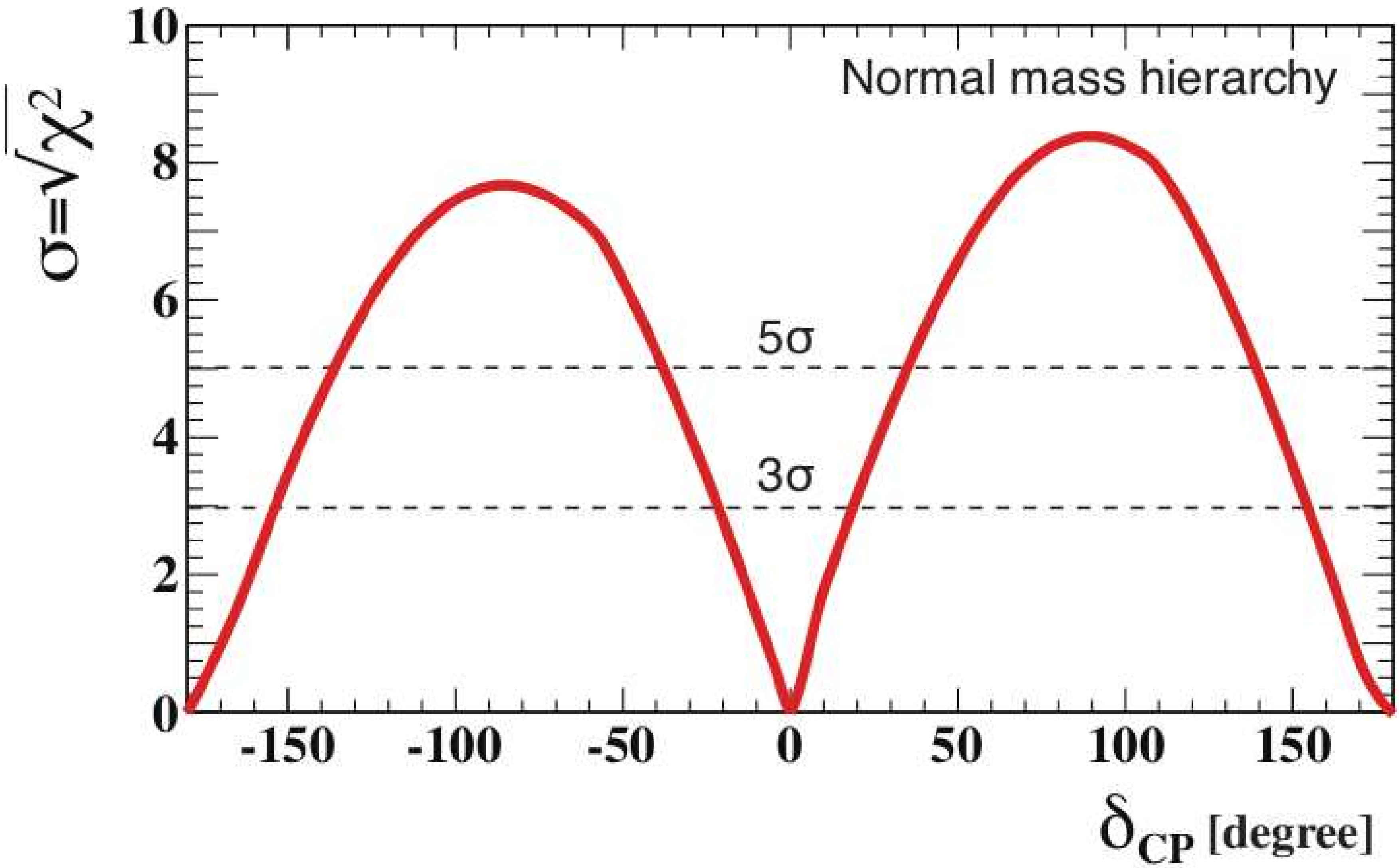
$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_F N^{man}_e \frac{2E}{\Delta m^2_{31}} \cong \frac{N^{man}_e}{N^{res}_e}.$$

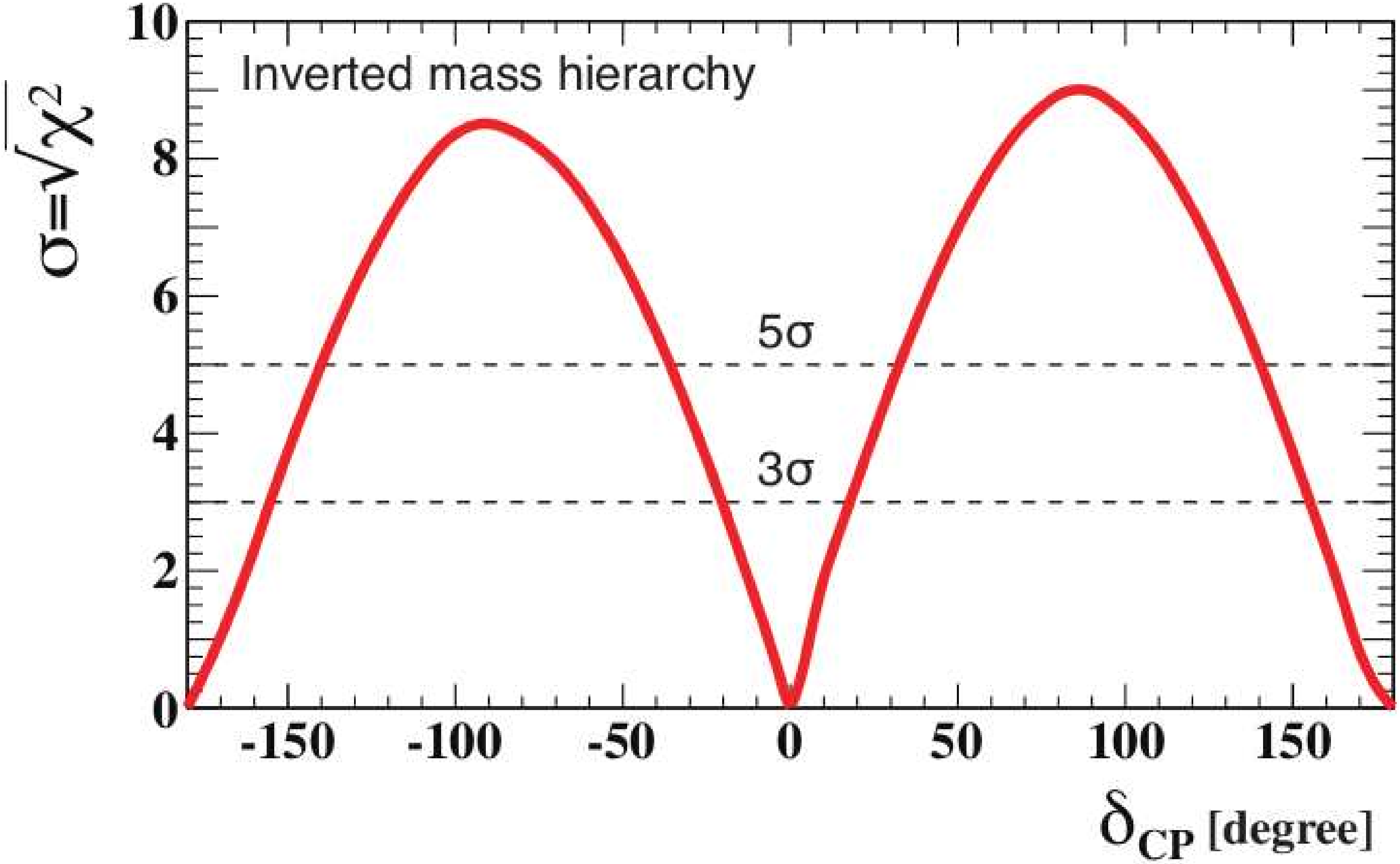
$\bar{\nu}_\mu \to \bar{\nu}_e$: $\delta,\;\; A \to (-\delta),\;\; (-A)$

The results on $\nu_{\mu} \rightarrow \nu_{e}$ oscillations from NOνA are compatible with, and strengthened, the hint that $\delta \cong 3\pi/2$ (A. Marone, talk at Nu2016 (July 8); F. Capozzi *et al.*, arXiv:1601.01777v1).

Expected T2HK sensitivity to CP violation

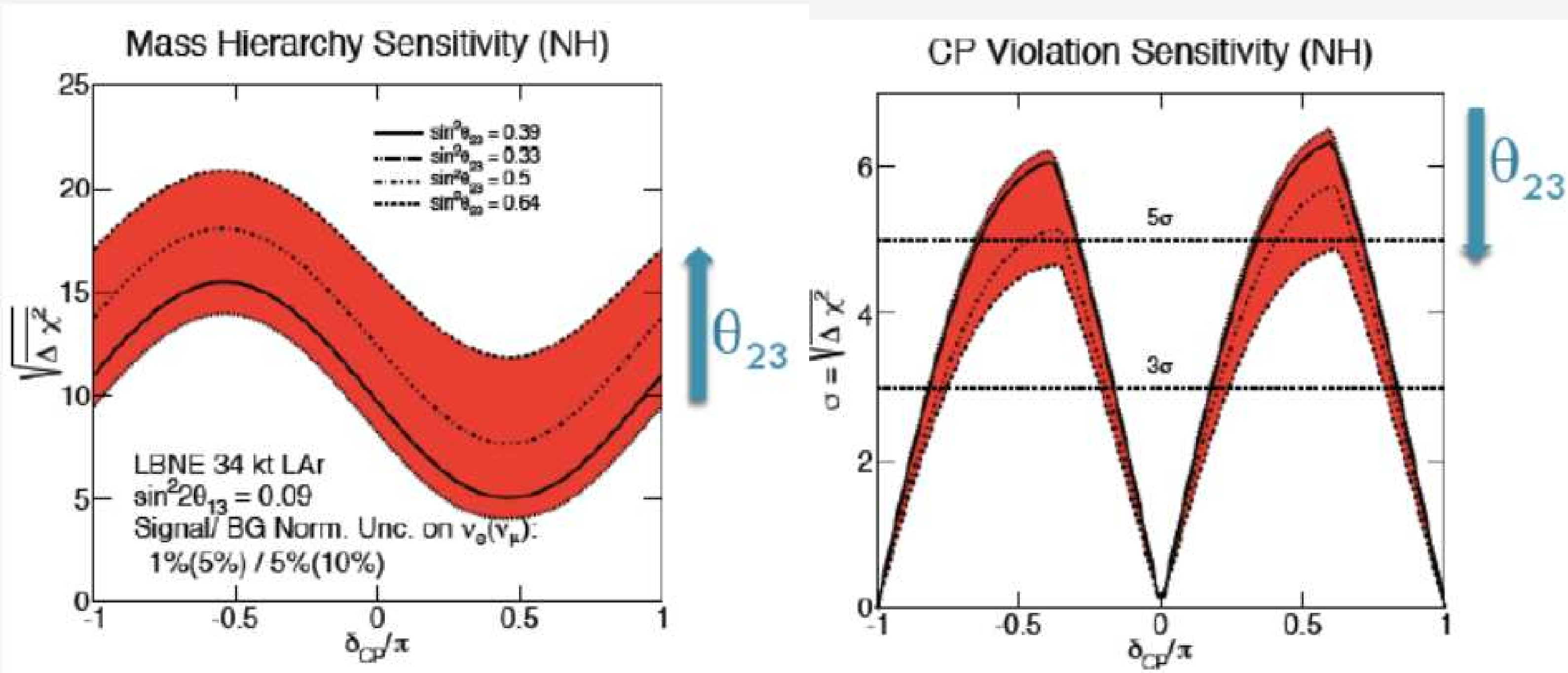


K. Abe et al., arXiv:1502.05199.



K. Abe et al., arXiv:1502.05199.

LBNE Sensitivity to MH & CPV



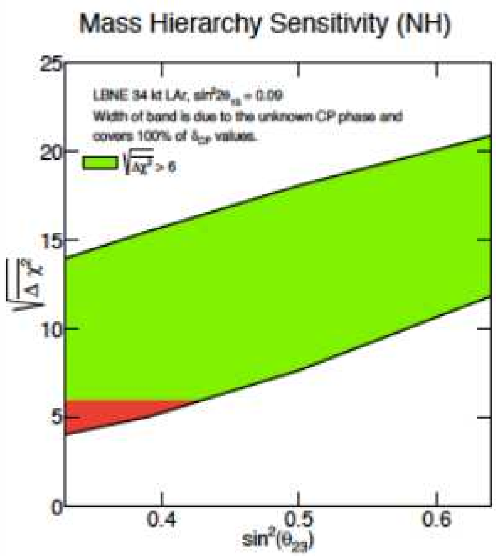
Width of the band indicates variation within the 2013 allowed rage for ϑ_{23}.

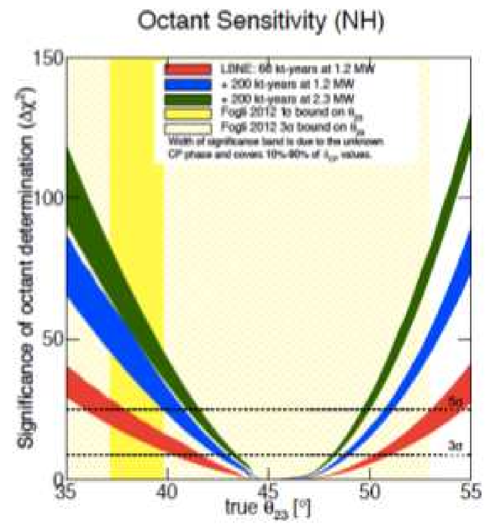
Exposure ~ 245kTon ~ 34 kT X 1.2 MW X (3v + 3vbar) years

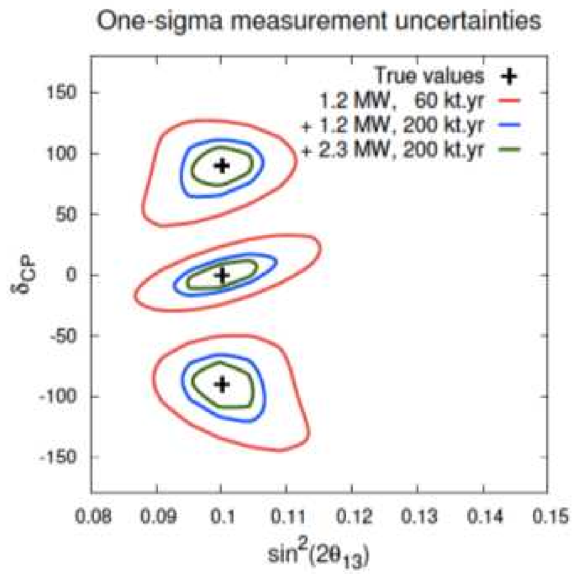
Elizabeth Worchester – NOW 2014

arXiv:1307.7335 – LBNE Document

Other Measurements for a Comprehensive Program







34kton, 6 yrs, 1.2MW

DUNE could also have very good sensitivity to CP-violation with a 60% coverage at 3σ in the allowed range of values of $\sin^2 2\theta_{13}$, for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

DUNE, for example, could achieve the determination of the mass ordering at 3σ in less then a year.

Determining the Nature of Massive Neutrinos:

Majorana Phases in $(\beta\beta)_{0\nu}$−Decay

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ \ P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j.

If $\nu_j -$ Majorana particles, U_{PMNS} contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \to \pi^- + \mu^+ + \mu^+$$

$$\mu^- + (\text{A}, \text{Z}) \to \mu^+ + (\text{A}, \text{Z} - 2)$$

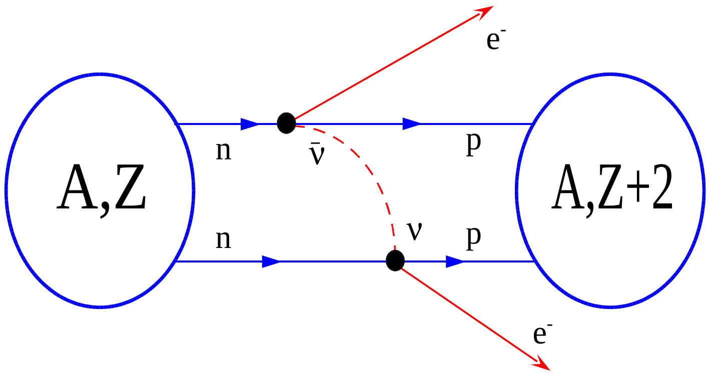
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\text{A}, \text{Z}) \to (\text{A}, \text{Z} + 2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

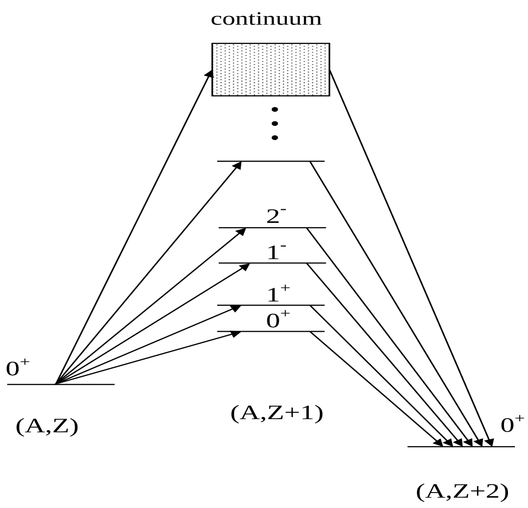
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \to uue^- e^- (\overline{\nu}_e \overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$A(\beta\beta)_{0\nu} \sim \; < m > $ M(A,Z), M(A,Z) - NME,

$$|< m >| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2 \; e^{i\alpha_{21}} + m_3|U_{e3}|^2 \; e^{i\alpha_{31}}\right|$$

$$= \left|m_1 \; c_{12}^2 \; c_{13}^2 + m_2 \; s_{12}^2 \; c_{13}^2 \; e^{i\alpha_{21}} + m_3 \; s_{13}^2 \; e^{i\alpha_{31}}\right|, \quad \theta_{12} \equiv \theta_{\odot}, \; \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} $((\alpha_{31} - 2\delta) \rightarrow \alpha_{31})$ - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi, \; \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_\odot}\,\sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}}\,\sin^2\theta_{13} e^{i\beta_M}\right|\,,\quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\,\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|<m>| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \ \ \text{(QD)},$$

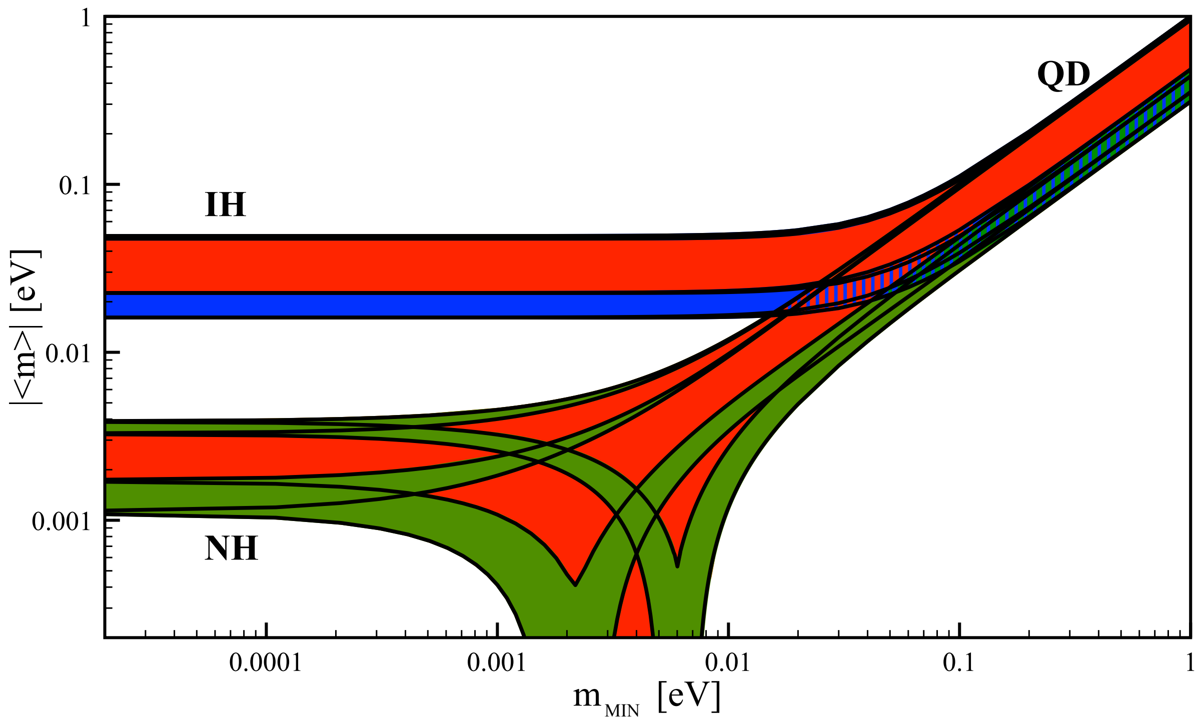
$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$|<m>| \ \lesssim 5 \times 10^{-3}$ eV, NH;

$\sqrt{\Delta m^2_{23}}\cos 2\theta_{12} \cong 0.013$ eV $\lesssim \ |<m>| \ \lesssim \sqrt{\Delta m^2_{23}} \cong 0.055$ eV, IH;

$m\cos 2\theta_{12} \lesssim \ |<m>| \ \lesssim m$, $m \gtrsim 0.10$ eV, QD .



S. Pascoli, RPP (PDG), 2016

$\sin^2\theta_{13} = 0.0214 \pm 0.0010$; $\delta = 0$.

$1\sigma(\Delta m^2_{21}) = 2.3\%$, $1\sigma(\sin^2\theta_{12}) = 5.6\%$, $1\sigma(|\Delta m^2_{31(23)}|) = 1.7\%$.

F. Capozzi et al. (Bari Group), arXiv:1601.07777

$2\sigma(|<m>|\ )$ used.

Results from IGEX (^{76}Ge), NEMO3 (^{100}Mo), CUORICINO+CUORE-0 (^{130}Te):

IGEX ^{76}Ge: $|{<}\, m \,{>}| \ < (0.33 - 1.35)$ eV **(90% C.L.).**

Data from NEMO3 (^{100}Mo), CUORICINO+CUORE-0 (^{130}Te):

$T(^{100}Mo) > 1.1 \times 10^{24}$ **yr,** $|{<}\, m \,{>}| \ <$**(0.3–0.6) eV;**
$T(^{130}Te) > 4.0 \times 10^{24}$ **yr.**

Best Sensitivity Results from 2012-2016:

$$T(^{136}Xe) > 1.6 \times 10^{25} yr \text{ at } 90\% \text{ C.L., EXO}$$

$$T(^{136}Xe) > 1.07 \times 10^{26} yr \text{ at } 90\% \text{ C.L., KamLAND-Zen}$$

$$|{<}\, m \,{>}| < (0.061 - 0.165) \text{ eV}.$$

$$T(^{76}Ge) > 5.2 \times 10^{25} yr \text{ at } 90\% \text{ C.L., GERDA II}$$

$$|{<}\, m \,{>}| < (0.16 - 0.26) \text{ eV}.$$

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$T(^{76}Ge) = 2.23^{+0.44}_{0.31} \times 10^{25} \text{ yr at } 90\% \text{ C.L.}$$

Large number of experiments: $|<m>|$ $\sim$ (0.01-0.05) eV

CUORE - ^{130}Te;
GERDA-II - ^{76}Ge;
MAJORANA - ^{76}Ge;
KamLAND-ZEN - ^{136}Xe;
(n)EXO - ^{136}Xe;
SNO+ - ^{130}Te;
PANDAX-III - ^{136}Xe;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca;
SuperNEMO - ^{82}Se, ^{150}Nd;
MAJORANA - ^{76}Ge;
NEXT - ^{136}Xe;
DCBA - ^{82}Se, ^{150}Nd;
XMASS - ^{136}Xe;
ZICOS - ^{96}Zr;
MOON - ^{100}Mo;
...

Majorana CPV Phases and $|<m>|$

CPV can be established provided

– $|<m>|$ measured with $\Delta \lesssim 15\%$;

– $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002