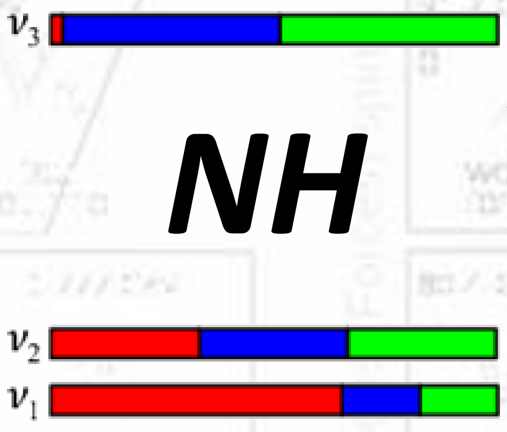
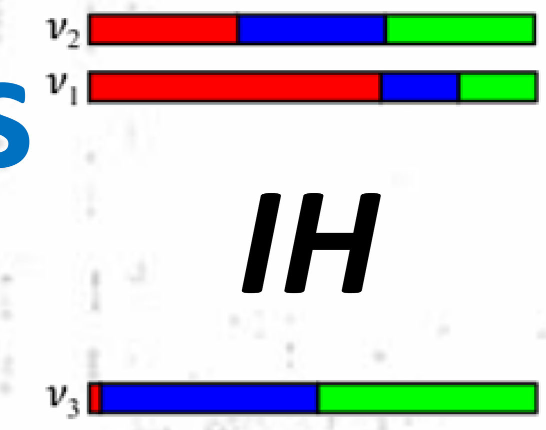
Determination of the **neutrino mass hierarchy** with a **new statistical methods**





L. Stanco, S. Dusini and **M. Tenti**

[Phys. Rev. D 95, 053002]

Motivation

The mixing of the 3 weak and 3 not degenerate mass neutrino eigenstates is well describing almost all neutrino oscillations phenomenology.

Few parameters remain to be determined, among them **neutrino mass ordering (MO)** has a crucial role in providing inputs for future studies and experimental proposals and in constraining analyses in other fields such as cosmology and astrophysics.

All the methods developed so far for establishing whether MO is **normal** (NH) or **inverted** (IH) are based on $\Delta\chi^2$ evaluation.

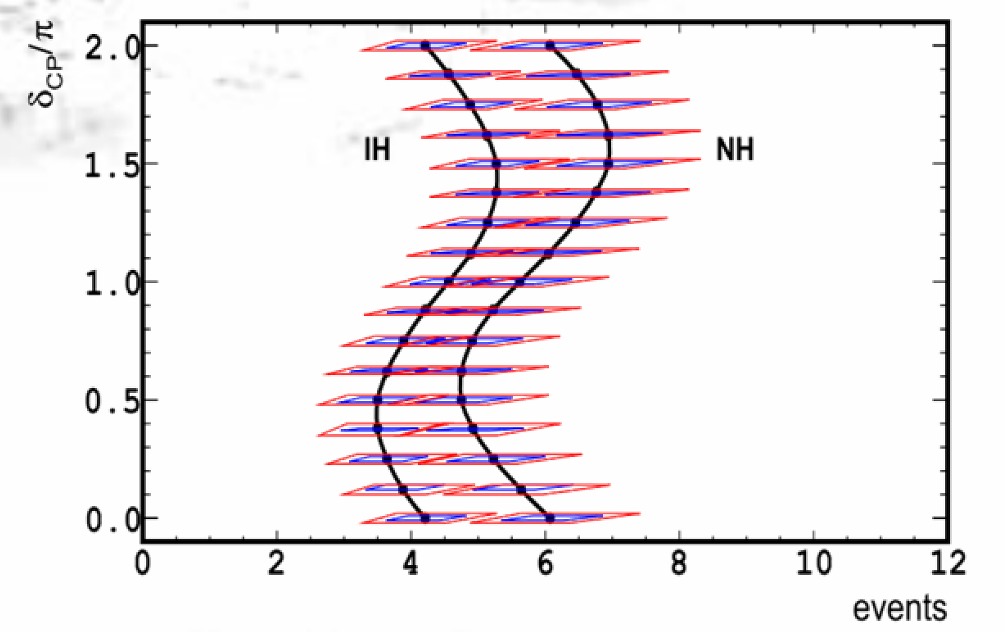
$$\Delta\chi^2 = \chi^2_{min}(IH) - \chi^2_{min}(NH)$$

Where the two minima are evaluated spanning the uncertainties of the 3 neutrino oscillation parameters.

The statistical significance in terms of standard deviations is usually computed as $\sqrt{\chi^2}$.

Given the current uncertainties of the oscillation parameters [1] from few percents to more than 10%, **the computation of the difference of the χ^2 best fits for NH and IH leads to almost null the sensitivity on mass ordering** [1].

As an example **NOvA 2015 results** has been used [2]. It has been re-obtained with GLoBES package.



No discrimination between IH and NH can be achieved if the χ^2 minimization is performed. Therefore a **more sophisticated test statistic should be introduced.**

New Statistical Method

A new test statistic q is defined, following a **Bayesian approach developed in a frequentist way**.

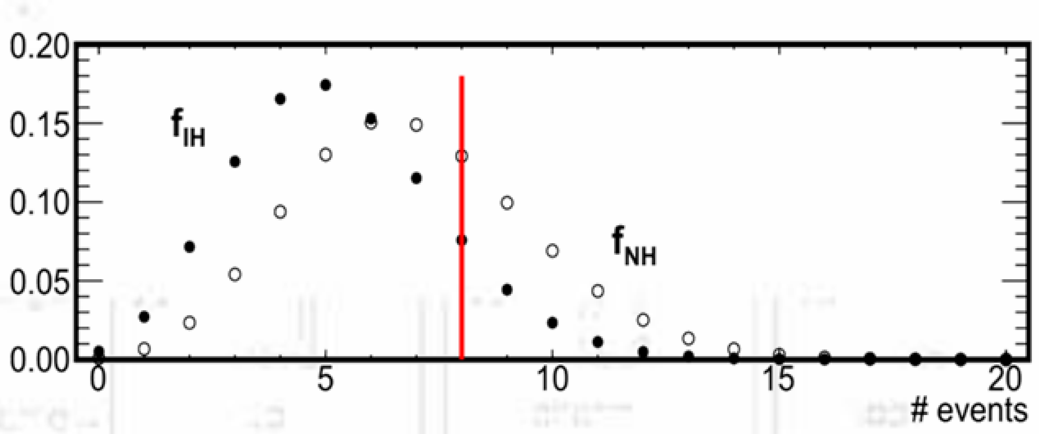
The new method takes into account the whole **shape of probability density function of test statistic** and exploits the intrinsic **statistical fluctuations**.

For each mass ordering, one considers the Poisson distributions

$$f_{MO}(n;\mu_{MO}|\delta_{CP})$$

where:

- n is the observed events
- MO is the neutrino mass ordering, i.e. NH or IH.
- μ_{MO} is its expectation given MO and δ_{CP}

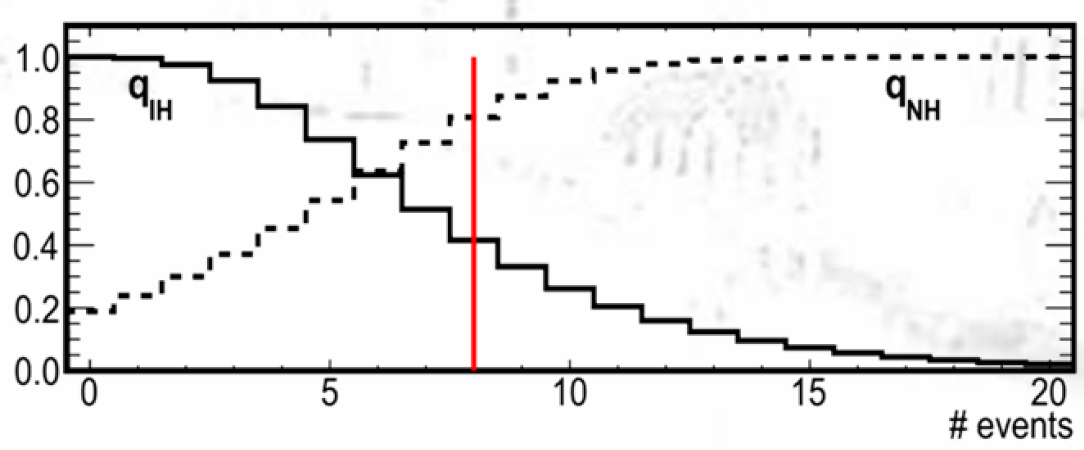


For a specific n the left and right **cumulative functions** of f_{IH} and f_{NH} are computed

The **ratios** q_{MO} are defined either for the NH or the IH case:

$$q_{NH}(n|\delta_{CP}) = \frac{\sum_{n_{NH}\leq n} f_{NH}(n_{NH}|\delta_{CP})}{\sum_{n_{IH}\leq n} f_{IH}(n_{IH}|\delta_{CP})}$$

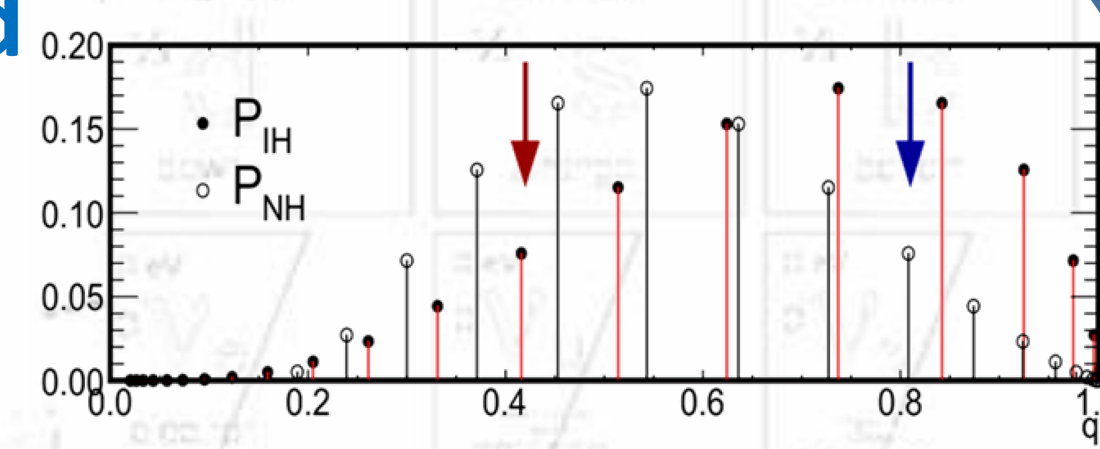
$$q_{IH}(n|\delta_{CP}) = \frac{\sum_{n_{NH}\geq n} f_{IH}(n_{IH}|\delta_{CP})}{\sum_{n_{IH}\geq n} f_{NH}(n_{NH}|\delta_{CP})}$$



The **probability mass functions** of q_{MO}, $P_{MO}(q_{MO})$, are computed and selecting the observed data n_D, the corresponding **p-values**, p_{MO} are evaluated as:

$$p_{IH}(n_D,\delta_{CP}) = \sum_{q'_{IH}<q_{IH}(n_D)} P_{IH}(q'_{IH};\delta_{CP})$$

$$p_{NH}(n_D,\delta_{CP}) = \sum_{q'_{NH}<q_{NH}(n_D)} P_{NH}(q'_{NH};\delta_{CP})$$



Finally, the **significance**, Z, is computed from the p_{MO} values with the one-sided option:

$$Z = \Phi^{-1}(1 - p_{MO})$$

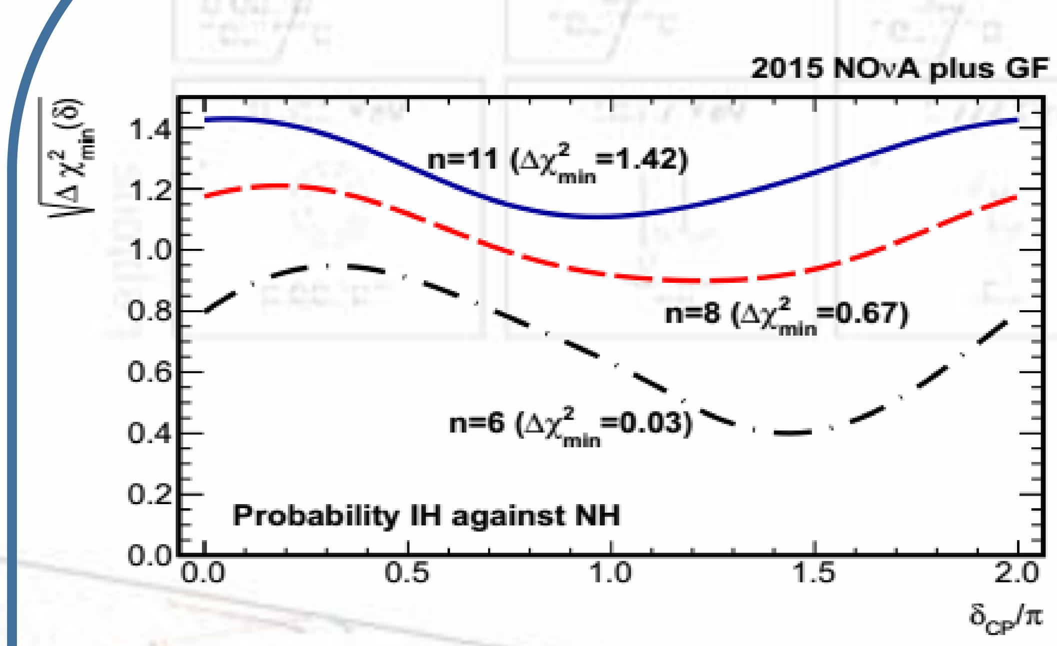
where Φ^{-1} is the quantile (inverse of the cumulative distribution) of the standard Gaussian.

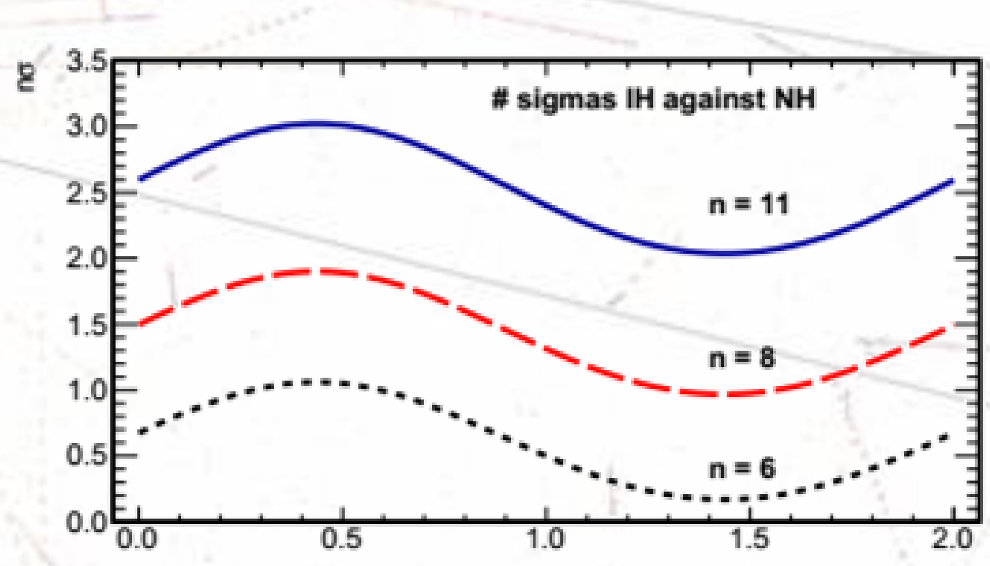
The **uncertainties on θ_{23} and θ_{13}, as well as the systematic errors**, let fluctuate the prediction of the median number of events. These uncertainties have been taken into account using two approaches:

A) convolution of the Poisson distributions with assumed Gaussian distributions [3] for the uncertainties on θ_{23}, θ_{13} (central values and standard deviations being given by the GF [1]) and the systematic errors on signal and background (as provided by NOvA).

B) evaluation of the error bands overlaying the significance, choosing a $\pm\sigma$ variation of the mixing angles and the systematic errors.

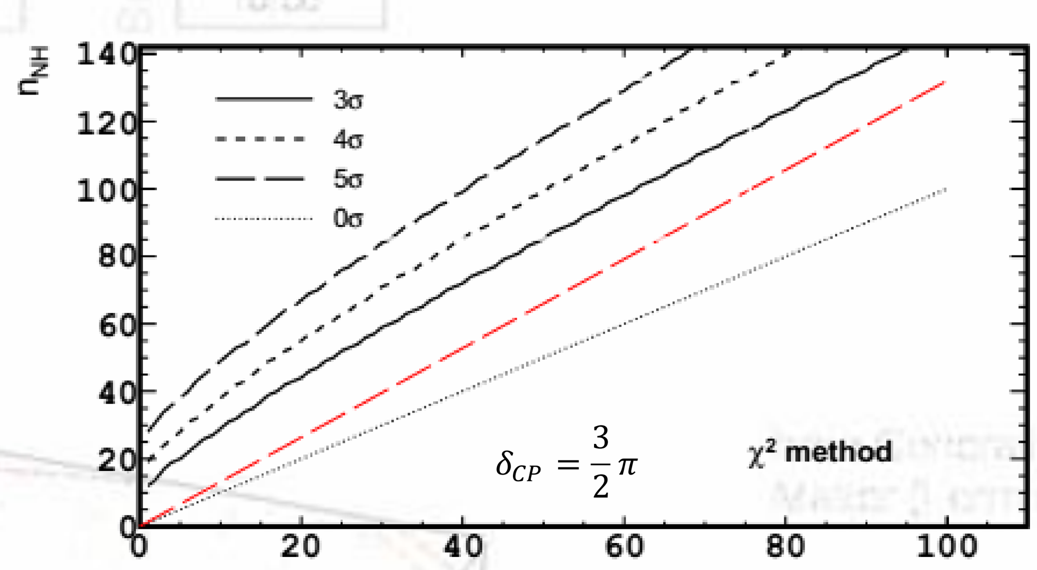
Results and Perspectives

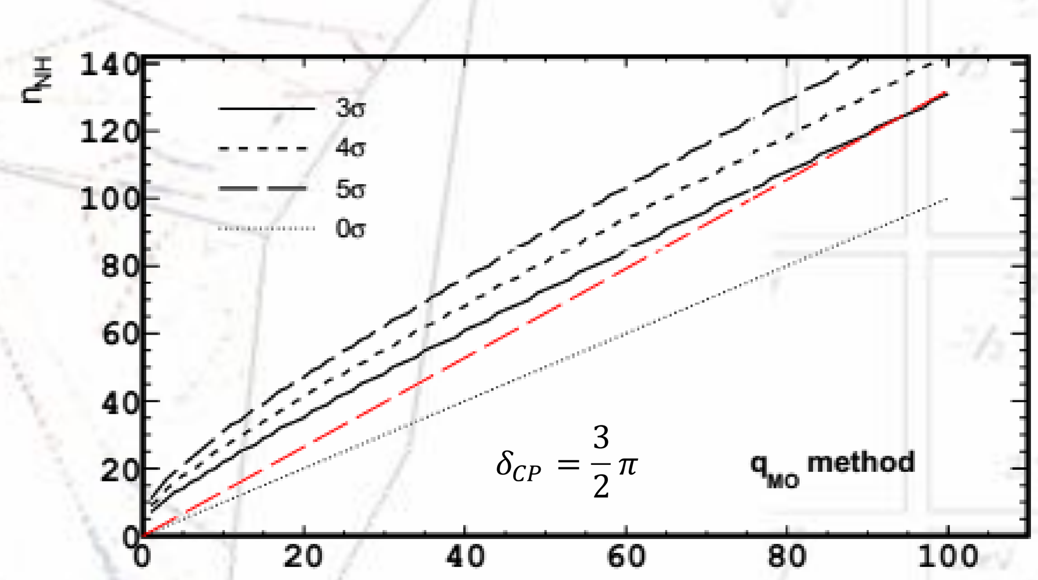




Overall, when n_D = 8 the new method provides an **increase in significance** of 0.5σ compared to the $\Delta\chi^2_{min}$ method for the 2015 NOvA results (much more for specific ranges of δ_{CP}).

The **power of the new method** can be also settled comparing the isoline corresponding to different levels of significance in the n_{NH} e n_{IH} plane, the predicted numbers of events in the NH and IH hypotheses, respectively.





When the q_{MO} estimator is used about a factor (**gaining factor**) two less is needed to get the 3σ separation. This corresponds to a net **gain in exposure** of q_{MO} against χ^2.

We also tried to apply the idea to allow some **fluctuation** of the data, mildly away from the median.

The table shows the average and spread over the δ_{CP} range of the gaining factor allowing statistical fluctuation.

fluctuation	average	spread
no fluctuation	2.27	+0.18, −0.12
32% fluctuation	2.75	+0.51, −0.19
10% fluctuation	3.78	+0.78, −0.39

We plan to **extend our technique** to other data, like JUNO and PINGU-like, and add T2K data.

[1] F. Capozzi et al., Nucl. Phys. B 908, 218 (2016)
[2] P. Adamson et al., Phys. Rev. Lett. 116, 151806 (2016)
[3] R.D. Cousins and V.L. Highland, Nucl. Instrum. Meth., A320, 33 (1992)
[5] P. Vahle (for the NOvA collaboration), talk at Neutrino2016, London (UK), 4-9 July 2016